Coherence properties of the radiation from FLASH

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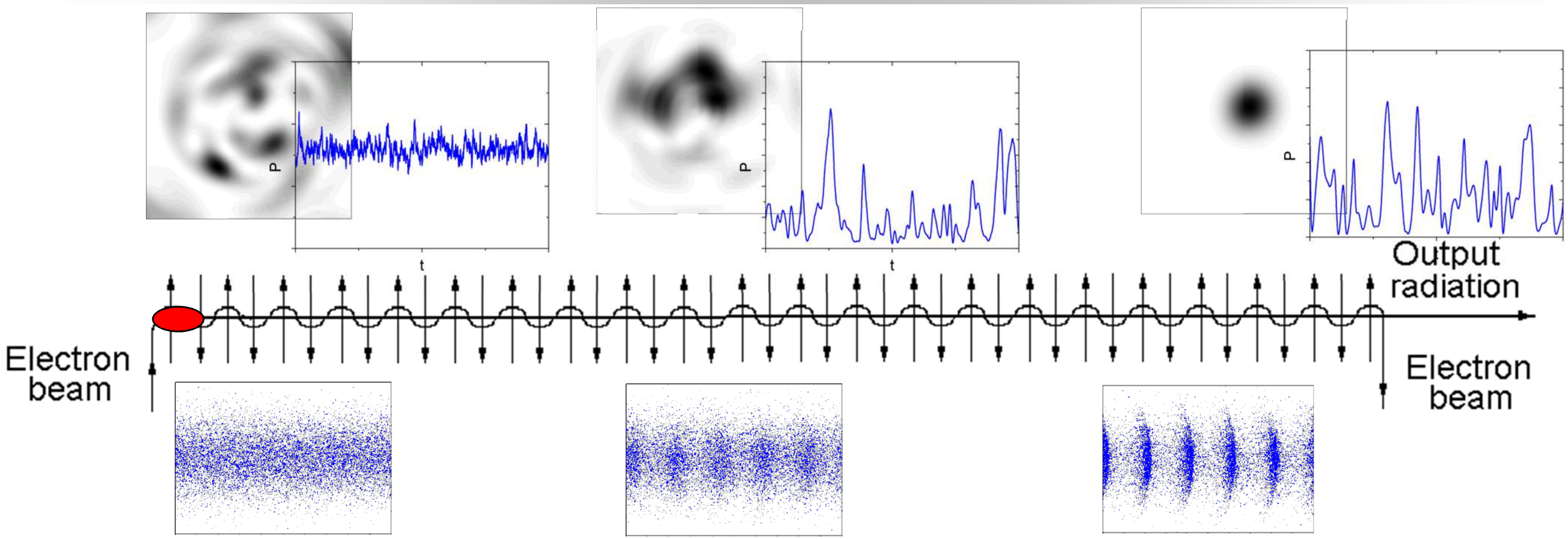
- Qualitative physical considerations.
- Strict definitions.
- Simulation results for FLASH: temporal and spatial coherence, higher harmonics.
- Discussions and conclusion.

Motivation

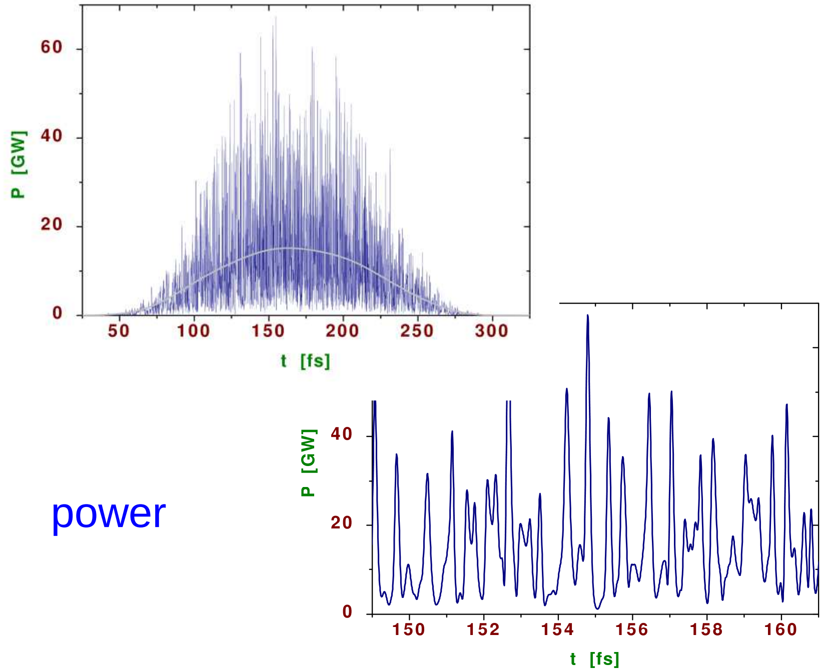
- Planning and analysis of user experiments will gain a lot with detailed knowledge of the radiation properties at the fundamental and higher harmonics: intensity, spatial profiles, temporal and spatial coherence.
- One of the difficulties to predict FEL properties at FLASH (and at other setups as well – FLASH is not exception) is big uncertainty of the electron beam parameters. In the present experimental situation many parameters of the electron beam at FLASH depend on practical tuning of the machine. Analysis of measurements and numerical simulations shows that depending on tuning of the machine emittance may change from about 1 to about 1.5 mm-mrad. Tuning at small charges may allow to reach smaller values of the emittance down to 0.5 mm-mrad. Peak current may change in the range from 1 kA to 2 kA depending on the tuning of the beam formation system. One (more or less) fixed parameter is average beta function (10 m).
- Recently we launched global task for analysis of the FLASH parameter space and calculation of FEL the characteristics for the fundamental, 3rd, and 5th harmonic.
- Here we illustrate the results of the studies of coherence properties of the radiation from FLASH operating at the radiation wavelength of 8.x nm at the fundamental harmonic, and higher odd harmonics (2.x nm and 1.x nm).
- We found that present configuration of FLASH free electron laser is not optimal for providing ultimate quality of the output radiation. The way for improving quality of the radiation is proposed.

Qualitative look at the radiation properties

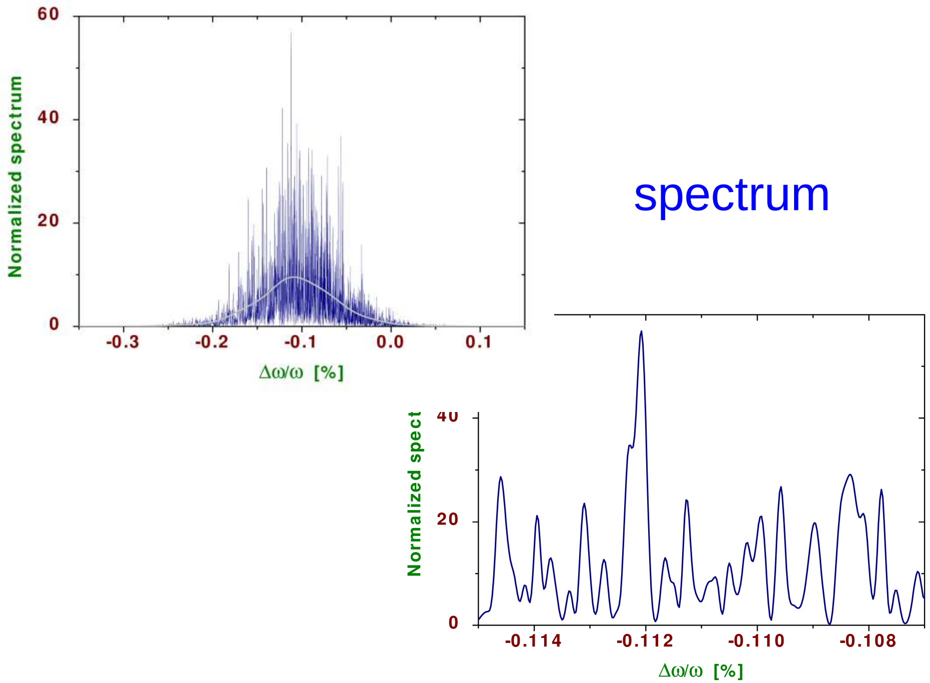


- Beam density modulations at frequencies close to the resonance frequency initiate the amplification process.
- Fluctuations of current density in the electron beam are uncorrelated not only in time but in space, too. Thus, a large number of transverse radiation modes are excited.
- Longitudinal coherence is formed due to slippage effects (electromagnetic wave advances electron beam by one wavelength while electron beam passes one undulator period). Thus, typical figure of merit is relative slippage of the radiation with respect to the electron beam on a scale of field gain length ➔ coherence time $\tau_c \sim \lambda L_g/(c\,\lambda_w)$.
- Transverse coherence is formed due to diffraction effects. Typical figure of merit is ratio of the transverse size of the electron beam σ^2 to the diffraction expansion of the radiation λL_g on a scale of the field gain length ➔ diffraction parameter B.

Qualitative look at the radiation properties



power



spectrum

- Radiation generated by SASE FEL consists of wavepackets (spikes). Typical duration of the spike is about coherence time τ_c.
- Spectrum also exhibits spiky structure. Spectrum width is inversely proportional to the coherence time, $\Delta\omega \sim 1/\tau_c$, and typical width of a spike in a spectrum is inversely proportional to the pulse duration T.
- Amplification process selects narrow band of the radiation, coherence time is increased, and spectrum is shrinked.

Simple estimates (1D)

- FEL parameter ρ and number of cooperating electrons N_{c}:

$$\rho = \frac{\lambda_{\mathrm{w}}}{4\pi}\left[\frac{4\pi^2 j_0 K^2 A_{\mathrm{JJ}}^2}{I_A \lambda_{\mathrm{w}} \gamma^3}\right]^{1/3} , \quad N_{\mathrm{c}} = I/(e\rho\omega) \ .$$

- Main properties of SASE FEL in the saturation can be quickly estimated in terms of ρ and N_{c}:

$$\begin{aligned}
\text{The field gain length :} \quad & L_{\mathrm{g}} \sim \frac{\lambda_{\mathrm{w}}}{4\pi\rho} , \\
\text{Saturation length :} \quad & L_{\mathrm{sat}} \sim \frac{\lambda_{\mathrm{w}}}{4\pi\rho}\left[3 + \frac{\ln N_{\mathrm{c}}}{\sqrt{3}}\right] \\
\text{Effective power of shot noise :} \quad & \frac{P_{\mathrm{sh}}}{\rho P_{\mathrm{b}}} \simeq \frac{3}{N_{\mathrm{c}}\sqrt{\pi \ln N_{\mathrm{c}}}} , \\
\text{Saturation efficiency :} \quad & \rho , \\
\text{The power gain at saturation :} \quad & G \simeq \frac{1}{3} N_{\mathrm{c}} \sqrt{\pi \ln N_{\mathrm{c}}} , \\
\text{Coherence time at saturation :} \quad & \tau_{\mathrm{c}} \simeq \frac{1}{\rho\omega}\sqrt{\frac{\pi \ln N_{\mathrm{c}}}{18}} . \\
\text{Spectrum bandwidth :} \quad & \sigma_{\omega} = \sqrt{\pi}/\tau_{\mathrm{c}} ,
\end{aligned}$$

SASE FEL: more detailed analysis

- Radiation from SASE FEL is statistical object:

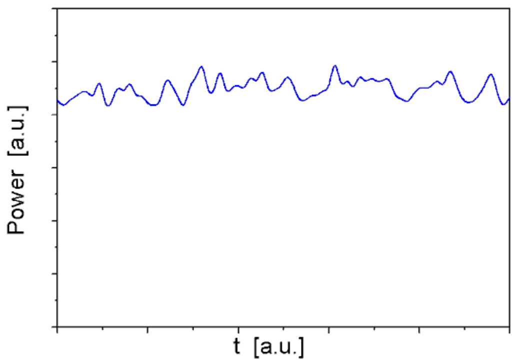
- Nature of statistics should be defined.
- Description in terms of correlation functions and probability distribution functions.
- Description of averaged characteristics (ensemble averaging).

