Field Map Calculation for the Fundamental Mode for a Single TESLA 3.9 GHz Cavity with Couplers

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Outline

- Motivation
- Computational Model
  - Geometry and mesh information
- Simulation results
  - Field components parallel to the cavity axis
  - FEM on tetrahedral meshes
  - Kirchhoff integral representation
- Summary / Outlook

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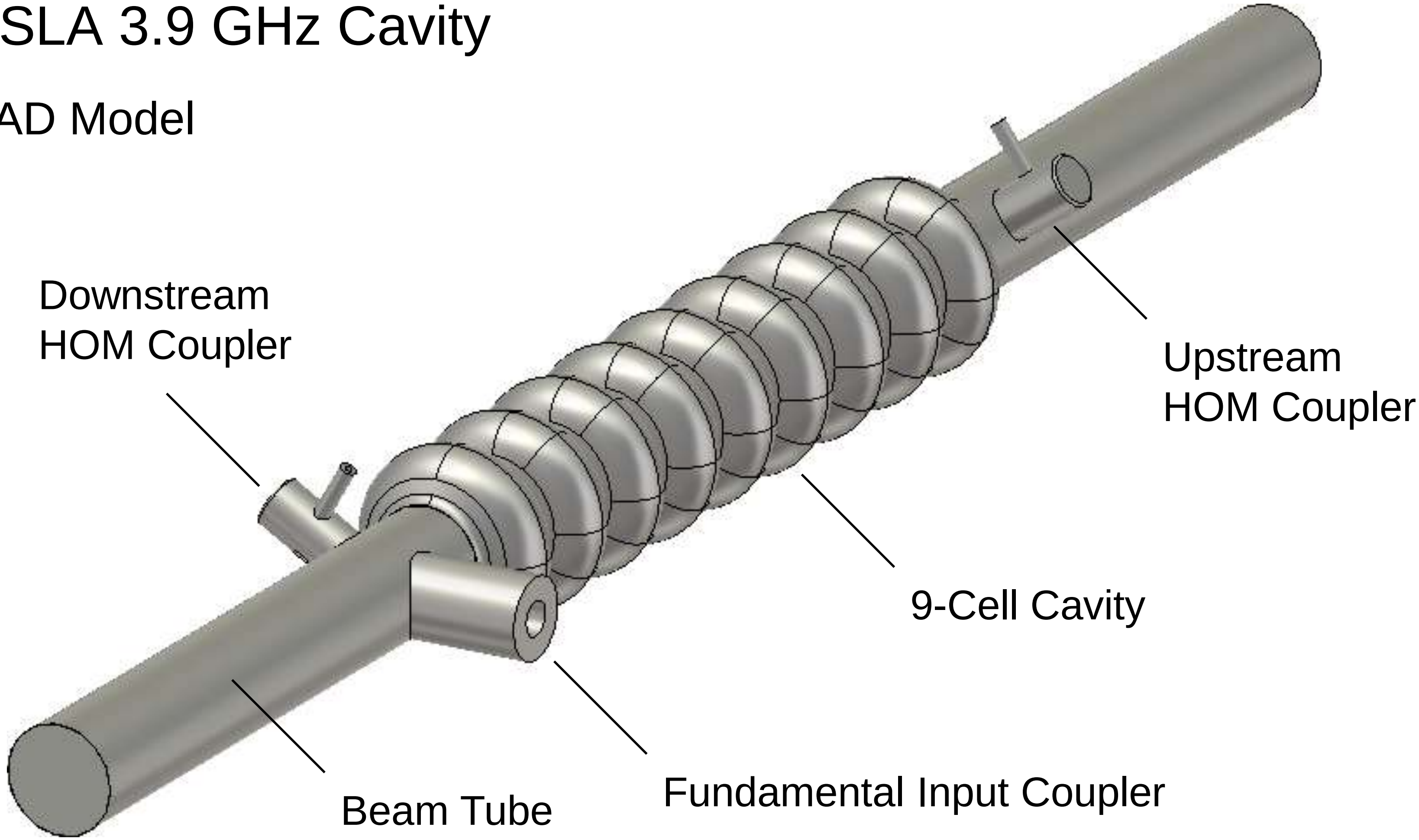
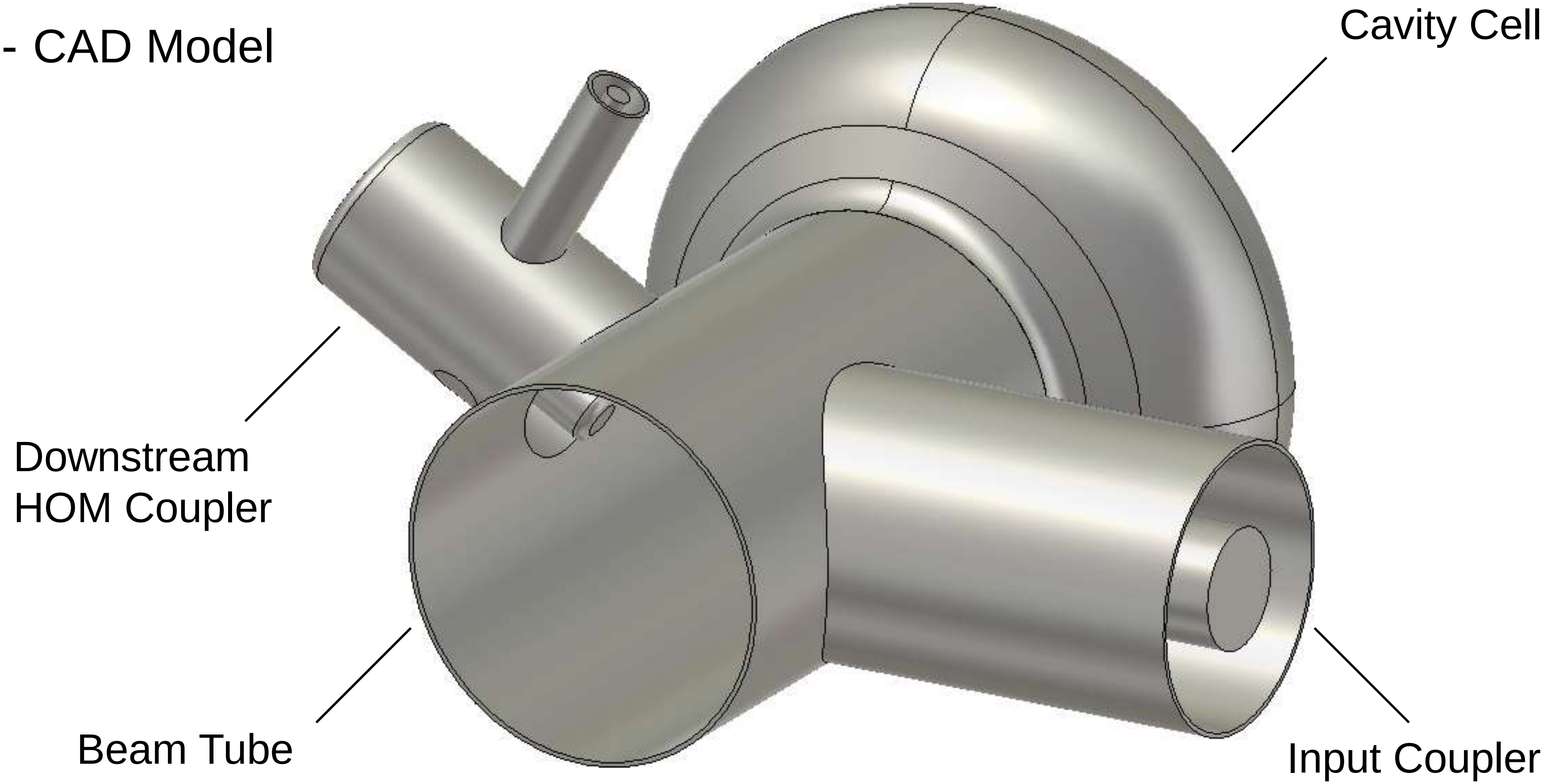
Motivation

- TESLA 3.9 GHz Cavity
  - CAD Model



Motivation

- TESLA 3.9 GHz Cavity
  - CAD Model



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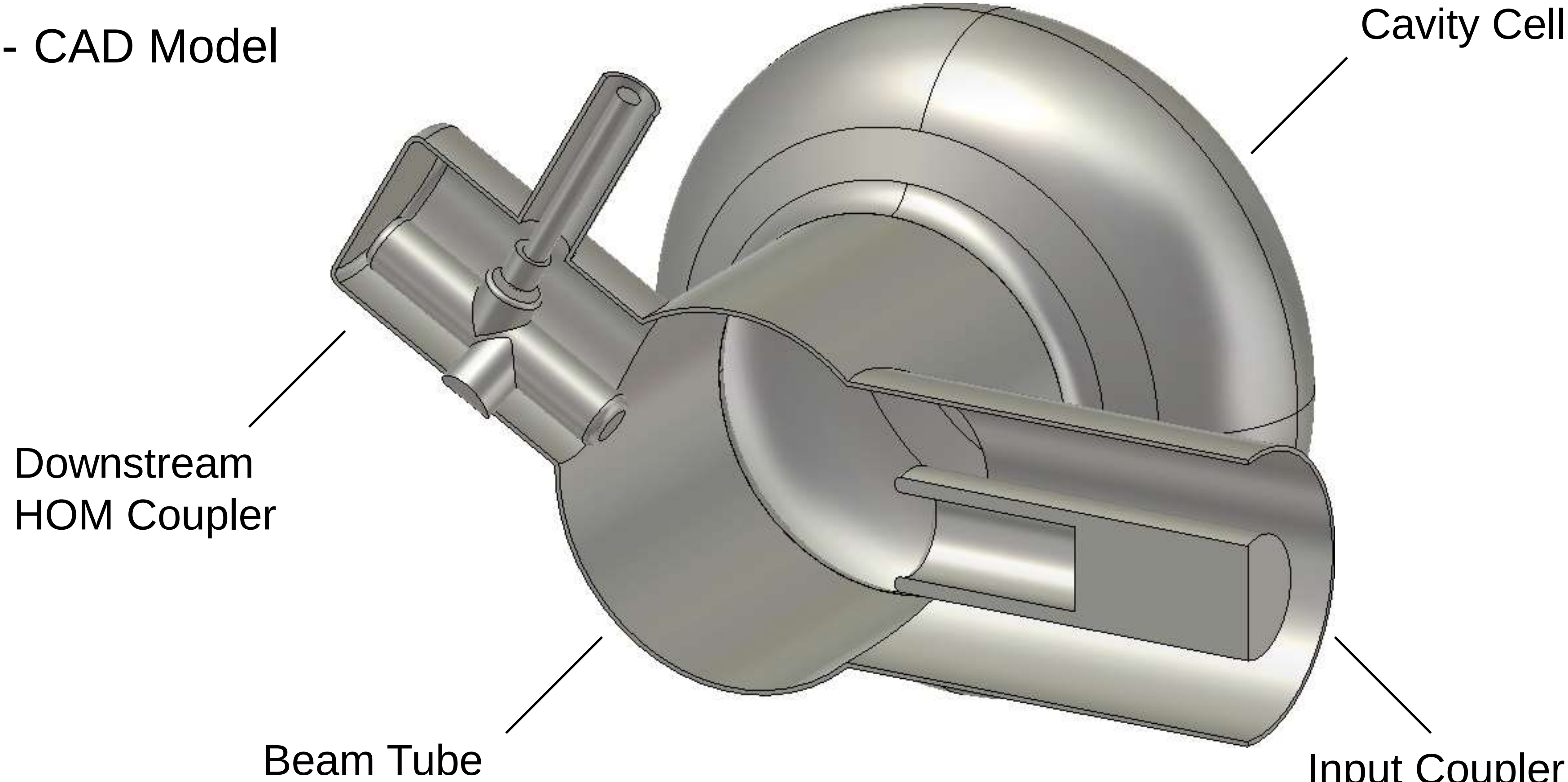

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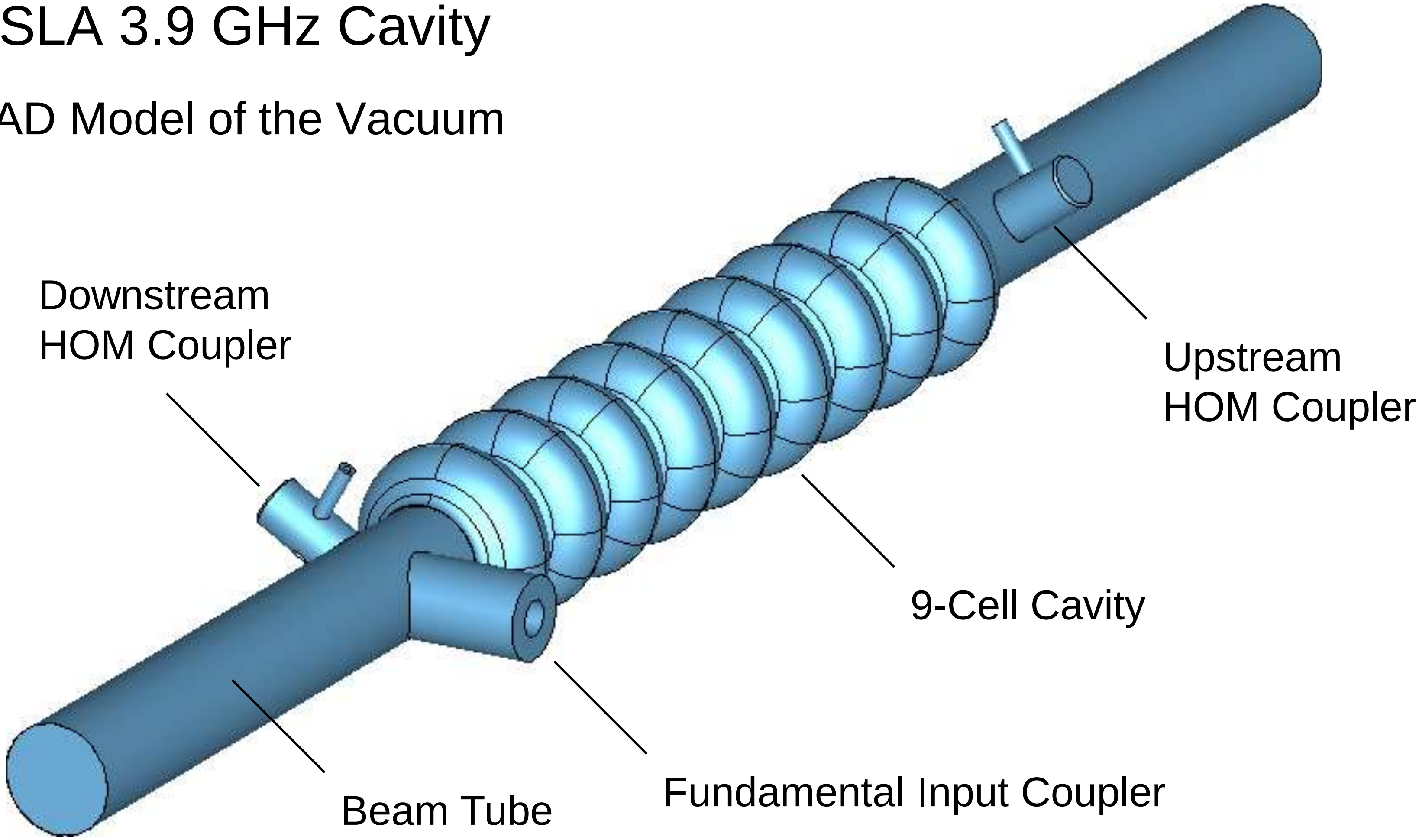
Computational Model

- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum



Computational Model

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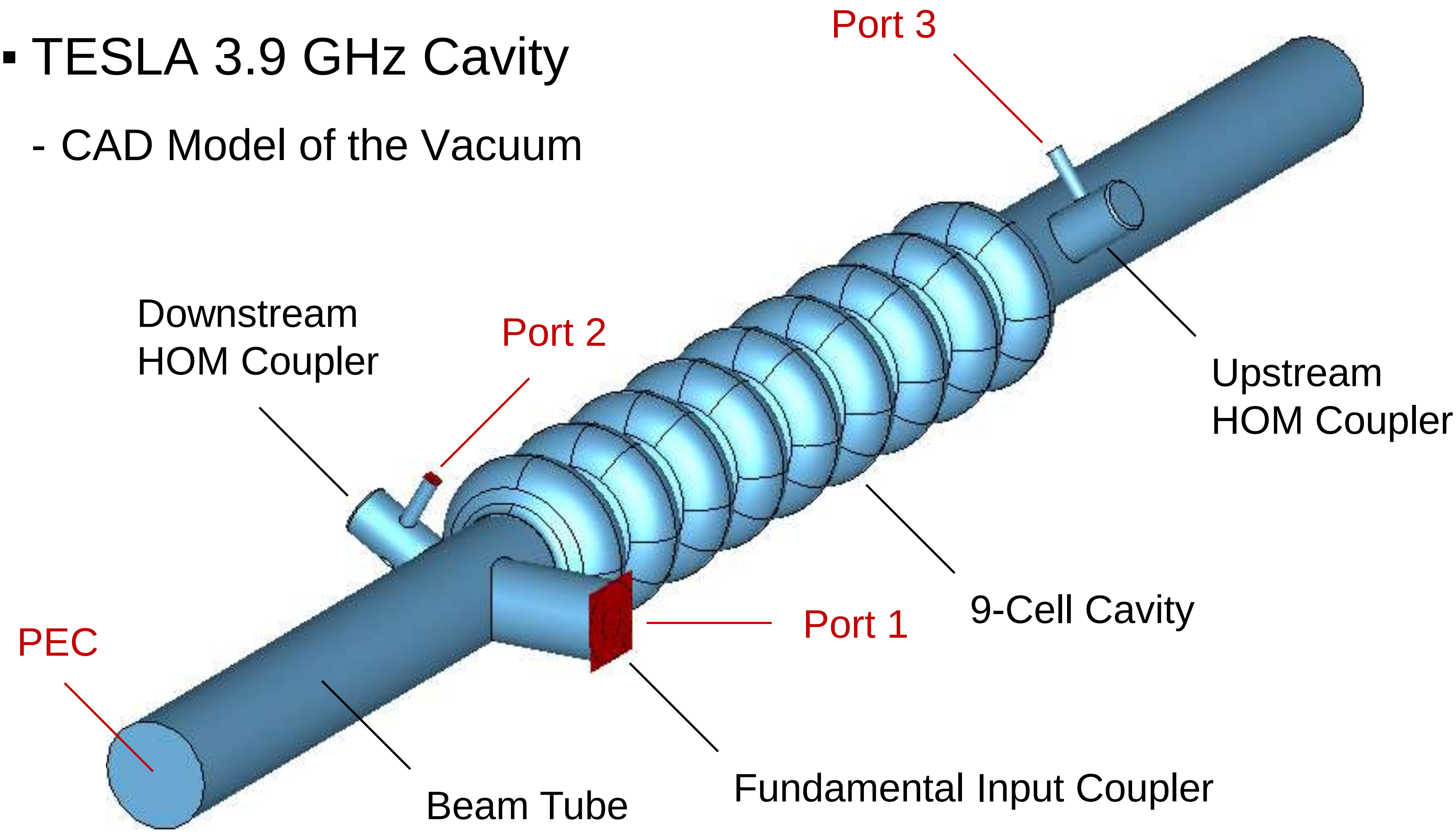

Computational Model

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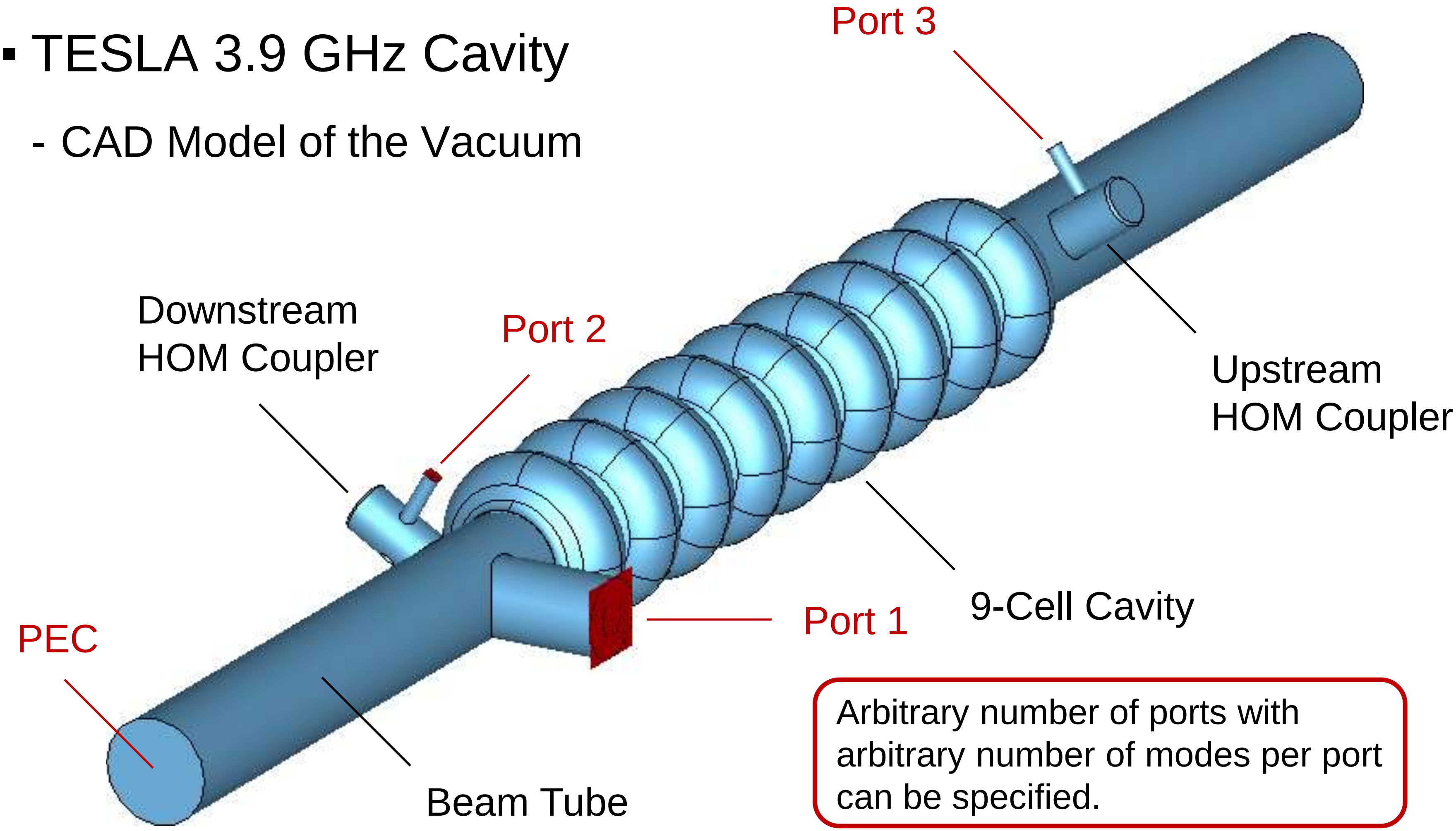
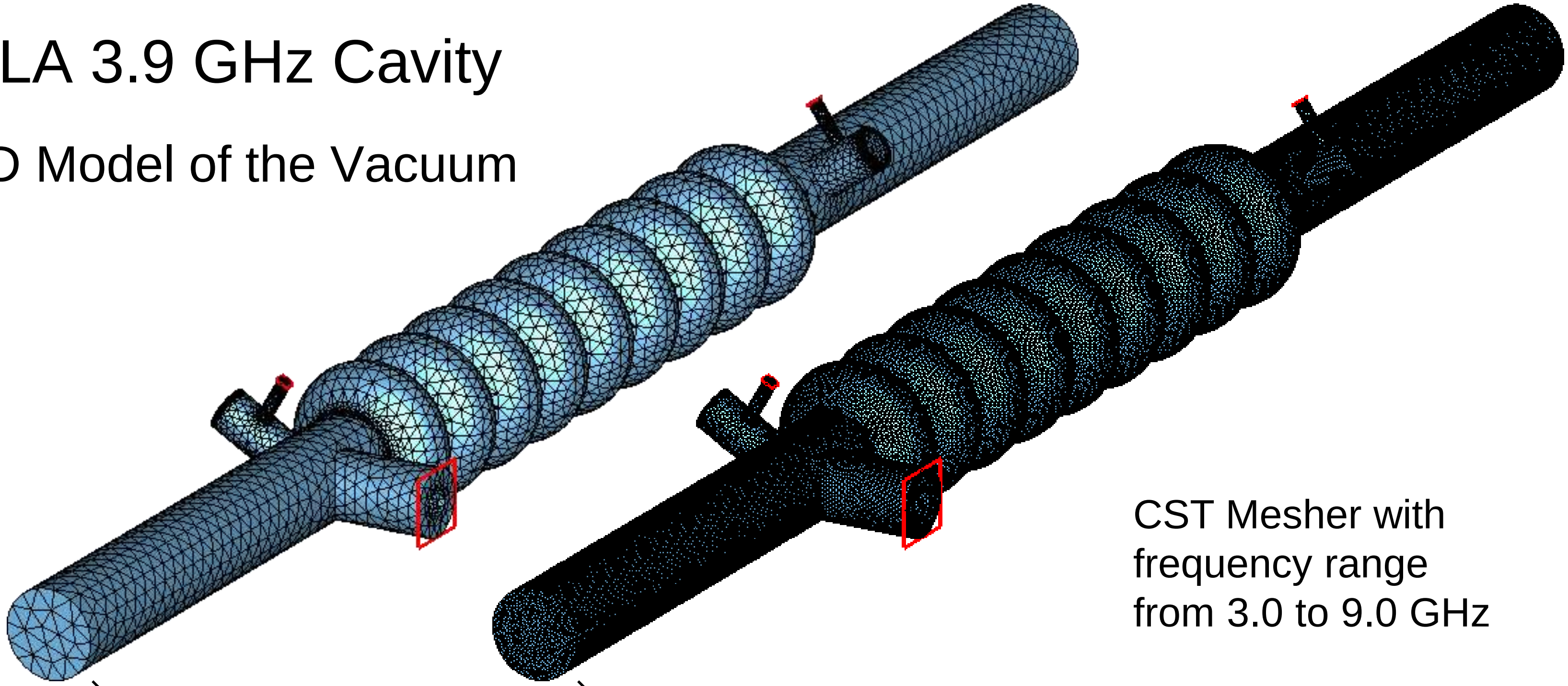


Arbitrary number of ports with arbitrary number of modes per port can be specified.

