Introduction to Theoretical Particle Physics

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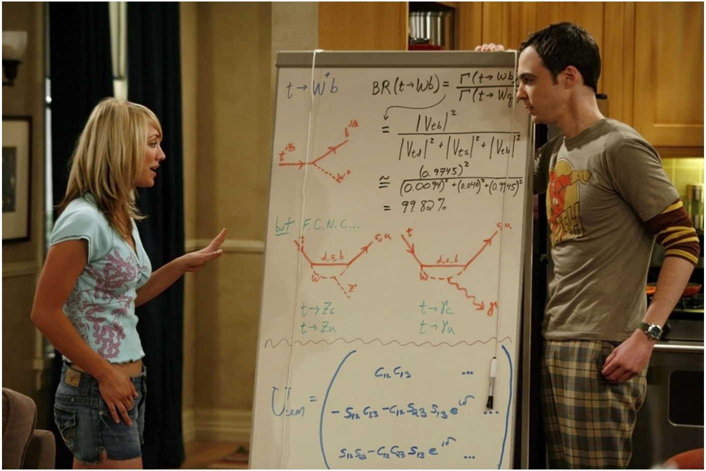
Hamburg, 08/2011

Literature

- Georgi: Weak Interactions and Modern Particle Physics, Dover, 2009
- Quigg: Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, Perseus 1997
- Peskin: An Introduction to Quantum Field Theory Addison Wesly, 1994
- Weinberg, The Quantum Theory of Fields, Vol. I/II/(III) Cambridge Univ. Press, 1995-98
- Itzykson/Zuber, Quantum Field Theory, McGraw-Hill, 1980
- Böhm/Denner/Joos, Gauge Theories of Strong and Electroweak Interactions, Springer, 2000
- Kugo, Eichtheorie, Springer, 2000 (in German)
- ...and many more

"I have nothing to offer but blood, toil, tears and sweat."

...but it is fun, though....



Part I (Vorabend)

From Lagrangians to Feynman Rules: Quantum Field Theory with Hammer and Anvil

Why Quantum Field Theory?

- Subatomic realm: typical energies and length scales are of order $(\hbar c) \sim 200$MeV $\cdot$ fm $\Rightarrow$ use of both special relativity and quantum mechanics mandatory
- Particles (quantum states) are created and destroyed, hence particle number not constant: beyond unitary time evolution of a single QM system
- Schrödinger propagator (time-evolution operator) violates microcausality
- Scattering on a potential well for relativistic wave equation leads to unitarity violation
- Use quantized fields: can be viewed as continuos limit of QM many-body system with many (discrete) degrees of freedom
- Least Action Principle leads to classical equations of motion (Euler-Lagrange equations)

$$S = \int dt L = \int dt d^3x \mathcal{L} = \int d^4x \mathcal{L}(\phi, \partial_\mu \phi) \qquad \Rightarrow \qquad \boxed{\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \frac{\partial \mathcal{L}}{\partial \phi}}$$

What kind of fields?

- Classical wave equations must be Lorentz-covariant
- Action and Lagrangian (density) are Lorentz scalars
- Fields classified according to irreps of Lorentz group
- Simplest case: Lorentz scalars (real/complex), Klein-Gordon equation

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - m^2 |\phi|^2 \quad \Rightarrow \quad \boxed{(\Box + m^2)\phi \equiv (\partial_t^2 - \vec{\nabla}^2 + m^2)\phi = 0}$$

- Spin 1/2 particles: Dirac equation

$$\mathcal{L} = i\overline{\Psi}(i\partial\!\!\!/ + m)\Psi \quad \Rightarrow \boxed{(i\partial\!\!\!/ - m)\Psi = 0} \qquad \text{where} \quad a\!\!\!/ \equiv a_\mu \gamma^\mu$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \sigma^\mu = (\mathbb{1}, \vec{\sigma}) \quad \bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma}) \quad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$$

$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is the chiral projector: $\mathcal{P}_{L/R} = \frac{1}{2}(1 \mp \gamma^5)$

The Dawn of gauge theories

- Spin 1 particles: Maxwell's equations

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \text{ with } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \Rightarrow$$

Invariant under (local) gauge transformation, G:

$$A_\mu \longrightarrow A'_\mu = \qquad - \frac{1}{g}(\partial_\mu G) G^{-1}$$

Electric and magnetic fields are defined via:

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla} A_0$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\boxed{\partial_\mu F^{\mu\nu} = 0} \quad \rightarrow$$

$$\begin{cases} 0 = D_\mu F^{\mu 0} = \vec{\nabla}\vec{E} \\ 0 = D_\mu F^{\mu i} = -\dot{E}^i + (\vec{\nabla} \times \vec{B})^i \end{cases}$$

The Dawn of gauge theories

- Spin 1 particles: Maxwell's equations

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \text{ with } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g\left[A_\mu, A_\nu\right] \quad \Rightarrow$$

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$$A_\mu \longrightarrow A'_\mu = G A_\mu G^{-1} - \frac{1}{g}(\partial_\mu G) G^{-1}$$

Electric and magnetic fields are defined via:

$$\begin{aligned} \vec{E} &= -\dot{\vec{A}} - \vec{\nabla} A_0 - g\left[A_0, \vec{A}\right] \\ \vec{B} &= \vec{\nabla} \times \vec{A} - \frac{g}{2}\left[\vec{A}\times, \vec{A}\right] \end{aligned}$$

$$\boxed{\partial_\mu F^{\mu\nu} = -ig\left[A_\mu, F^{\mu\nu}\right]} \quad \rightarrow$$

$$\begin{cases} 0 = D_\mu F^{\mu 0} = \vec{\nabla}\vec{E} + g\left[\vec{A}\cdot, \vec{E}\right] \\ 0 = D_\mu F^{\mu i} = -\dot{E}^i + (\vec{\nabla} \times \vec{B})^i - g\left[A_0, E^i\right] + g\left[\vec{A}\times, \vec{B}\right]^i \end{cases}$$

...for the curious...