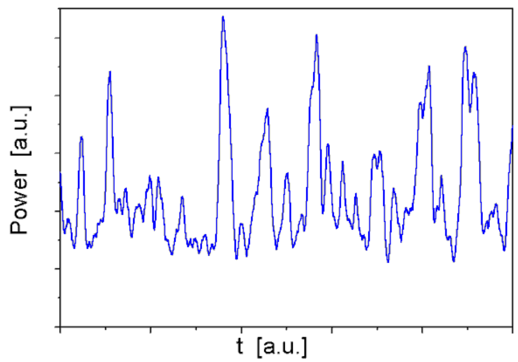
- Physical effects related to coherence properties:

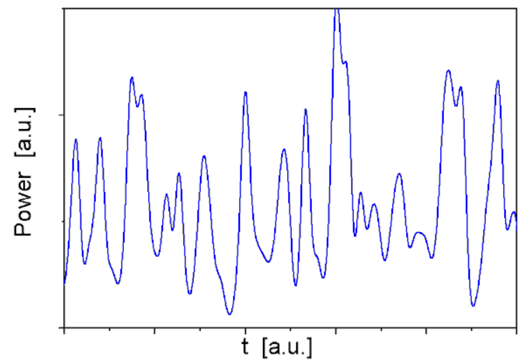
- Longitudinal coherence: slippage effects – can be studied in the framework of 1D approximation.
- Transverse coherence: diffraction effects – can be studied in the framework of full 3D model only.

Statistics and probability distributions

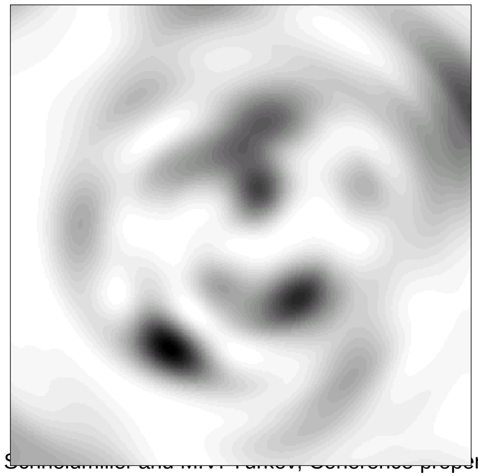
$z = 0.1\ z_{sat}$ $z = 0.5\ z_{sat}$ $z = z_{sat}$

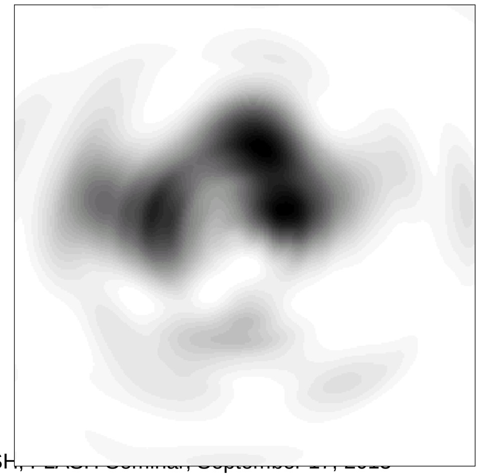


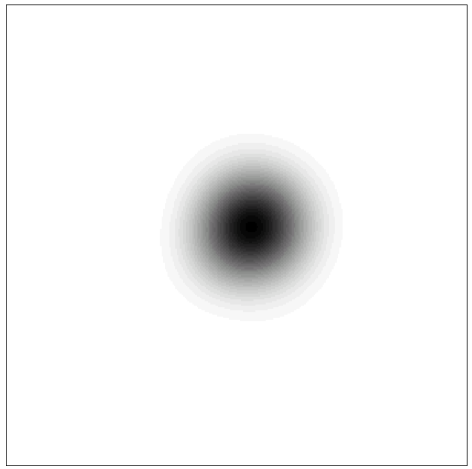




- Transverse (bottom) and longitudinal (top) distributions of the radiation intensity exhibit rather chaotic behaviour.

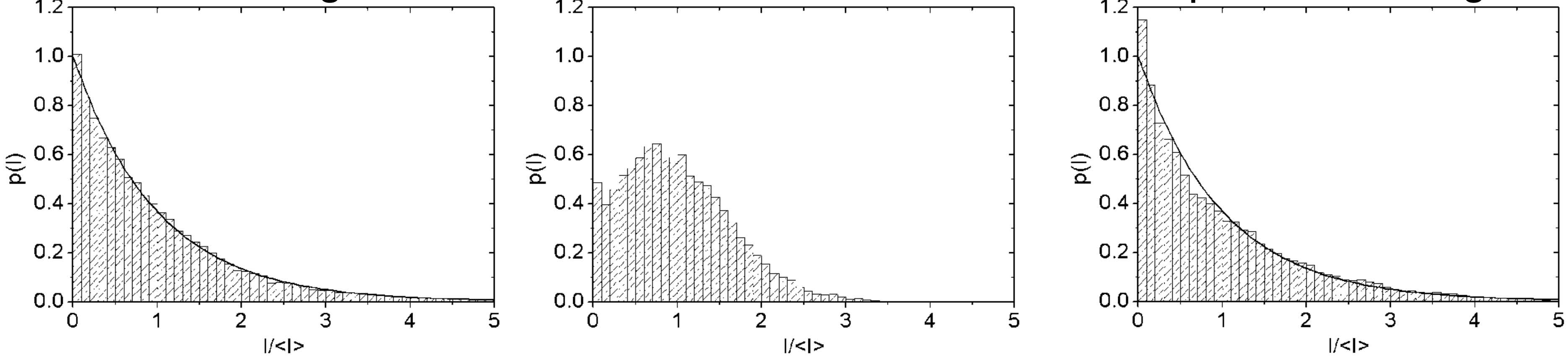




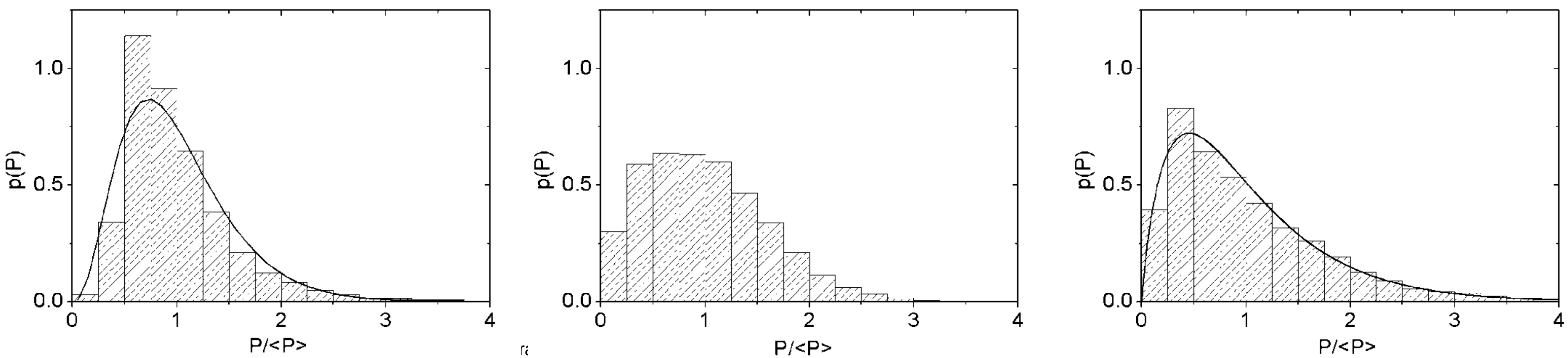


Statistics and probability distributions

Linear regime — Saturation — Deep nonlinear regime



• Probability distributions of the instantaneous power density (top) and of the instantaneous radiation power (bottom) look more elegant and seem to be described by simple functions.



SASE FEL, linear regime: Completely chaotic polarized light

- The higher order correlation functions are expressed via the first order correlation function

$$g_2(t-t') = 1 + |g_1(t-t')|^2$$

$$g_2(\Delta\omega) = 1 + |g_1(\Delta\omega)|^2$$

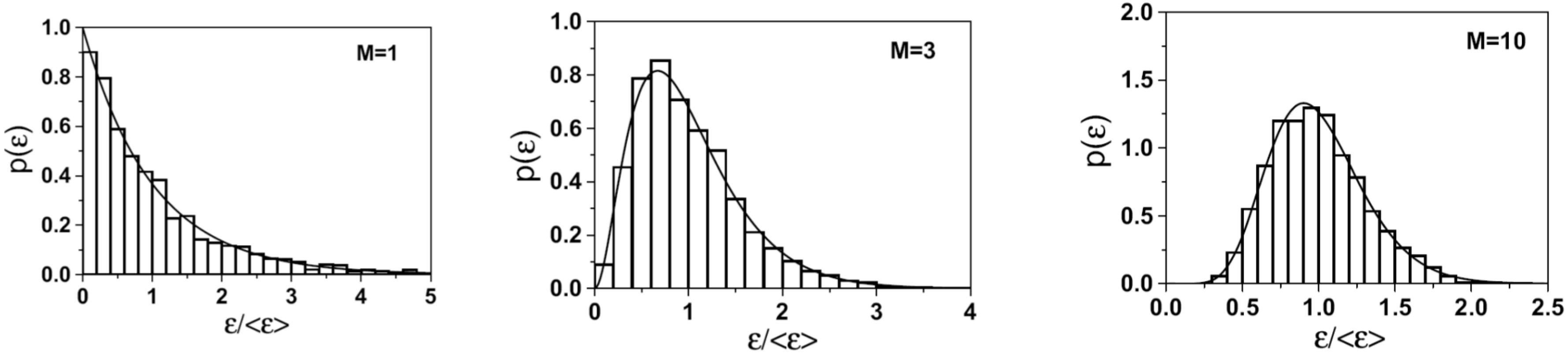
- The probability density distribution of the instantaneous radiation power follows the negative exponential distribution

$$p(P) = \frac{1}{<P>} \exp\left(-\frac{P}{<P>}\right)$$

- The probability density function of the finite-time integrals of the instantaneous power and of the radiation energy after monochromator follows the gamma distribution