Computational Model

- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum



CST Mesher with frequency range from 3.0 to 9.0 GHz

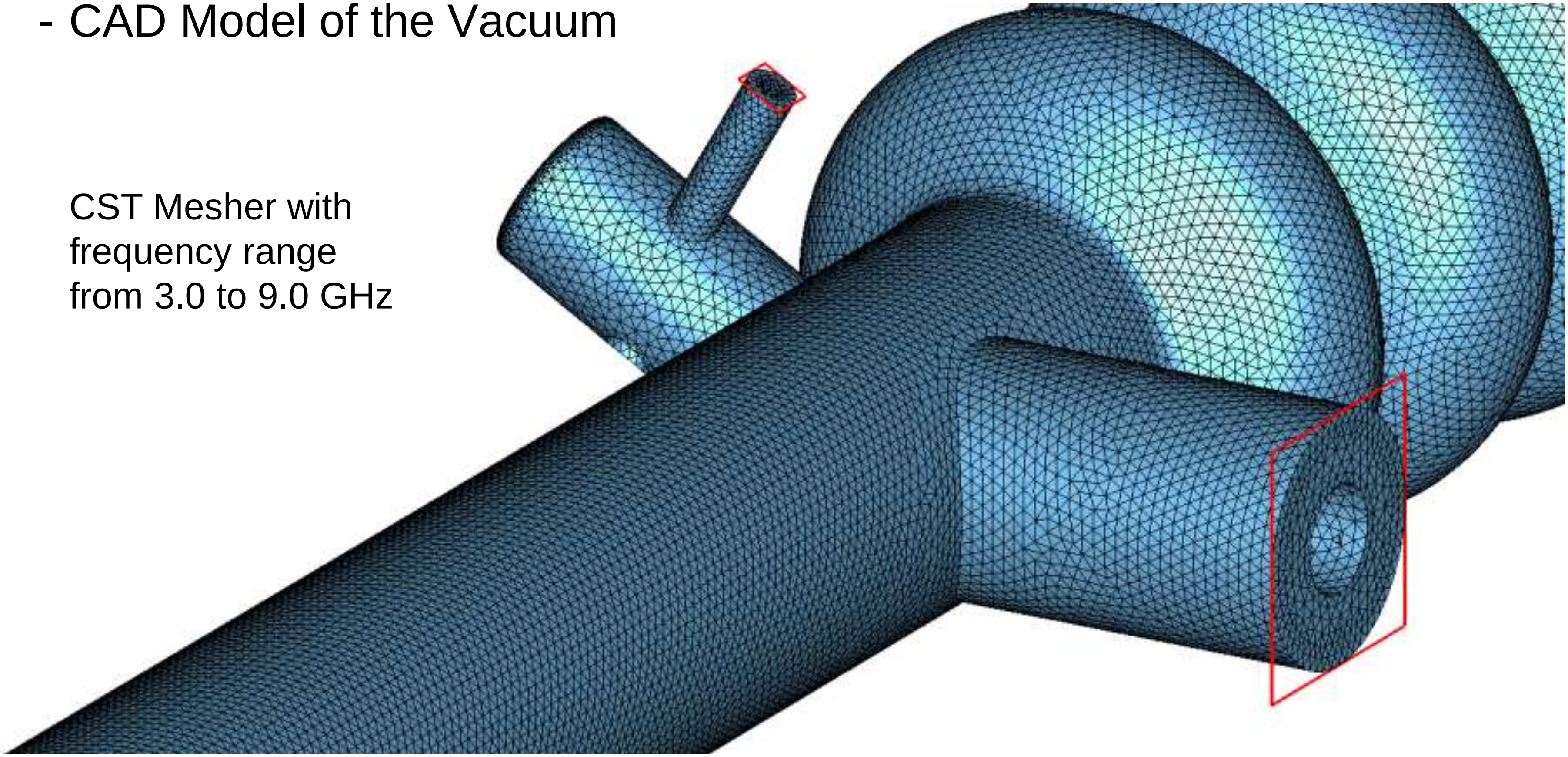
LPW	4	6	8	10	12	14	16	18	20
Tetrahedrons	136.443	187.435	304.833	480.376	767.271	1.177.883	1.704.528	2.432.978	3.337.736
Complex DOF	761.820	1.079.488	1.802.314	2.885.154	4.668.072	7.227.096	10.509.404	15.064.232	20.721.334

Computational Model

- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum

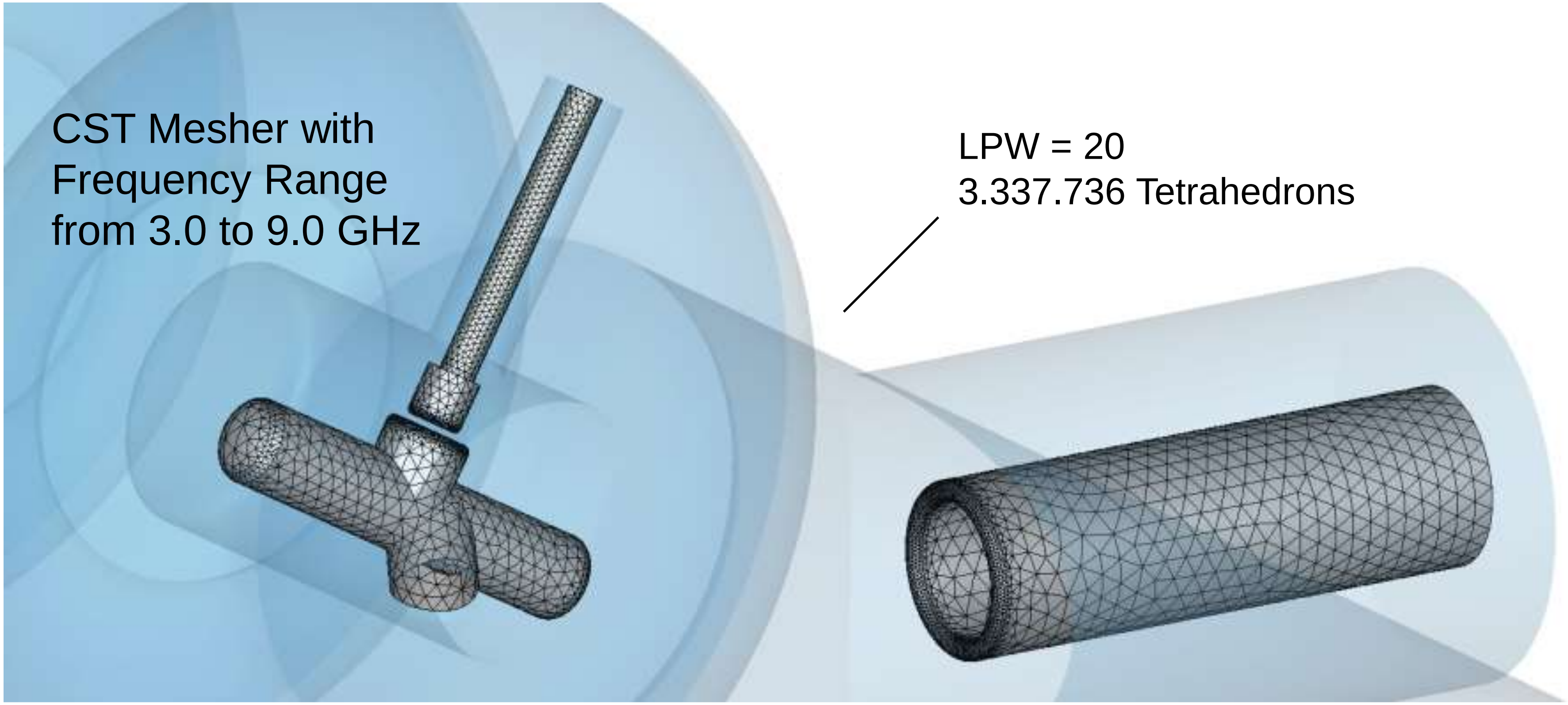
LPW = 20
3.337.736 Tetrahedrons

CST Mesher with
frequency range
from 3.0 to 9.0 GHz



Computational Model

- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum with surface mesh on the PEC couplers



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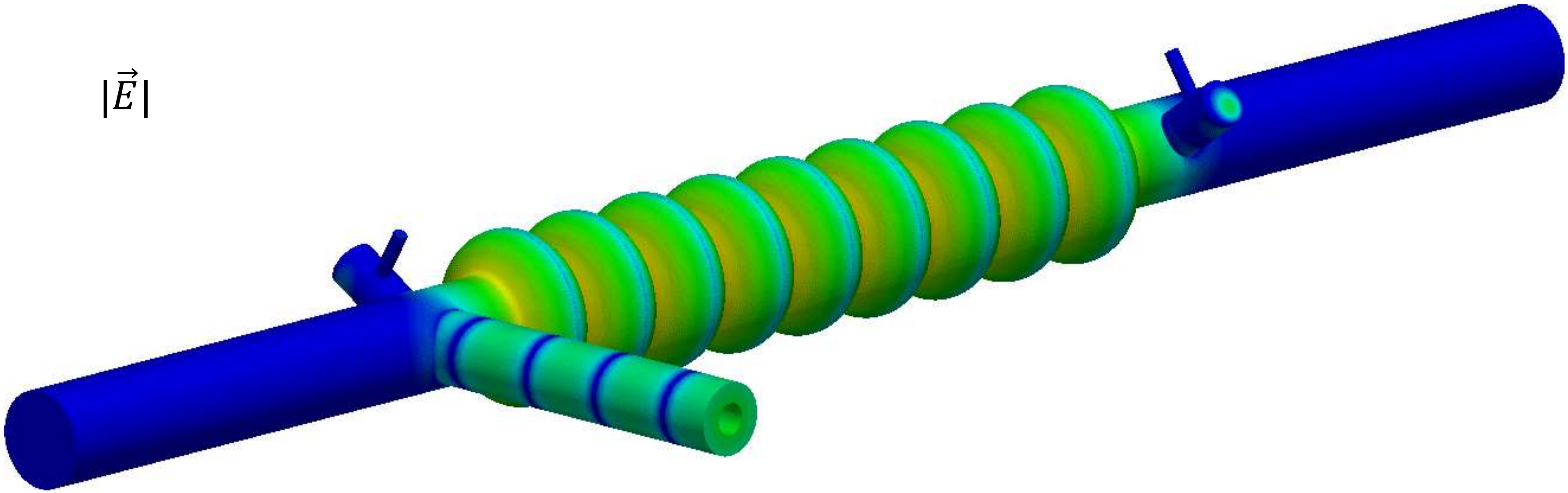
Simulation Results

- TESLA 3.9 GHz Cavity
  - Fundamental mode

    Absolute value of the electric field strength

$|\vec{E}|$

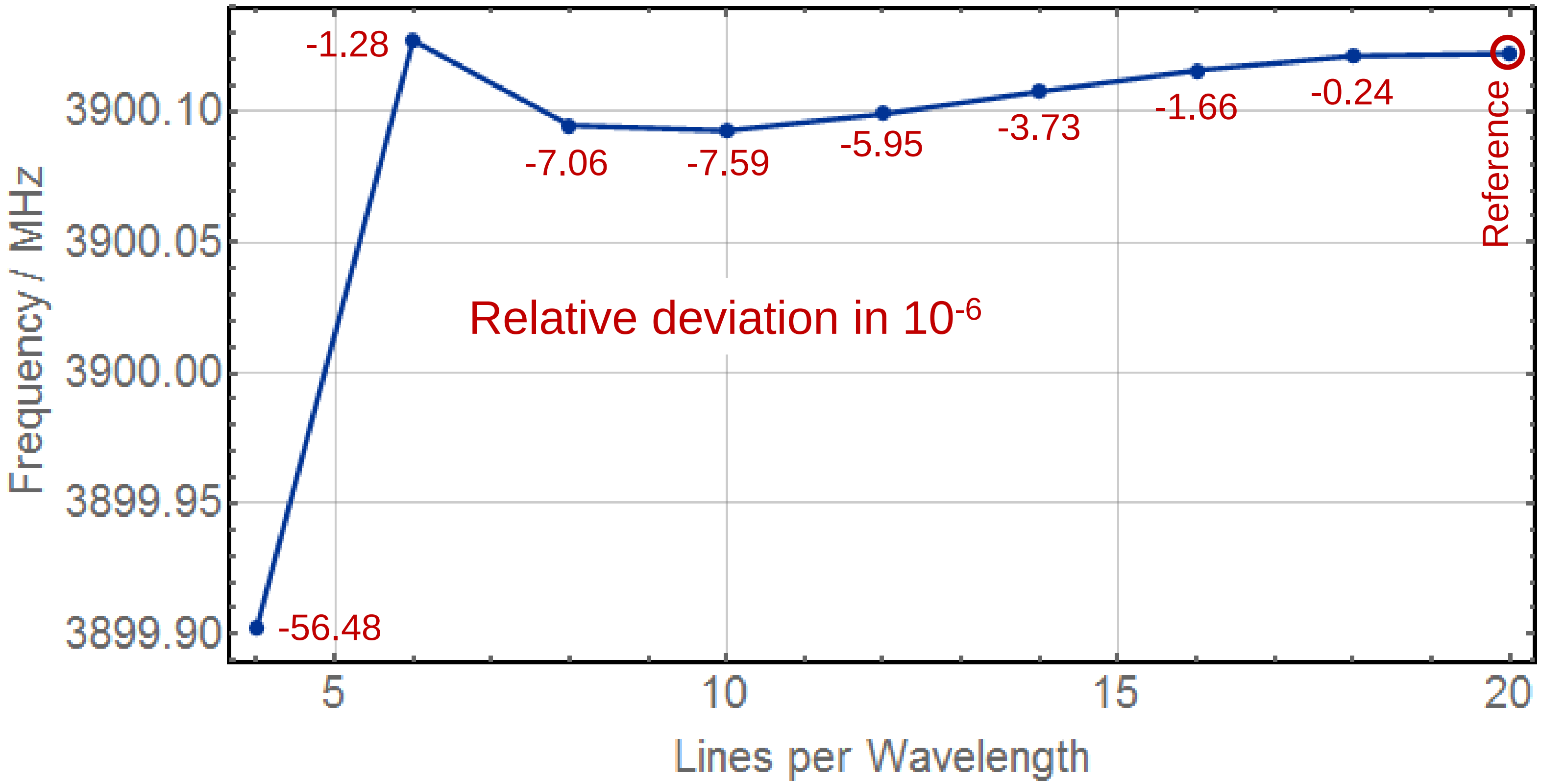


Logarithmic scale from 1e4 to 1e7 V/m

LPW = 20
3.337.736 Tetrahedrons

Simulation Results

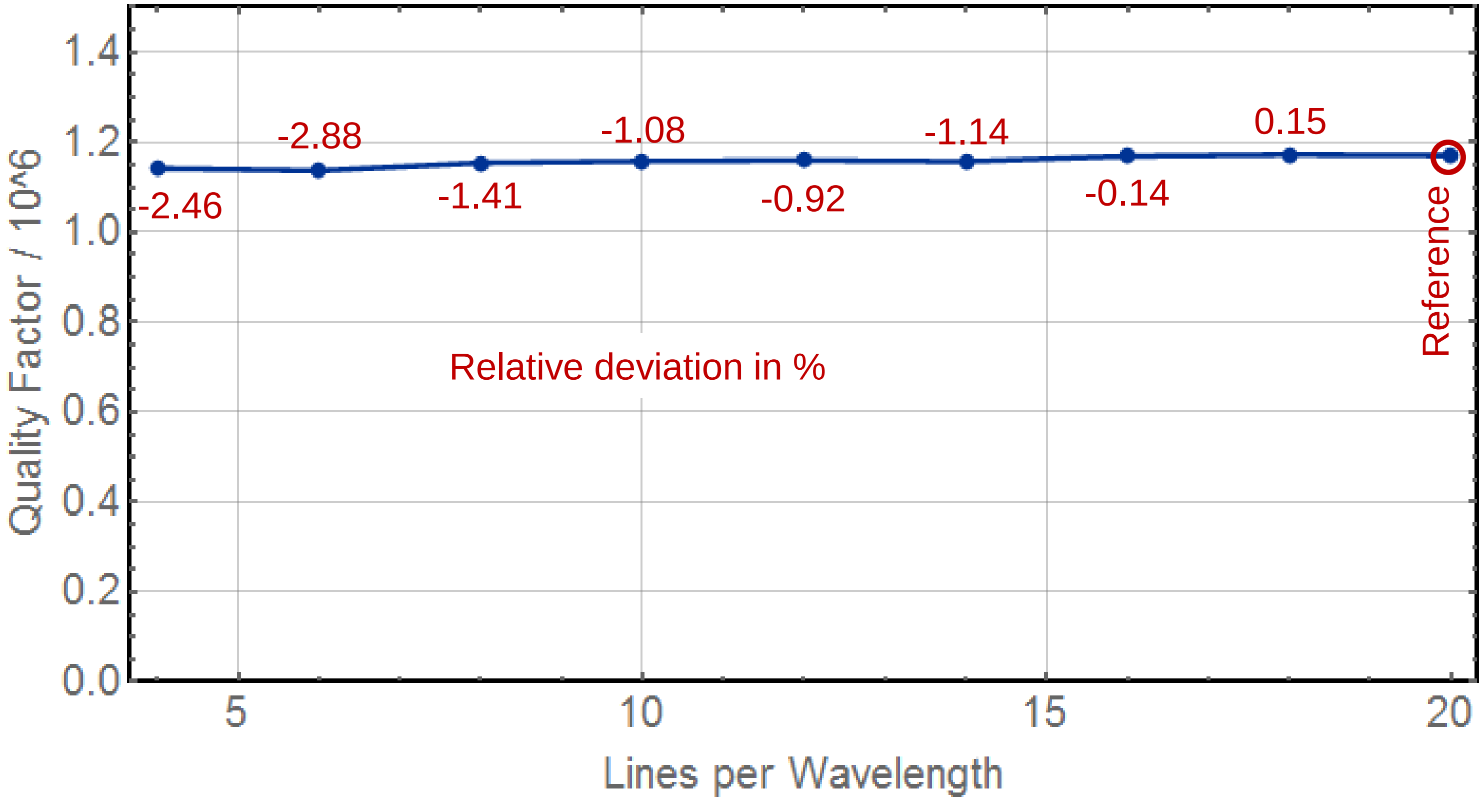
- Convergence study for global quantities
  - Resonance frequency



Simulation Results

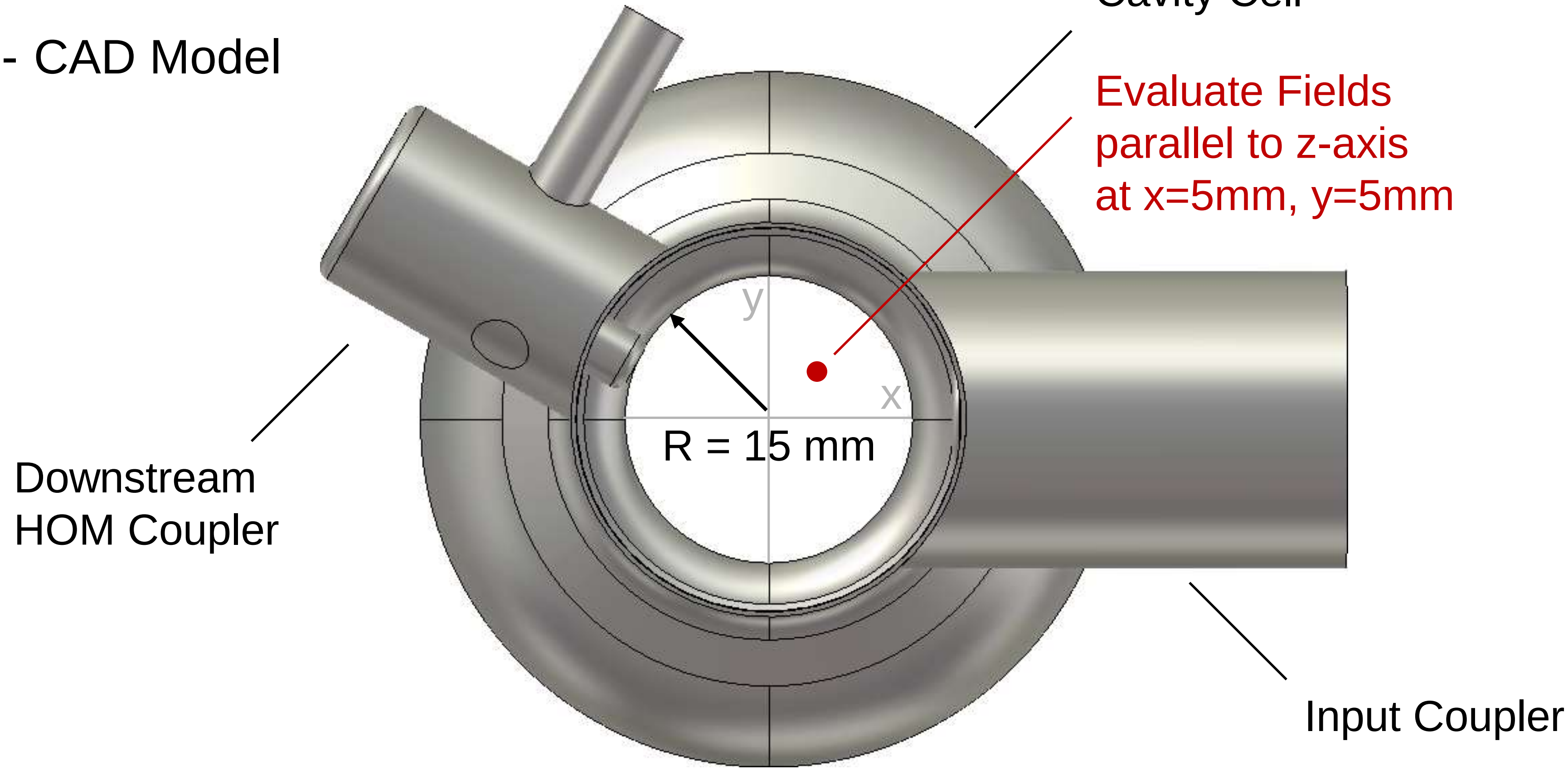
▪ Convergence study for global quantities

- Quality factor



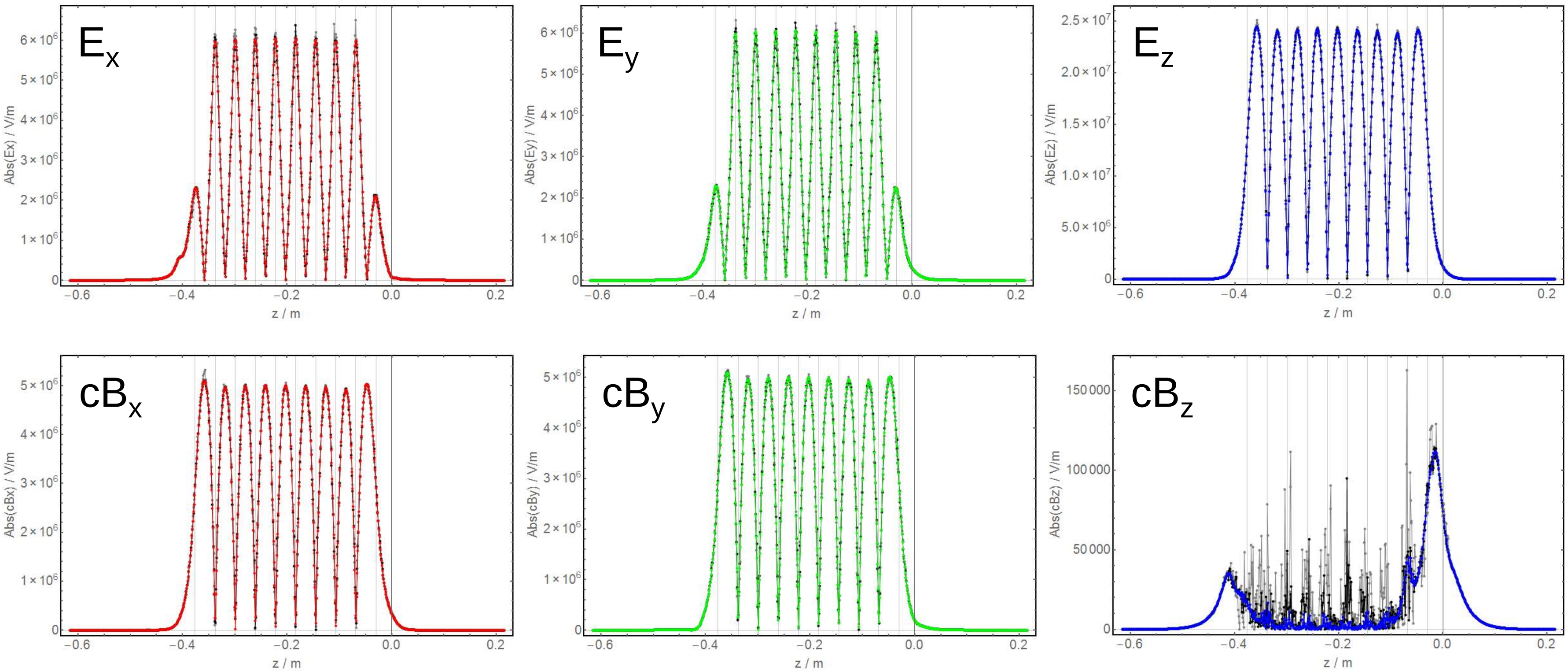
Simulation Results

- TESLA 3.9 GHz Cavity
  - CAD Model



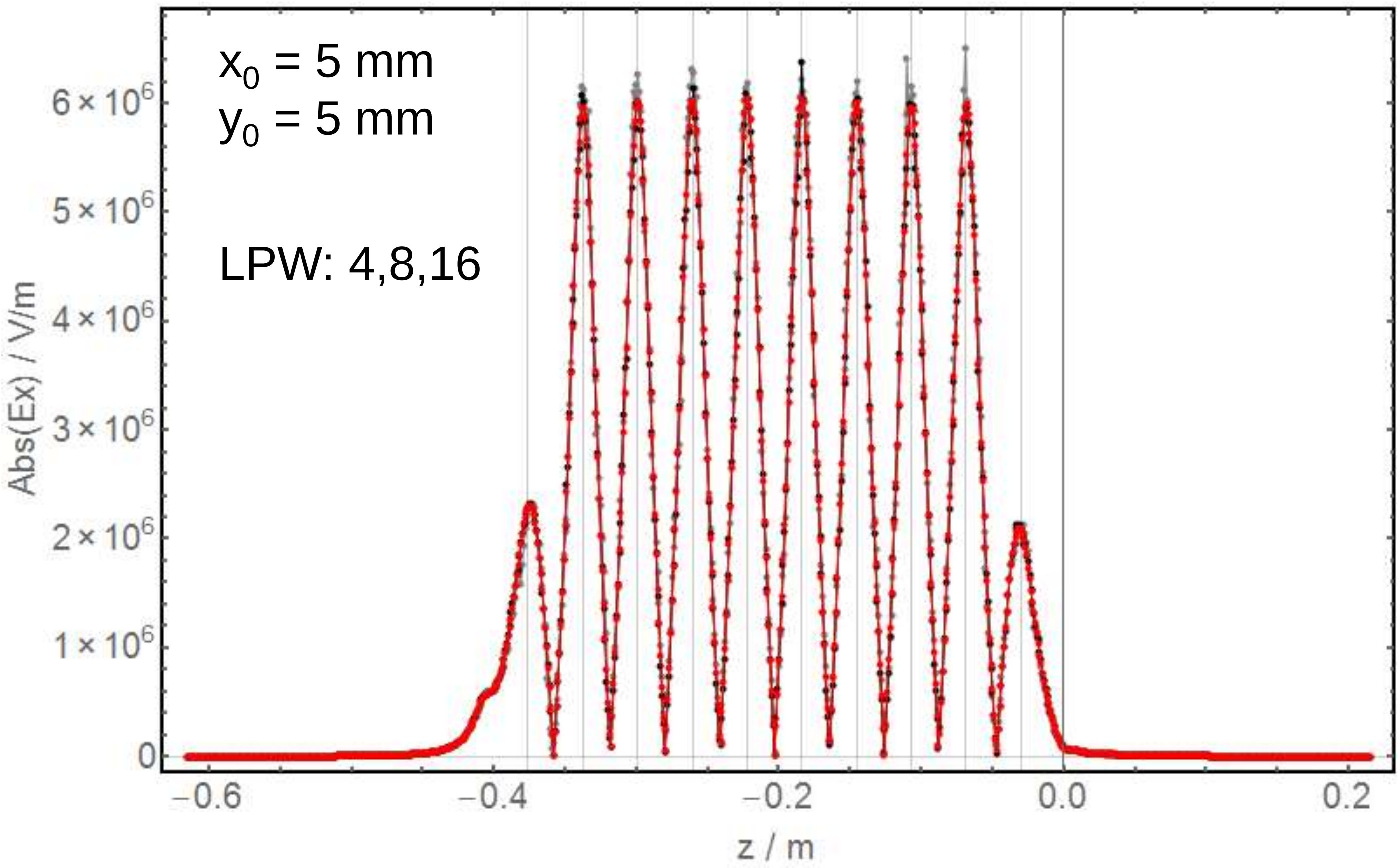
Simulation Results

- Field components parallel to the cavity axis (LPW 4,8,16)
  - Transversal offset at x_0 = 5 mm, y_0 = 5 mm



Simulation Results

- Field component E_x parallel to the cavity axis



Simulation Results

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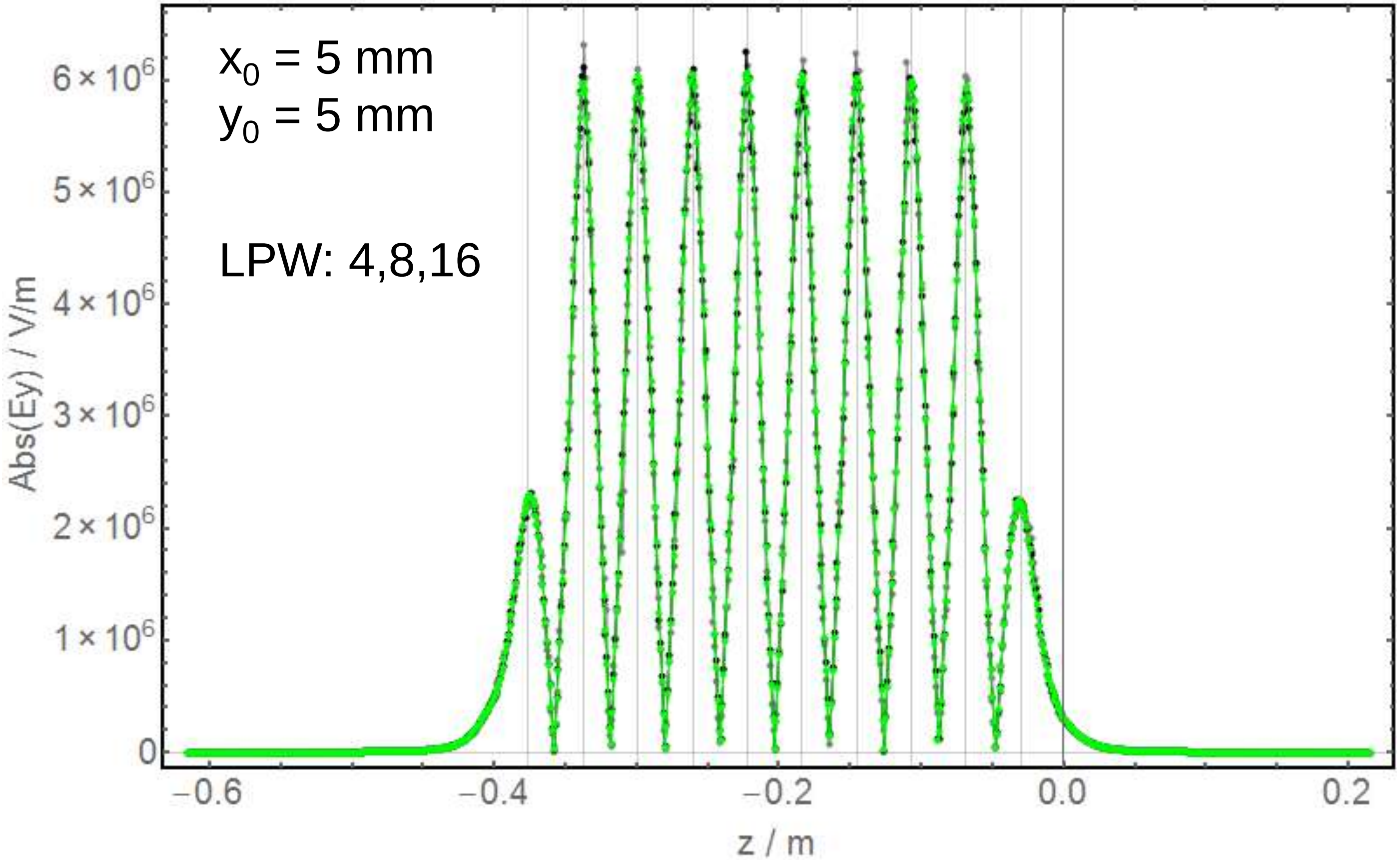
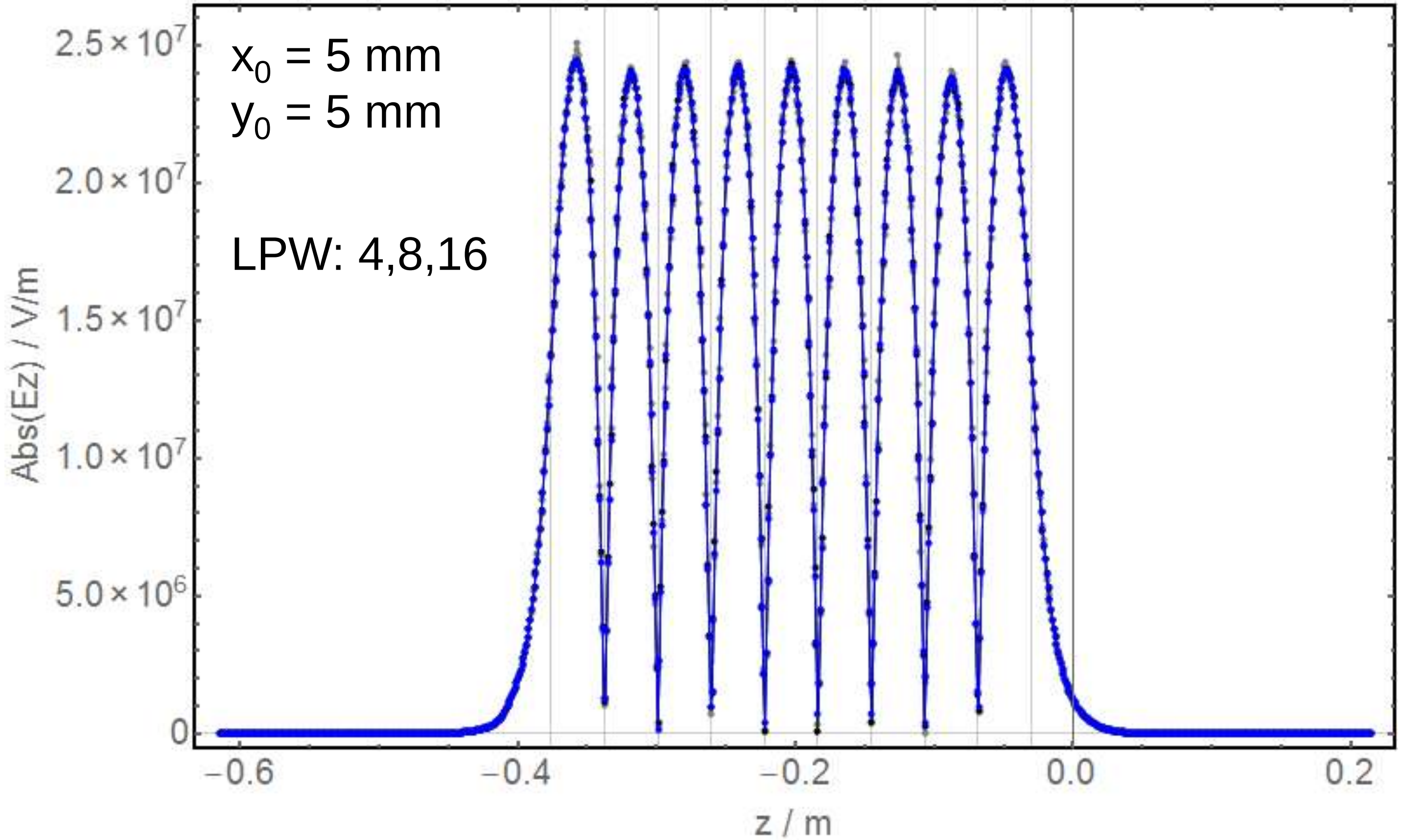
- Field component E_y parallel to the cavity axis



