## Calculation of Complex Eigenmodes for TESLA 1.3 GHz and BC0 structures



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# Outline



- Eigenvalue calculation for the 1.3 GHz TESLA 9-cell structure
  - Acceleration mode below cut-off frequency of the beam tubes, calculation of field maps for various main coupler penetration depths
  - Higher order modes above the cut-off frequency of the beam tubes, influence of different boundary conditions to terminate the beam tubes
- Postprocessing of the field data calculated for BC0
  - Longitudinal loss parameter
  - Kick parameter



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- Linac: Cavities
  - Photograph



- Numerical model

http://newsline.linearcollider.org



CST Studio Suite 2014











 Superconducting resonator modeling Beam tube 0 mm Input coupler Parameter variation 0, 2, 4, 6, 8, 10 mm Downstream Upstream higher order mode coupler higher order mode coupler





Superconducting resonator modeling







- Field reconstruction using the Kirchhoff integral
  - Field values inside a closed surface can be determined once the surface field components are available
  - Kirchhoff integral

C -	$e^{-ik \vec{r}-\vec{r}' }$	k =	$2\pi f$
G –	$\overline{4\pi  \vec{r} - \vec{r'} }$	h - f	$c_0$

$$\vec{E}(\vec{r}) = \int \left( k(\vec{n}' \times ic_0 \vec{B}') \ G - (\vec{n}' \times \vec{E}') \times \nabla G - (\vec{n}' \cdot \vec{E}') \ \nabla G \right) dA'$$
$$ic_0 \vec{B}(\vec{r}) = \int \left( k(\vec{n}' \times \vec{E}') \ G - (\vec{n}' \times ic_0 \vec{B}') \times \nabla G - (\vec{n}' \cdot ic_0 \vec{B}') \ \nabla G \right) dA'$$





Superconducting resonator modeling







#### Port boundary condition







#### Port boundary condition













# **Numerical Results**



# Superconducting resonator

- Convergence study

Main coupler depth	Number of tets	Resonance freqency	Quality factor
8 mm	366 955	1.300055 GHz	2.832 10 <sup>6</sup>
8 mm	720 324	1.300032 GHz	2.753 10 <sup>6</sup>
8 mm	1 320 954	1.300012 GHz	2.735 10 <sup>6</sup>
8 mm	2 131 752	1.300005 GHz	2.670 10 <sup>6</sup>
8 mm	3 284 180	1.300002 GHz	2.667 10 <sup>6</sup>

**Estimated accuracy** 

$$\Delta f \approx 10^{-5}$$
$$\Delta Q \approx 10^{-3}$$



# **Numerical Results**



# Superconducting resonator

- Eigenvalue calculation

Main coupler depth	Number of tets	Resonance freqency	Quality factor
0 mm	3 290 937	1.300003 GHz	12.605 10 <sup>6</sup>
2 mm	3 288 250	1.300002 GHz	8.304 10 <sup>6</sup>
4 mm	3 286 608	1.300002 GHz	5.571 10 <sup>6</sup>
6 mm	3 285 819	1.300002 GHz	3.812 10 <sup>6</sup>
8 mm	3 284 180	1.300002 GHz	2.667 10 <sup>6</sup>
10 mm	3 282 786	1.300002 GHz	1.909 10 <sup>6</sup>



Complex-valued field components are available within the FEM or the Kirchhoff's integral formulation for any specified main coupler penetration depth. File format for ASTRA input has been provided by DESY (thanks to Martin).



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Electric BC

**Magnetic BC** 



Electric BC

Magnetic BC

Influence of boundary conditions at the beam tube

- Close tube using PEC material

 $^{\}$  HOM and MAIN couplers always modeled using port BC

- Close tube using PMC material





- Influence of boundary conditions at the beam tube
  - Infinite tube using port boundary conditions



- "Cavity" boundary condition

# "Cavity" BC "Cavity" BC





- Influence of boundary conditions at the beam tube
  - Infinite tube using port boundary conditions



- "Cavity" boundary condition
- Port BC
   "Cavity" BC
   "Cavity" BC
   Port BC

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#### Quality factor versus frequency (1.3 GHz structure)