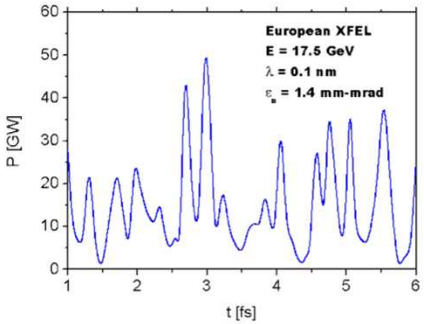
$$p(W) = \frac{M^M}{\Gamma(M)} \left(\frac{W}{<W>}\right)^{M-1} \frac{1}{<W>} \exp\left(-M\frac{W}{<W>}\right) ,$$

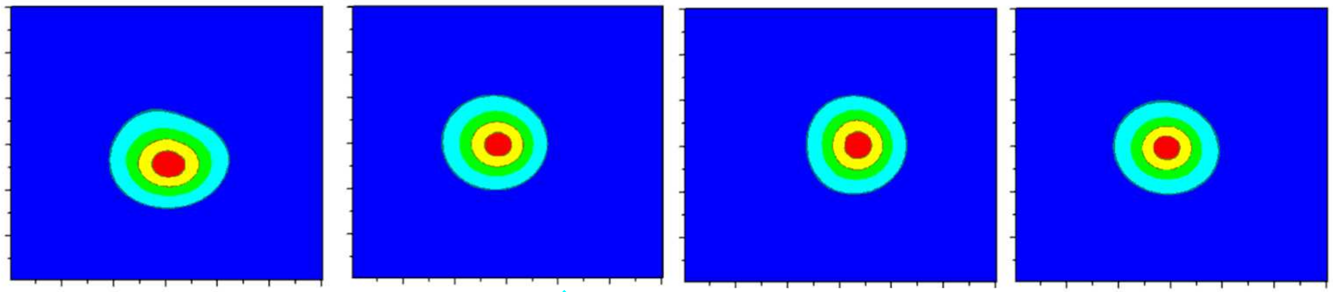
$$M^{-1} = \sigma_W^2 = <(W-<W>)^2> / <W>^2$$



E.A. Schneidmiller and M.V. Yurkov, Coherence properties of the radiation FLASH, FLASH Seminar, September 17, 2013

Qualitative look at the transverse coherence

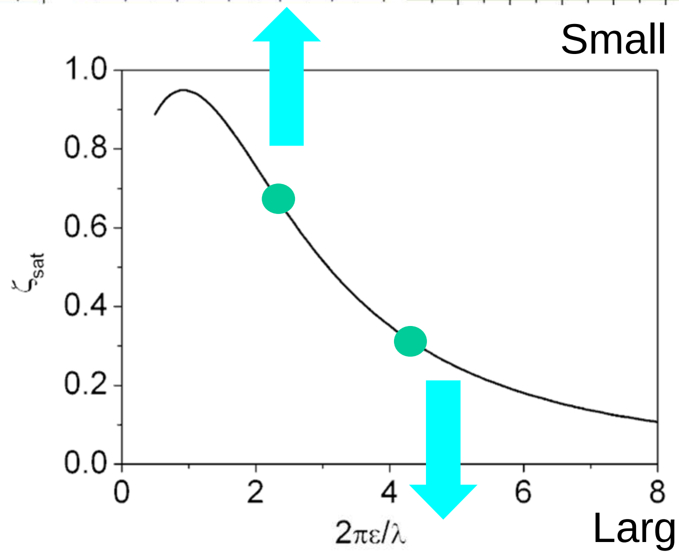




Small diffraction parameter

$2\pi\varepsilon/\lambda = 2.5$, $\zeta = 0.65$

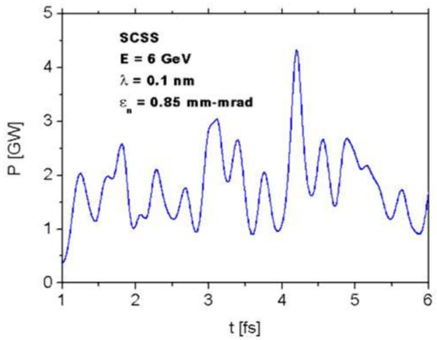
t = 2 fs, 2.7 fs, 3 fs, 4.1 fs

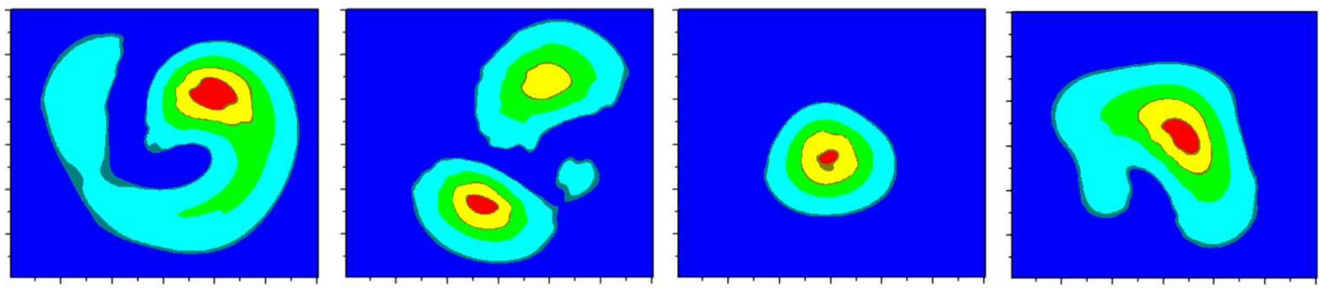


$2\pi\varepsilon/\lambda = 4.5$, $\zeta = 0.4$

t = 1.7 fs, 2.4 fs, 3.1 fs, 4.2 fs

Large diffraction parameter





Free electron laser: dimensional parameters and physical parameters of the problem

- Parameters of the undulator: λ_{w} , H_{w} [L_{w}, technology constrains]
- Parameters of the electron beam: $E(s,z)$, $I(s)$, $\epsilon(s)$, $\sigma_{\mathrm{E}}(s,z)$, β_{f}
- Parameters of the radiation: λ [power, spectrum, spatial properties, longitudinal and transverse coherence]
- Parameters of 3D FEL theory:

$$
\begin{aligned}
\Gamma &= \left[\frac{I}{I_{\mathrm{A}}}\frac{8\pi^2K^2A_{\mathrm{JJ}}^2}{\lambda\lambda_{\mathrm{w}}\gamma^3}\right]^{1/2} , \\
B &= 2\Gamma\sigma^2\omega/c , \\
\rho &= \frac{\lambda_{\mathrm{w}}\Gamma}{4\pi} , \\
\hat{\Lambda}_{\mathrm{T}}^2 &= (\sigma_{\mathrm{E}}/\mathcal{E})^2/\rho^2 , \\
\hat{k}_\beta &= 1/(\beta\Gamma) .
\end{aligned}
$$

- The gain parameter Γ defines the scale of the FEL gain, and diffraction parameter B is the figure of merit for diffraction effects. For diffraction limited electron beams the value of diffraction parameter is $B \lesssim 1$. Large values of the diffraction parameter B is indication on bad transverse coherence of SASE FEL.

Strict definitions of statistical characteristics

- The first order time correlation function and coherence time:

$$g_1(\vec{r}, t-t') = \frac{\langle \tilde{E}(\vec{r},t)\tilde{E}^*(\vec{r},t')\rangle}{\left[\langle|\ \tilde{E}(\vec{r},t)\ |^2\rangle\langle|\ \tilde{E}(\vec{r},t')\ |^2\rangle\right]^{1/2}}\ , \qquad \tau_c = \int\limits_{-\infty}^{\infty} |g_1(\tau)|^2 \,\mathrm{d}\,\tau\ .$$

- The first-order transverse correlation function and degree of transverse coherence:

$$\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp, z, t) = \frac{\langle \tilde{E}(\vec{r}_\perp, z, t)\tilde{E}^*(\vec{r\prime}_\perp, z, t)\rangle}{\left[\langle|\tilde{E}(\vec{r}_\perp, z, t)|^2\rangle\langle|\tilde{E}(\vec{r\prime}_\perp, z, t)|^2\rangle\right]^{1/2}}\ .$$

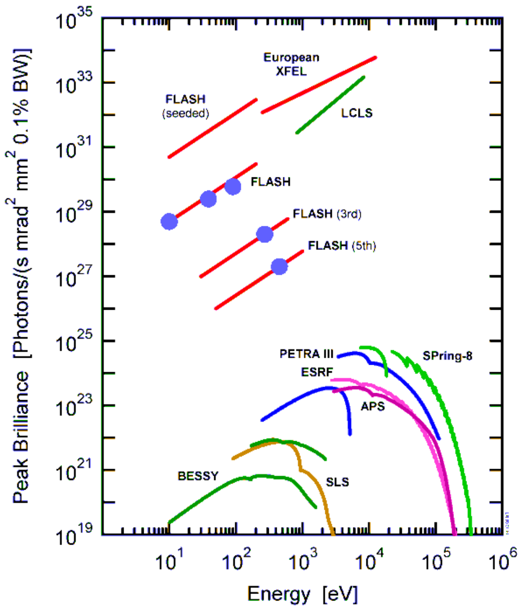
$$\zeta = \frac{\int\int |\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp)|^2 \langle I(\vec{r}_\perp)\rangle\langle I(\vec{r\prime}_\perp)\rangle \,\mathrm{d}\,\vec{r}_\perp \,\mathrm{d}\,\vec{r\prime}_\perp}{\left[\int\langle I(\vec{r}_\perp)\rangle \,\mathrm{d}\,\vec{r}_\perp\right]^2}\ .$$

- Degeneracy parameter – the number of photons per mode

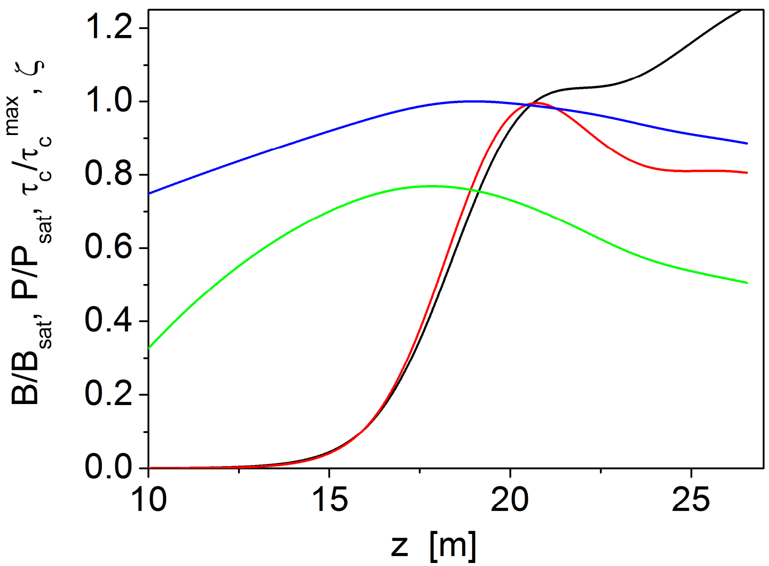
$$\delta = \dot{N}_{ph}\tau_c\zeta\ .$$

- Peak brilliance is defined as a transversely coherent spectral flux:

$$B_r = \frac{\omega\,\mathrm{d}\,\dot{N}_{ph}}{\mathrm{d}\,\omega}\,\frac{\zeta}{\left(\frac{\lambda}{2}\right)^2} = \frac{4\sqrt{2}c}{\lambda^3}\,\delta\ .$$



