Discrete Flavor Symmetries

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Outline

1. Introduction
   - The Standard Model
   - Beyond the Standard Model
   - Discrete Flavor Symmetries
2. The Discrete Flavor Symmetry D_5
   - Properties of the Dihedral Group
   - D_5 Invariant Masses
3. Summary & Outlook

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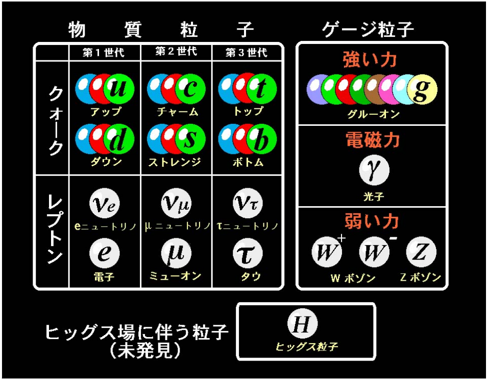
Particle Content of the Standard Model



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$$SU(2)_L \otimes U(1)_Y \xrightarrow{SSB} U(1)_{em}$$

Not only physical mumbo-jumbo!

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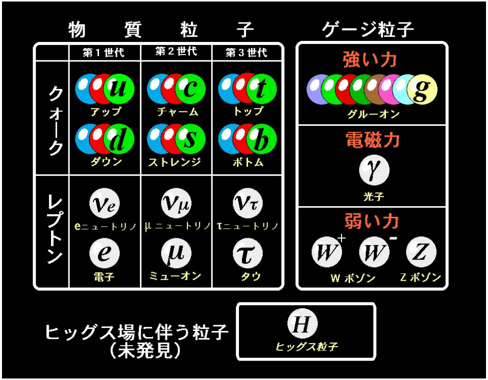

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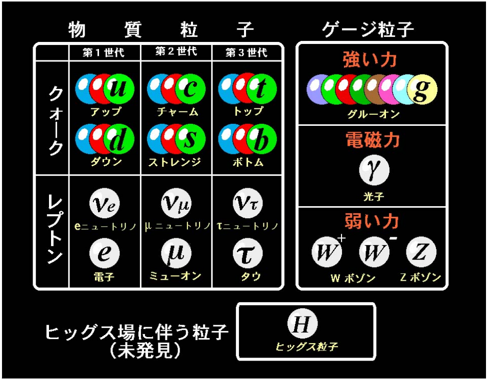

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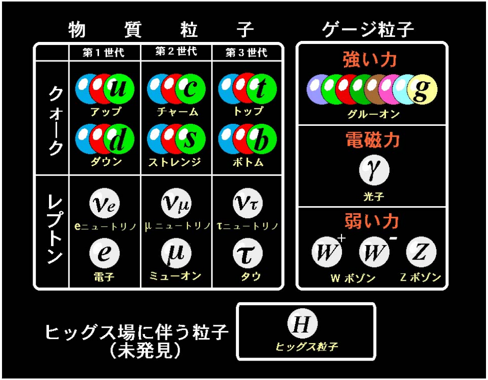

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The standard model is already very successful in explaining the world of particles we are living in.

Still open questions:

- Why do we have 3 families of fermions?
- Why do they have such a strong mass hierarchy?
- How many Higgs bosons do we have?
- What are the values of the neutrino masses?
- ...

Discrete Flavor Symmetries

Definition

A *flavor-*, *family-*, *generation-* or also called *horizontal-symmetry* is a symmetry which acts at the generation space.

Why should we take them?

- In the SM is no difference between the 3 families.
- A discrete flavor symmetry makes this possible.

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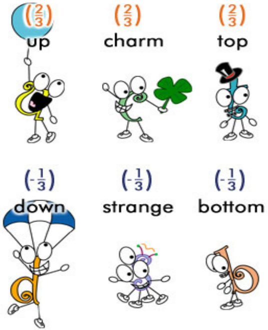

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Properties of the Dihedral Group

Discrete Symmetries in General

- They are in general broken at low energies.
- They commute with the gauge groups.
- A discrete symmetry can be assumed in addition to a GUT, i.e. $SO(10) \times D_n$

Illustration for $n > 1$

Symmetry group of an n-sided regular polygon.

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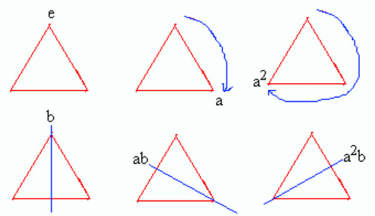

D_3, the Symmetry Group of a Triangle

Illustration for $n > 1$

Symmetry group of an n-sided regular polygon.

Mass Terms

- Mass terms: Mixes right (R) and left (L)-handed states.
  $\bar{\psi}\psi = \bar{R}L + \bar{L}R$

- Problem: L,R different quantum numbers and so simple mass terms violate gauge invariance.

- Solution: Higgs mechanism

Dirac terms for charged leptons and down-type quarks:

$$\lambda_{ij} L_i \phi R_j$$

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D_5 Invariant Masses

Transformation Behavior

$\phi_1 \sim \mathbf{1}_1$, $\phi_2 \sim \mathbf{1}_2$,
$\psi_1 \sim \mathbf{2}_1$ under D_5.
$L = \{L_1, L_2, L_3\}$, where L_i is the i^{th} left-handed generation.

