Outlook on SIBC Implementation in Time-Domain Wakefield Calculations

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TEMF – DESY Collaboration Meeting

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Introducion

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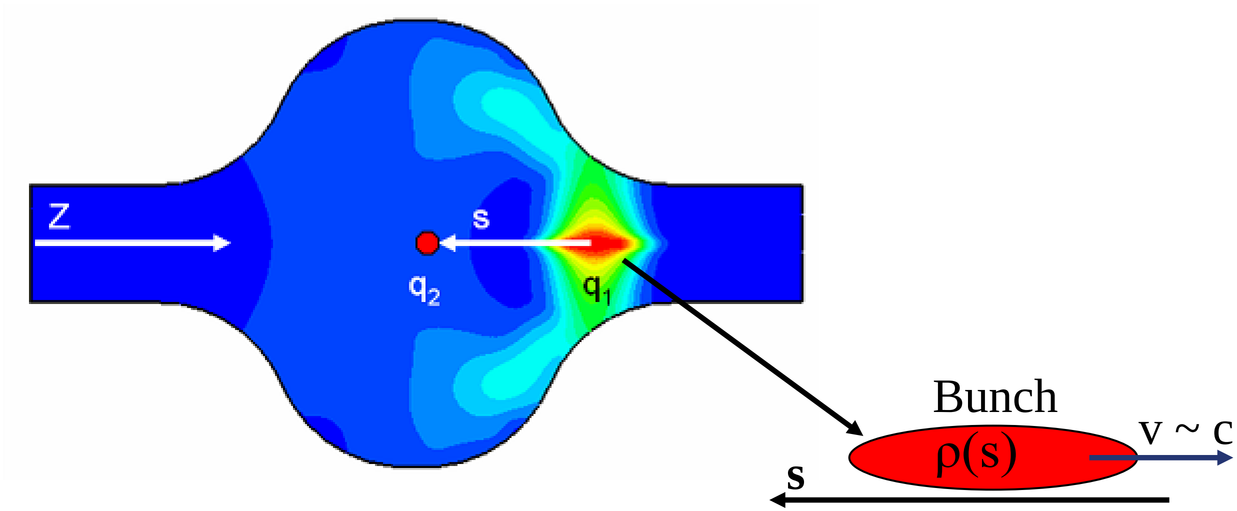
Wakefields and wake potentials



$$\vec{W}(s,\vec{r}_\perp^{(q_2)}) = \frac{1}{q_1}\int_{-\infty}^{+\infty}(\vec{E}+\vec{v}\times\vec{B})\Big|_{t=\frac{z+s}{c}}\,dz$$

Wakefields

- Geometric
- Resistive

$$\Delta\vec{p}^{(q_2)} \sim \vec{W}(s,\vec{r}_\perp^{(q_2)})$$ ⇒

- Energy Loss
- Transverse Kick

Physical Motivation

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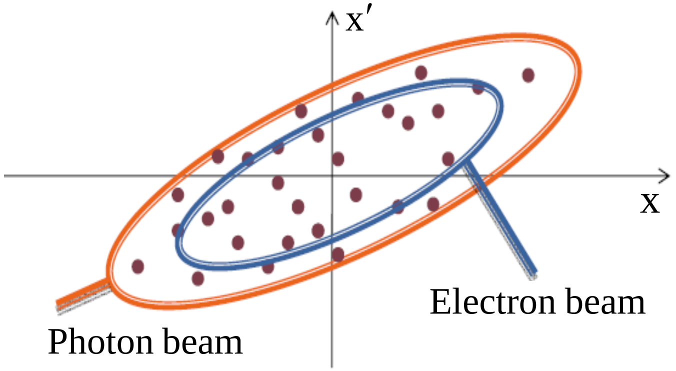
Example: **X-Ray SASE FELs**

Requirements on electron beam

- Electron beam emittance $\varepsilon < \dfrac{\lambda_r}{4\pi}$

(Overlapping of electron and photon beams in phase space)



- Relative energy spread (typical) $\dfrac{\Delta E}{E} \sim 10^{-4}$

(To prevent widening of the spontaneous radiation line)

European XFEL

$\varepsilon_n \sim 1.4\ \mu m$
$E \sim 17.5\ GeV$
$\sigma_b \sim 25\ \mu m$
$\lambda_r \sim 0.1\ nm$

- High peak current ⇨ Short bunches

The wakefields of short bunches

- Emittance growth
- Extra induced energy spread

⇨ • Degradation of FEL process

Numerical Motivation

Why dispersion free numerical method?