Evolution of the radiation properties along the undulator



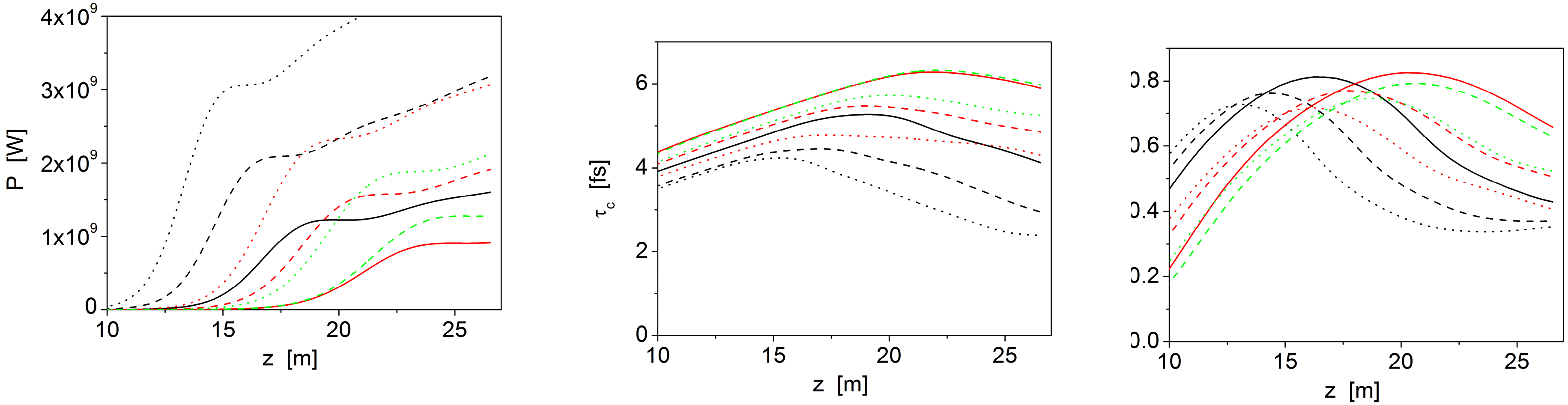
Radiation power
Coherence time
Degree of transverse coherence
Brilliance

FLASH: 8 nm, 1.5kA, 1 mm-mrad

- Radiation power grows continuously along the undulator length.
- Brilliance reaches maximum value at the saturation point.
- Degree of transverse coherence and coherence time reach their maximum values in the end of exponential regime.

In the following we traced parameter space of FLASH: emittance from 0.5 mm-mrad to 1.5 mm-mrad, peak beam current from 1 kA to 2 kA. Detailed radiation properties were calculated for the 1st, 3rd, and 5th harmonic.

Fundamental harmonic: radiation power, coherence time, and degree of transverse coherence



In the nonlinear regime coherence time scales inversely proportionally to harmonic number.

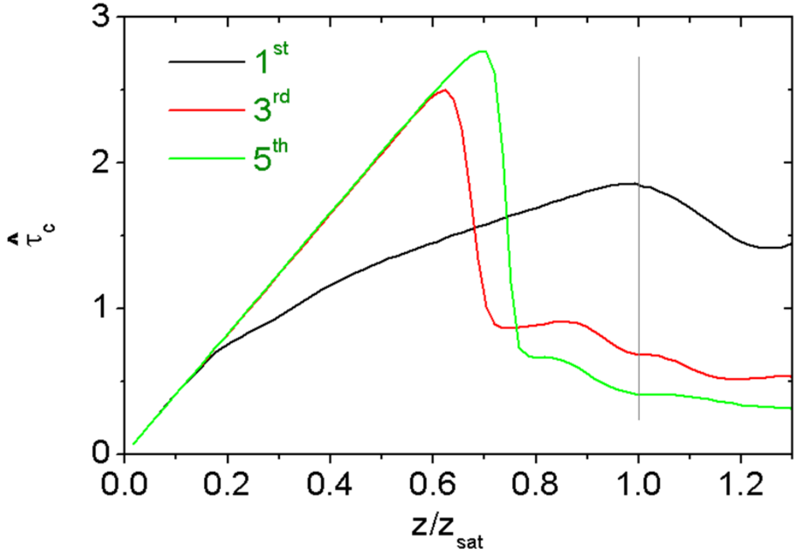
FLASH: Radiation wavelength is 8 nm.

Color codes (black, red and green) refer to different emittance 0.5, 1, and 1.5 mm-mrad. Line styles (solid, dash, and dot) refer to different values of peak current 1 kA, 1.5 kA, and 2 kA.

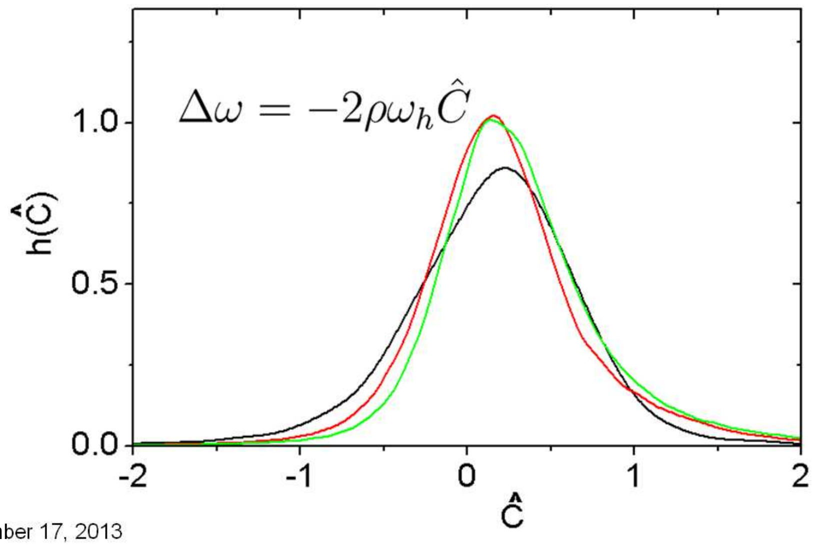
XFEL with planar undulator: odd harmonics

- Contributions of the higher odd harmonics to the FEL power for SASE FEL operating at saturation are universal functions of the undulator parameter K.
- Power of higher harmonics is subjected to larger fluctuations than that of the fundamental one.
- Probability distributions of the instantaneous power of higher harmonics in saturation regime is close to the negative exponential distribution.
- The coherence time in saturation falls inversely proportional to harmonic number: $\tau_c \propto 1/h$.
- Relative spectrum bandwidth remains constant with harmonic number.

Coherence time: 1st, 3rd, 5th

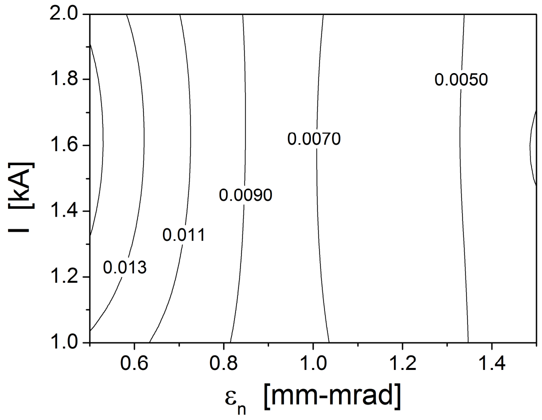


Average spectra: 1st, 3rd, 5th



3^{rd} and 5^{th} harmonic: relative contribution to the radiation power in saturation and at full undulator length

3^{rd}



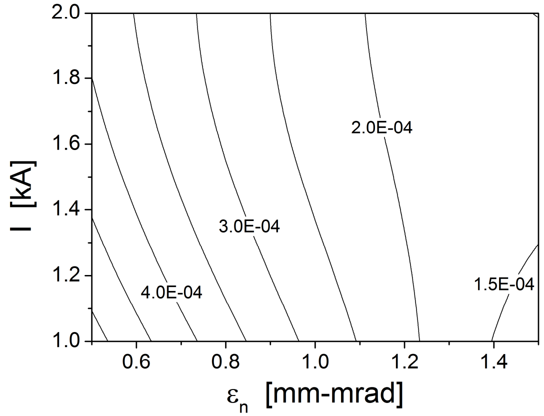
Saturation

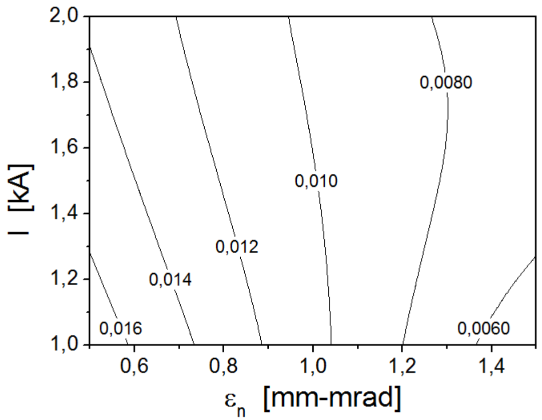
$$\frac{\langle W_3 \rangle}{\langle W_1 \rangle}\Big|_{\mathrm{sat}} = 0.094 \times \frac{K_3^2}{K_1^2}$$

$$\frac{\langle W_5 \rangle}{\langle W_1 \rangle}\Big|_{\mathrm{sat}} = 0.03 \times \frac{K_5^2}{K_1^2}$$

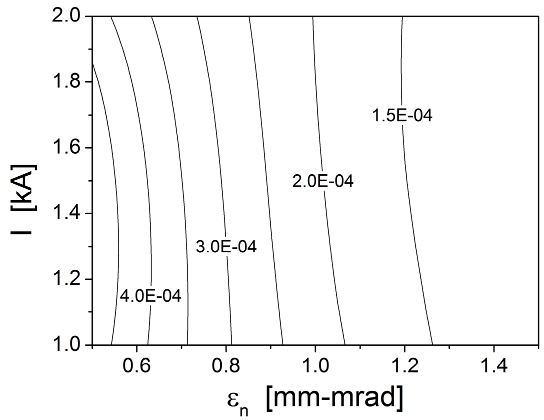
(with relevant correction for beam quality)