Example

$L \sim (1_2, 1_1, 1_1)$
$R \sim (2_1, 1_1)$

$$M \sim \begin{pmatrix} \kappa_1 \psi_2^1 & -\kappa_1 \psi_1^1 & \kappa_4 \phi^2 \\ \kappa_2 \psi_2^1 & \kappa_2 \psi_1^1 & \kappa_5 \phi^1 \\ \kappa_3 \psi_2^1 & \kappa_3 \psi_1^1 & \kappa_6 \phi^1 \end{pmatrix}$$

To get no zero eigenvalue:

- At least 3 Higgs have to acquire a VEV.

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Higgs Mechanism

Summary & Outlook

- A discrete flavor symmetry can serve as an explanation why we have three families and such a strong mass hierarchy.
- If they are broken, no further Goldstone bosons appear.
- We can use them in addition to a GUT, e.g. $SO(10) \times D_n$.
- They make predictions about the structure of the CKM and PMNS Matrix.

Based on C. Hagedorn, M. Lindner, F.P. (in preparation)

Fermions			Quantum Numbers			
1. Family	2. Family	3. Family	T	T_3	Y	Q
ν_e	ν_μ	ν_τ	$\frac{1}{2}$	$\frac{1}{2}$	-1	0
e^- $)_L$	μ^- $)_L$	τ^- $)_L$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e_L^+	μ_L^+	τ_L^+	0	0	2	1
u	c	t	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
d $)_L$	s $)_L$	b $)_L$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
u_L^c	c_L^c	t_L^c	0	0	$-\frac{4}{3}$	$-\frac{2}{3}$
d_L^c	s_L^c	b_L^c	0	0	$\frac{2}{3}$	$\frac{1}{3}$

Principle of The Higgs Mechanism

We introduce the potential

$$V(\phi^\dagger\phi) = \frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2, \text{ where } \lambda > 0.$$

The symmetry in the ground state is broken for negative μ^2, i.e.it has two minima, $\pm v = \pm\sqrt{\frac{|\mu|^2}{\lambda}} \neq 0$.

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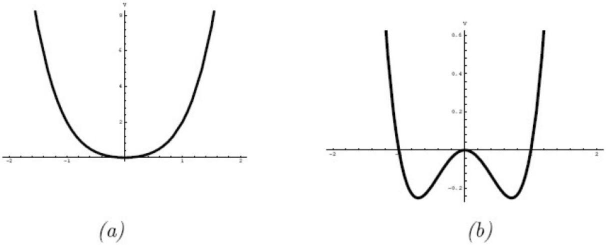

Scalar potential for $\mu^2 > 0$ (a) and for $\mu^2 < 0$ (b)

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We shift the field about v,

$$\phi = v + \eta \text{ , where } \langle 0|\eta|0\rangle = 0$$

and apply it on our scalar field ϕ, where $\mathscr{L}$ is invariant under

$$\phi(t, \vec{x}) \rightarrow e^{i\alpha(t,\vec{x})}\phi(t, \vec{x})$$

(local $U(1)$ gauge symmetry).

Lagrangian

$$\mathscr{L} = \frac{1}{2}(D_\mu\phi)^\dagger D^\mu\phi - V(\phi^\dagger\phi) + \mathscr{L}^{A_\mu}_{free},$$

$$\text{with } D_\mu = \partial_\mu + igA_\mu \text{ .}$$

A_μ = $U(1)$ invariant gauge field.
g = coupling constant.

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Unitary Gauge, i.e. $\alpha = -\xi$

$$\phi = \underbrace{e^{i\alpha} e^{i\xi}}_{=1} (v + \eta)$$

$e^{i\xi}$ = general phase under which our Lagrangian is invariant.

From the first term of $\mathscr{L}$ we get

$$\frac{1}{2}(vg)^2 A_\mu A^\mu =: \frac{1}{2} m_A^2 A_\mu A^\mu \ .$$

This is a mass term and therefore it is often said that the ξ field is "eaten" by A_μ and got as a result massive with mass $m_A = gv$.

Before SSB		After SSB	
A_μ, Spin 1, mass=0:	2	A_μ, Spin 1, mass≠0:	3
ξ, η:	2	η:	1

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Higgs Mechanism in the Standard Model

Lagrangian

$$\mathscr{L}_{scalar} = D_\mu \mathbf{\Phi}^\dagger D^\mu \mathbf{\Phi} - V(\mathbf{\Phi}^\dagger \mathbf{\Phi}), \text{ with}$$

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$D_\mu = \partial_\mu + ig\frac{\tau^i}{2} W_\mu^i + i\frac{g'}{2} Y B_\mu$.

$\mathbf{\Phi}$ is a complex scalar doublet.

$$\langle \mathbf{\Phi} \rangle_0 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix} \text{, with } v = \sqrt{\frac{\mu^2}{\lambda}},$$

Result of the SSB, $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$:

The photon γ remains massless while the $W^\pm$ and Z bosons acquire mass.

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Conclusion