Longitudinal dispersion

Reduction → Conventional FDTD

$$c\Delta t < \Delta z << \sqrt{\frac{\sigma^3}{L}}$$

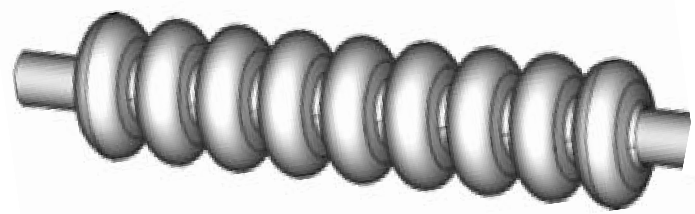
$$\Delta z << 30\ \text{nm}$$

Free → Dispersion free scheme

$$c\Delta t = \Delta z < \sigma$$

$$\Delta z < 10\ \mu\text{m}$$

Example: σ=10 μm, L=1m



TESLA cavities

Numerical Motivation

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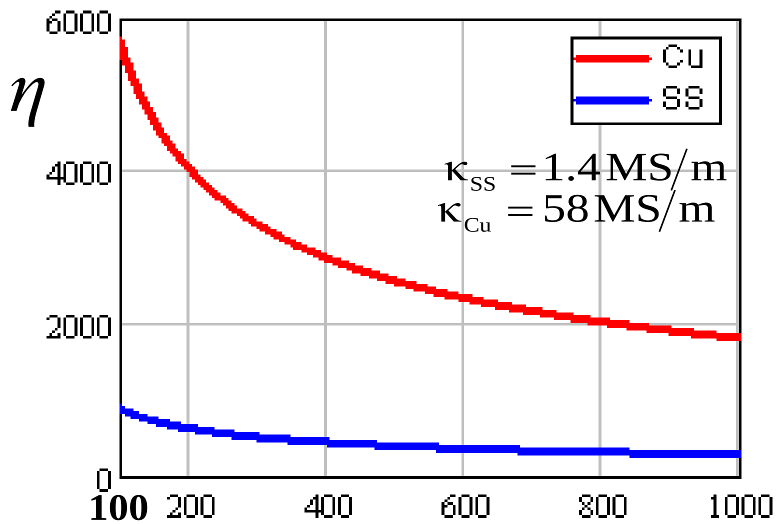
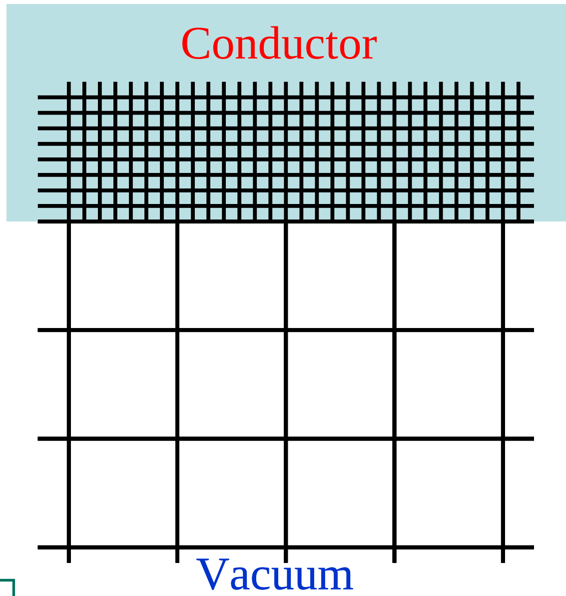
Why we need surface impedance model ?

$$\eta \equiv \frac{\lambda_{vacuum}}{\lambda_{metal}} \sim \sqrt{\frac{c}{\omega} Z_0 \kappa}$$





Volume conductivity → Mesh and time step reduction → Computational time increases $\sim \eta^4$

