Grand Unification and Orbifolds

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DESY Summer Student Programme 2005

Student Session

Standard Model

- chiral gauge theory $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

- Higgs mechanism: SM $\rightarrow$ QCD & QED

Some open questions

- charge quantization ?

- unification of couplings ?

- number of generations ?

- ...

Unification

- Simplest appropriate group which contains G_{SM} is $SU(5)$

- $\Rightarrow$ Try to embed SM in a $SU(5)$ gauge theory

Some facts about $SU(5)$

- contains G_{SM} as subgroup, rank 4
- simple Lie group $\Rightarrow$ charge quantization $\left(\operatorname{tr} Q\left(\mathbf{5}^*\right) \overset{!}{=} 0\right)$
- complex representations to describe Standard Model particles

$$\text{anti-fund. } \mathbf{5}^* = (\mathbf{1}, \mathbf{2}) \oplus (\mathbf{3}^*, \mathbf{1}) = (\nu_e, e^-)_L + d_R^c$$
$$\mathbf{10} = (\mathbf{1}, \mathbf{1}) \oplus (\mathbf{3}^*, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{2}) = e_R^c + u_R^c + (u, d)_L$$

$$\text{adjoint } \mathbf{24} = \underbrace{(\mathbf{8}, \mathbf{1})}_{\text{gluons}} \oplus \underbrace{(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{1})}_{W^\pm, Z, \gamma} \oplus \underbrace{(\mathbf{3}, \mathbf{2}) \oplus (\mathbf{3}^*, \mathbf{2})}_{X, Y \text{bosons}}$$
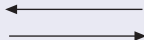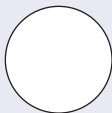
# Orbifolds as extra (compact) space dimension(s)

Definition

Orbifold := Manifold modulo a non freely acting group

Analogy instead of Definition

- Manifold $\leftrightharpoons$ *locally* isomorphic to $\mathbb{R}^d$

- Orbifold $\leftrightharpoons$ *locally* isomorphic to $\mathbb{R}^d/\mathbb{Z}_n$

- Orbifold $\cong$ manifold modulo discrete symmetry

- Manifold: "smooth" $\leftrightharpoons$ Orbifolds: "smooth" with some singularities

- $O = S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ with its fixed points at the two ends
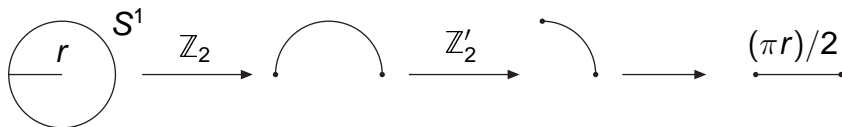
- fixed points given by orbifold action $\mathbb{Z}_2 \times \mathbb{Z}_2'$
- Fields on the bulk $\mathbb{R}^4 \times O$
  - scalar fields $\Phi(x^\mu, y)$
  - gauge fields $A^a(x^\mu, y)$
- matter fields $\Psi(x^\mu, 0)$ restricted to a $\mathbb{R}^4$ brane
  - chirality requires even dimension
  - brane located at one of the fixed points

Orbifold Compactification

Parity assignment

- $\mathbb{Z}_2 \times \mathbb{Z}_2'$ also acts on field space by
  $P = \text{diag}(1, 1, 1, 1, 1), P' = \text{diag}(-1, -1, -1, 1, 1)$

- parity assignment $(\pm\pm)$ according to transformation behaviour of fields

Selection mechanism

- only states with $(++)$ parity contain zero mass modes

- higher order modes already at GUT scale M_{GUT}

- $\Rightarrow$ zero mass modes needed to describe SM

- $\Rightarrow$ Mechanism to reduce number of particles

Symmetry Breaking

- Mechanism to break $SU(5)$ to G_{SM} needed

- $T^a = P' T^a P'^{-1}$ only for SM generators!

- Non SM generators are taken out by orbifold action!

- Symmetry breaking $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$

- Orbifold as additional compact space dimension $\Rightarrow$ bulk $\mathbb{R}_4 \times O$

- Orbifold compactification reduces both gauge symmetry and space time

Predictions

- Standard Model

- 2 Higgs bosons

- coupling unification at GUT scale $M_{GUT} = 1/R$

- perhaps even proton decay could be suppressed

Going further...

- heterotic string theories with 6-dim. orbifolds

- each matter generation (**16**) close to a fixed point

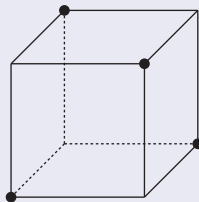- $\implies$ my project: analyzing orbifold given by $SU(3) \times SO(8)$

$SU(3)$: $2D$ non orthonormal root lattice

- $\mathbb{Z}_3$ action
- lattice shifts
- $\longrightarrow$ 3 fixed points
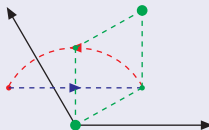
$SO(8)$ $4D$ non orthonormal root lattice

- Symmetry ? ($\mathbb{Z}_6$)
- Fixed Points ? (4)
- Coxeter element!!
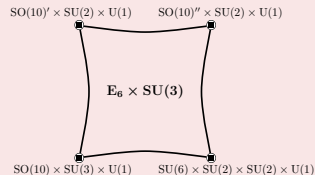 - determined by lattice
  - gives symmetry
  - gives fixed points

projection of fixed points to

orthonormal sublattice

Summary

Things to remember...

- **GUT attractive**

- **extra dimensions attractive**

- **Present technique: Orbifolds**

$SO(10)' \times SU(2) \times U(1)$ $SO(10)'' \times SU(2) \times U(1)$

$E_6 \times SU(3)$

$SO(10) \times SU(3) \times U(1)$ $SU(6) \times SU(2) \times SU(2) \times U(1)$

Thanks to...

- DESY for the great programme & organisation

- my supervisors for their kind and excellent help

  - Wilfried Buchmüller

  - Koichi Hamaguchi

  - Christoph Lüdeling

- the other students for the nice atmosphere