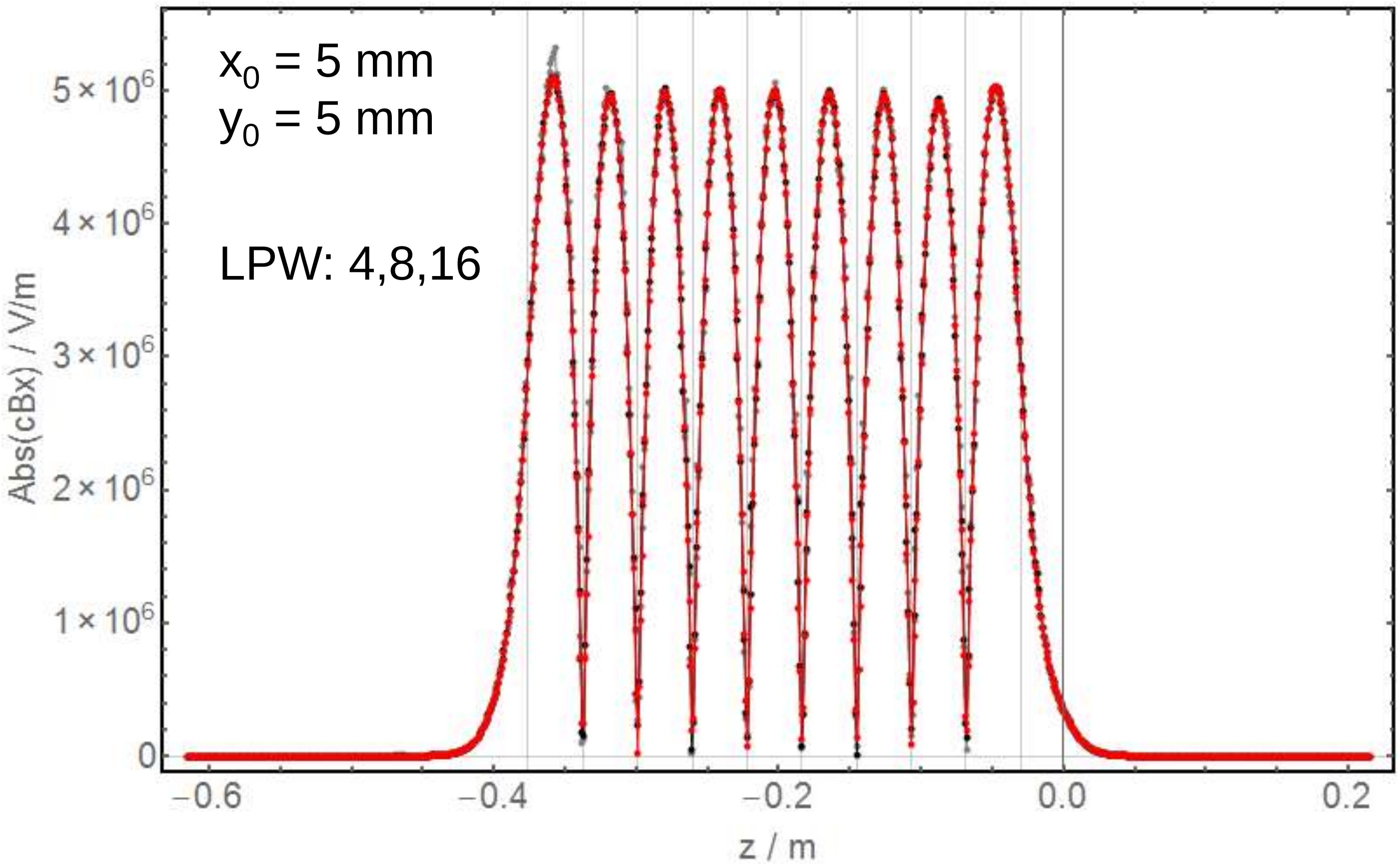
Simulation Results

- Field component E_z parallel to the cavity axis



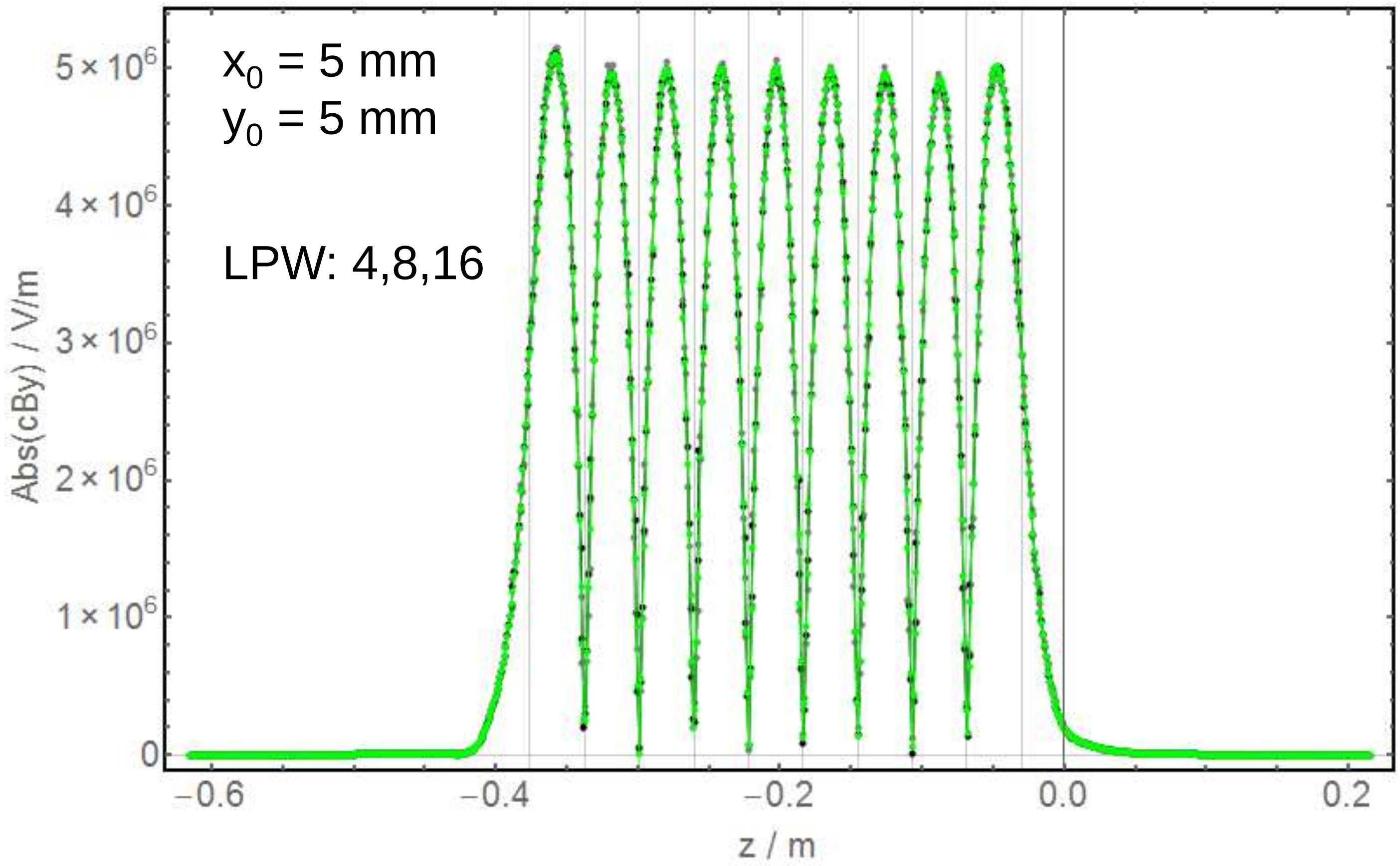
Simulation Results

- Field component cB_x parallel to the cavity axis



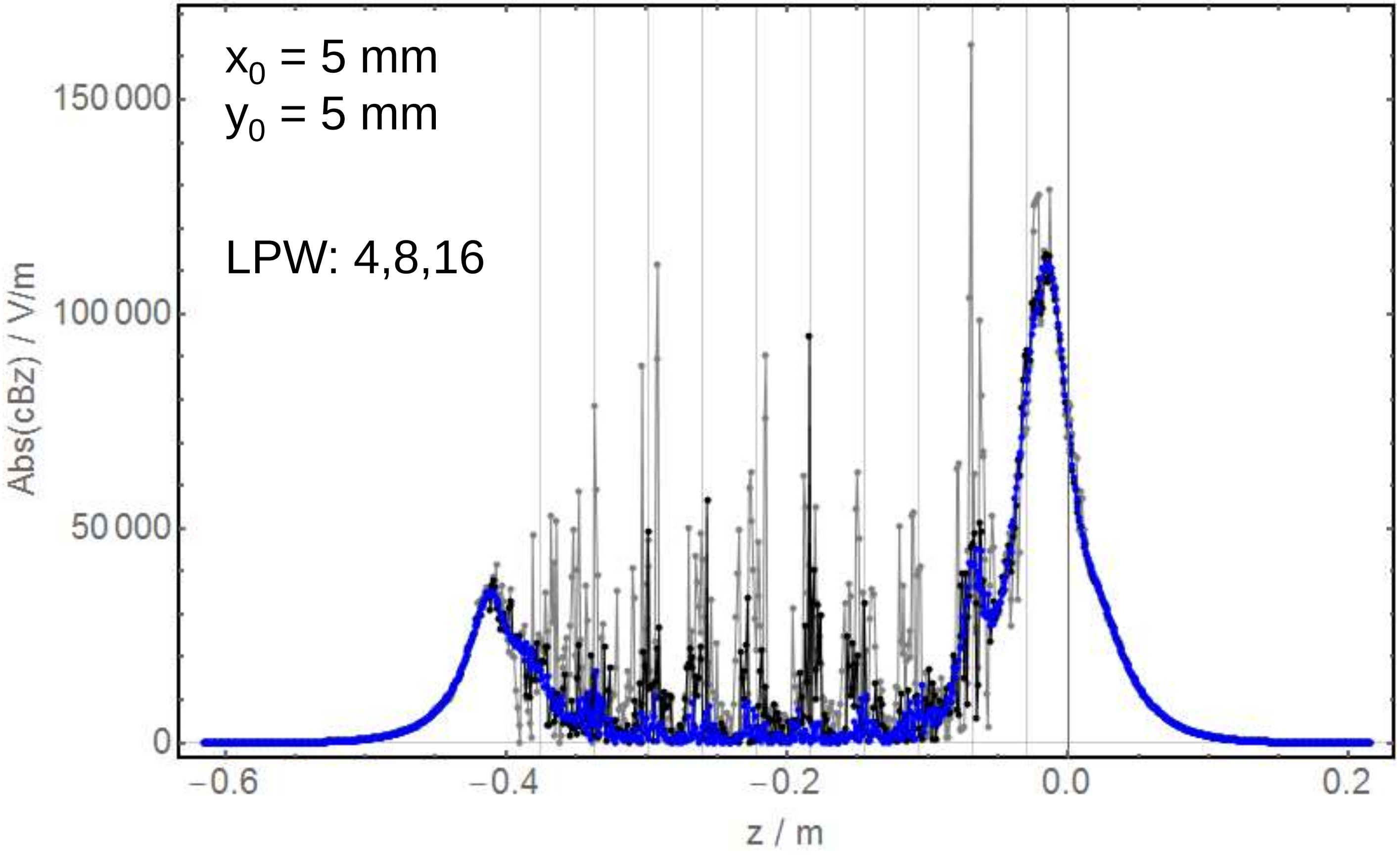
Simulation Results

- Field component cB_y parallel to the cavity axis



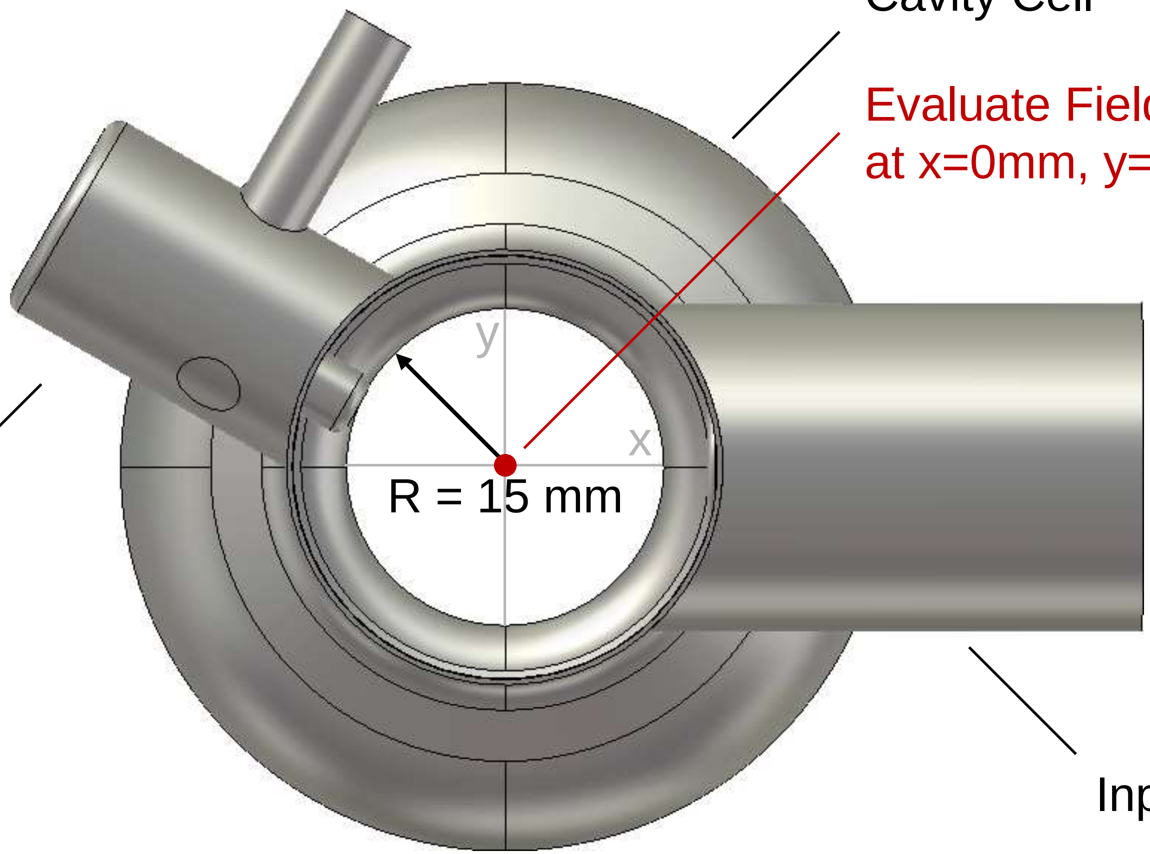
Simulation Results

- Field component cB_z parallel to the cavity axis



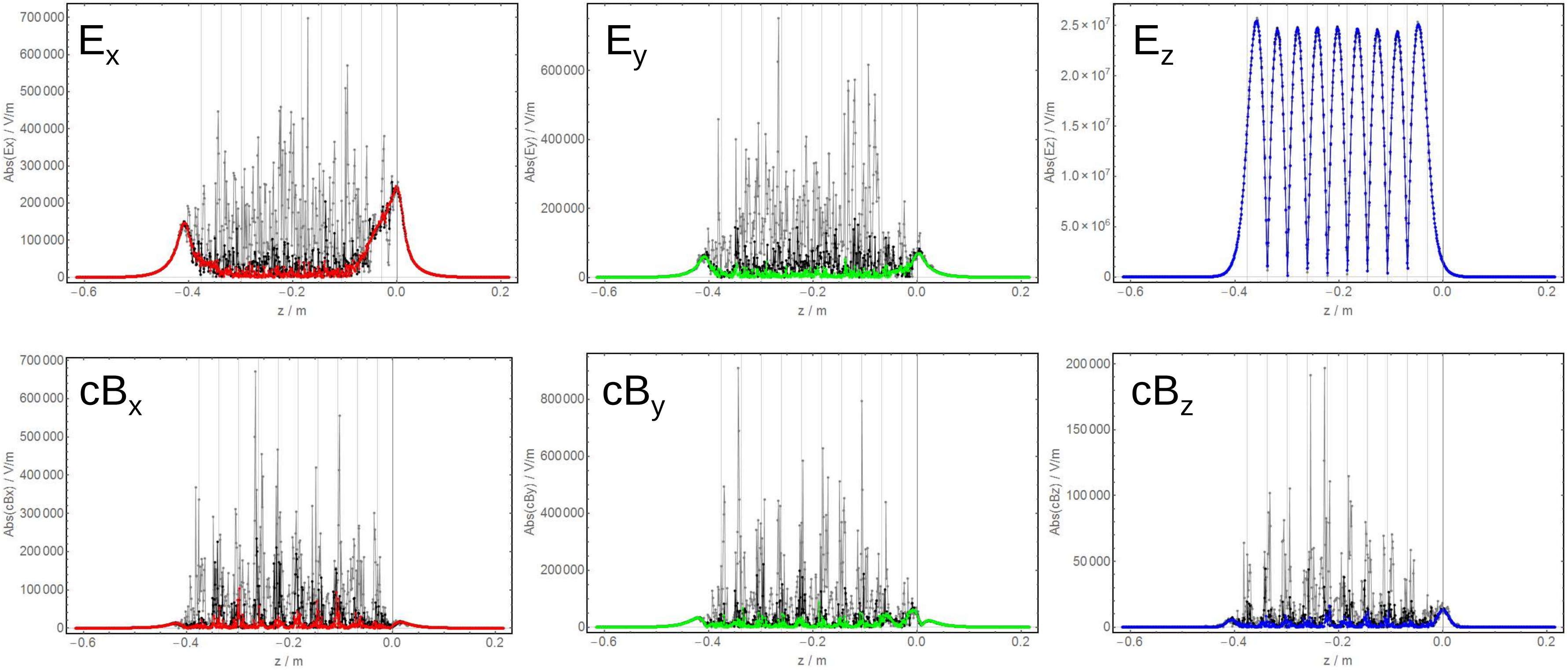
Simulation Results

- TESLA 3.9 GHz Cavity
  - CAD Model



Simulation Results

- Field components parallel to the cavity axis (LPW 4,8,16)
  - Transversal offset at x_0 = 0 mm, y_0 = 0 mm



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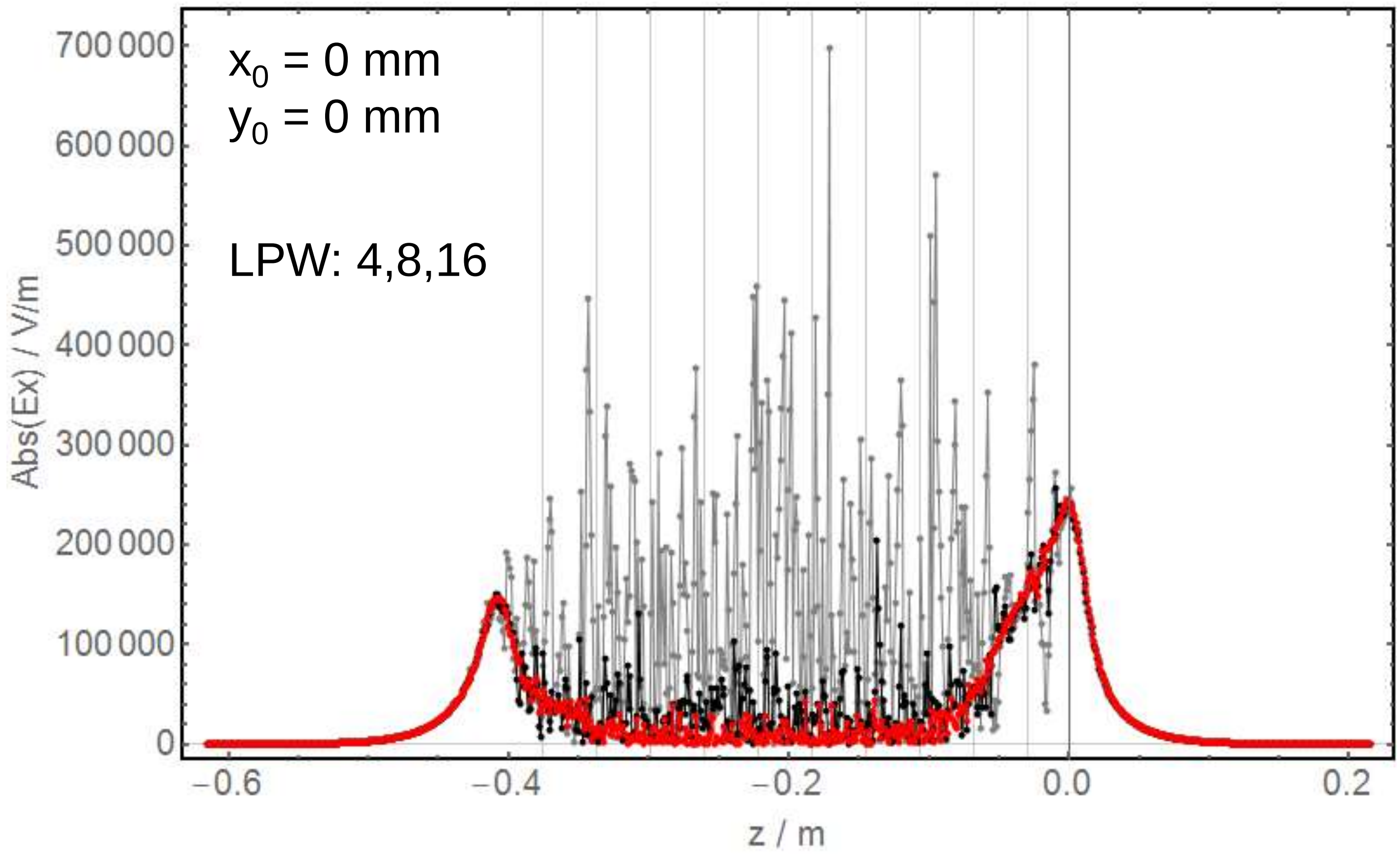
- Field component E_x parallel to the cavity axis



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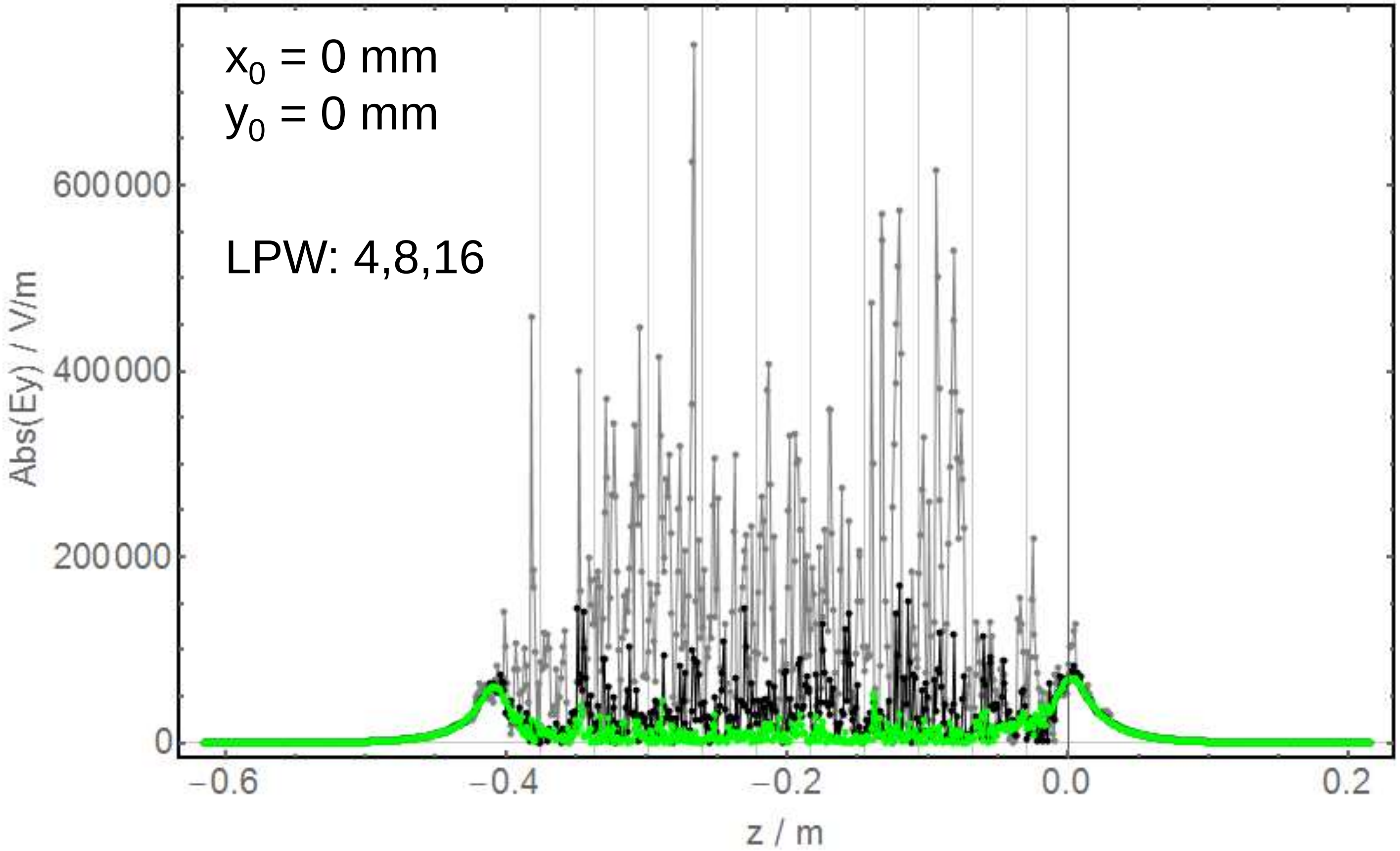
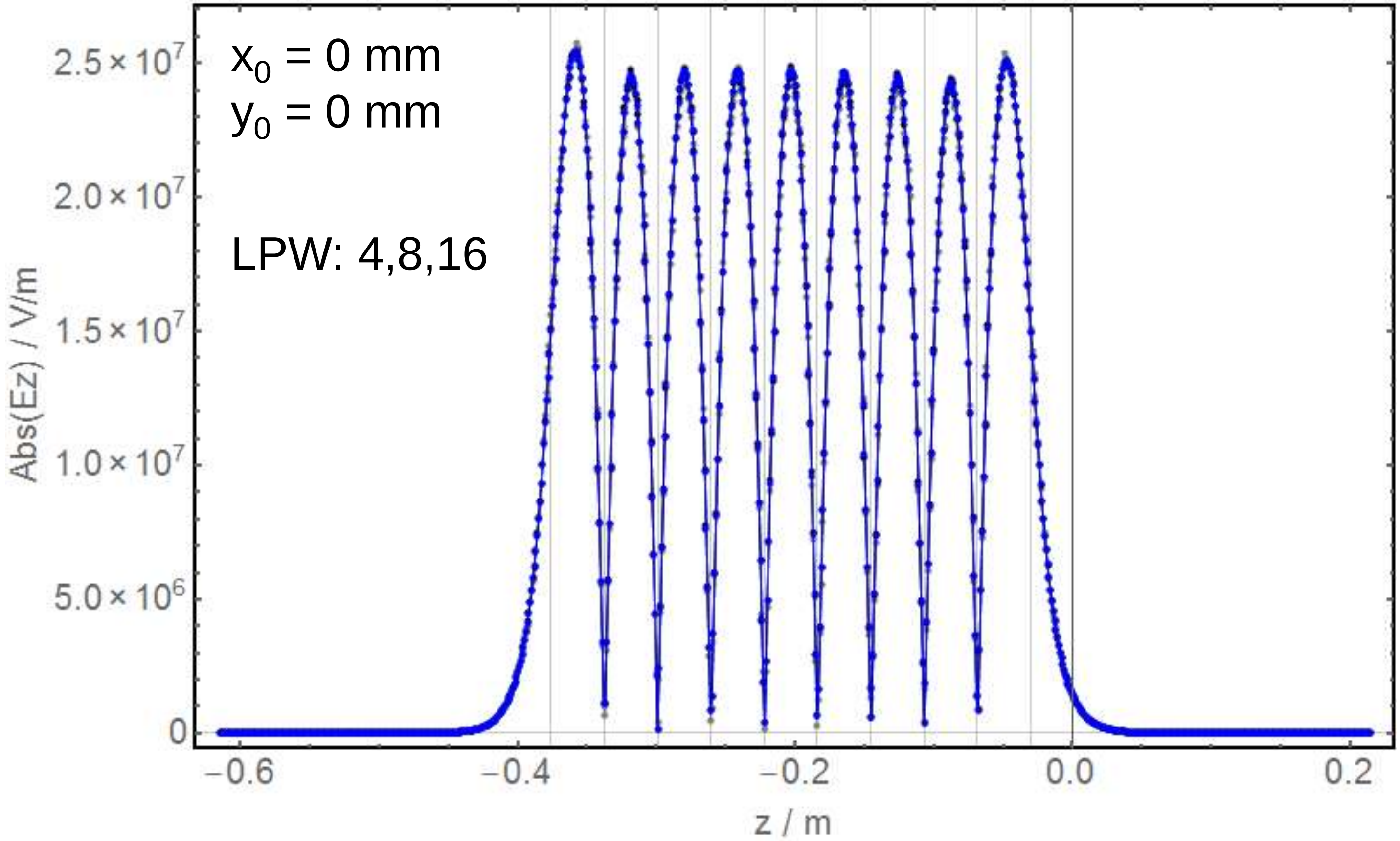