Surface conductivity → No mesh and time step reduction

SIBC in Time Domain

SIBC in Frequency Domain

$$\vec{E}_\tau(\omega) = Z_s(\omega)\,[\vec{n} \times \vec{H}_\tau(\omega)]$$

Transformation to TD

SIBC in Time Domain

$$\vec{E}_\tau(t) = \sum_{i=0}^{N} \vec{G}_i(t)$$

Rational function approximation

$$Z_s(\omega) \approx \alpha_0 + \sum_{i=1}^{N} \frac{\alpha_i}{j\omega - \beta_i}$$

Auxiliary Differential Equations

$$\vec{G}_0 = \alpha_0\,[\vec{n} \times \vec{H}_\tau]$$

$$\frac{d}{dt}\vec{G}_i - \beta_i \vec{G}_i = \alpha_i\,[\vec{n} \times \vec{H}_\tau]$$

- K. S. Oh and J. E. Schutt-Aine, An Efficient Implementation of SIBC for the FDTD Method, IEEE Trans. Antennas Propagat., vol. 43, pp. 660–666, 1995.
- Riku M. Mäkinen, An Efficient SIBC for Thin Wires of Finite Conductivity, IEEE *Trans.* Antennas Propagat, vol. 52, pp. 3364-3372 , 2004
- R. Mäkinen, T. Lau, E. Gjonaj, T. Weiland, Computation of Resistive Wakefield with the PBCI Code, Proceedings of EPAC08, Genoa, Italy, 2008, pp. 1753-1755

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Semi-Discrete Maxwell's Equations with SIBC

$$\frac{d}{dt}\begin{pmatrix} \widehat{e} \\ \widehat{h} \\ 0 \\ G_1 \\ \vdots \\ G_N \end{pmatrix} = \begin{pmatrix} 0 & M_\varepsilon^{-1}C^T & 0 & 0 & \cdots & 0 \\ -M_\mu^{-1}C & 0 & C_B & C_B & \cdots & C_B \\ 0 & -\alpha_0 & 1 & 0 & \cdots & 0 \\ 0 & \alpha_1 & 0 & \beta_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \alpha_N & 0 & 0 & \cdots & \beta_N \end{pmatrix} \begin{pmatrix} \widehat{e} \\ \widehat{h} \\ G_0 \\ G_1 \\ \vdots \\ G_N \end{pmatrix}$$

Boundary Effects

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$$Z_s(\omega) = Z_s^{\sigma}(\omega) + Z_s^{L}(\omega)$$

- ***Finite Resistivity***

$$Z_s^{\sigma}(\omega) \approx \sqrt{\frac{j\omega\mu}{\sigma(\omega) + j\omega\varepsilon}}$$

$$\sigma(\omega) \approx \frac{\sigma_0}{1 + j\omega\tau}$$

- ***Surface Roughness***
- ***Metal Oxidation***

$$Z_s^{L}(\omega) \approx j\omega L$$

$$L \approx \mu_0 \left[\frac{\varepsilon_r - 1}{\varepsilon_r} \cdot \Delta_{oxide} + 0.01 \cdot \Delta_{rough} \right]$$

$$\varepsilon_r \sim 10$$
$$\Delta_{oxide} \sim 7\,nm$$

$$\Delta_{rough} \sim 500\,nm$$

- M. Dohlus. TESLA report 2001-26, 2001
- K. Bane, G. Stupakov, SLAC-PUB-10707, 2004
- A. Tsakanian, M. Dohlus, I. Zagorodnov, TESLA-FEL-2009-05, 2009

Dispersion-Free Numerical Methods

LT Splitting Scheme

FIT

$$\oint_{\partial S} \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \mu\vec{H}\cdot d\vec{A}$$

$$\oint_{\partial S} \vec{H}\cdot d\vec{l} = \int_S \left[\vec{J} + \frac{\partial}{\partial t}\varepsilon\vec{E}\right]\cdot d\vec{A}$$

$$\oint_{\partial V} \varepsilon\vec{E}\cdot d\vec{A} = \int_V \rho\, dV$$

$$\oint_{\partial V} \mu\vec{H}\cdot d\vec{A} = 0$$

Strang Splitting

$$\begin{pmatrix} \hat{e} \\ \hat{h} \end{pmatrix}^{n+1} = G_t\left(\frac{\Delta t}{2}\right) G_l(\Delta t) G_t\left(\frac{\Delta t}{2}\right) \begin{pmatrix} \hat{e} \\ \hat{h} \end{pmatrix}^{n}$$