#### Quality factor versus frequency

- Comparison using different beam-tube boundary conditions







# Quality factor versus frequency

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#### Quality factor versus frequency

- Comparison using different beam-tube boundary conditions











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- Bunch compressor 0
  - Vacuum chamber (construction)













#### Bunch compressor 0

- Particle trajectories
  - Hard-edged magnets: analytical calculation of particle trajectories
  - Determination of local coordinates aligned to tangential direction
  - Specification of sample points to evaluate eigenmode fields





# Implementation



- Eigenvalue solver and auxiliary programs
  - Postprocessing
    - Change point evaluation from existing 'line' to new 'arbitrary list'
    - Specify list of points along selected trajectory
    - Evaluate field components for all determined modes and all points
    - Transform field components to the particles coordinate system
    - Perform path integration to determine loss and kick parameter





## Implementation



- Eigenvalue solver and auxiliary programs
  - Definition of the longitudinal loss and kick parameter
    - Resulting energy

Kick parameter

$$\vec{k}_{\nu} = \frac{1}{q^2} \int_{t_0}^{t_1} \frac{\mathrm{d}\vec{p}}{\mathrm{d}t'} \, \mathrm{d}t' = \int_{t_0}^{t_1} \left( \frac{a_{\nu(t')}}{q} \ \vec{e}_{\nu(t')} - \vec{v} \times \vec{h}_{\nu(t')} \ \mu \ \omega_{\nu} \ \frac{A_{\nu(t')}}{q} \right) \, \mathrm{d}t'$$

$$c_{\nu(t)} = q \ \dot{\vec{r}}_{(\vec{r}_{(t)})} \cdot \vec{e}_{\nu(\vec{r}_{(t)})}$$

$$a_{\nu(t)} = -\operatorname{Re}\left( e^{i\omega_{\nu}t} \int_{-\infty}^{t} e^{i\omega_{\nu}\tau} c_{\nu(\tau)} \ \mathrm{d}\tau \right)$$

$$A_{\nu(t)} = c_0 \int_{-\infty}^{t} a_{\nu(t)} \ \mathrm{d}\tau \quad \land \text{Amplitude}$$





- Eigenvalue solver using real-value arithmetic
  - Computational mesh







#### Eigenvalue solver

- Field pattern of the electric field strength







#### Eigenvalue solver

#### - Field pattern of the electric field strength

Mode 6

... more than 1000 modes have been examined (all modes up to 4 GHz).







- Eigenvalue solver
  - Frequency distribution













Eigenvalue solver



- Loss parameter



# Comparison to the TESLA 1.3 GHz structure

mode	f /GHz	$k^{(0)}/$
		V/(pC)
MM-28	-3.0971	0.02
MM-29	3.0971	$0.377 \ 10^{-04}$
Band 4		
MM-30	3.3898	$0.981 \ 10^{-02}$
MM-31	3.3921	$0.388 \ 10^{-02}$
MM-32	3.4055	$0.592 \ 10^{-02}$
MM-33	3.4261	$0.977 \ 10^{-02}$
MM-34	3.4541	$0.144 \ 10^{-02}$
MM-35	3.4885	$0.689 \ 10^{-02}$
MM-36	3.5283	$0.538 \ 10^{-02}$
MM-37	3.5719	$0.163 \ 10^{-05}$
MM-38	3.6174	$0.954 \ 10^{-02}$
MM-39	3.6650	0.03
	TEOL	1 0 0 0 1 0 0

TESLA 2001-33 R. Wanzenberg























- Eigenvalue solver
  - Accuracy estimation path "min"

Mode 914

2000







# Summary / Outlook



- Summary:
  - Eigenvalue calculations concentrating on the accelerating mode for the TESLA 1.3 GHz structure performed based on various main coupler penetration depth
  - Field maps for the various setups provided
  - Comparison of the eigenvalue distribution (fourth dipole passband) using different beam-tube boundary conditions
  - Postprocessing of the BC0 vacuum chamber modes, determination of longitudinal loss and kick parameter























#### HOM Coupler

- Poynting