Simulation Results

- Field component E_z parallel to the cavity axis



Simulation Results

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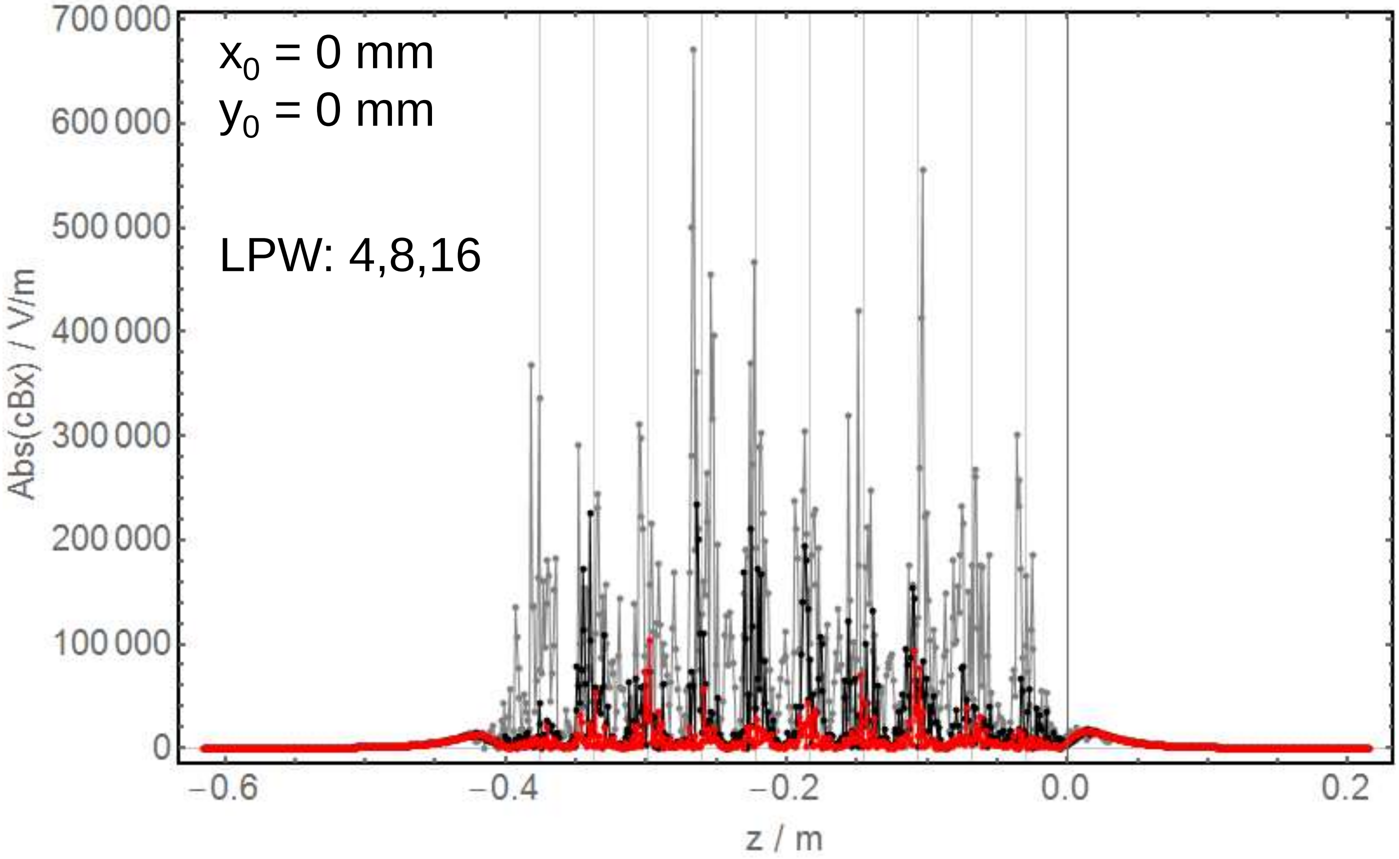
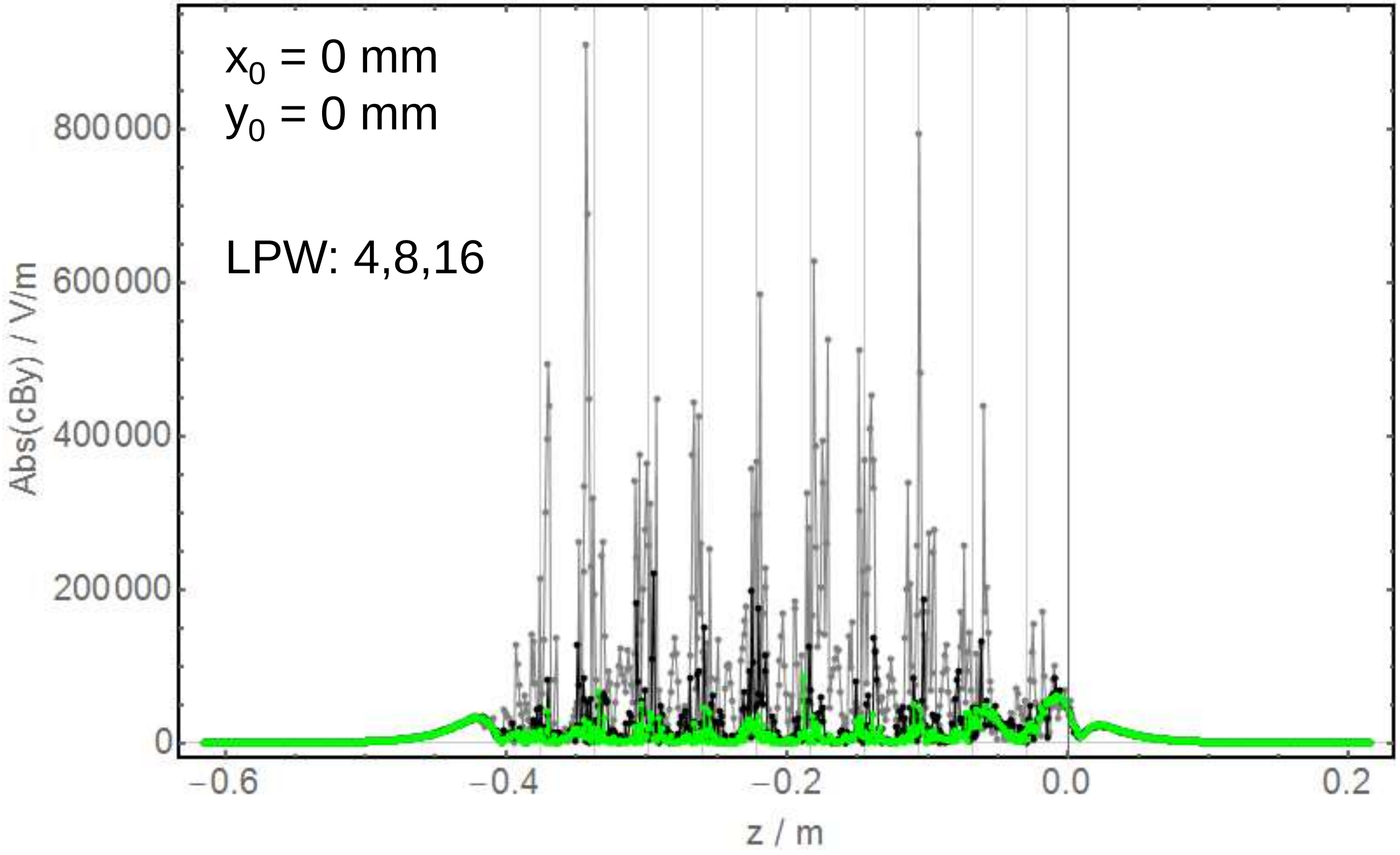
- Field component cB_x parallel to the cavity axis



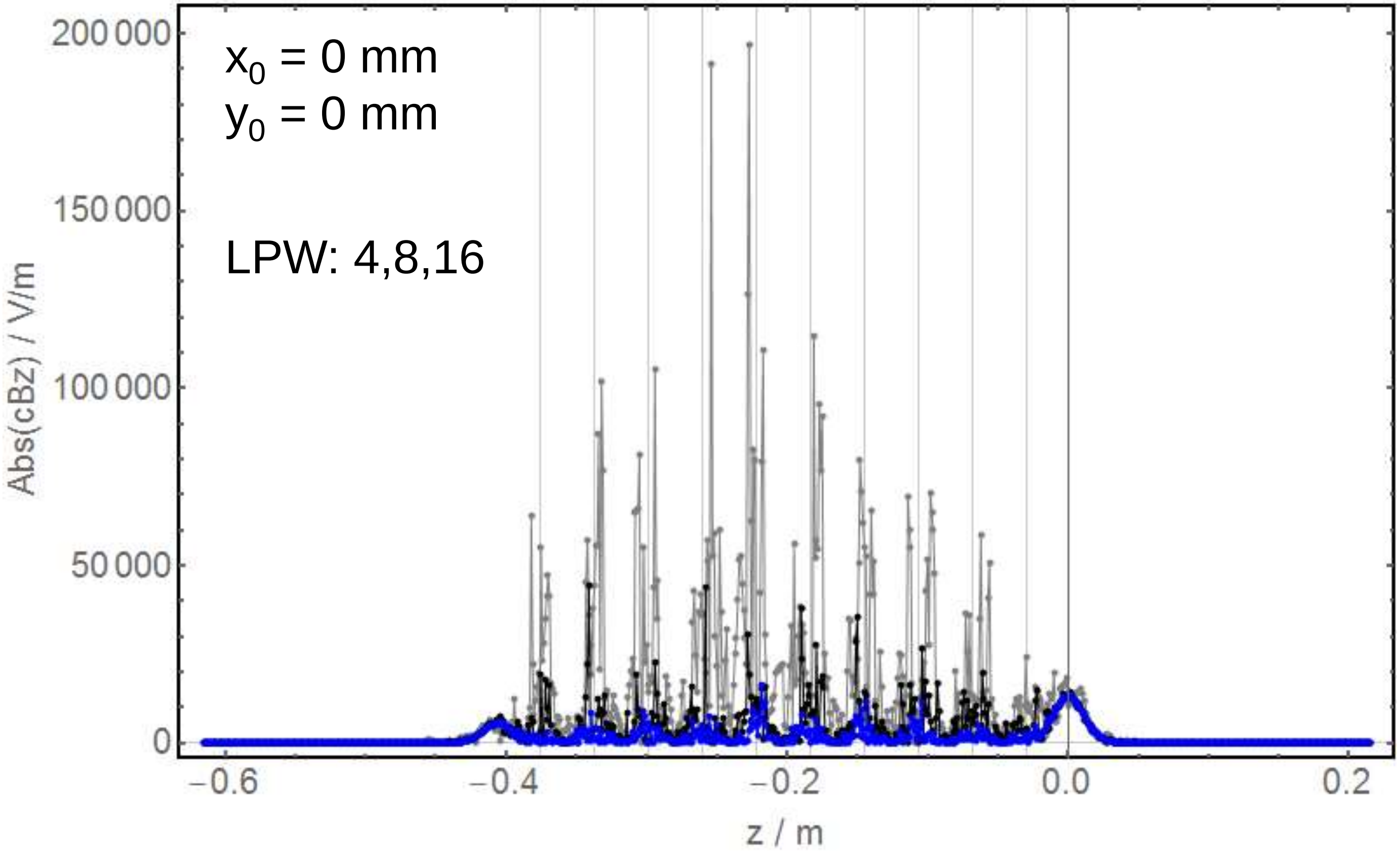
Simulation Results

- Field component cB_y parallel to the cavity axis



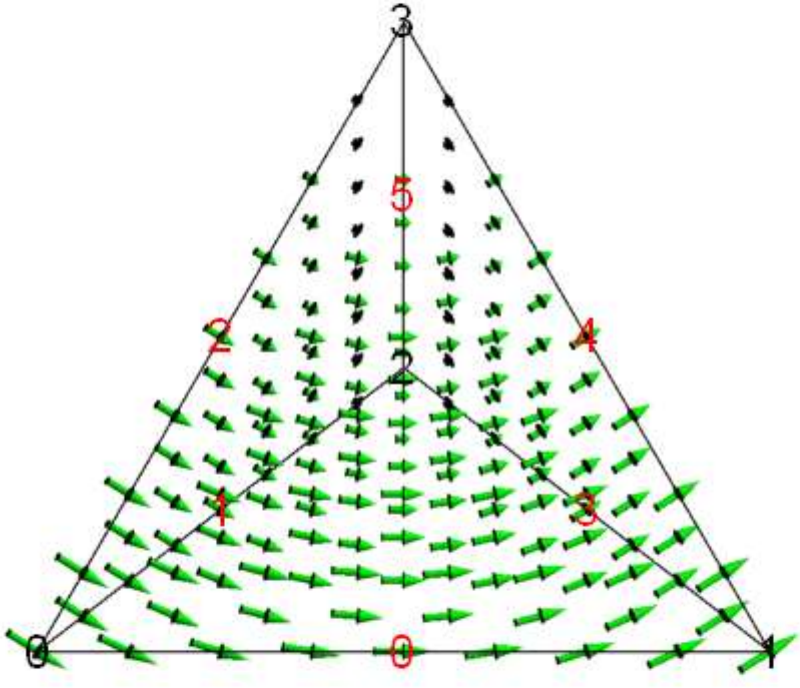
Simulation Results

- Field component cB_z parallel to the cavity axis



Simulation Results

- Field Representation in the Finite Element Method
  - Vector Basis Funktion $\vec{w}_0(\vec{r})$



Example:
Equilateral tetrahedron

Point	x	y	z
0	0	0	0
1	1	0	0
2	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	0
3	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\sqrt{2/3}$

Simulation Results

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- Field Representation in the Finite Element Method
  - Representation of vector fields

$$\vec{f}(\vec{r}) = \sum_{i=0}^{N-1} a_i \, \vec{w}_i(\vec{r})$$

  - Projection of an arbitrary vector field $\vec{f}$ on the basis $\vec{w}_i$

$$\sum_{i=0}^{N-1} a_i \underbrace{\iiint_\Omega \vec{w}_i \cdot \vec{w}_j \, d\Omega}_{\text{mat}} = \underbrace{\iiint_\Omega \vec{f} \cdot \vec{w}_j \, d\Omega}_{\text{vec}}$$

⇒ Solve linear system to obtain the weighting coefficients a_i

Simulation Results

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- Field Representation in the Finite Element Method

  - Residuals of vector fields

$$\vec{R}(\vec{r}) = \sum_{i=0}^{N-1} a_i \, \vec{w}_i(\vec{r}) - \vec{f}(\vec{r})$$

  - Fundamental field components

$$\vec{f}(\vec{r}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix} x^2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} (x-1)x + (33 + 10\sqrt{3}y + 5\sqrt{6}z)/180 \\ 0 \\ 0 \end{pmatrix}$$

Example:
FEM with full linear basis

Order	DOF per cell
0.5	6
1	12
1.5	20
2	30
2.5	45
3	60
3.5	84
4	105

Simulation Results

- Field Representation in the Finite Element Method

  - Residuals of vector fields

$$\vec{R}(\vec{r}) = \sum_{i=0}^{N-1} a_i \, \vec{w}_i(\vec{r}) - \vec{f}(\vec{r})$$

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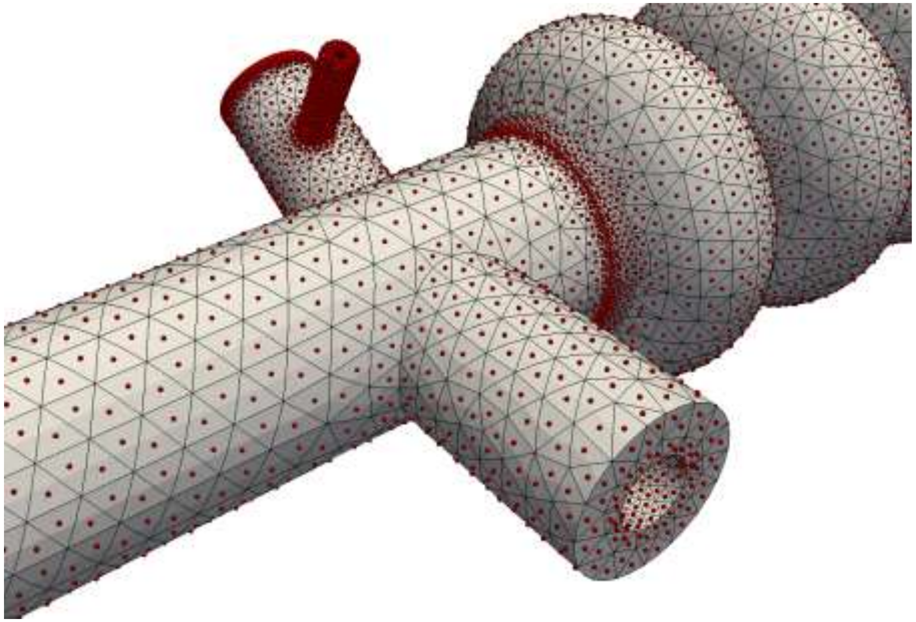
$$\vec{f}(\vec{r}) = \begin{pmatrix} x^2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} (x-1)x + (33 + 10\sqrt{3}y + 5\sqrt{6}z)/180 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix} y^2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} -7y/(6\sqrt{3}) + y^2 + (13 + 5\sqrt{6}z)/180 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix} z^2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} 2/45 - 2/3\sqrt{2/3}z + z^2 \\ 0 \\ 0 \end{pmatrix}$$

Simulation Results

- Field reconstruction using the Kirchhoff integral
  - Field values inside a closed surface can be determined once the surface field components are available
  - Kirchhoff integral



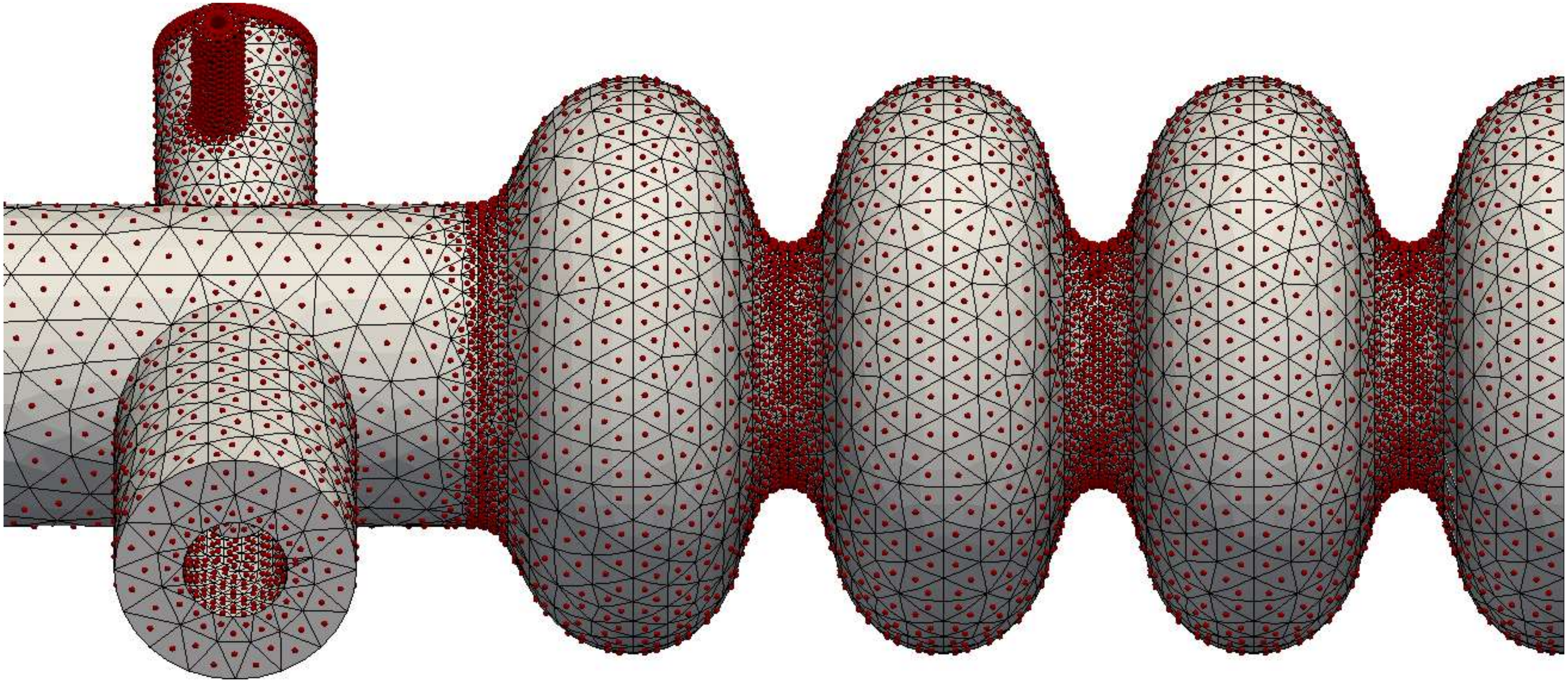
$$G = \frac{e^{-ik|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|} \qquad k = \frac{2\pi f}{c_0}$$

$$\vec{E}(\vec{r}) = \int \left(k(\vec{n}' \times ic_0\vec{B}')\, G - (\vec{n}' \times \vec{E}') \times \nabla G - (\vec{n}' \cdot \vec{E}')\, \nabla G \right) \mathrm{d}A'$$

$$ic_0\vec{B}(\vec{r}) = \int \left(k(\vec{n}' \times \vec{E}')\, G - (\vec{n}' \times ic_0\vec{B}') \times \nabla G - (\vec{n}' \cdot ic_0\vec{B}')\, \nabla G \right) \mathrm{d}A'$$

Simulation Results

- Field reconstruction using the Kirchhoff integral
  - Surface selection



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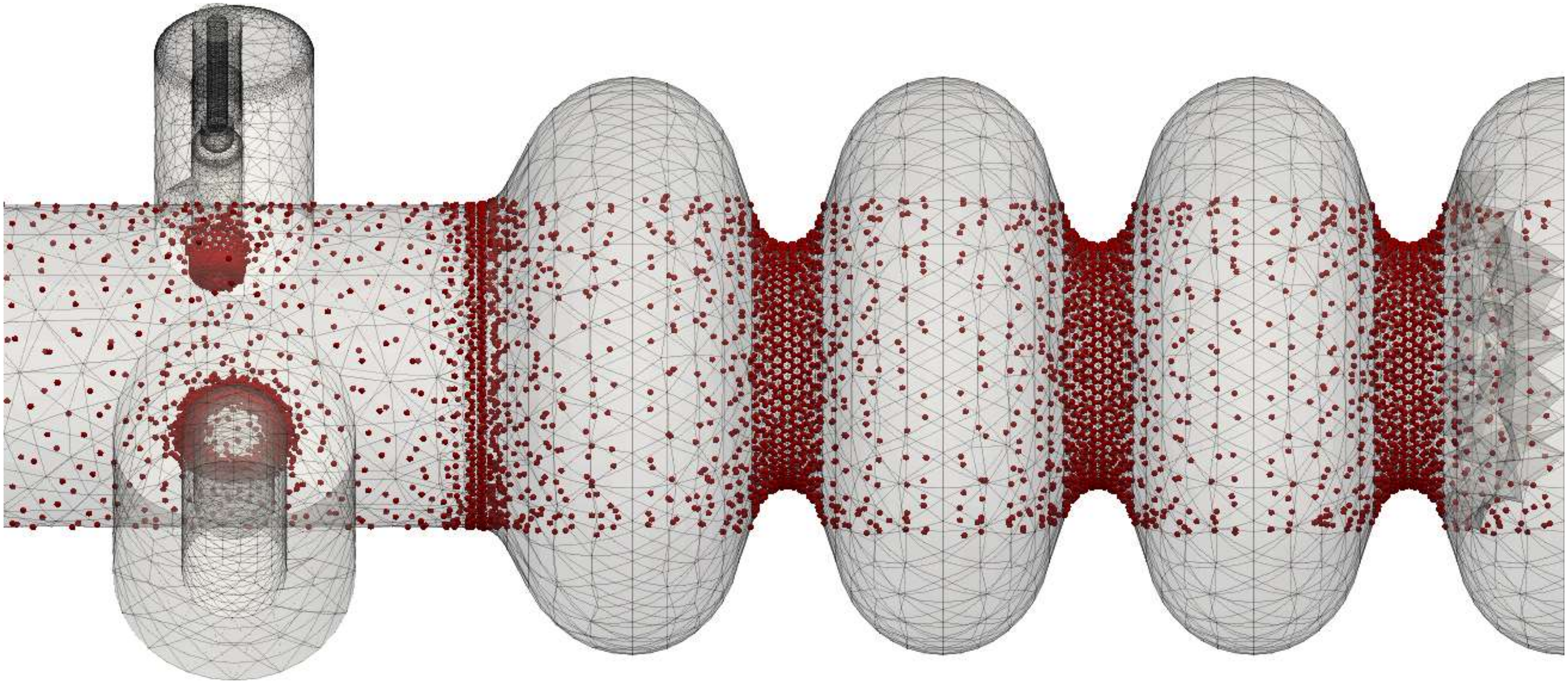