Dispersion-free in longitudinal direction

FVTD Scheme

$$\oint_{\partial V} \vec{E}\times d\vec{A} = -\frac{\partial}{\partial t}\int_V \mu\vec{H}\, dV$$

$$\oint_{\partial V} \vec{H}\times d\vec{A} = \int_V \left[\vec{J} + \frac{\partial}{\partial t}\varepsilon\vec{E}\right] dV$$

$$\oint_{\partial V} \varepsilon\vec{E}\cdot d\vec{A} = \int_V \rho\, dV$$

$$\oint_{\partial V} \mu\vec{H}\cdot d\vec{A} = 0$$

$$\begin{pmatrix} e \\ h \end{pmatrix}^{n+1} = G(\Delta t) \begin{pmatrix} e \\ h \end{pmatrix}^{n}$$

No Splitting

- E. Gjonaj, T. Lau, W. Muller, T. Weiland and el., Large Scale Wake Field Computations for 3D-Accelerator Structures with the PBCI Code, Proceed. Of ICAP 06, Chamonix, France, 2006, pp. 9-34
- E. Gjonaj, T. Lau, T. Weiland, Wakefield Computation with the PBCI Code using a Non-Split Finite Volume Method, Proceedings of PAC09, Vancouver, Canada, 2009, pp. 4516-4518

Finite Volume Time Domain Method

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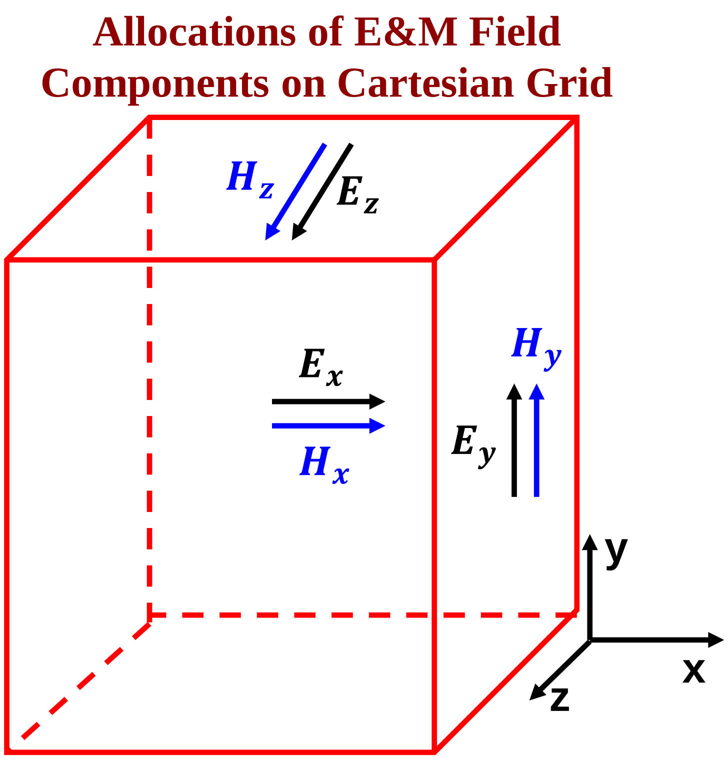