5^{th}



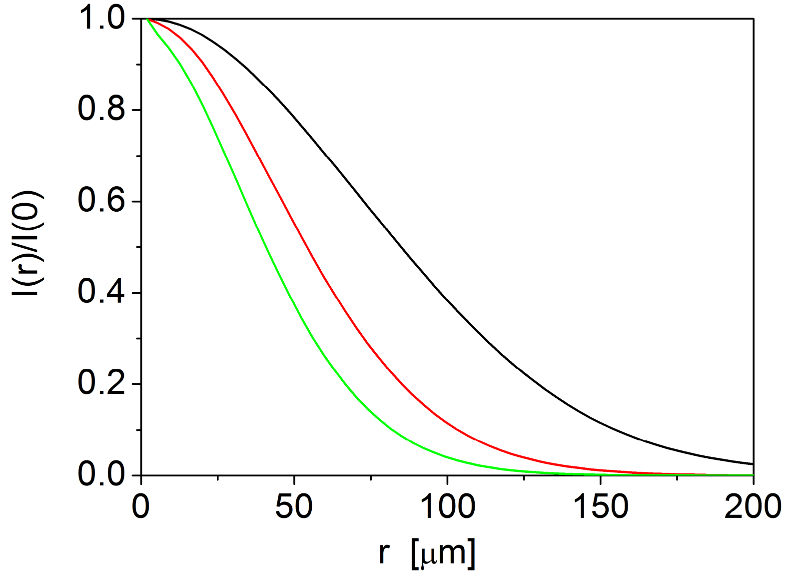


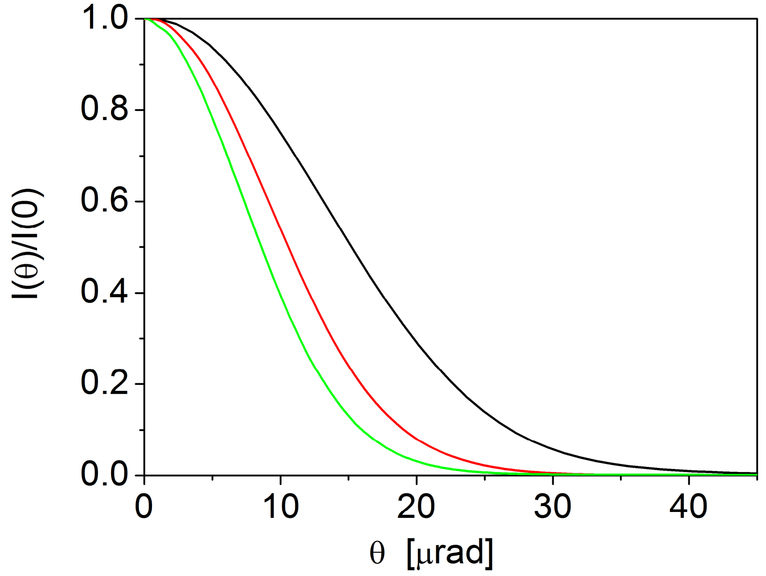
27 m



FLASH: Radiation wavelength is 8 nm.

1st, 3rd and 5th harmonic: Intensity distributions in the near and far zone in saturation

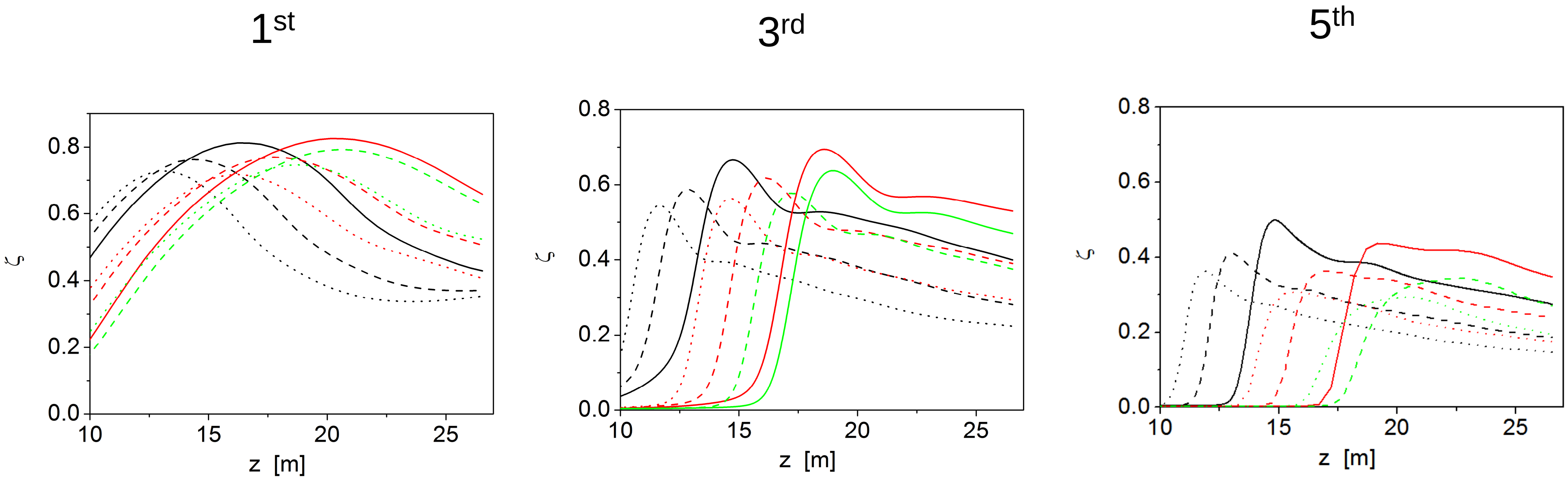




FLASH: Radiation wavelength is 8 nm.

Color codes (black, red and green) refer to the 1st, 3rd, and 5th harmonic.

1^{st}, 3^{rd} and 5th harmonic: Degree of transverse coherence

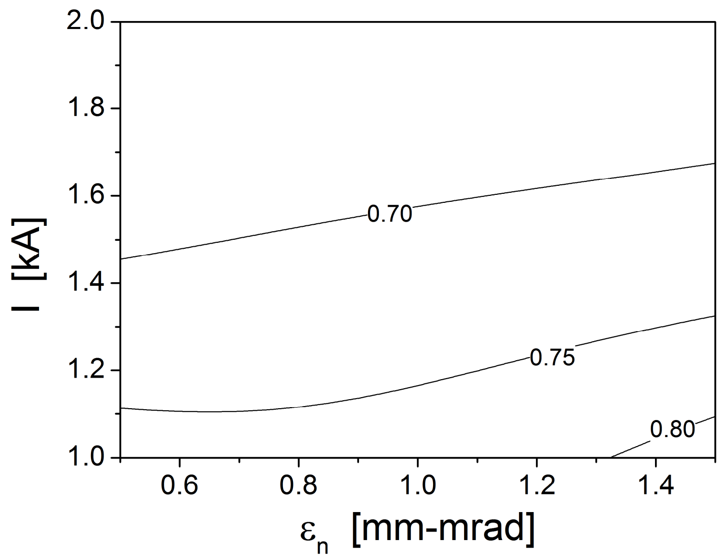


FLASH: Radiation wavelength is 8 nm.

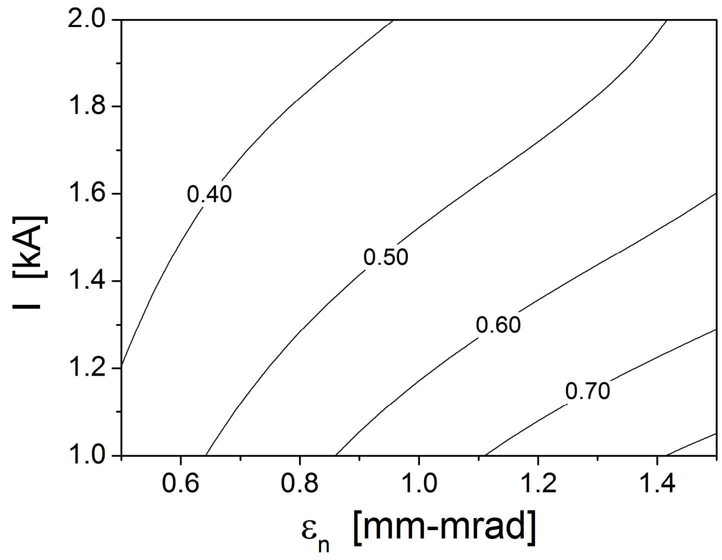
Color codes (black, red and green) refer to different emittance 0.5, 1, and 1.5 mm-mrad. Line styles (solid, dash, and dot) refer to different values of peak current 1 kA, 1.5 kA, and 2 kA.

Fundamental harmonic: degree of transverse coherence in saturation and at full undulator length

Saturation



27 m



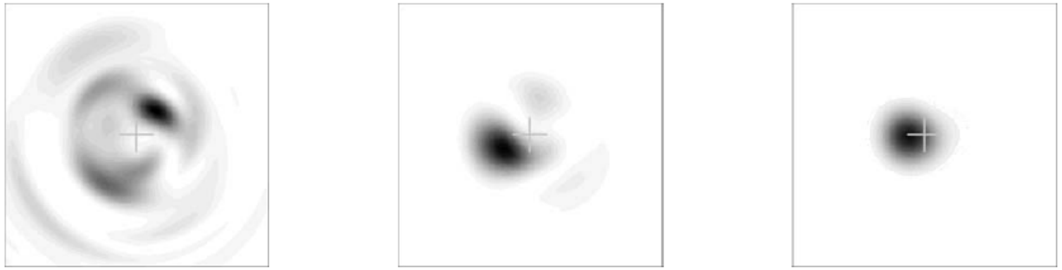
FLASH: Radiation wavelength is 8 nm.

Where is origin of poor transverse coherence?

Mode selection in the linear mode of operation

Slippage effect in the nonlinear regime

Self-reproducing beam radiation modes



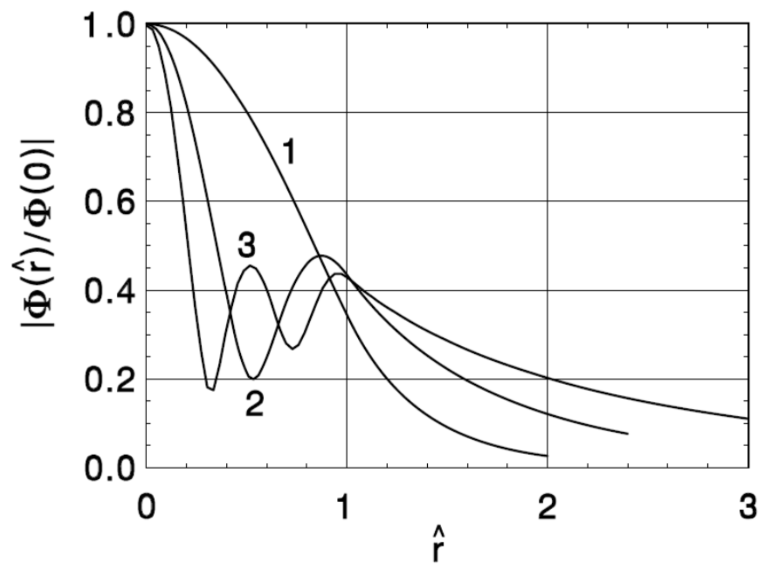
- In the linear high-gain limit the radiation emitted by the electron beam in the undulator can be represented as a set of modes:

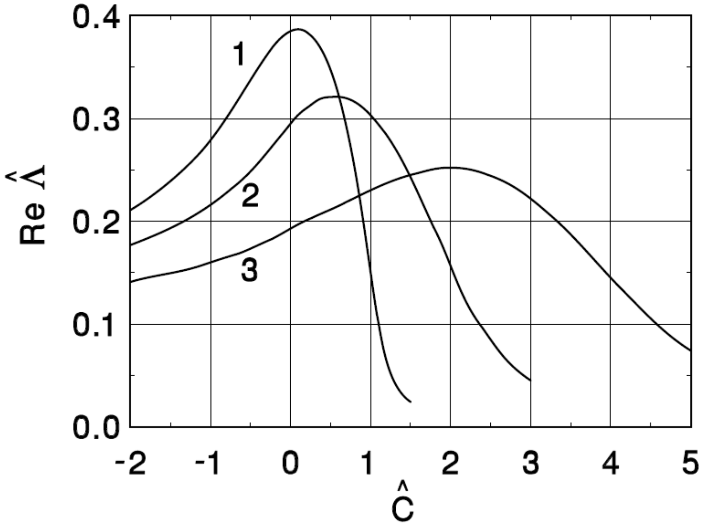
$$E_{\mathrm{x}} + iE_{\mathrm{y}} = \int \mathrm{d}\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r, \omega) \exp[\Lambda_{nk}(\omega)z + in\phi] \ .$$

- A large number of transverse radiation modes are excited when the electron beam enters the undulator.
- These radiation modes have different gain. As undulator length progresses, the high gain modes will predominate more and more and we can regard the XFEL as filter, in the sense that it filters from arbitrary radiation field those components corresponding to the high gain modes. At an appropriate optimization of the XFEL parameters it is possible to obtain a high degree of trnasverse coherence.
- In the nonlinear regime of SASE FEL operation higher modes (suppressed in the high gain linear regime) grow dramatically, and transverse coherence is degraded.