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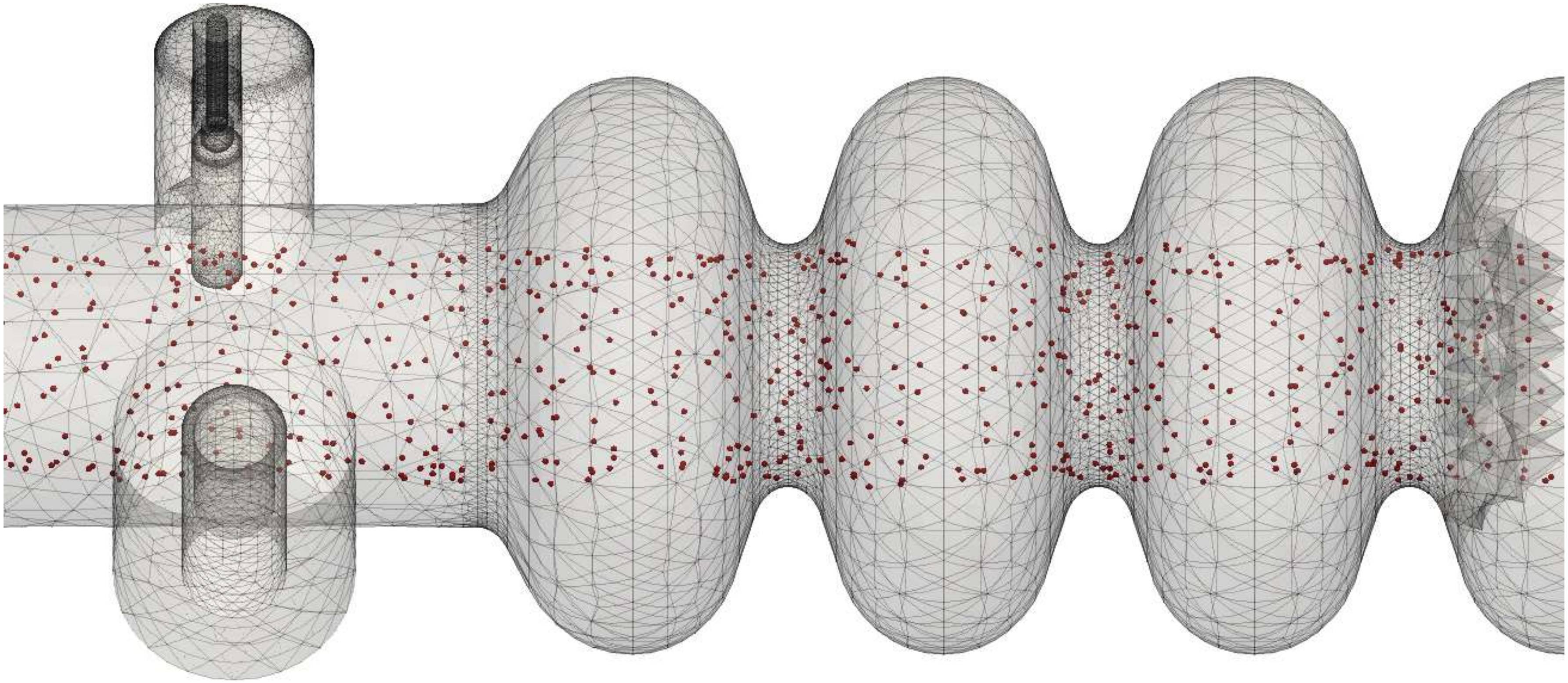
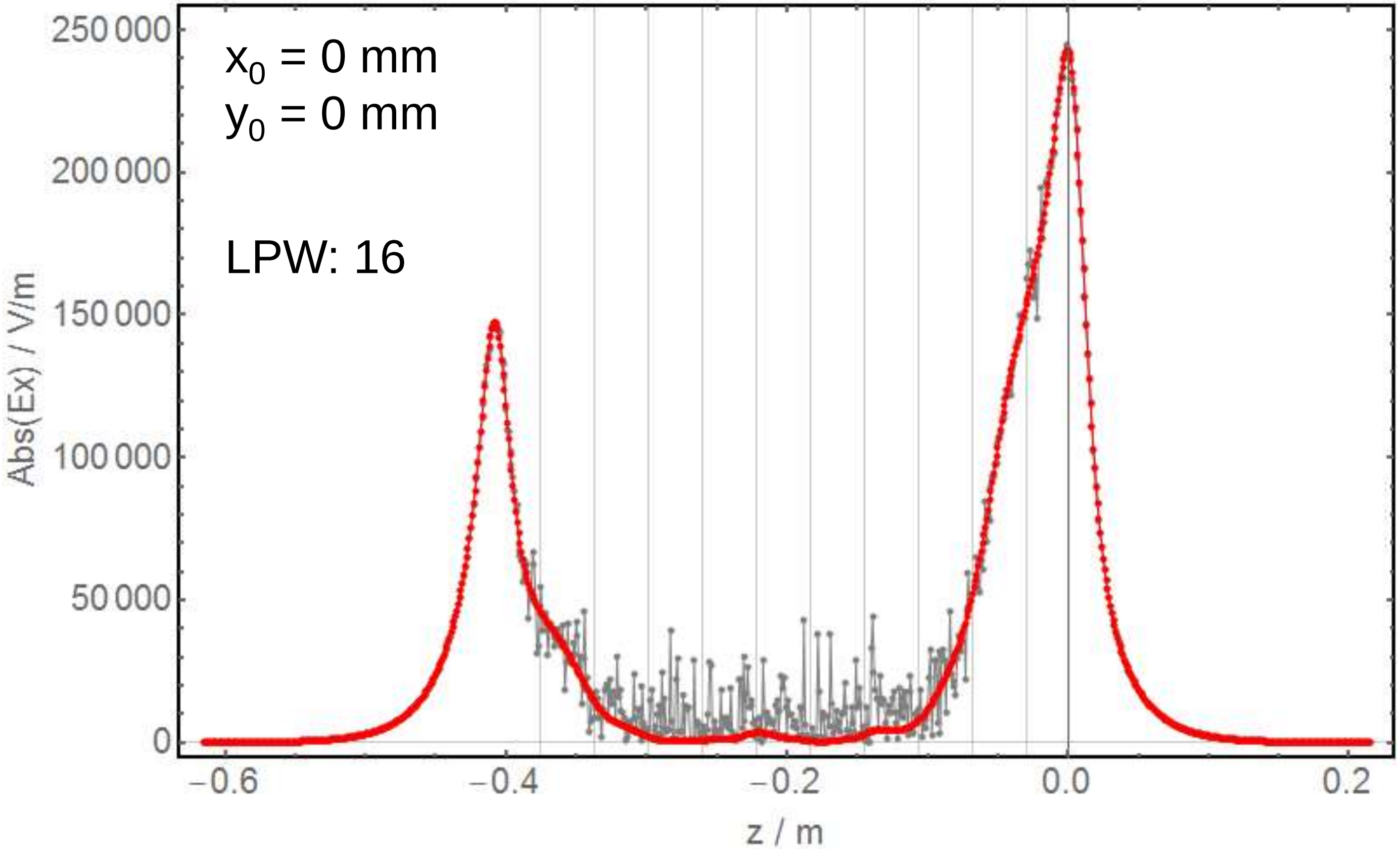


Simulation Results

- Field component E_x parallel to the cavity axis



Simulation Results

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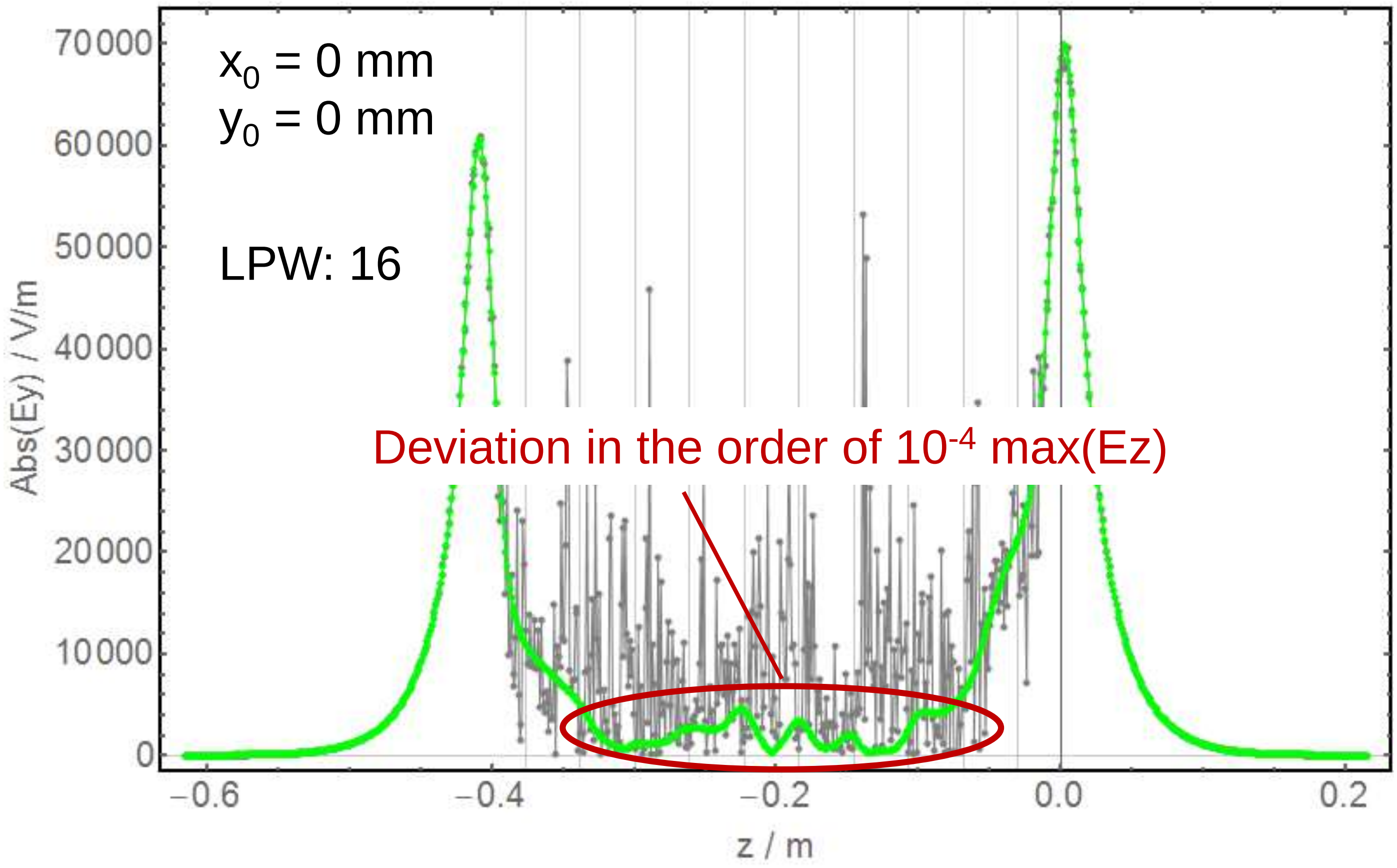
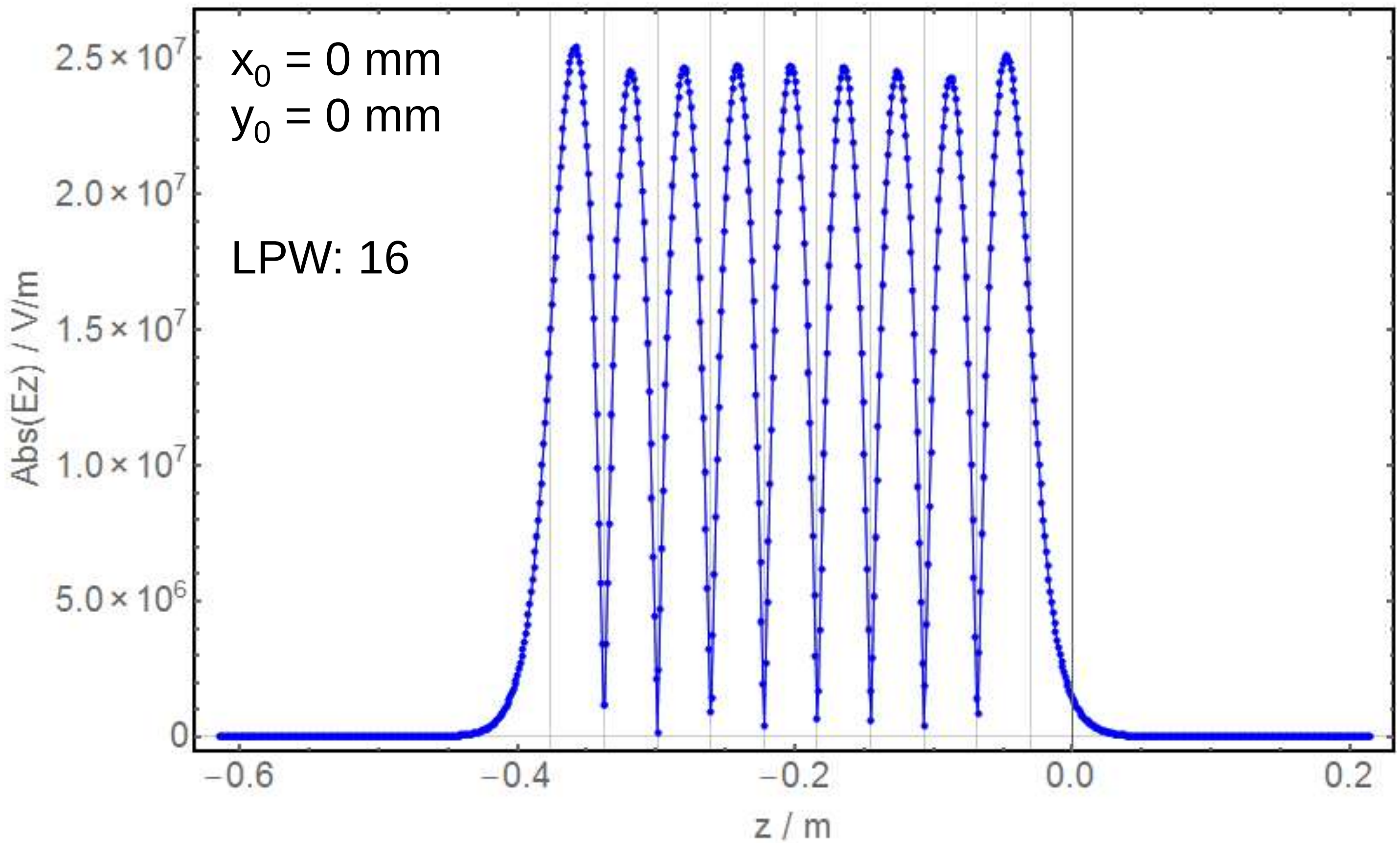
- Field component E_y parallel to the cavity axis



Simulation Results

- Field component E_z parallel to the cavity axis



Simulation Results

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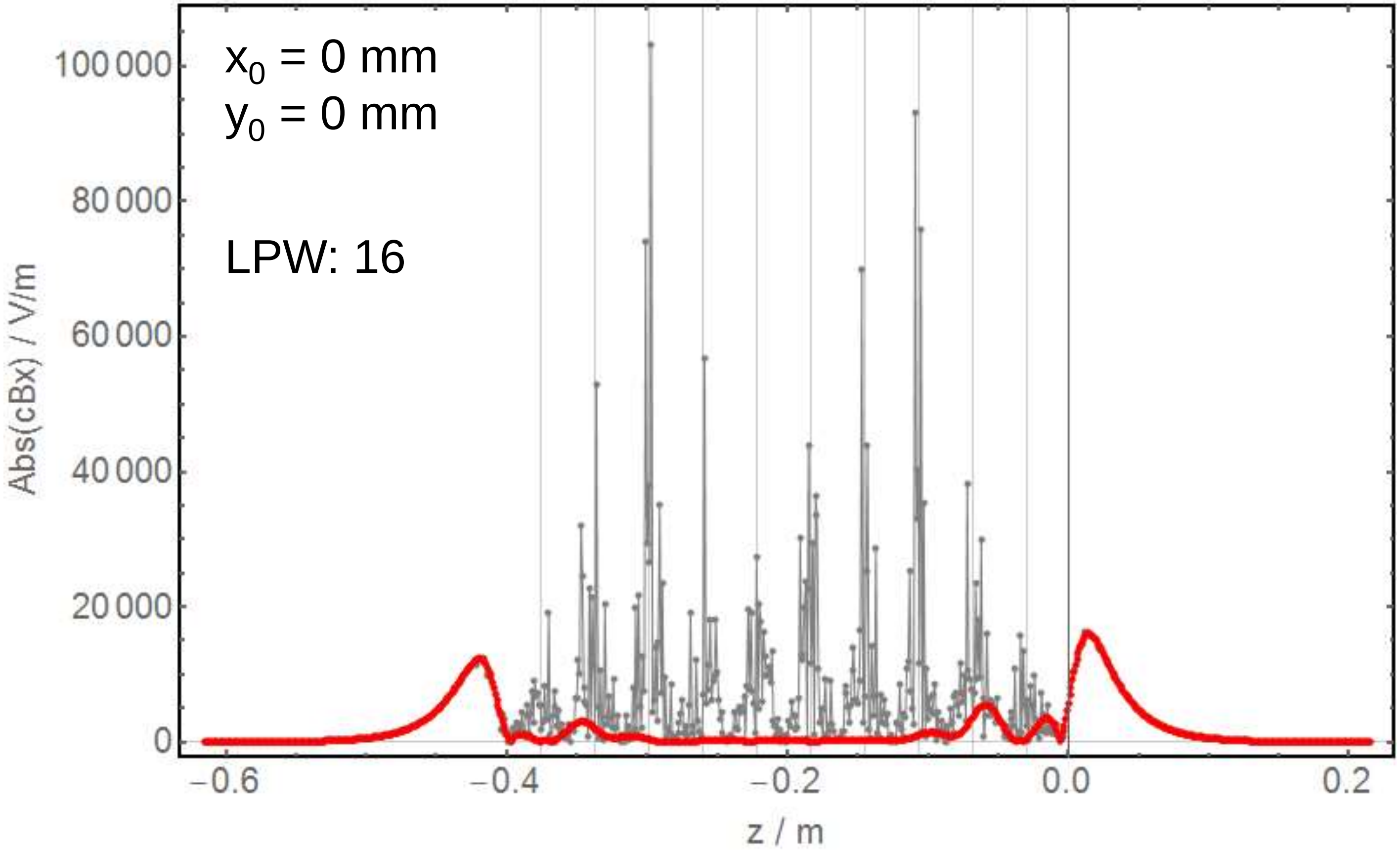
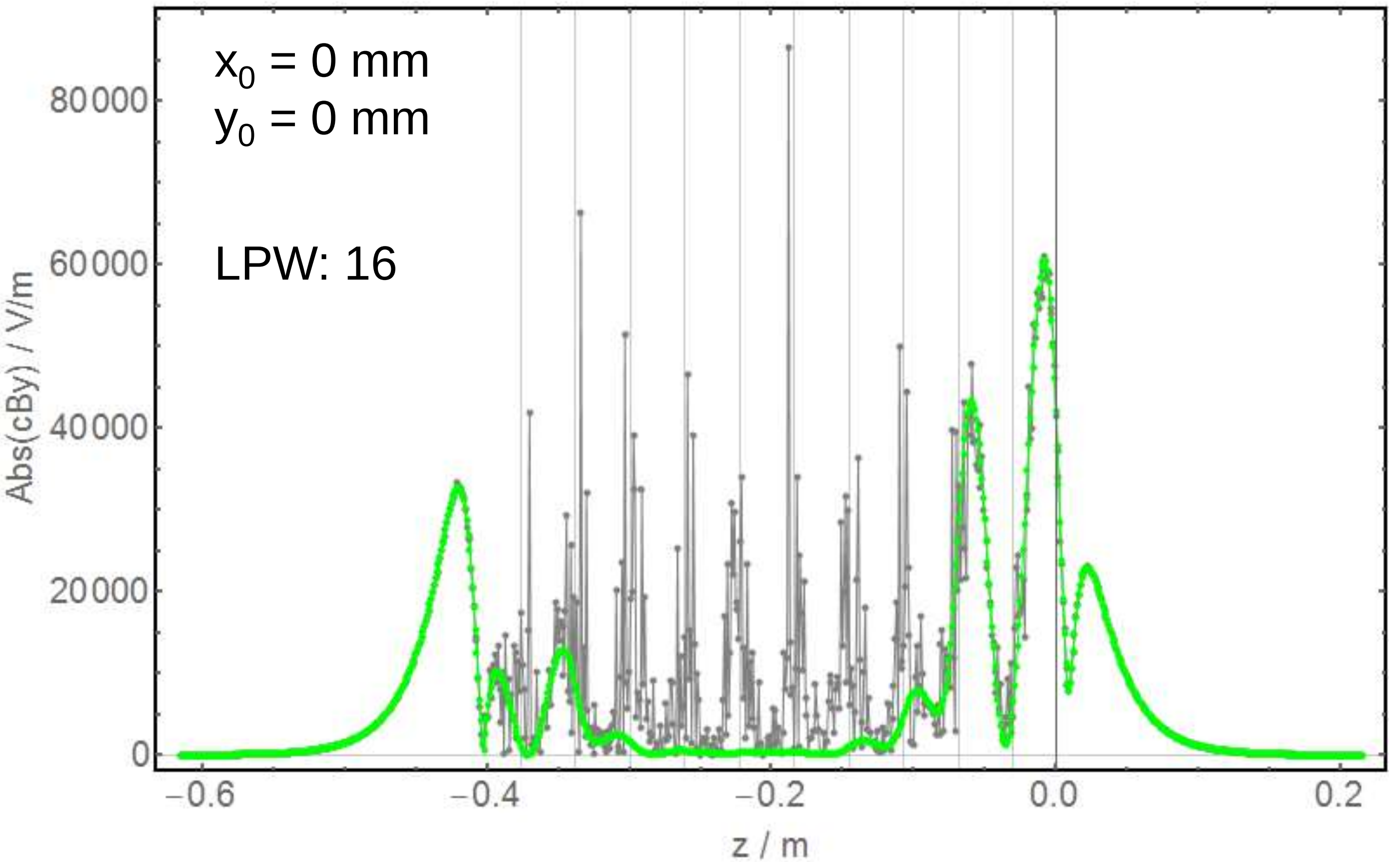
- Field component cB_x parallel to the cavity axis



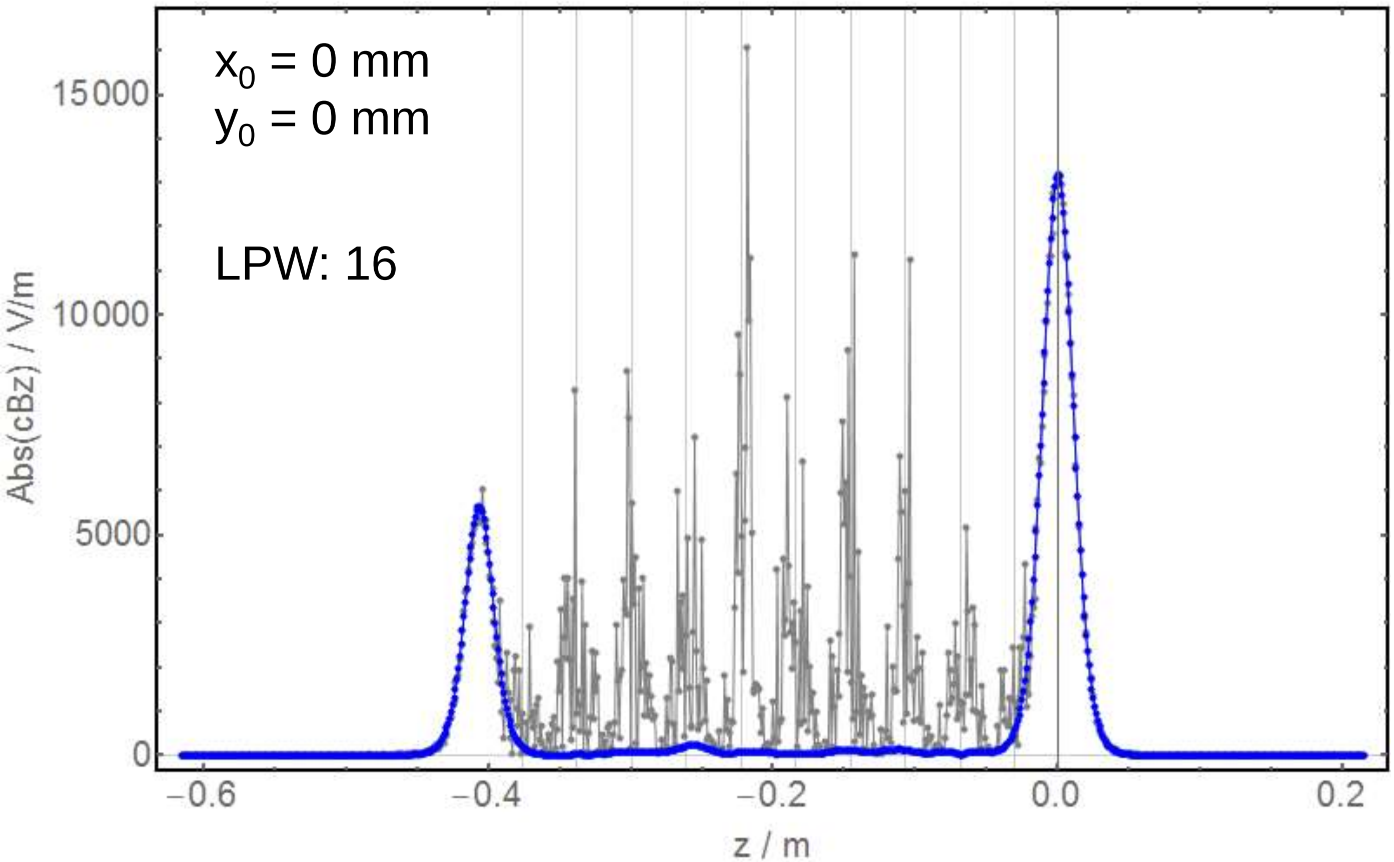
Simulation Results

- Field component cB_y parallel to the cavity axis



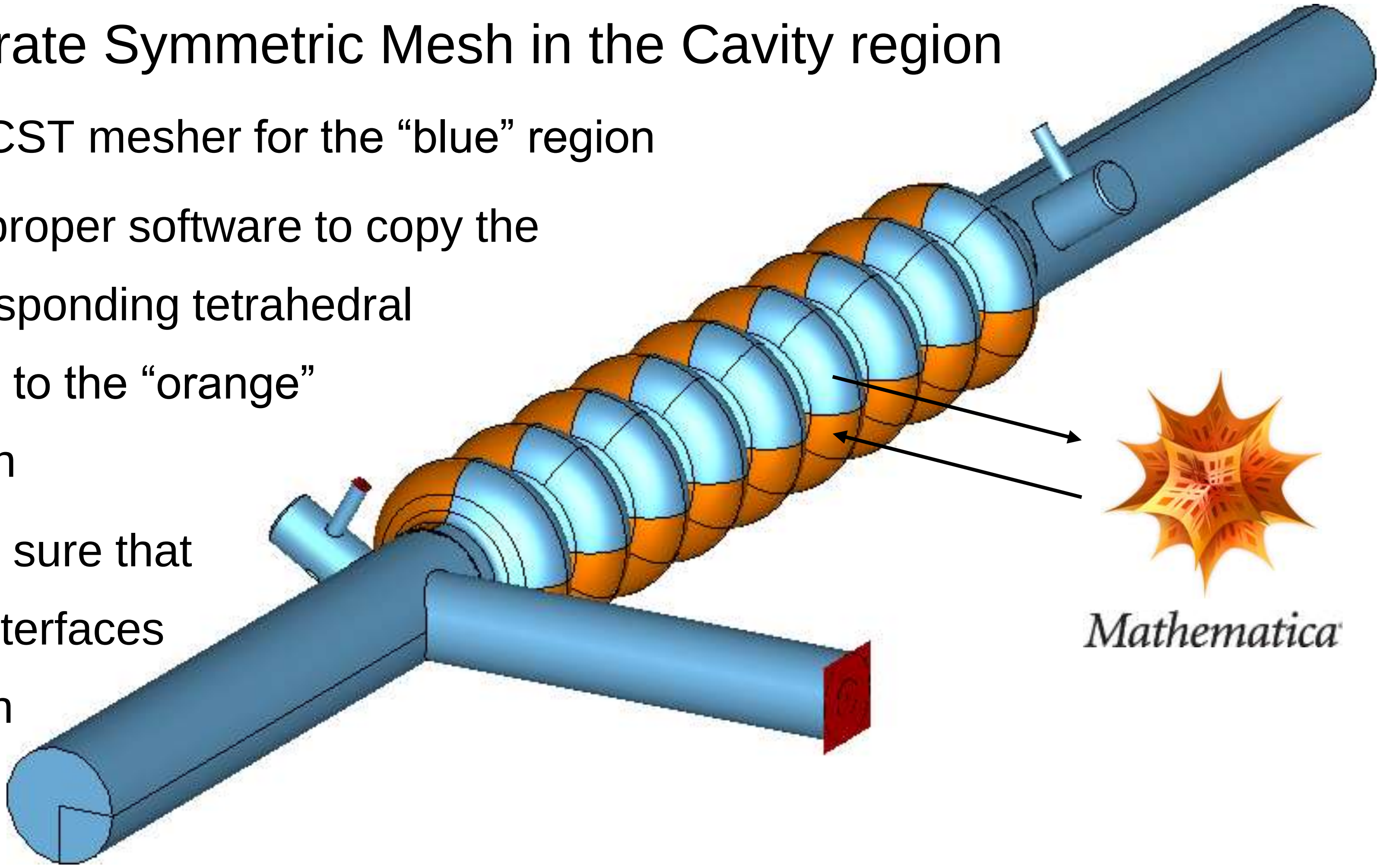
Simulation Results

- Field component cB_z parallel to the cavity axis



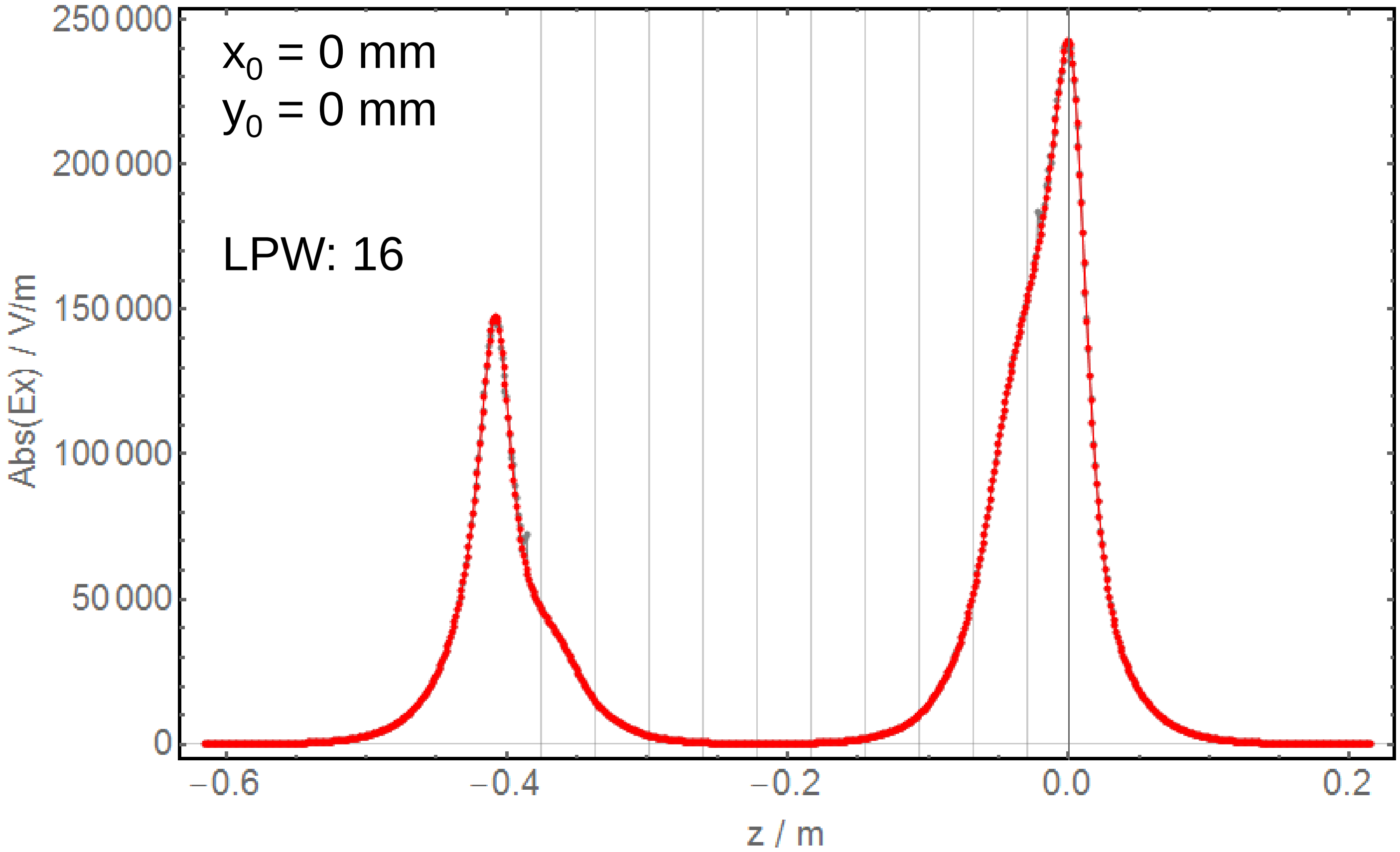
Simulation Results

- Generate Symmetric Mesh in the Cavity region
  - Use CST mesher for the “blue” region
  - Use proper software to copy the corresponding tetrahedral mesh to the “orange” region
  - Make sure that the interfaces match