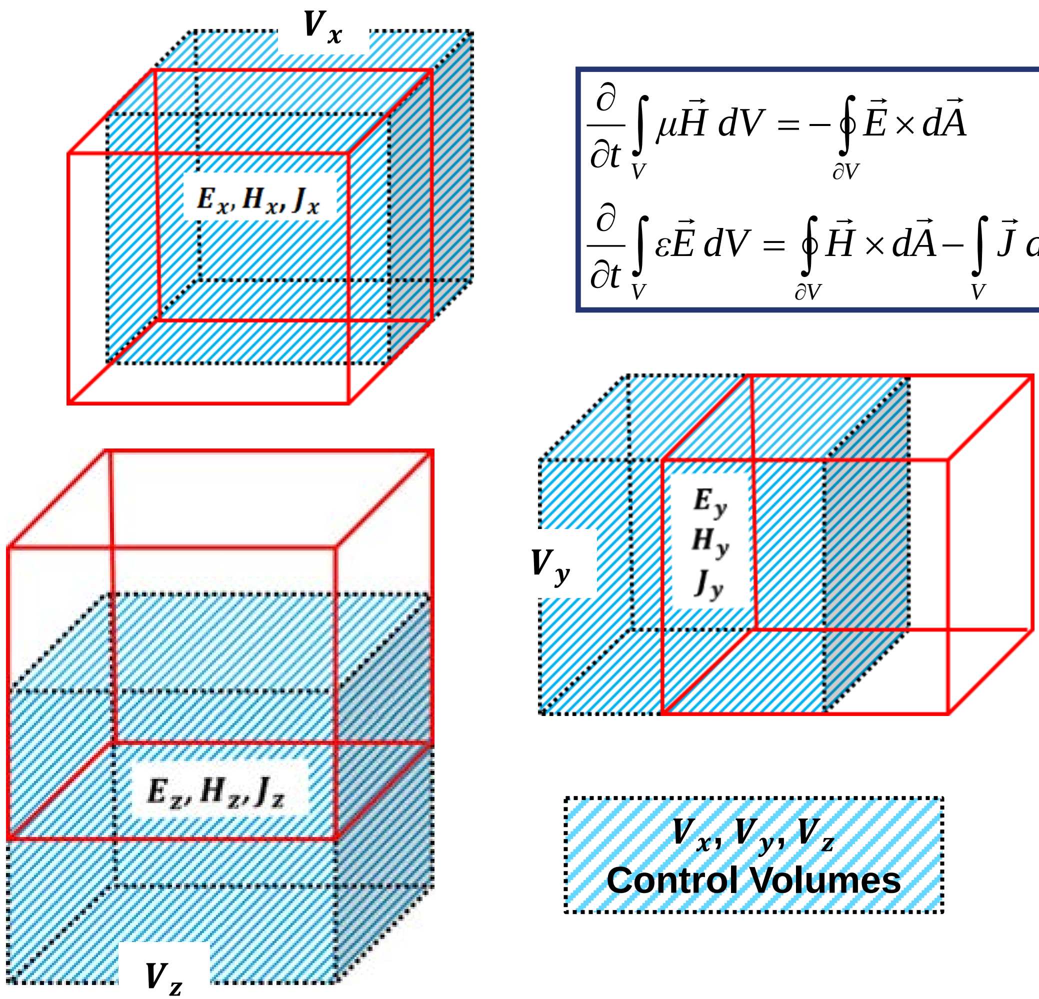

$$\frac{\partial}{\partial t}\int_V \mu\vec{H}\,dV = -\oint_{\partial V}\vec{E}\times d\vec{A}$$

$$\frac{\partial}{\partial t}\int_V \varepsilon\vec{E}\,dV = \oint_{\partial V}\vec{H}\times d\vec{A} - \int_V \vec{J}\,dV$$