Self-reproducing beam radiation modes

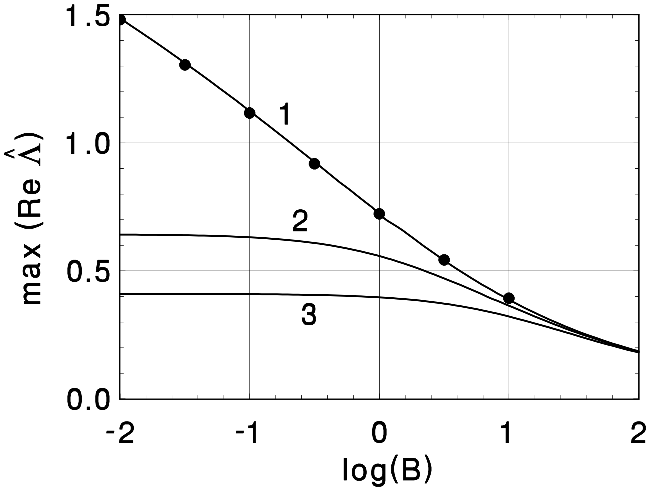
$$
\begin{aligned}
&\left[\frac{\mathrm{d}^2}{\mathrm{d}\hat{r}^2}+\frac{1}{\hat{r}}\frac{\mathrm{d}}{\mathrm{d}\hat{r}}-\frac{n^2}{\hat{r}^2}+2\mathrm{i}B\hat{\Lambda}\right]\Phi_n(\hat{r}) \\
&= -4\int_0^\infty d\hat{r}'\hat{r}'\,\{\Phi_n(\hat{r}') \qquad (4) \\
&\quad +\frac{\hat{\Lambda}_{\mathrm{p}}^{2}}{2}\left[\frac{\mathrm{d}^2}{\mathrm{d}\hat{r}'^2}+\frac{1}{\hat{r}'}\frac{\mathrm{d}}{\mathrm{d}\hat{r}'}-\frac{n^2}{\hat{r}'^2}+2\mathrm{i}B\hat{\Lambda}\right]\Phi_n(\hat{r}')\Big\} \\
&\quad \times \int_0^\infty \mathrm{d}\zeta\frac{\zeta}{\sin^2(\hat{k}_\beta\zeta)}\exp\left[-\frac{\hat{\Lambda}_T^{2}\zeta^2}{2}-(\hat{\Lambda}+\mathrm{i}\hat{C})\zeta\right] \\
&\quad \times \exp\left[-\frac{(1-\mathrm{i}B\hat{k}_\beta^2/2)(\hat{r}^2+\hat{r}'^2)}{\sin^2(\hat{k}_\beta\zeta)}\right] \\
&\quad \times I_n\left[\frac{2(1-\mathrm{i}B\hat{k}_\beta^2\zeta/2)\hat{r}\hat{r}'\cos(\hat{k}_\beta\zeta)}{\sin^2(\hat{k}_\beta\zeta)}\right]
\end{aligned}
$$





Analytical techniques are used to calculate radiation fields in the linear mode of operation, and time-dependent numerical simulation codes are used in the general case.

Degradation of the transverse coherence at large values of diffraction parameter B



Growth rates of TEM_{00}, TEM_{10}, and TEM_{01} modes versus the value of diffraction parameter B.

- In the linear high-gain limit the radiation of the electron beam in the undulator can be represented as a set of modes:

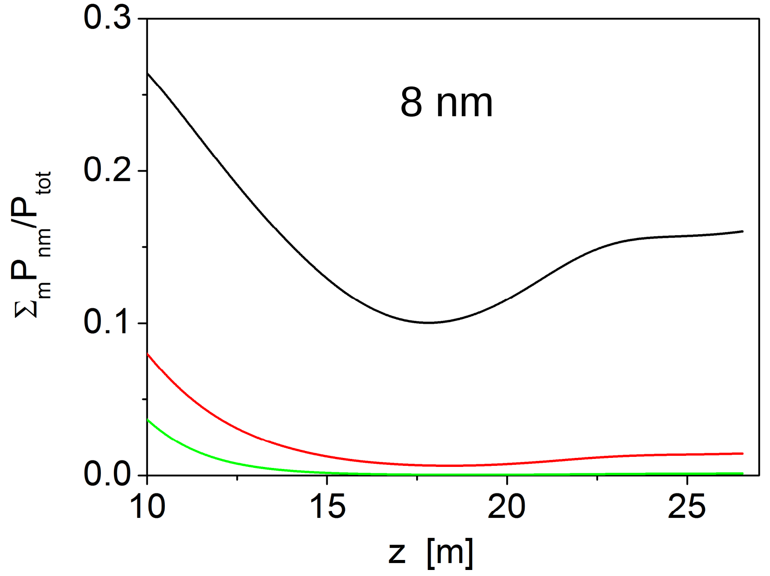
$$E_{\mathrm{x}} + iE_{\mathrm{y}} = \int \mathrm{d}\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r, \omega) \exp[\Lambda_{nk}(\omega)z + in\phi] \ .$$

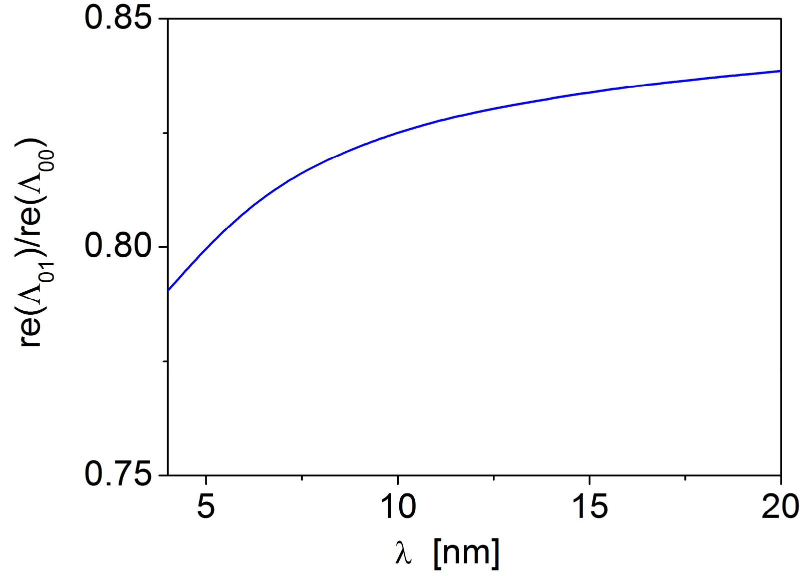
- A large number of transverse radiation modes are excited when the electron beam enters the undulator. As undulator length progresses, the high gain modes predominate, and we can regard the XFEL as filter, in the sense that it filters from arbitrary radiation field those components corresponding to the high gain modes. At an appropriate optimization of the XFEL parameters it is possible to obtain a high degree of trnasverse coherence.

- However, when diffraction expansion of the radiation on a scale of the field gain length is small, growth rates of different radiation modes become to be comparable. In other words, mode degenertion takes place. This range of parameters is characterized with large value of diffraction parameter B.

- Length of the SASE FEL undulator to saturation is in the range from 9 to 11 in the units of the field gain length. Starting from some value of the diffraction parameter, this length is not sufficient to provide mode selection. This results in the degradation of the transverse coherence.

Mode degeneration at FLASH



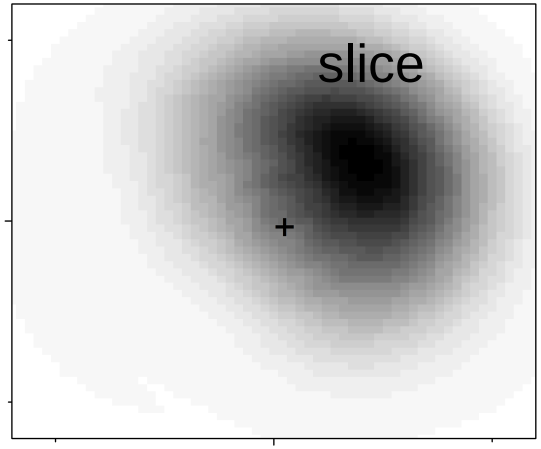


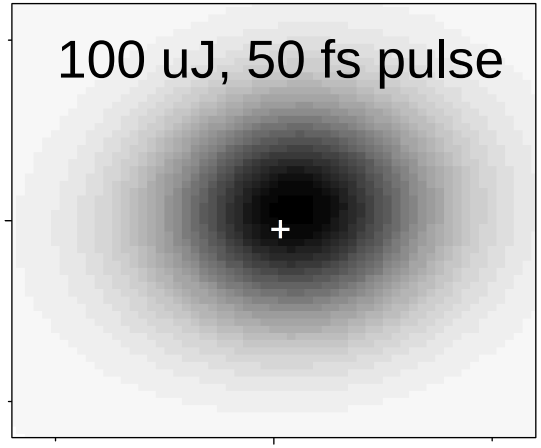
Partial contribution of the higher azimuthal modes of the fundamental harmonic to the total radiation power. Peak current is 1.5 kA, rms normalized emittance is 1 mm-mrad. Black, red, and green curves refer to the modes with n = +/- 1, n = +/- 2, and n = +/- 3.

Ratio of the field gain of the first azimuthal mode TEM_01 to the gain of the ground FEL mode TEM_00 versus radiation wavelength.

Peak current is 1.5 kA, rms normalized emittance is 1 mm-mrad. Focusing beta function is 10 m.