Simulation Results

- Field component E_x parallel to the cavity axis



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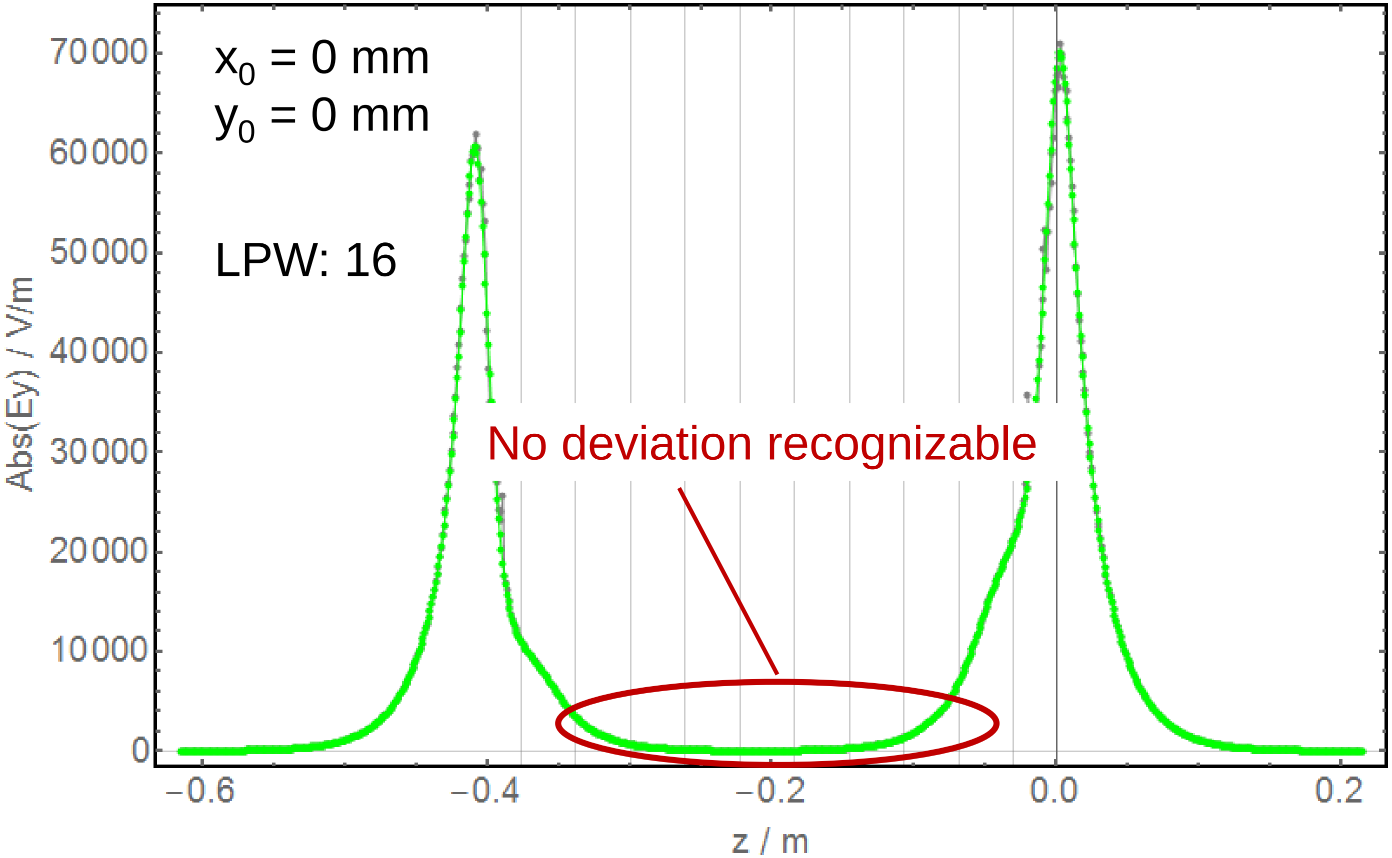

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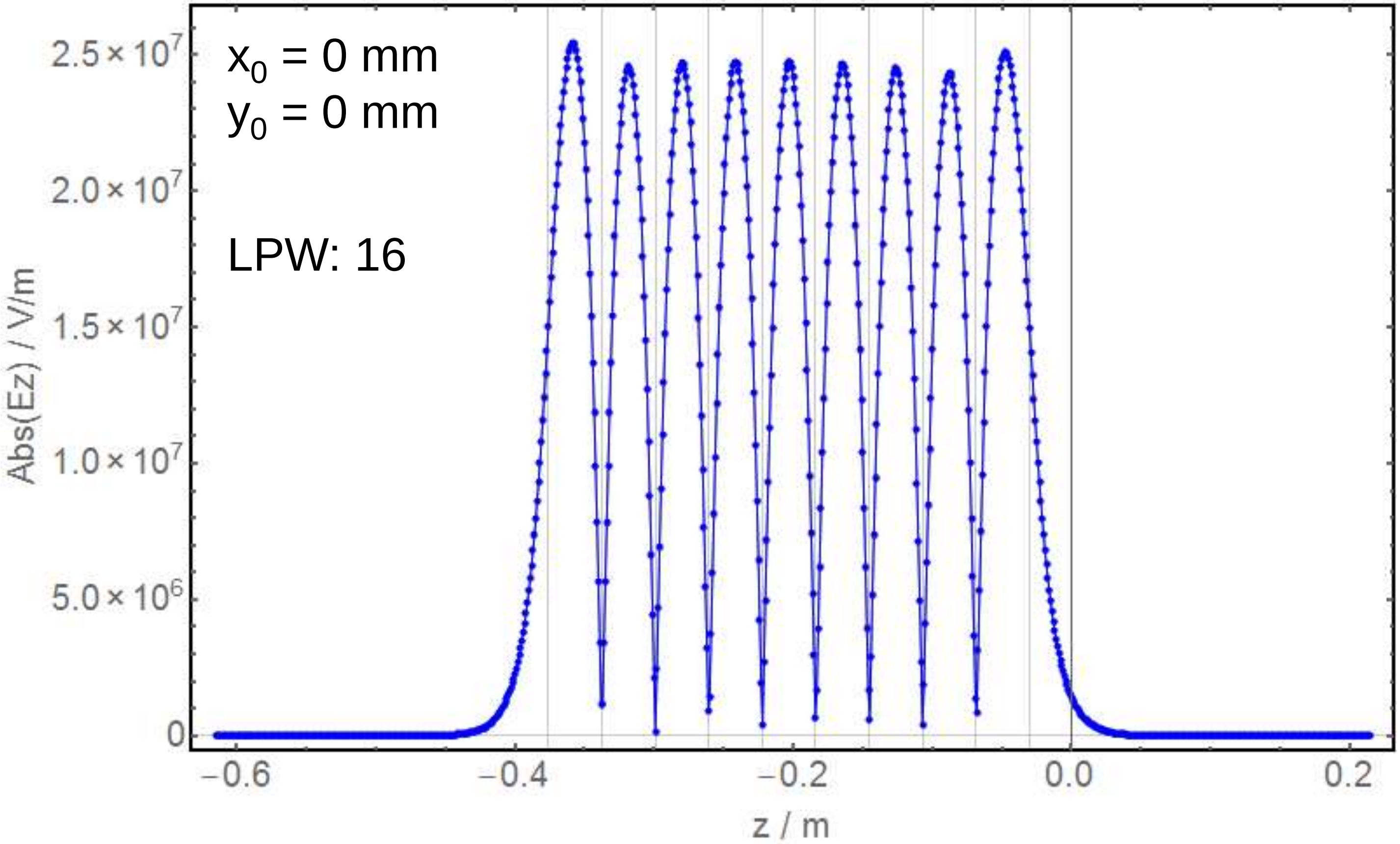

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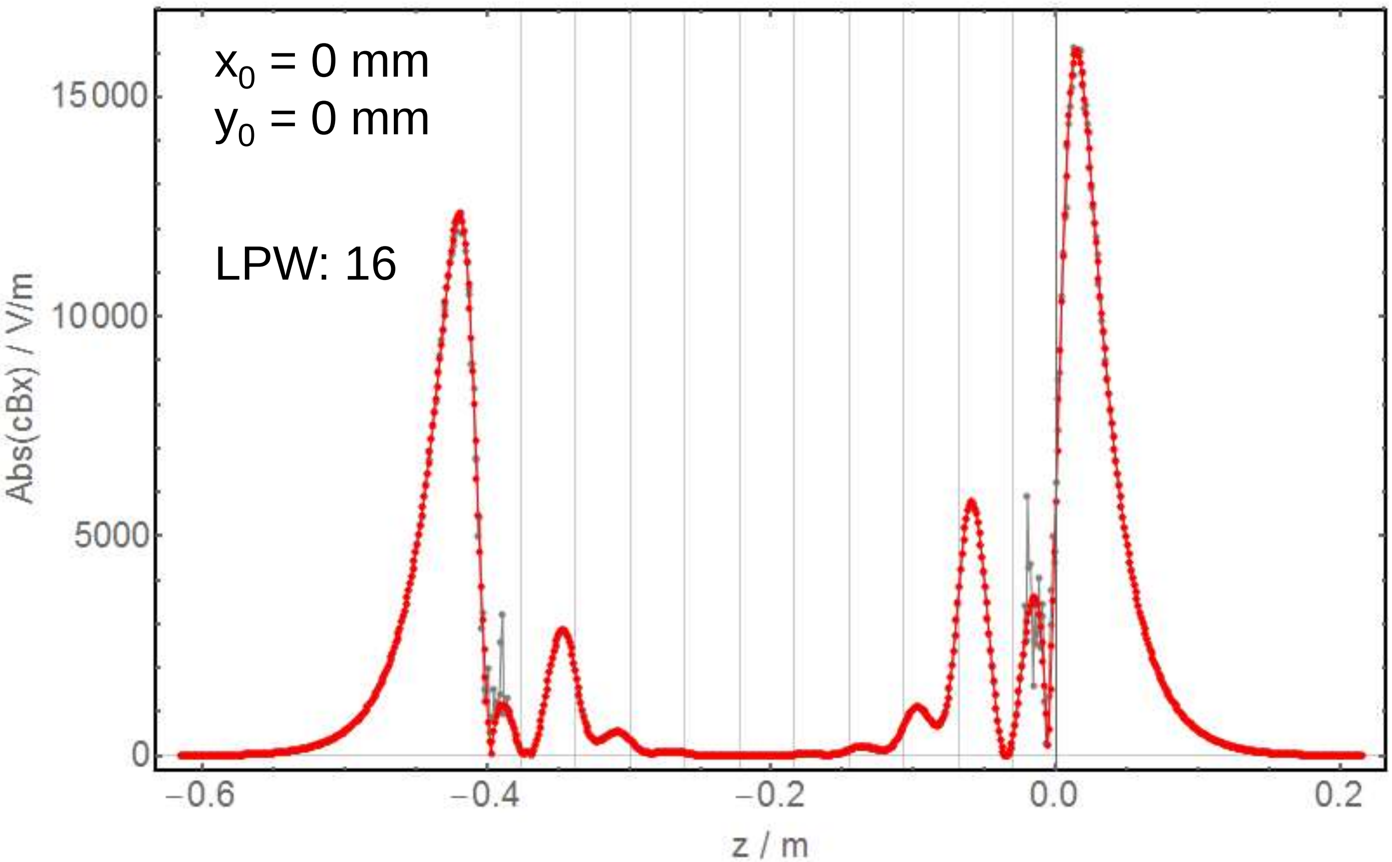

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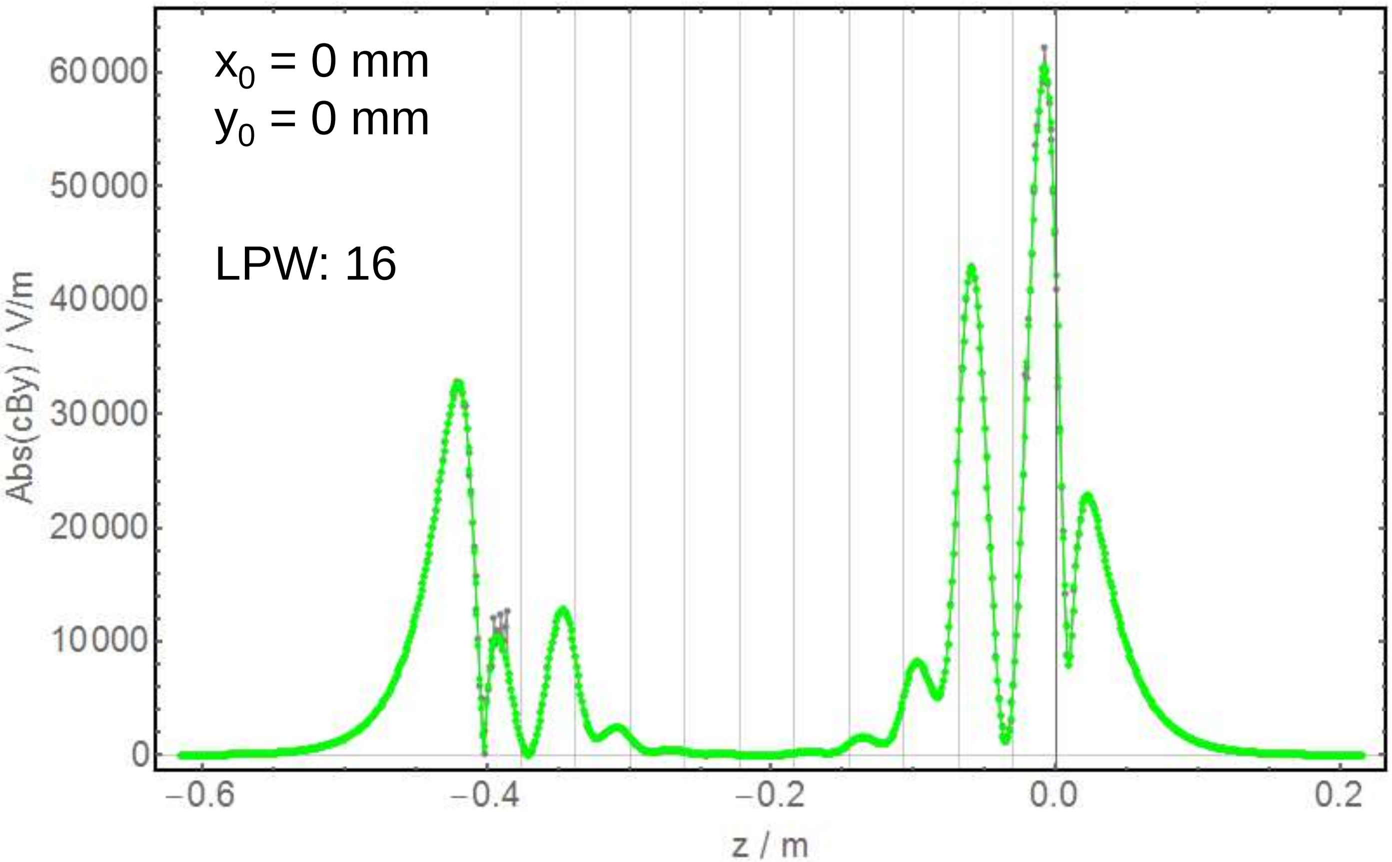

Simulation Results

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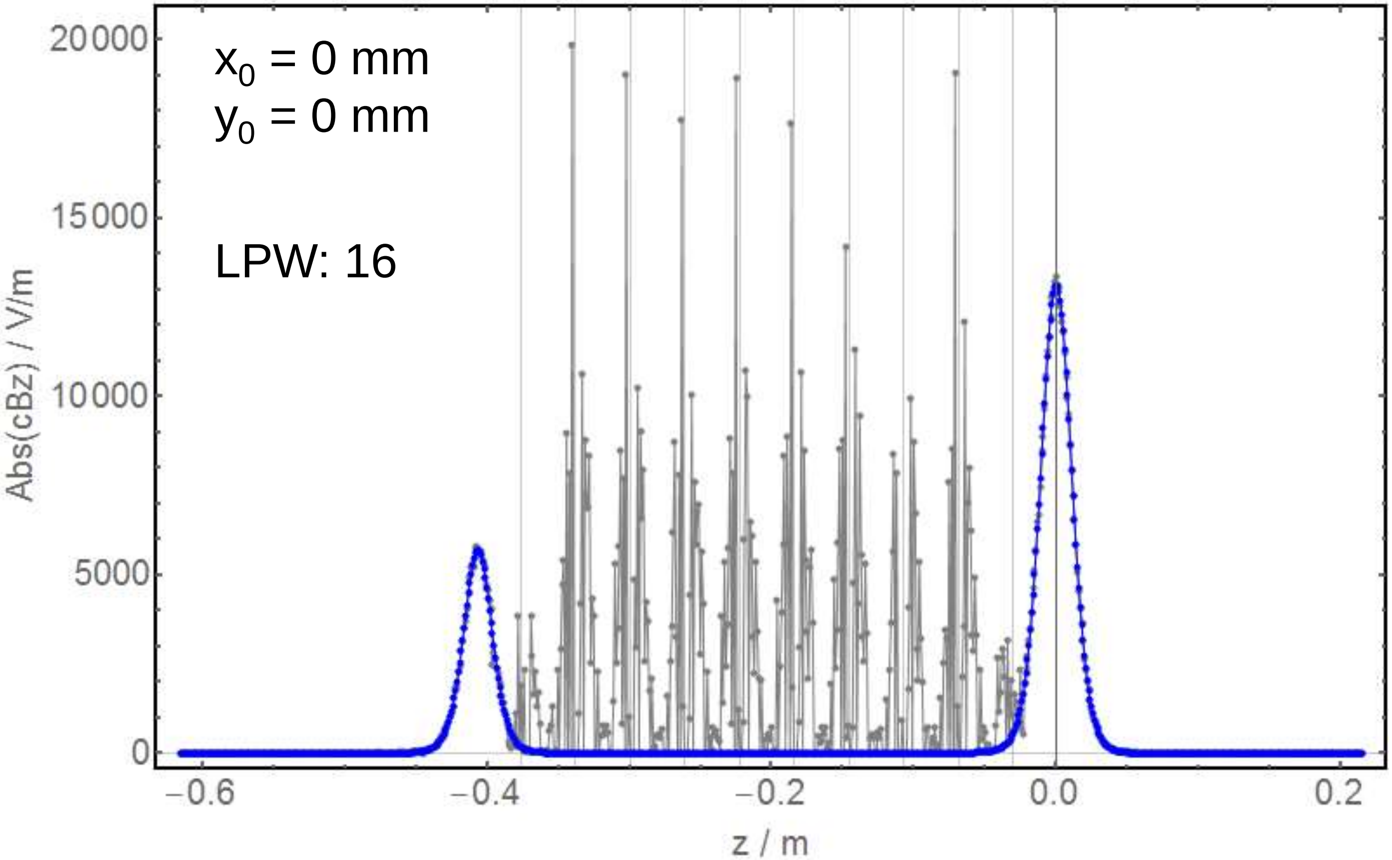
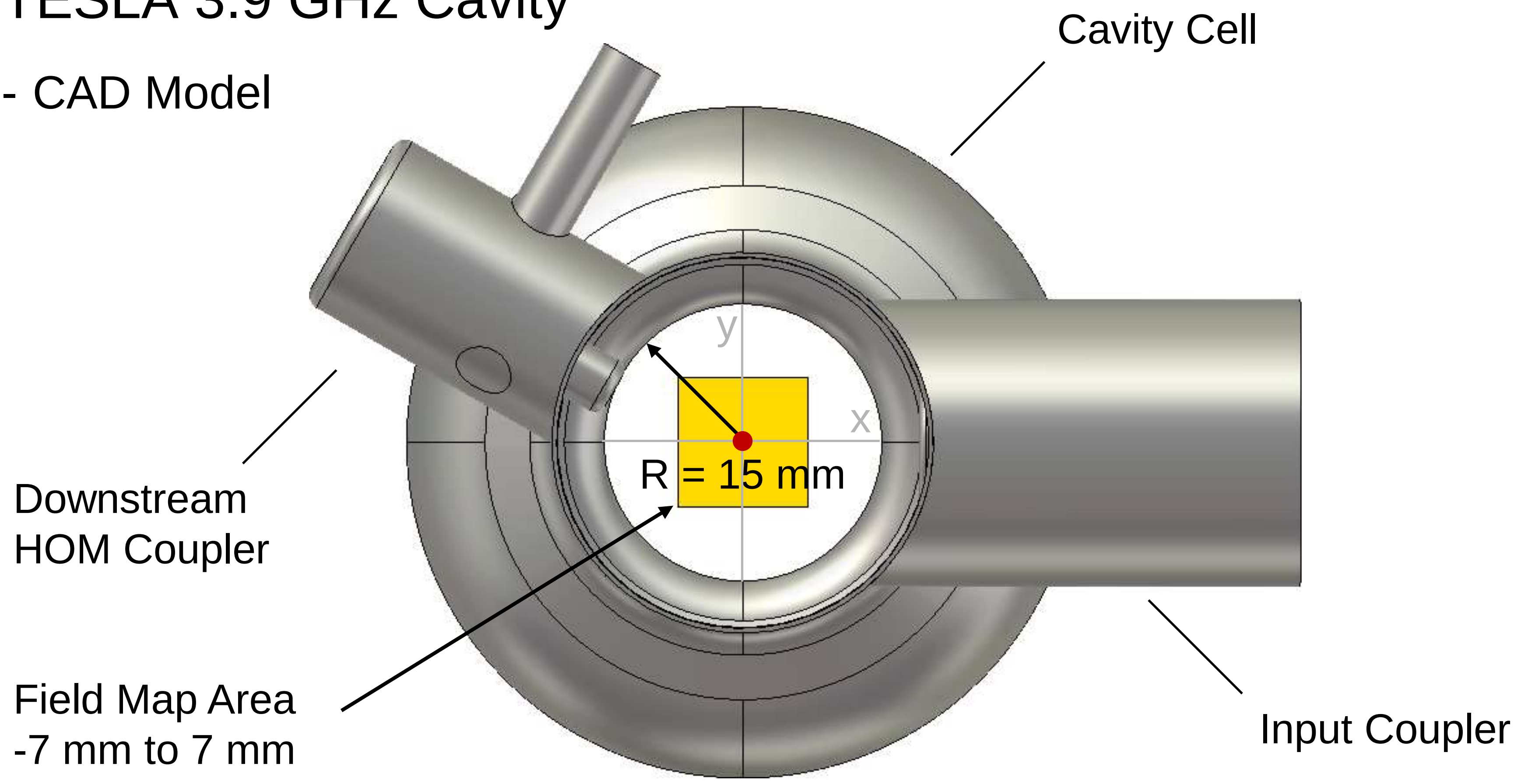