Finite Volume Time Domain Method

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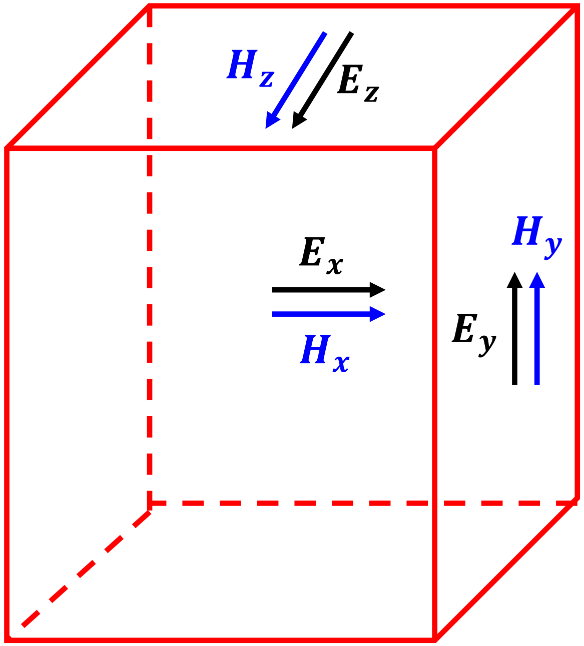
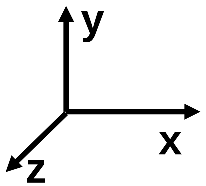
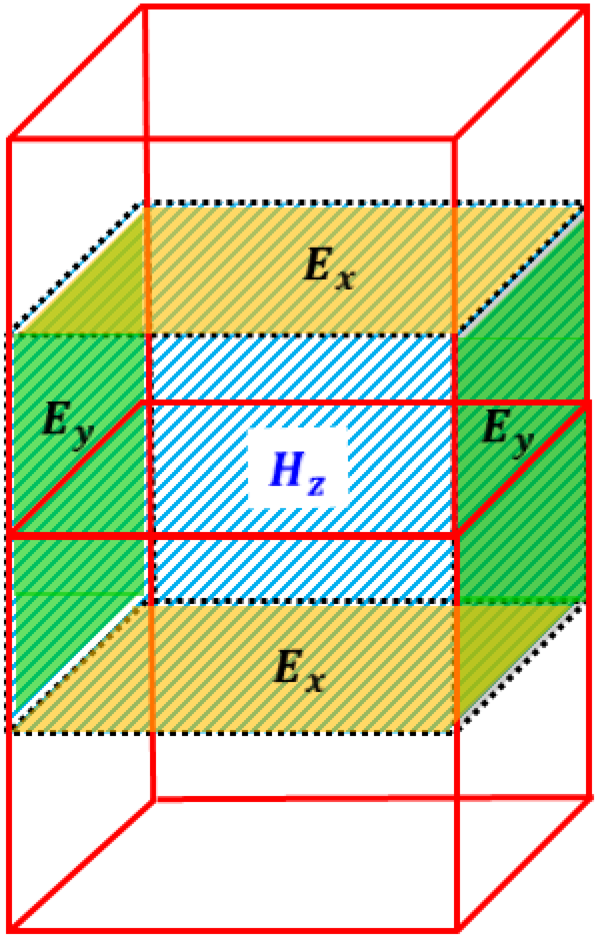
Allocations of E&M Field Components on Cartesian Grid







$$\frac{\partial}{\partial t}\int_V \mu\vec{H}\,dV = -\oint_{\partial V}\vec{E}\times d\vec{A}$$

Dispersion Properties of Numerical Methods

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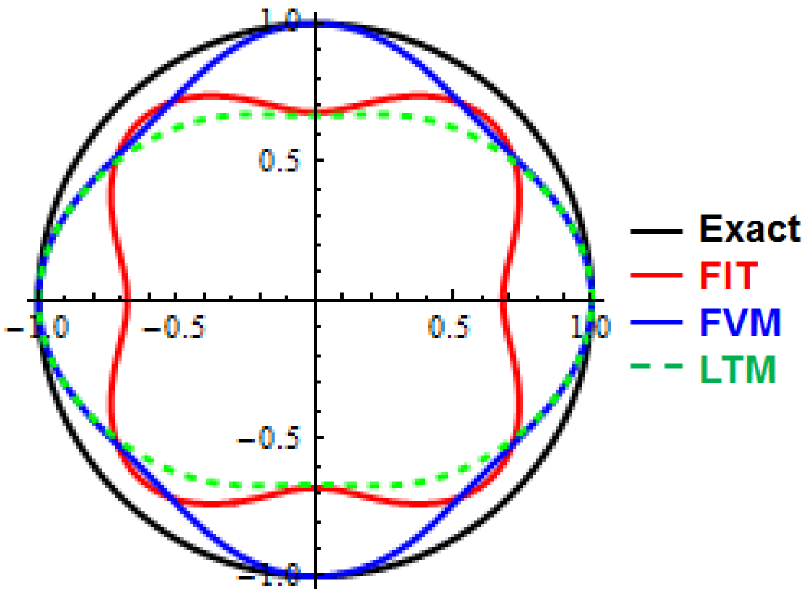
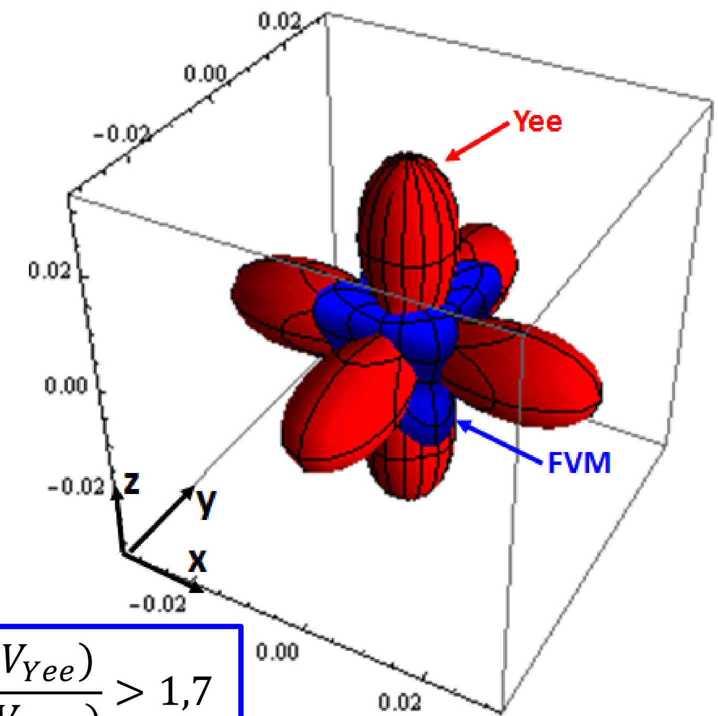
Numerical phase velocity vs propagation direction