Mode degeneration at FLASH: pointing stability

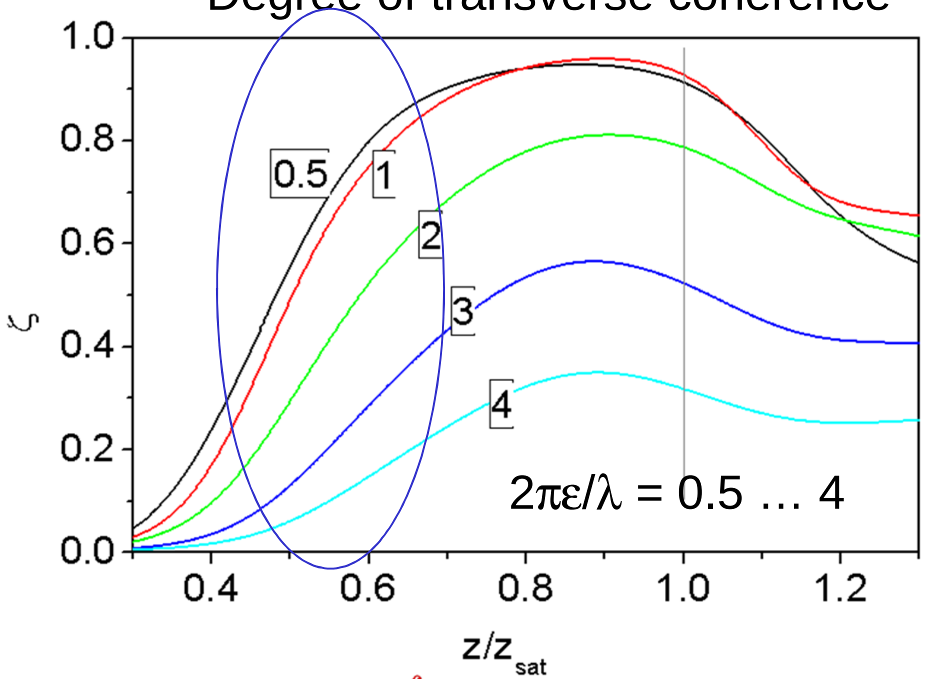




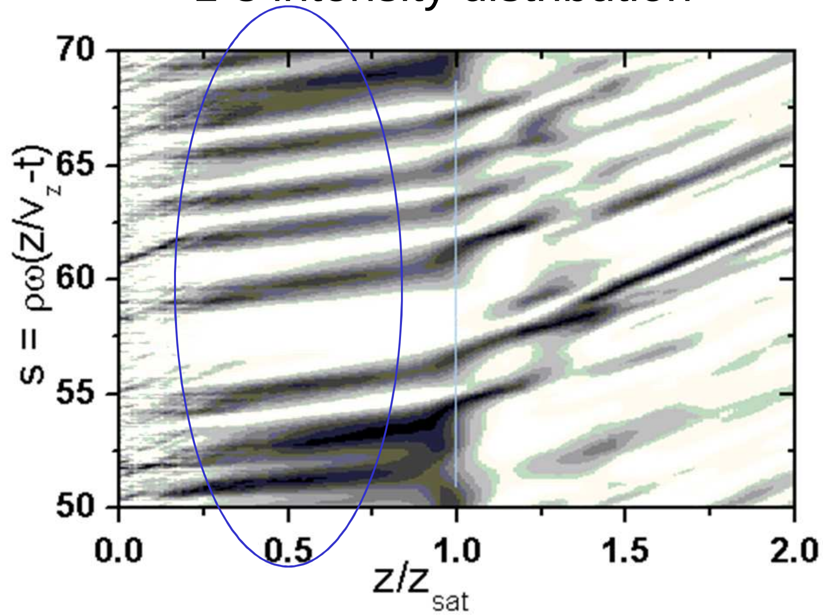
Crosses at these plots show the center of gravity of the radiation intensity averaged over many pulses. We see that the center of gravity of single shot is visibly shifted off axis. Spot shape of a short radiation pulse changes from pulse to pulse. Position of the pulse also jumps from shot to shot which is frequently referred as bad pointing stability. However, in our case bad pointing stability has fundamental origin in poor transverse coherence, but not in unstable operation of the accelerator systems. It is our practical experience from FLASH that an effect of poor pointing stability becomes more pronouncing for shorter pulses.

Formation of the transverse coherence in the linear regime

Degree of transverse coherence



z-s intensity distribution

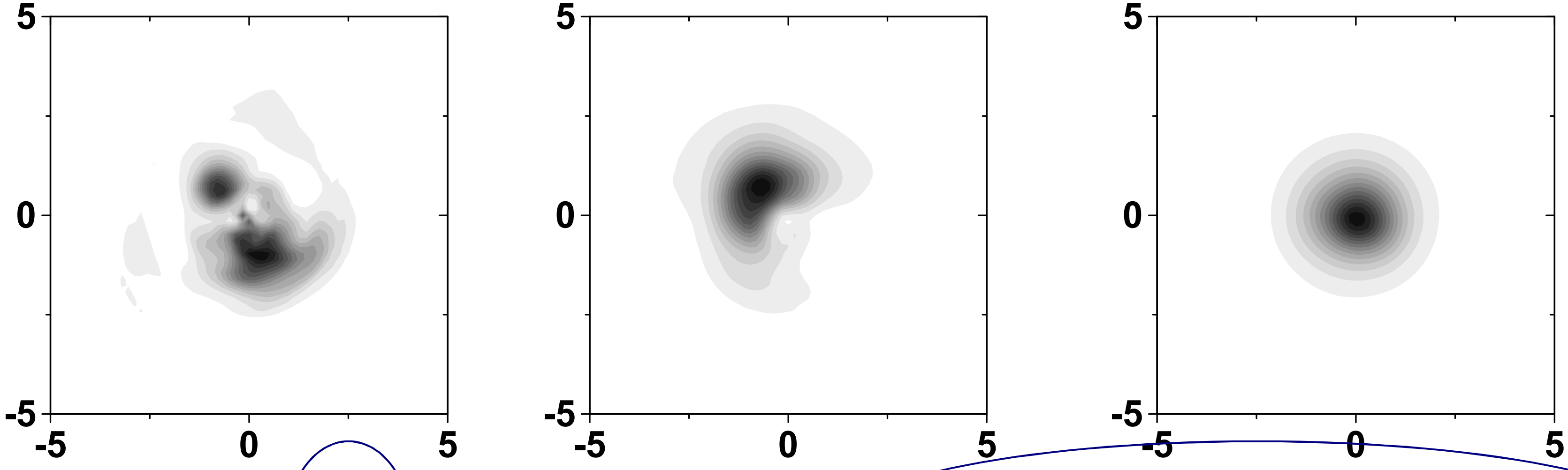


$$E_x + iE_y = \int \mathrm{d}\omega \, \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z) \Phi_{nk}(r, \omega) \exp[\Lambda_{nk}(\omega) z + in\phi]$$

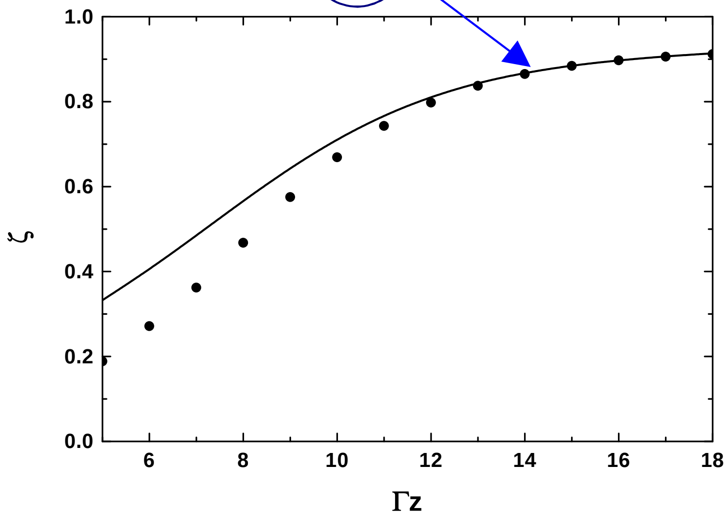
- In the case of large emittance the degree of transverse coherence degrades due to poor mode selection.
- For small emittances the degree of transverse coherence visibly differs from unity. This happens due to poor longitudinal coherence: radiation spikes move forward along the electron beam, and interact with those parts of the beam which have different amplitude/phase.
- Longitudinal coherence develops slowly with the undulator length thus preventing full transverse coherence.

E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 186(2000)185

SASE FEL: limitation of transverse coherence



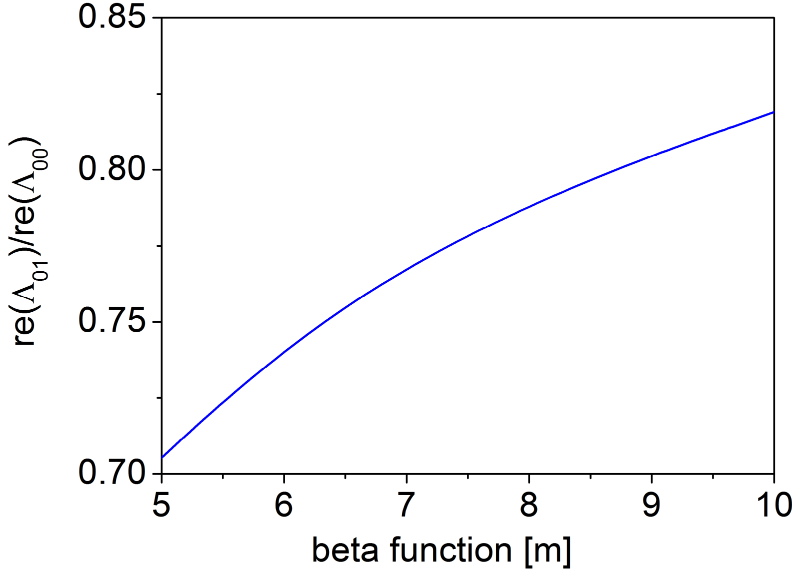
$$E_x + iE_y = \int d\omega \exp[i\omega(z/c-t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega)\exp[\Lambda_{nk}(\omega)z + in\phi]$$

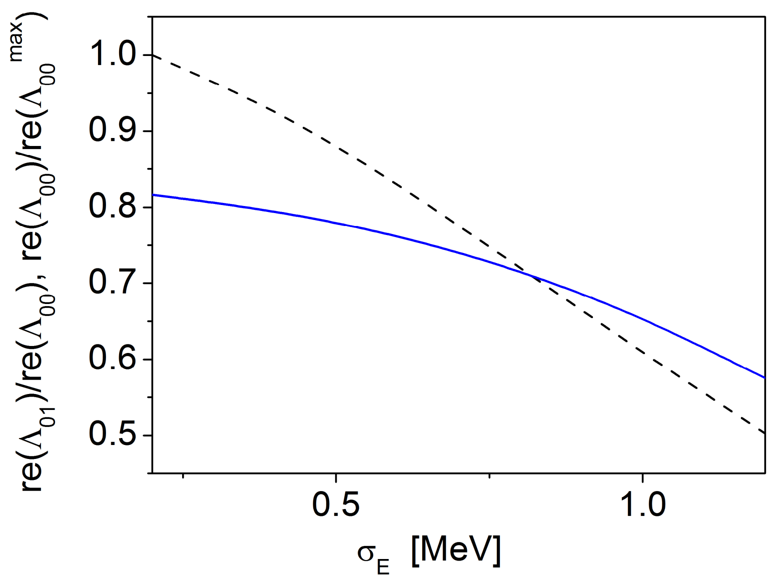


Even after finishing the transverse mode selection process the degree of transverse coherence of the radiation from SASE FEL visibly differs from unity. This is consequence of the interdependence of the longitudinal and transverse coherence. The SASE FEL has poor longitudinal coherence which develops slowly with the undulator length thus preventing a full transverse coherence.