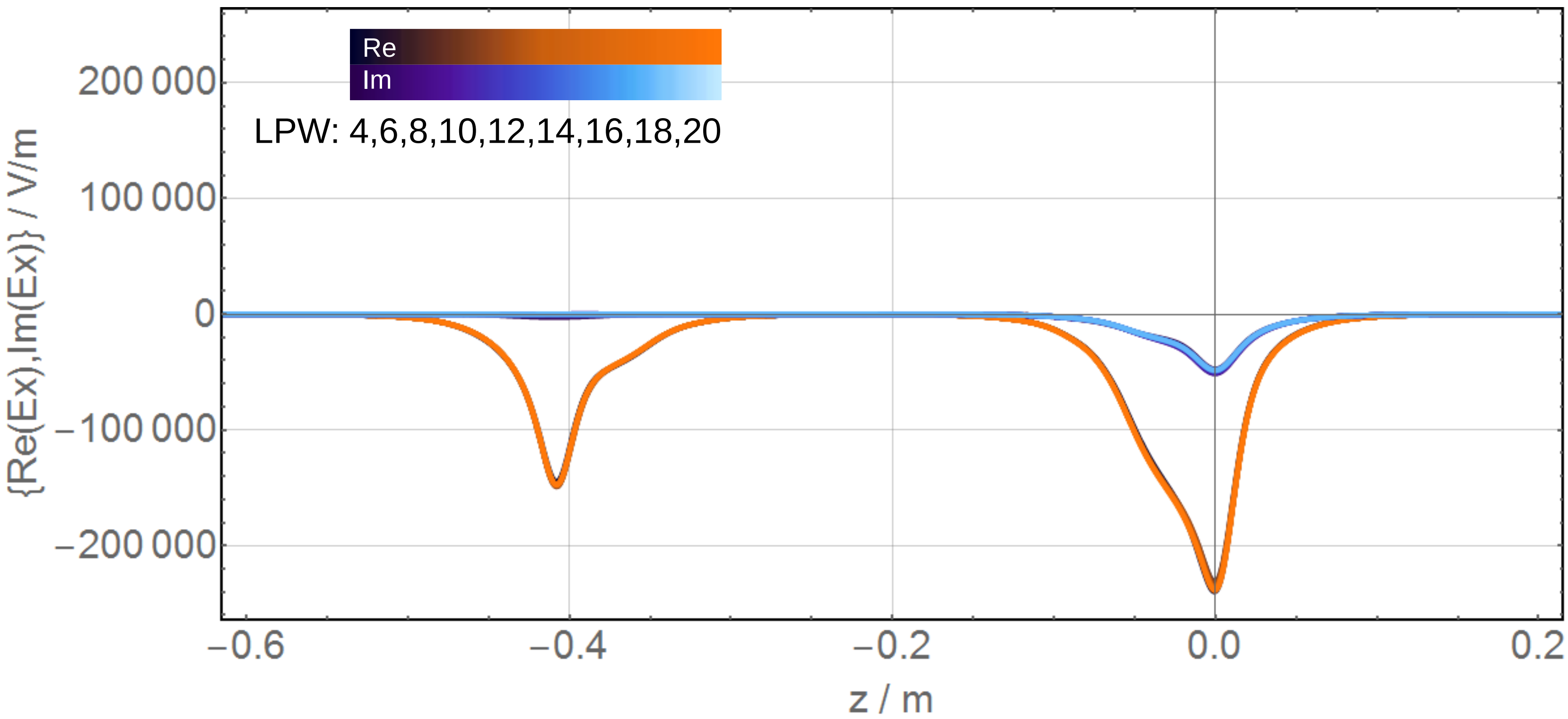
Simulation Results

- TESLA 3.9 GHz Cavity
  - CAD Model



Simulation Results

- Field component E_x along the cavity axis
  - Real and imaginary parts



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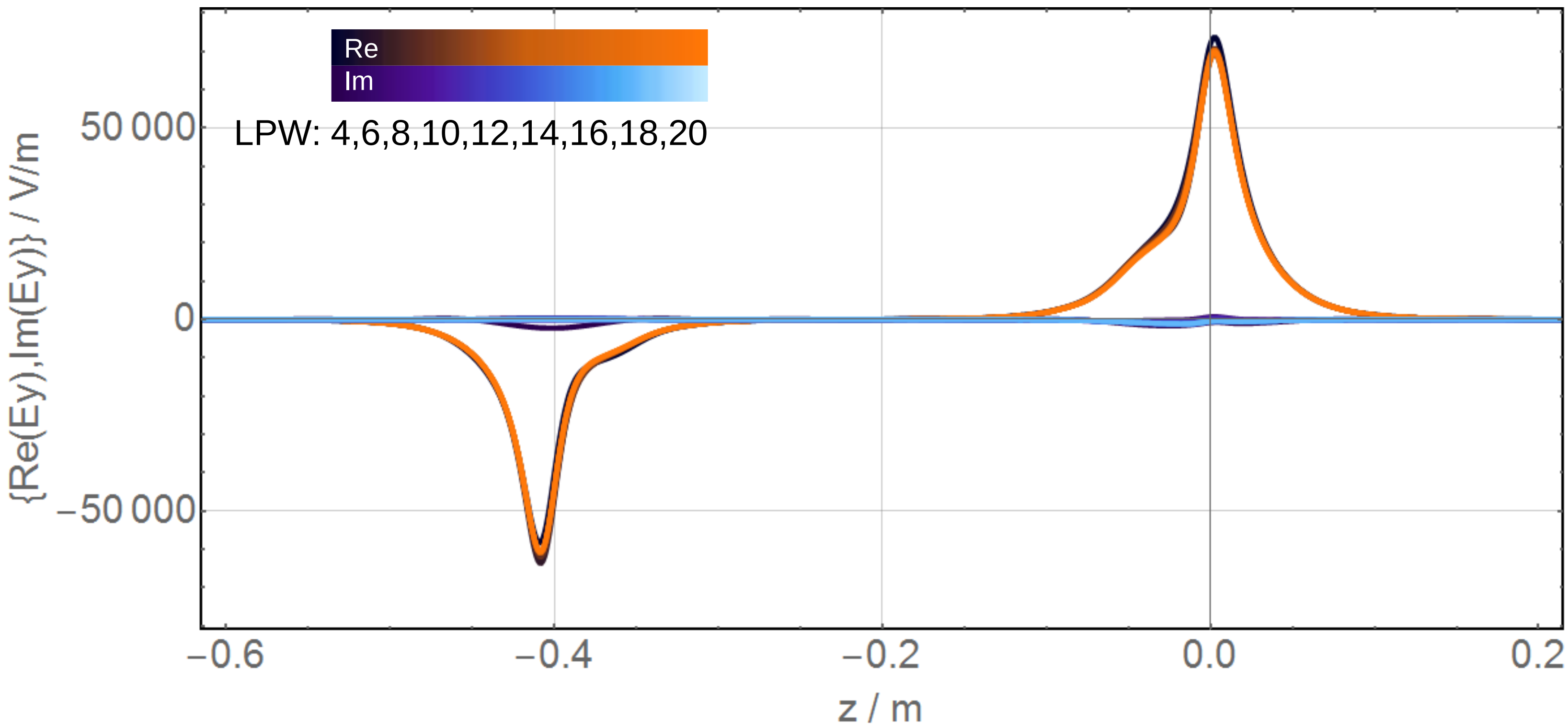
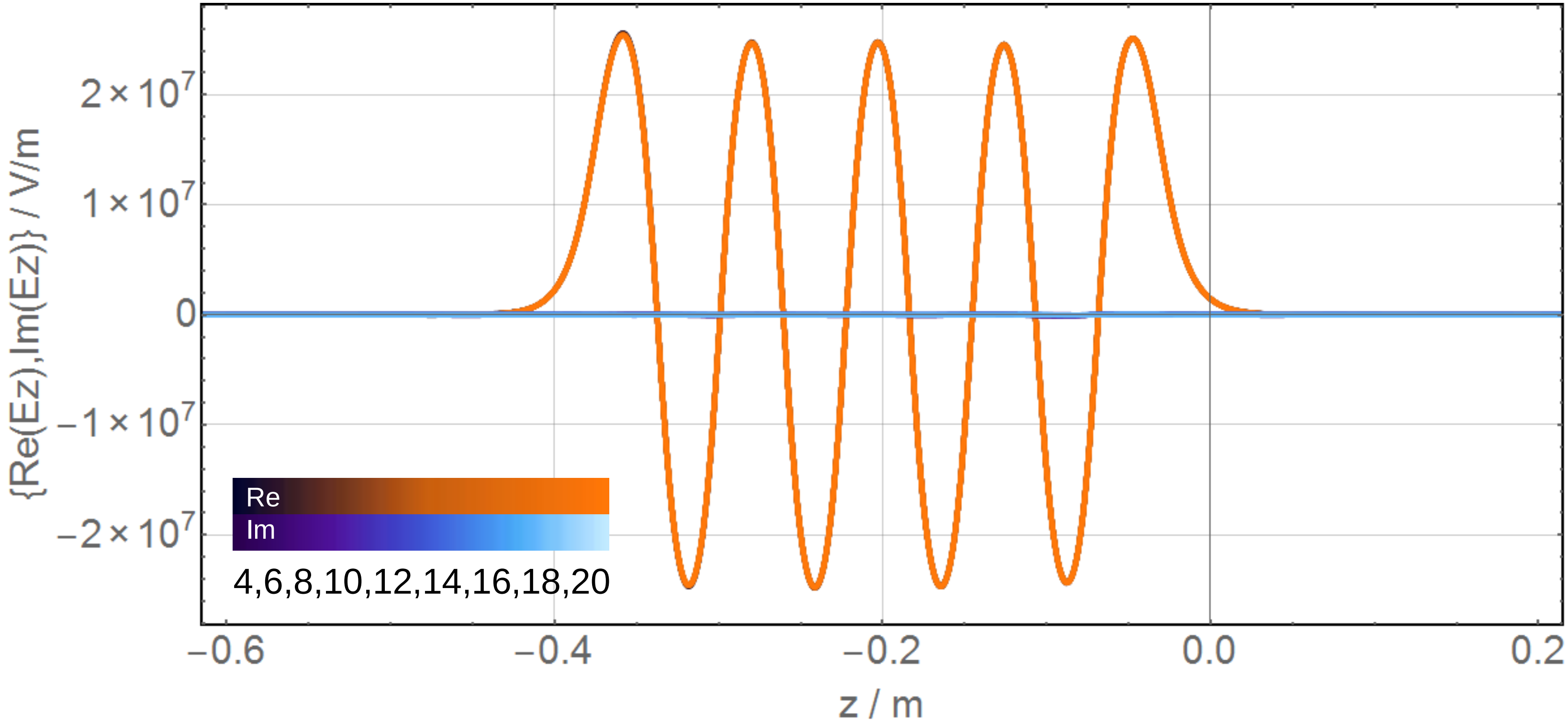
- Field component E_y along the cavity axis
  - Real and imaginary parts



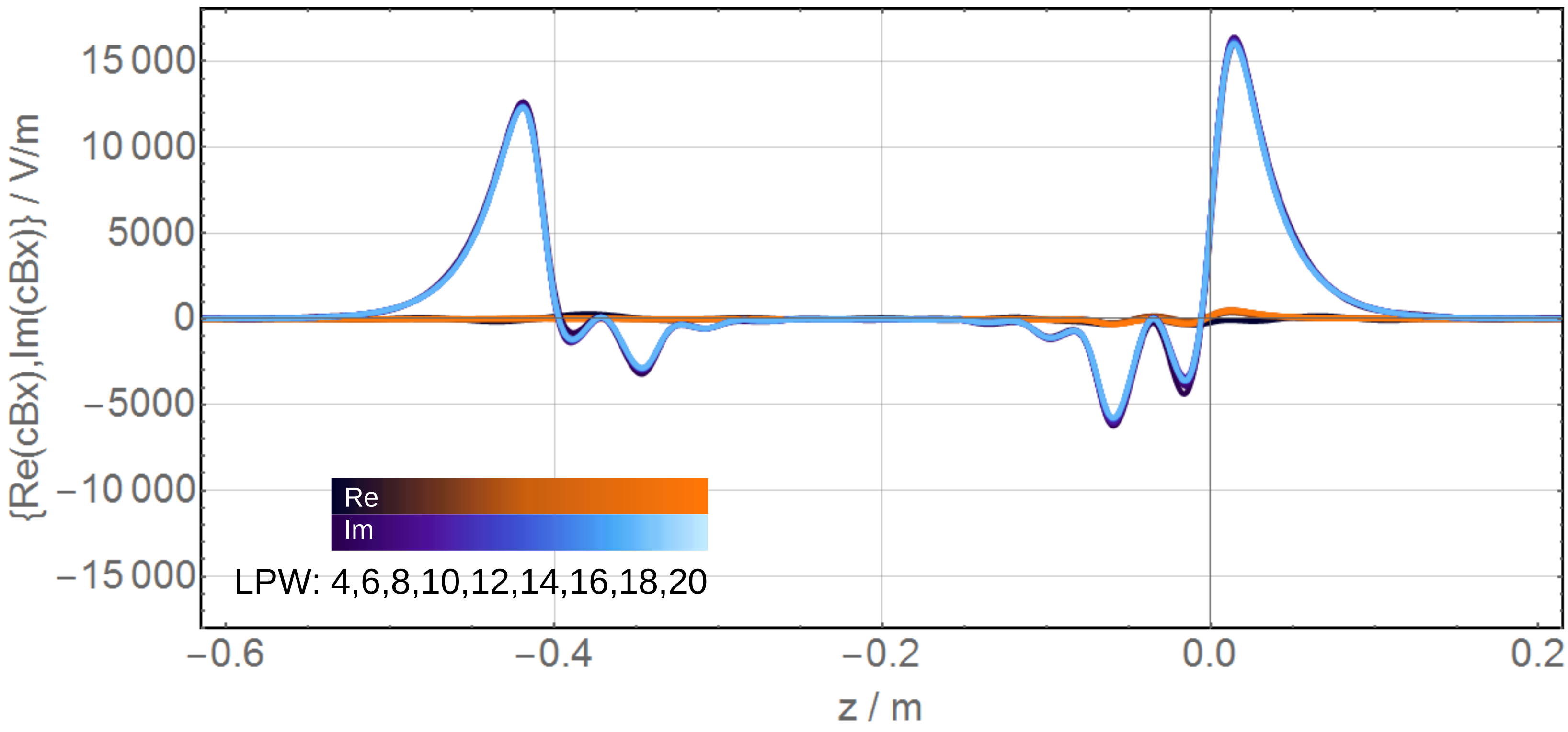
Simulation Results

- Field component E_z along the cavity axis
  - Real and imaginary parts



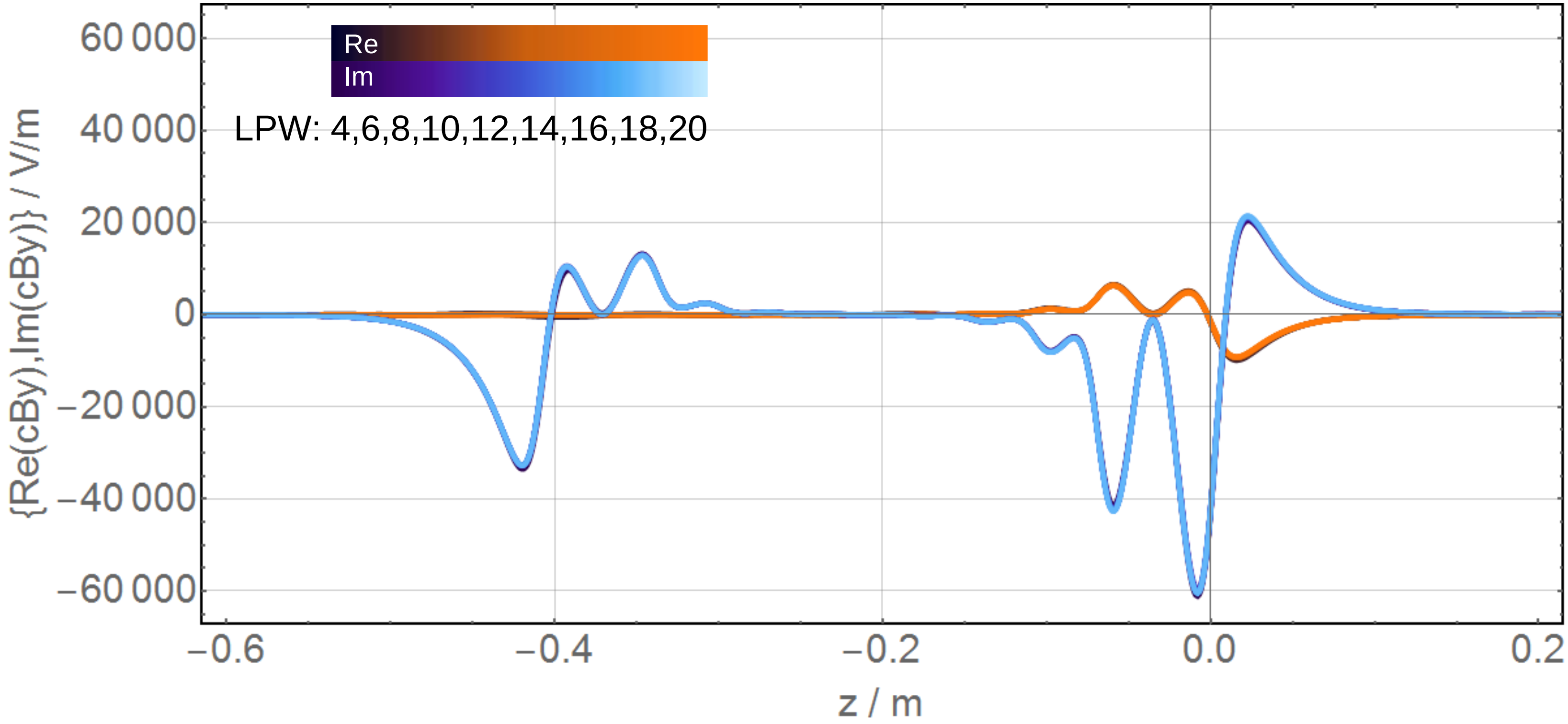
Simulation Results

- Field component cB_x along the cavity axis
  - Real and imaginary parts



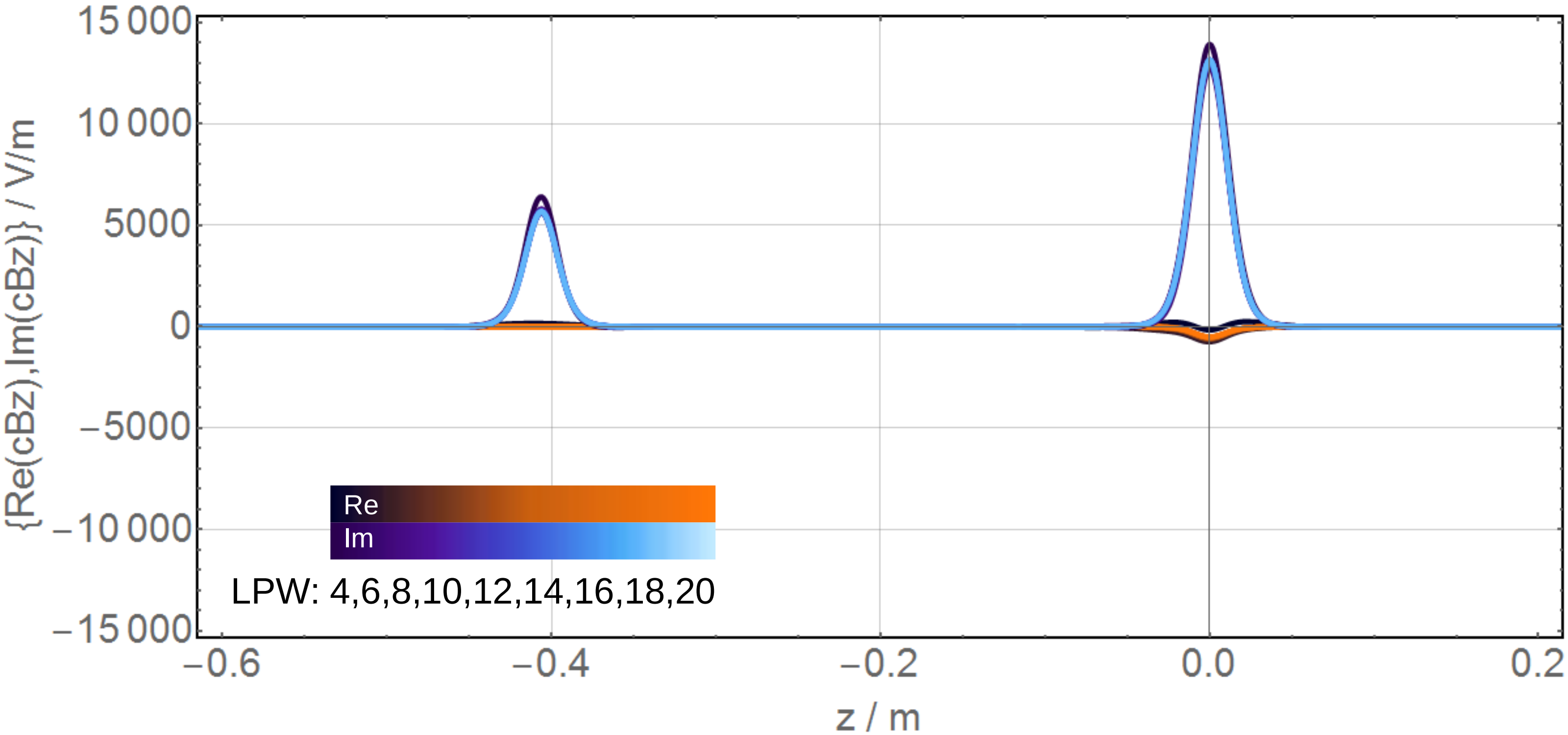
Simulation Results

- Field component cB_y along the cavity axis
  - Real and imaginary parts



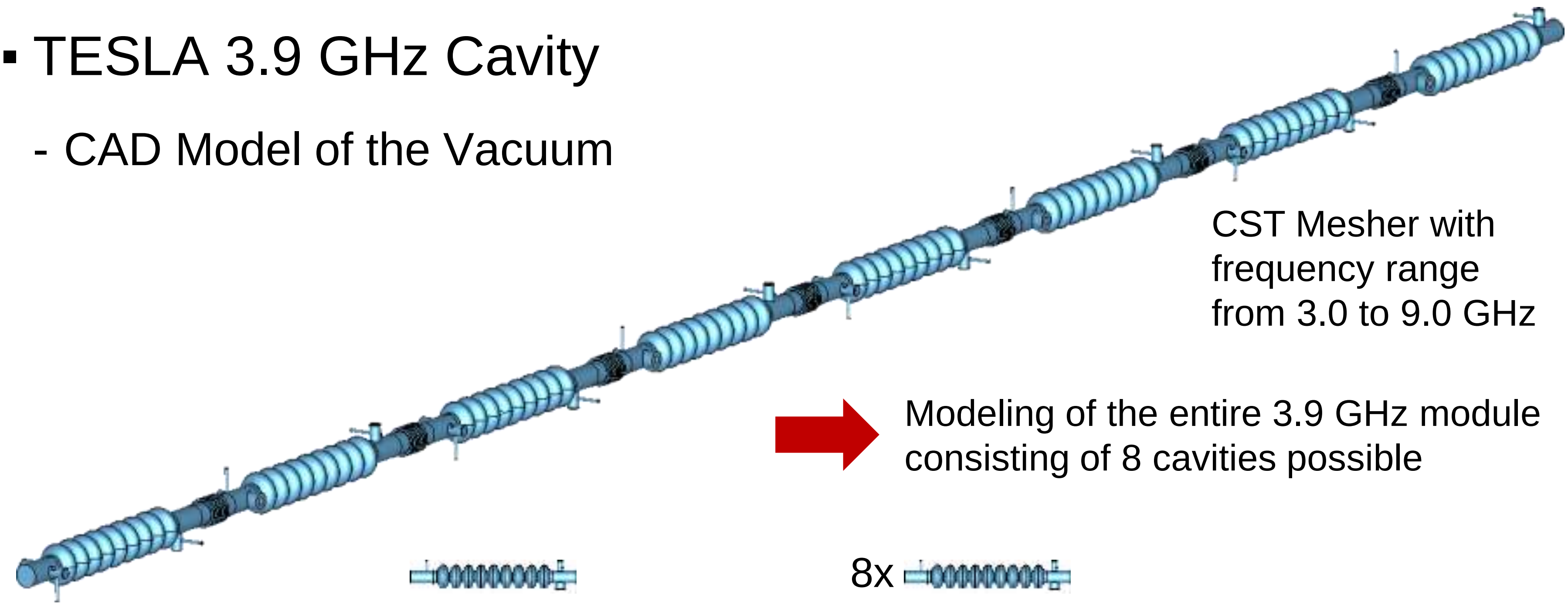
Simulation Results

- Field component cB_z along the cavity axis
  - Real and imaginary parts



Simulation Results

- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum



CST Mesher with frequency range from 3.0 to 9.0 GHz

Modeling of the entire 3.9 GHz module consisting of 8 cavities possible

8x

LPW	4	6	8	10	12	14	16	18	20
Tetrahedrons	136.443	187.435	304.833	480.376	767.271	1.177.883	1.704.528	2.432.978	3.337.736
Complex DOF	761.820	1.079.488	1.802.314	2.885.154	4.668.072	7.227.096	10.509.404	15.064.232	20.721.334

Outline

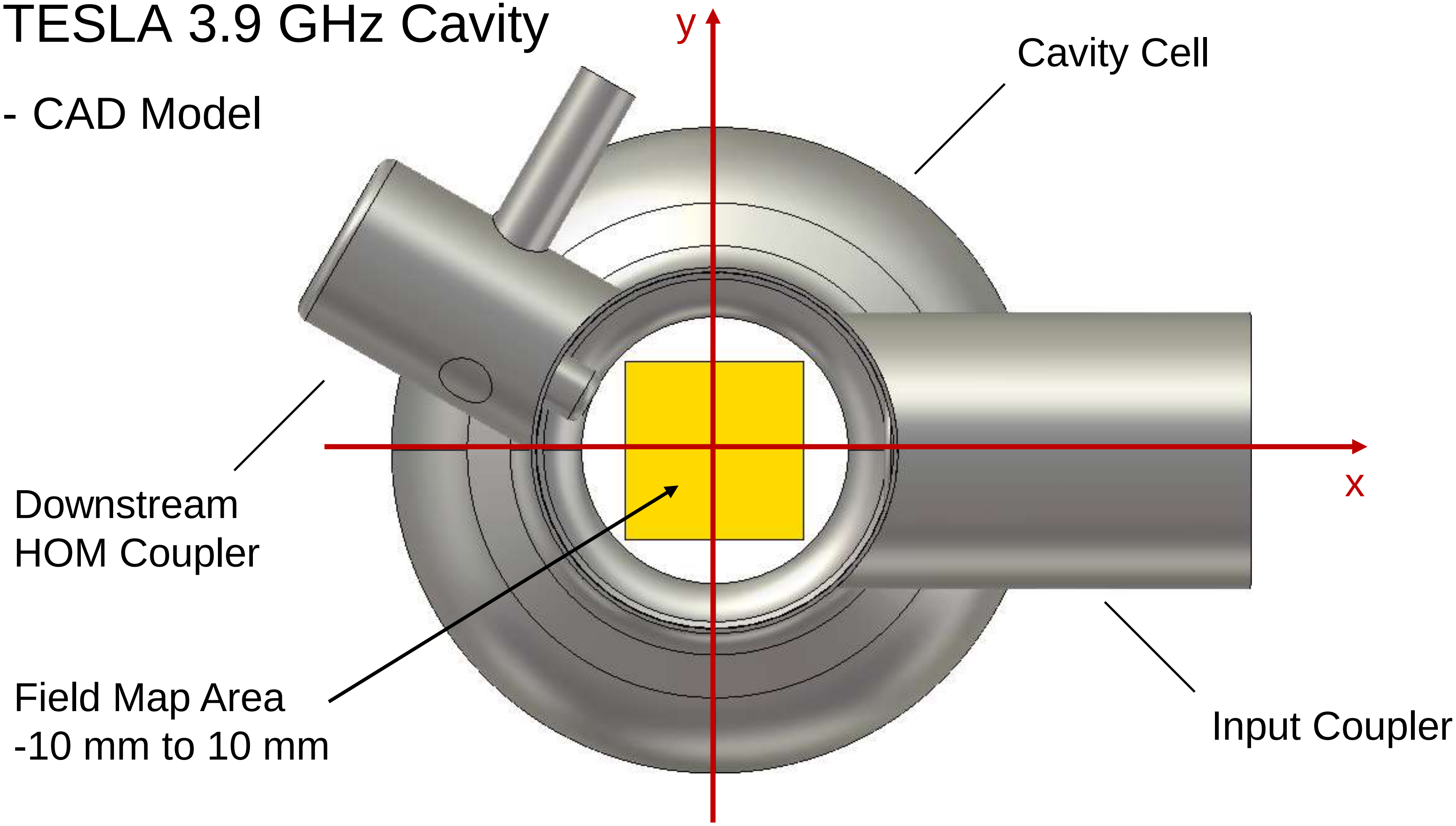
- Motivation
- Computational Model
  - Geometry and mesh information
- Simulation results
  - Field components parallel to the cavity axis
  - FEM on tetrahedral meshes
  - Kirchhoff integral representation
- Summary / Outlook

Summary / Outlook

- Summary:
  - Precise modeling of a single TESLA 3.9 GHz cavity including the input coupler and two HOM couplers
  - Eigenmode analysis performed for the accelerating mode
  - Electromagnetic field extraction based on unsymmetric/symmetric tetrahedral meshes for classical FEM solutions and the Kirchhoff integral representation
- Outlook:
  - Application to a chain of cavities
  - Vinh Pham-Xuan will improve the eigenvalue solver

Numerical Modeling

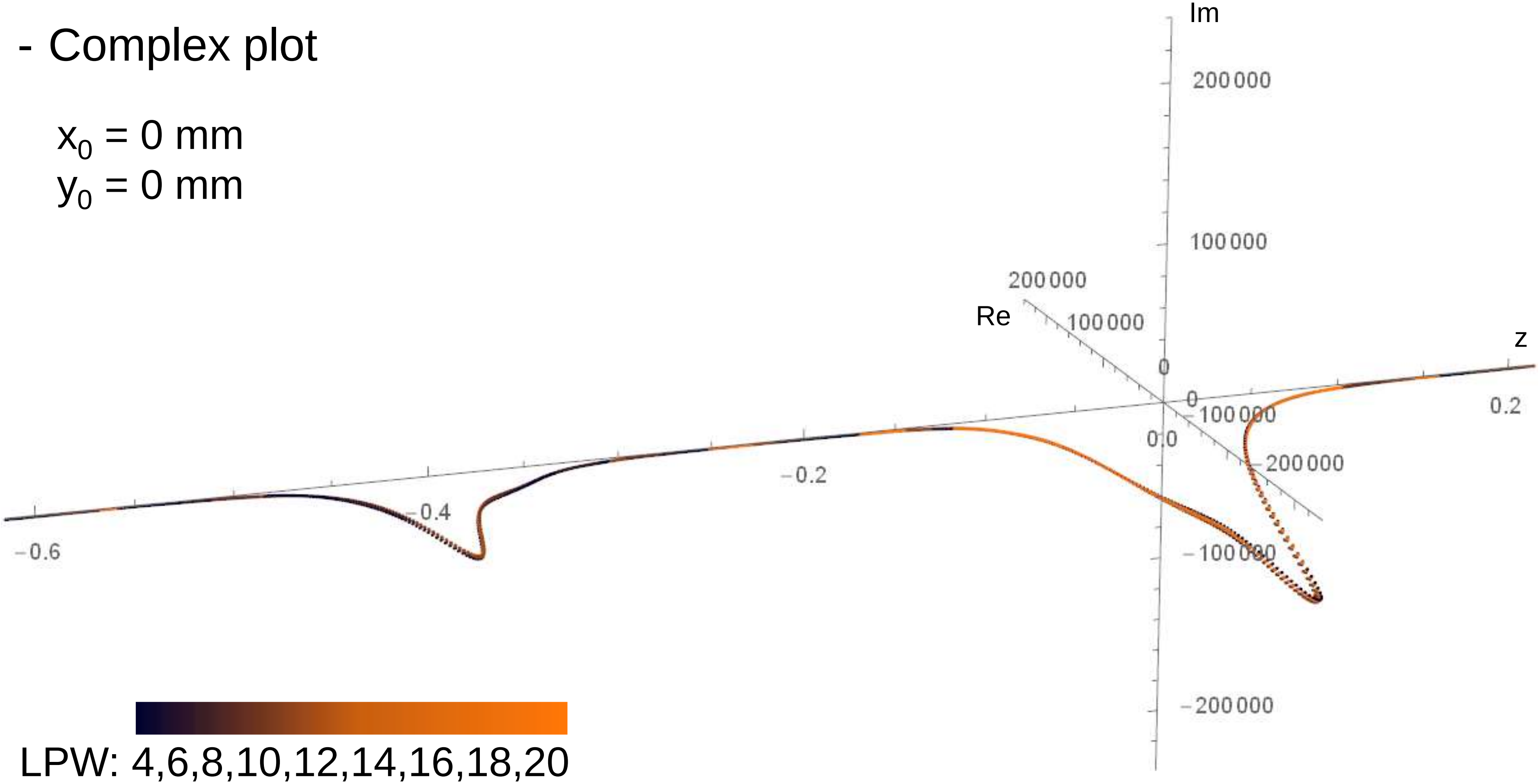
- TESLA 3.9 GHz Cavity
  - CAD Model



Numerical Modeling

- Field component E_x parallel to the cavity axis
  - Complex plot

    $x_0 = 0$ mm
    $y_0 = 0$ mm



LPW: 4,6,8,10,12,14,16,18,20

Numerical Modeling

- Field component E_x parallel to the cavity axis
  - Complex plot

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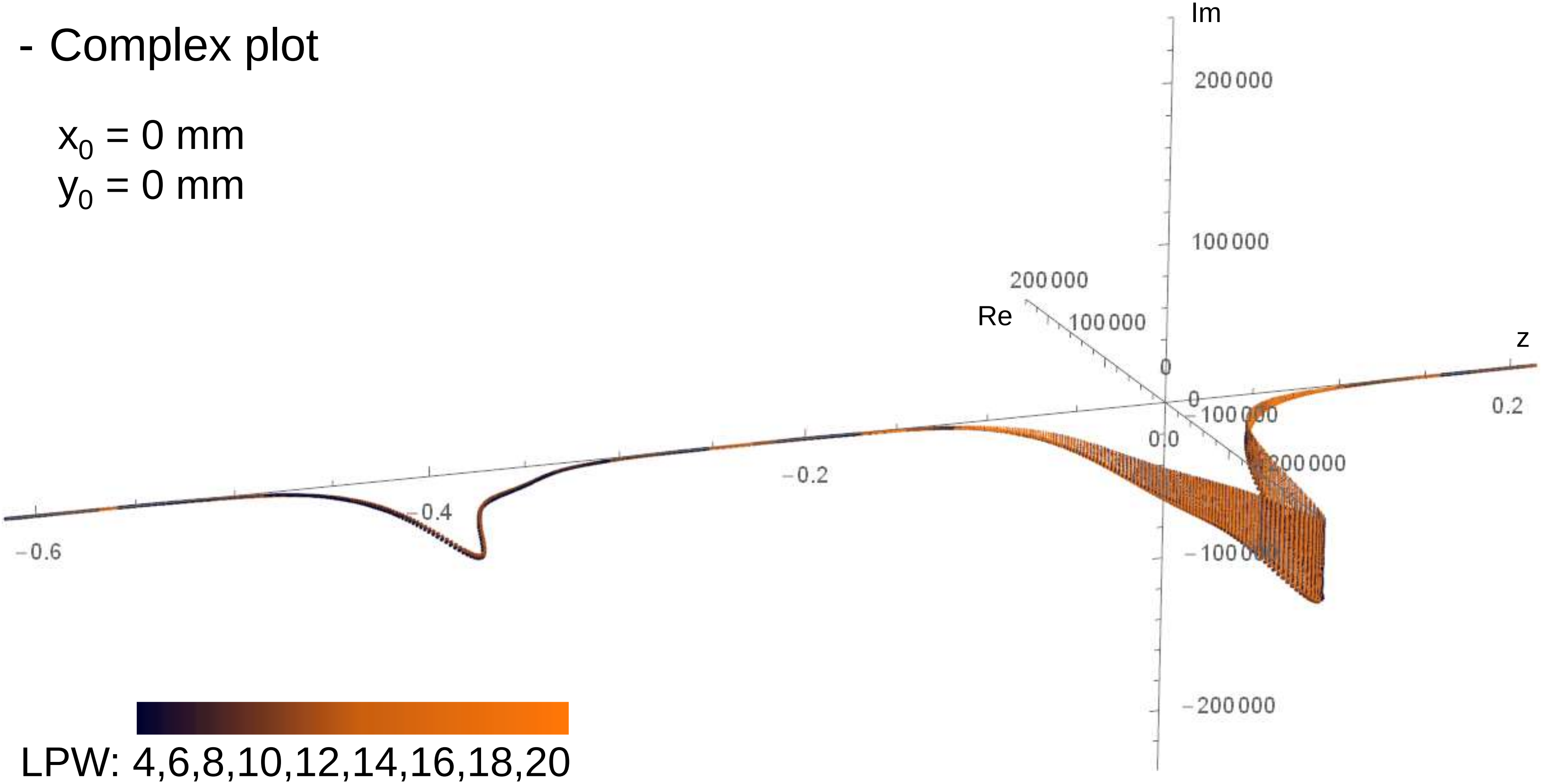
y_0 = 0 mm