2D – Δ=λ/2



Numerical phase velocity error vs propagation direction

3D – Δ=λ/6



$$\frac{\max(\Delta V_{Yee})}{\max(\Delta V_{FVM})} > 1{,}7$$

Applications

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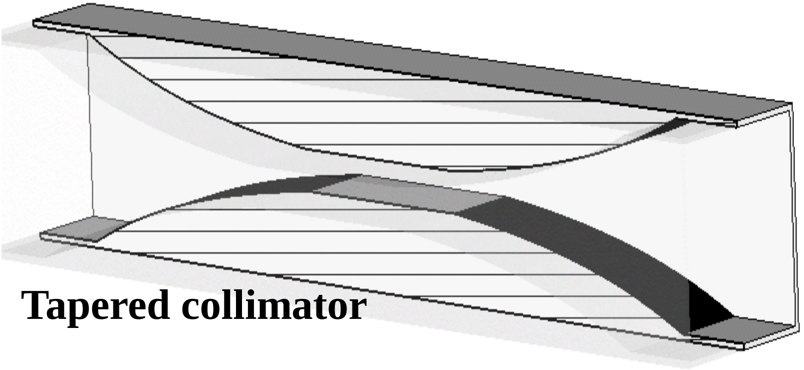
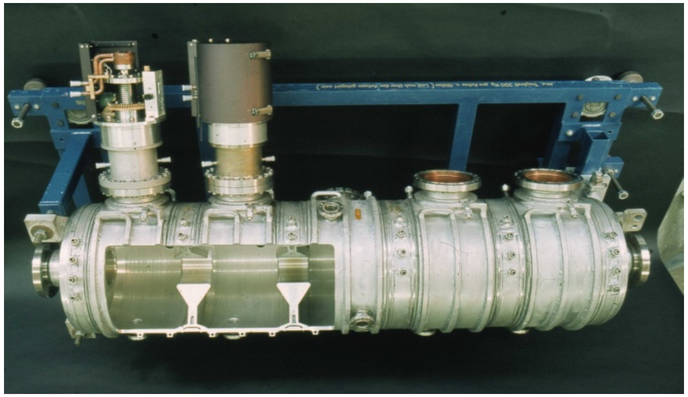
Resistive wake field calculations of ultra short bunches in various structures:



Tapered collimator

- Collimators
- Undulator beampipe (Elliptical)
- Undulator intersections
- Warm accelerating structures
- Multi-layer structures (check SIBC model)
- Etc.



PETRA cavity

Thank You for Your Attention!