SSY, Opt. Comm. 186(2000)185

Suppression of mode degeneration at FLASH

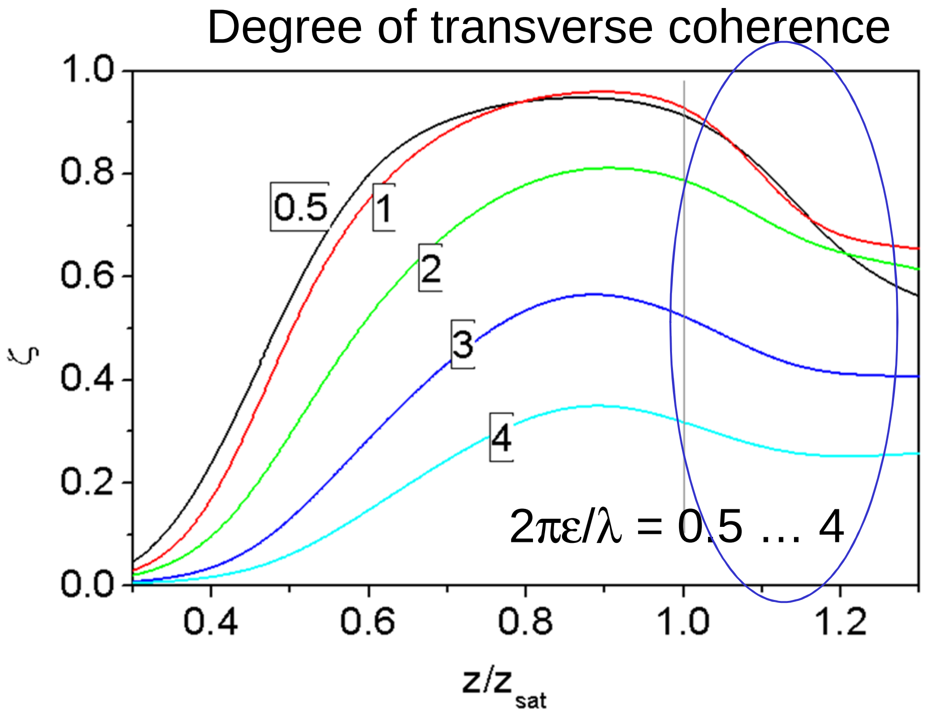


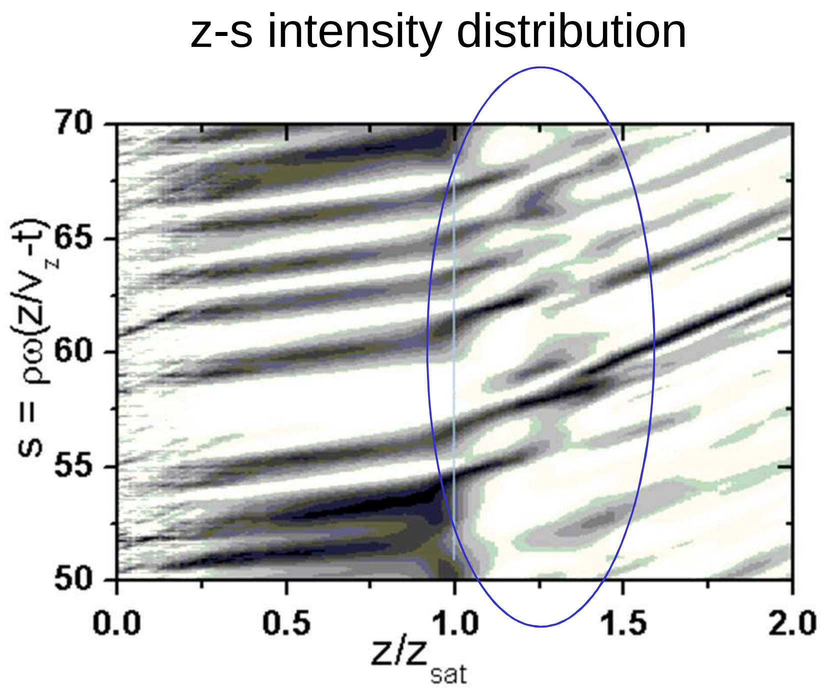


Ratio of the field gain of the first azimuthal mode TEM_01 to the gain of the ground FEL mode TEM_00 versus focusing beta function. Radiation wavelength is 8 nm, peak current is 1.5 kA, rms normalized emittance is 1 mm-mrad

Ratio of the field gain of the first azimuthal mode TEM_01 to the gain of the ground FEL mode TEM_00 versus rms energy spread in the electron beam (blue curve). Black dashed curve shows relative reduction of the gain of the fundamental TEM_00 mode. Radiation wavelength is 8 nm, peak current is 1.5 kA, rms normalized emittance is 1 mm-mrad. Focusing beta function is 10 m.

Degradation of ransverse coherence in the nonlinear regime

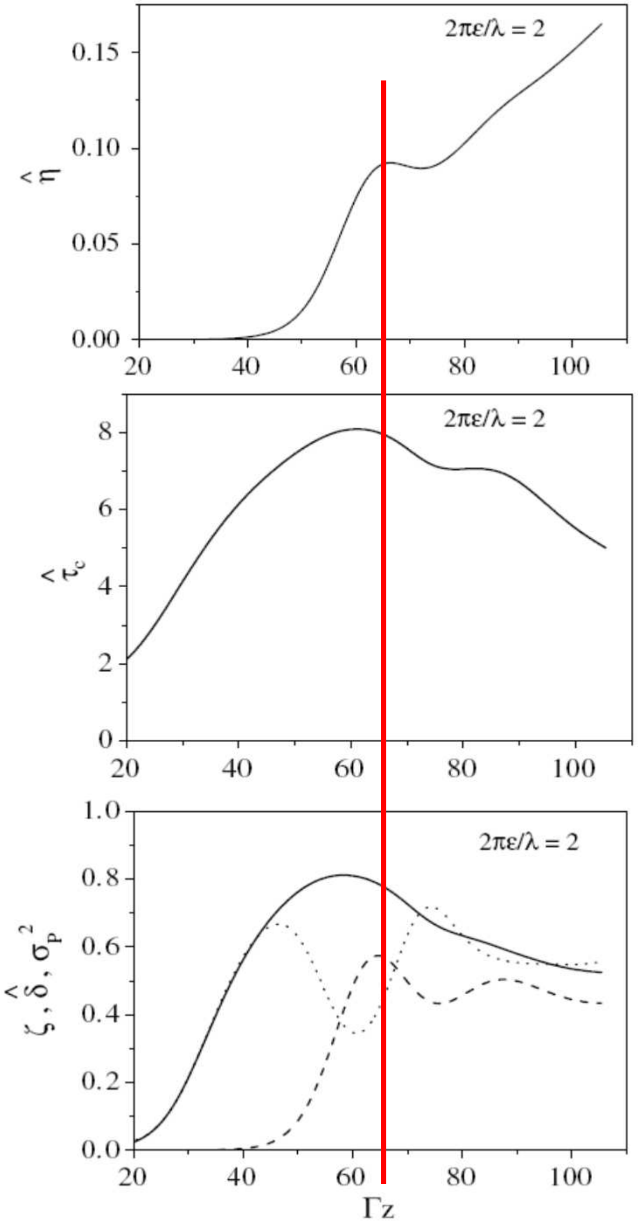




- Poor longitudinal coherence is responsible for the fast degradation of the transverse coherence in the nonlinear regime.
- In the linear exponential regime group velocity of spikes is visibly less than the velocity of light due to strong interaction with the electron beam. In the nonlinear regime group velocity of spikes approaches velocity of light due to weak interaction with the electron beam.
- Radiation spikes move forward faster along the electron beam and start to interact with those parts of the beam which were formed due to interaction with different wavepackets.
- This process develops on the scale of the field gain length.

Summary (1)

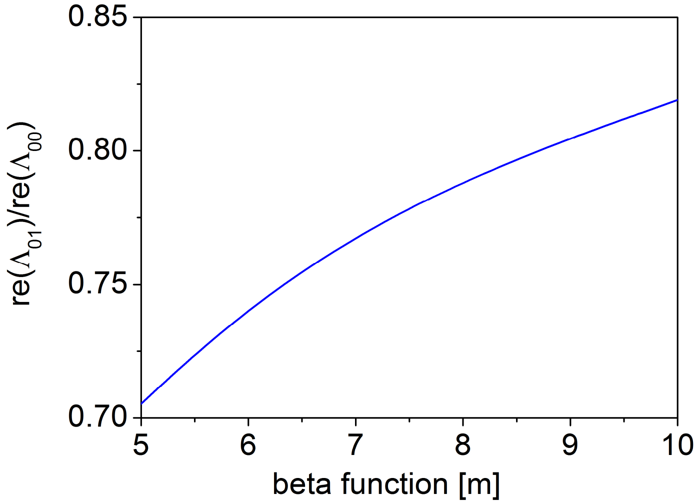
- Maximum degree of transverse coherence and maximum coherence time are achieved in the end of linear regime.
- Maximum brilliance is achieved in the saturation.
- Further increase of the undulator length results only in moderate increase of the radiation power while coherent properties of the radiation degrade significantly.



Summary (2)

- Simulations presented in this paper trace nearly complete range of parameter space of FLASH in terms of emittance and peak current.
- We found that the radiation has relatively poor transverse coherence. This happens because FLASH FEL operates in the parameter space when different radiation modes have close values of the gain. In other words, an effect of mode degeneration takes place.
- Figure of merit here is the value of the diffraction parameter, the ratio of the electron beam size to the diffraction expansion of the radiation on a scale of the field gain length. In the parameter space of FLASH diffraction parameter is in the range between 10 and 20.
- In this case gain of the first azimuthal mode TEM_01 approaches to the gain of the ground TEM_00 mode. Quality of the electron beam is high (small longitudinal velocity spread due to emittance and energy spread) which is not sufficient to suppress the gain of the higher modes.
- One more harmful consequence of poor transverse coherence is bad pointing stability of the radiation for short pulses.

Summary (3)



- Achieving of an ultimate degree of transverse coherence at FLASH is prevented by poor mode selection. Situation can be improved with the following steps:

(1) Reduction of the focusing beta function in the undulator to 5 meters.

(2) Operation at smaller emittance and peak current.

(3) Careful tuning of the saturation regime avoiding operation in the deep nonlinear regime.

- Future developments (like design of a new undulator for FLASH) should also take into account this problem and provide relevant technical solutions for keeping small size of the electron beam in the undulator.

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Thank you for your attention!