Numerical Modeling

- Field component E_y parallel to the cavity axis
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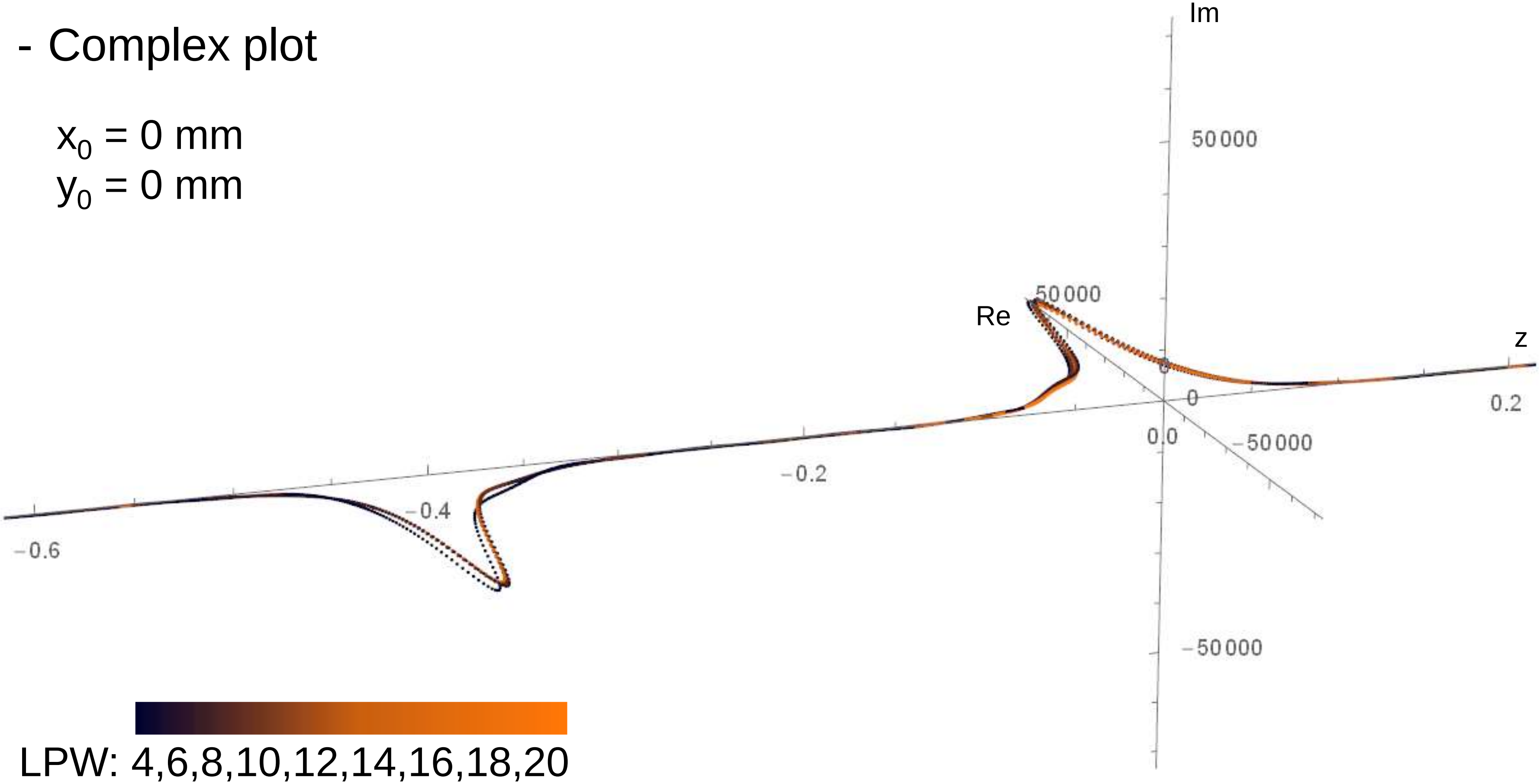
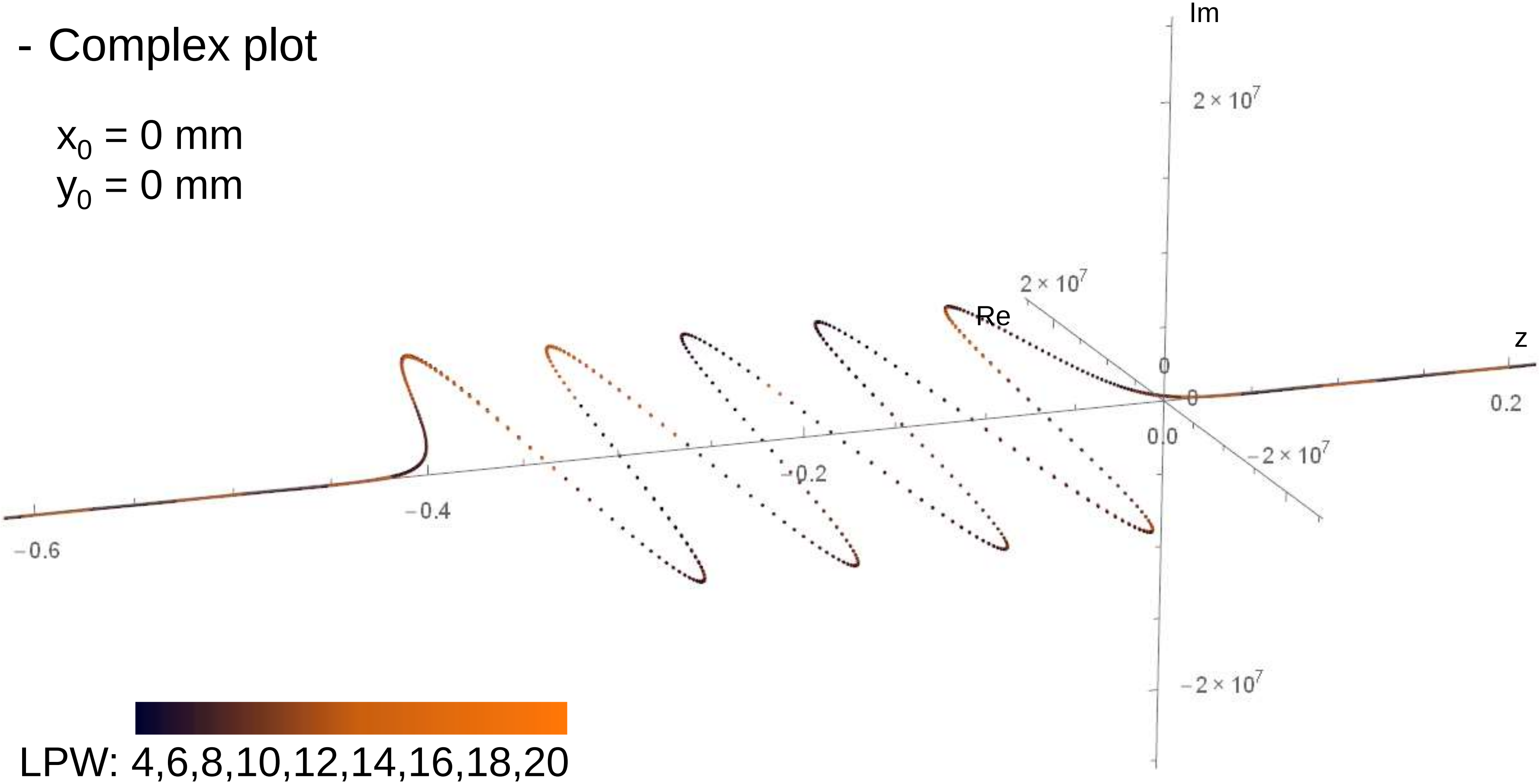


LPW: 4,6,8,10,12,14,16,18,20

Numerical Modeling

- Field component E_z parallel to the cavity axis
  - Complex plot

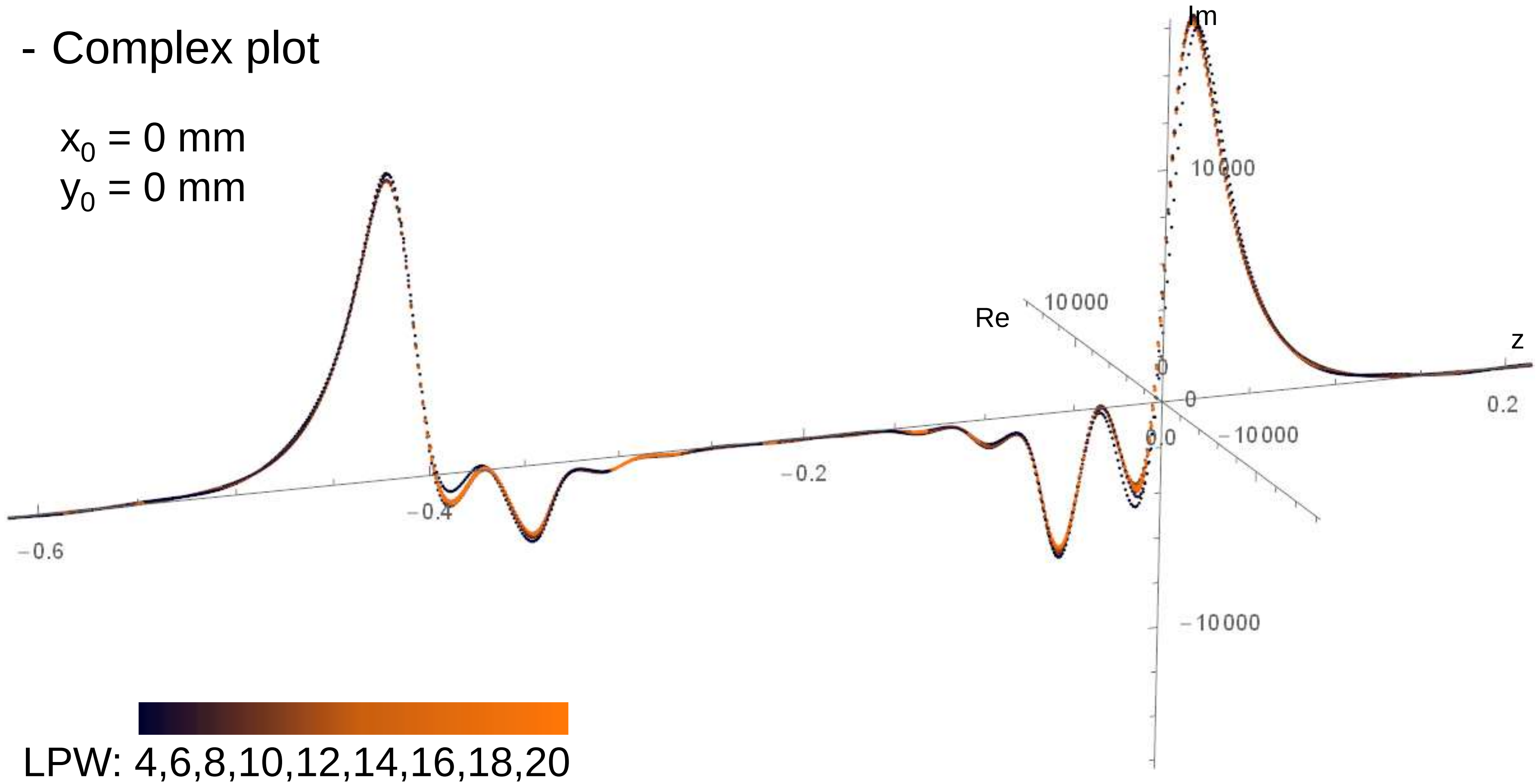
$x_0 = 0$ mm
$y_0 = 0$ mm



LPW: 4,6,8,10,12,14,16,18,20

Numerical Modeling

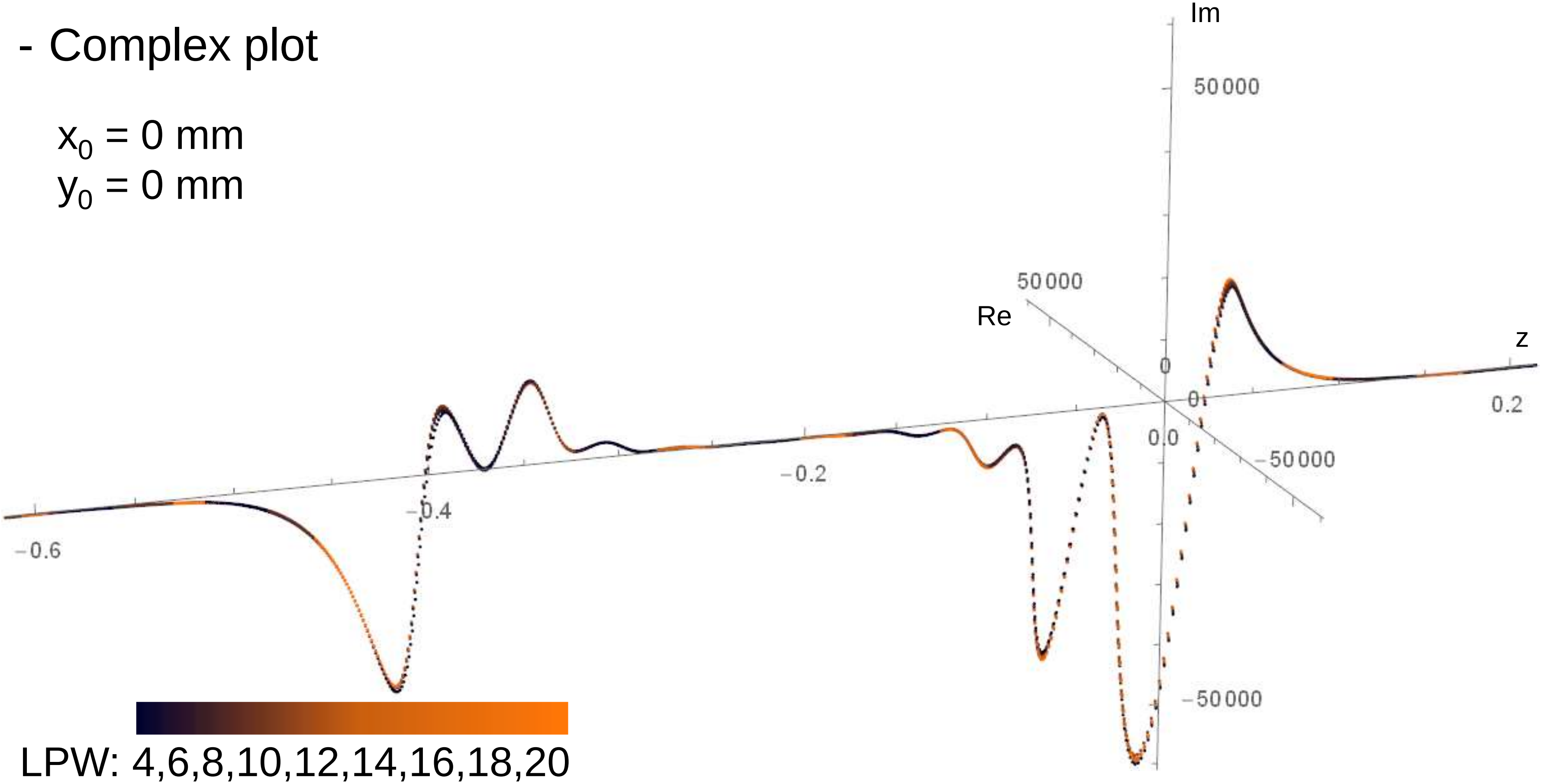
- Field component cB_x parallel to the cavity axis
  - Complex plot



Numerical Modeling

- Field component cB_y parallel to the cavity axis
  - Complex plot

x_0 = 0 mm
y_0 = 0 mm



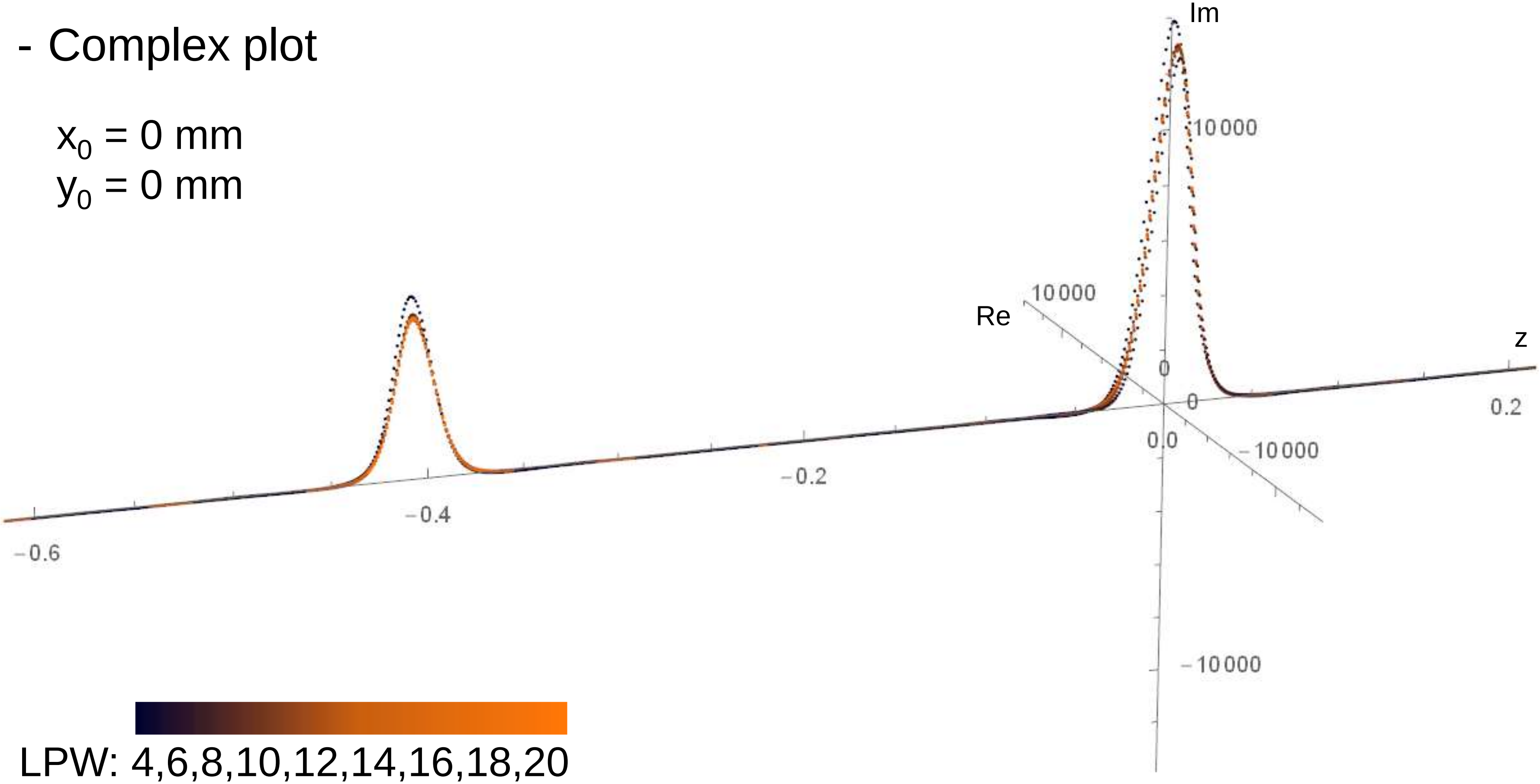
LPW: 4,6,8,10,12,14,16,18,20

Numerical Modeling

TECHNISCHE UNIVERSITÄT DARMSTADT

- Field component cB_z parallel to the cavity axis
  - Complex plot

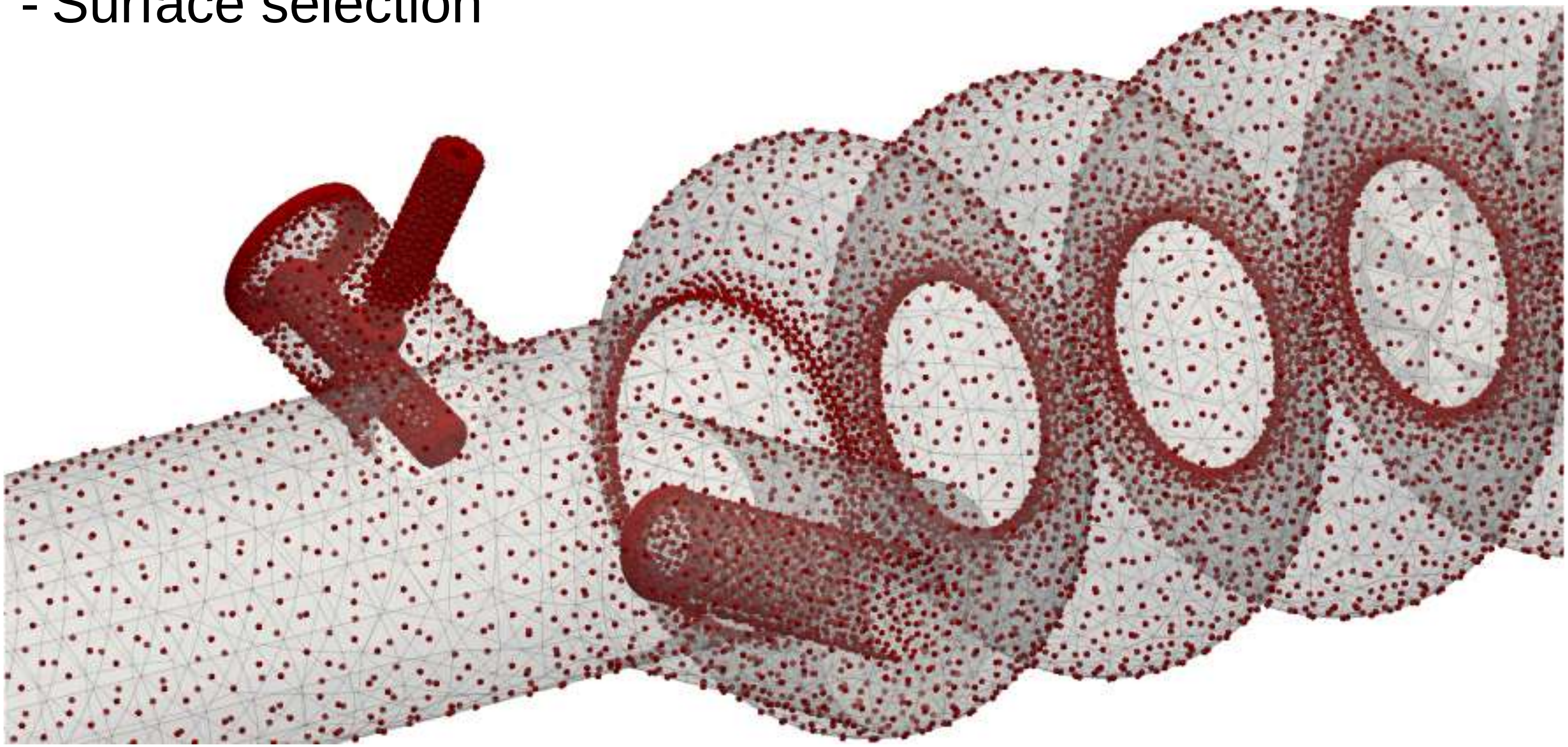
x_0 = 0 mm
y_0 = 0 mm



LPW: 4,6,8,10,12,14,16,18,20

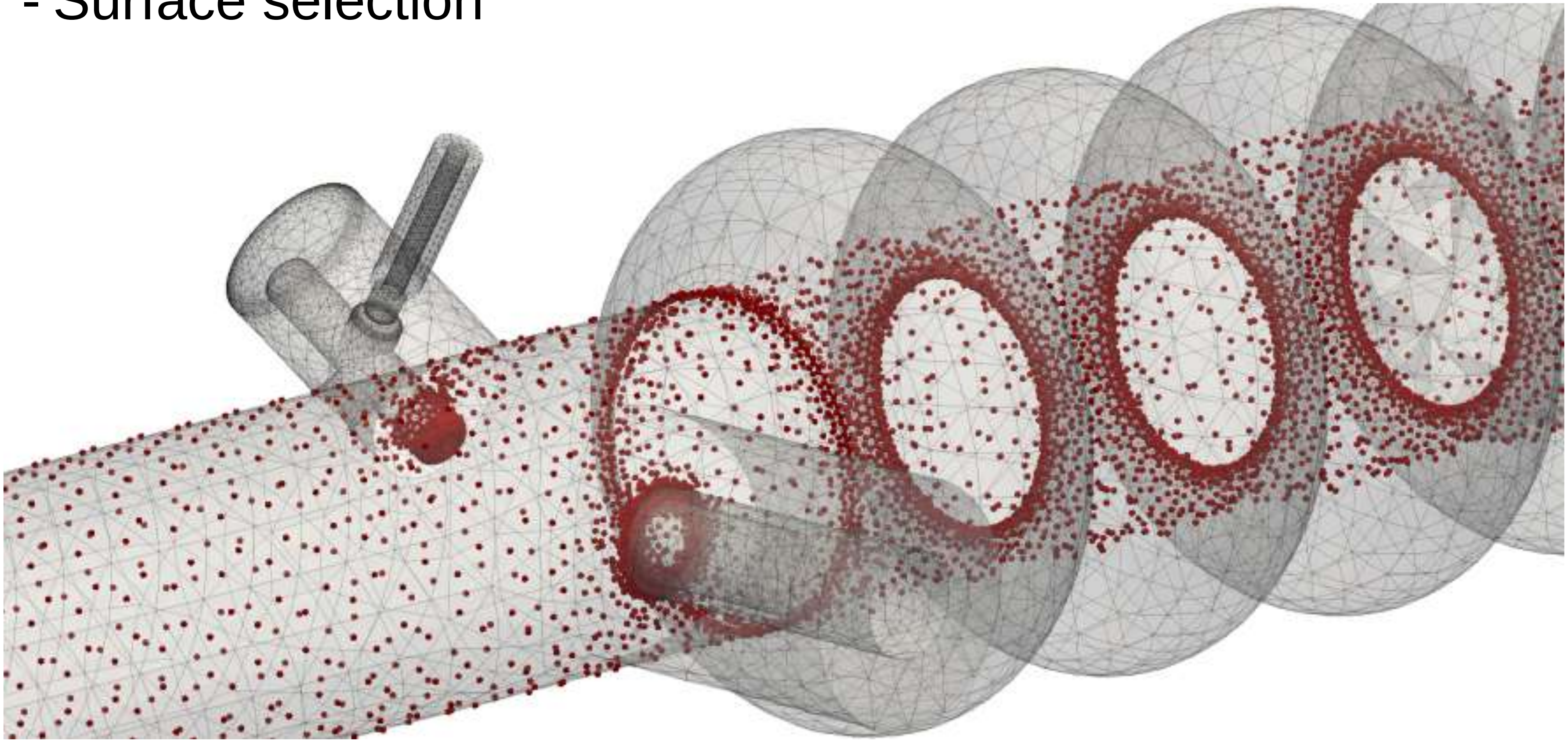
Numerical Modeling

- Field reconstruction using the Kirchhoff integral
  - Surface selection



Numerical Modeling

- Field reconstruction using the Kirchhoff integral
  - Surface selection



Numerical Modeling

- Field reconstruction using the Kirchhoff integral
  - Surface selection