- Spin 3/2 particles: Rarita-Schwinger equation

$$\mathcal{L} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\Psi}_\mu\gamma_5\gamma_\nu\partial_\rho\Psi_\nu - \frac{1}{4}m\overline{\Psi}_\mu\left[\gamma^\mu,\gamma^\nu\right]\Psi_\nu \quad \Rightarrow$$

$$\boxed{\begin{array}{c} m\left(\partial\!\!\!/\gamma^\nu\Psi_\nu - \gamma^\nu\partial\!\!\!/\Psi_\nu\right) = 0 \\ \\ \gamma^\mu\Psi_\mu = 0 \qquad \partial^\mu\Psi_\mu = 0 \qquad \left(i\partial\!\!\!/ - m\right)\Psi_\mu = 0 \end{array}}$$

Leads only to sensible theory in supergravity

- Spin 2 particles: de Donder equation

$$\mathcal{L} = -\frac{4\pi G_N}{c^4}\sqrt{|\det g|}\, R \qquad \text{with} \quad R = R^{\mu}{}_{\mu}{}^{\nu}{}_{\nu} \quad \text{and}$$

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho{}_{\nu\sigma} - \partial_\nu\Gamma^\rho{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\lambda}\Gamma^\lambda{}_{\nu\sigma} - \Gamma^\rho{}_{\nu\lambda}\Gamma^\lambda{}_{\mu\sigma}$$

$$\Gamma^\rho{}_{\mu\nu} = \tfrac{1}{2}g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} - \partial_\nu g_{\mu\sigma} - \partial_\rho g_{\mu\nu}\right) \quad \text{define} \quad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{16\pi G_N}\, h_{\mu\nu}$$

$$\boxed{\Box\left(h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}h^\rho{}_\rho\right) = \partial_\mu\partial^\rho\left(h_{\rho\nu} - \tfrac{1}{2}\eta_{\rho\nu}h^\sigma{}_\sigma\right) + (\mu \leftrightarrow \nu)}$$

How to quantize a field (Analogous to QM)

- Field and its canonically conjugate momentum: $\pi = \partial\mathcal{L}/\partial\dot{\phi}$
- Canonically transform to the Hamiltonian of the system:

$$\mathcal{H} = \pi \frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \mathcal{L}\Big|_{\dot{\phi}=\dot{\phi}(\phi,\pi)} \to \tfrac{1}{2}\pi^2 + \tfrac{1}{2}\left(\vec{\nabla}\phi\right)^2 + \tfrac{1}{2}m^2\phi^2$$

- Impose equal-time commutation relations:

$$[\phi(\vec{x}), \pi(\vec{y})] = i\delta^3(\vec{x}-\vec{y}) \qquad [\phi(\vec{x}), \phi(\vec{y})] = [\pi(\vec{x}), \pi(\vec{y})] = 0$$

- Creation and annihilation operators to diagonalize the Hamiltonian:

$$[a_p, a^\dagger_{p'}] = 2E_p(2\pi)^3\delta^3(\vec{p}-\vec{p}') \qquad [a_{\vec{p}}, a_{\vec{p}'}] = 0$$

- Heisenberg/interaction picture: (field) operators time dependent
- Creation/annihilation operators Fourier coefficients of field operators:

$$\phi(x) = \int \widetilde{dk}\left(a_k e^{-ik\cdot x} + a^\dagger_k e^{+ik\cdot x}\right) \qquad \int \widetilde{dk} \equiv \int \frac{d^3k}{(2\pi)^3(2E_k)}$$

- Hamiltonian is a sum of harmonic oscillators:

$$\mathcal{H} = \int \frac{d^3p}{(2\pi)^3} E_p \left(a_p^\dagger a_p + const. \right)$$

- Infinite constant is abandoned by normal ordering renormalization

$$:\mathcal{H} := \int \frac{d^3p}{(2\pi)^3} E_p : a_p^\dagger a_p : \qquad \text{Note:} \qquad \langle 0| : \mathcal{O} : |0\rangle = 0$$

- Creation operator creates a 1-particle state with well-defined momentum (plane wave): $a_p^\dagger |0\rangle = |p\rangle$
- Field operator creates a 1-particle state at x: $\phi(x)|0\rangle = \int \widetilde{dp}\, e^{ip\cdot x} |p\rangle$
- Consider a complex field:

$$\phi(x) = \int \widetilde{dk} \left(a_k e^{-ik\cdot x} + b_k^\dagger e^{+ik\cdot x} \right), \qquad \phi^\dagger(x) = \int \widetilde{dk} \left(b_k e^{-ik\cdot x} + a_k^\dagger e^{+ik\cdot x} \right)$$

  Field operator: positve frequency modes ($e^{-ik\cdot x}$) have annihilation operator for particles, negative frequency modes ($e^{-ik\cdot x}$) have creation operator for anti-particles
  a, b are independent, commute (independent Fourier components!)
- **Got rid of negative energies as in classical wave equations**

Microcausality and the Feynman propagator

- Amplitude for a particle to propagate from y to x:

$$D(x-y) \equiv \langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip(x-y)} \quad \overset{|\vec{x}-\vec{y}|\to\infty}{\sim} \quad e^{-m|\vec{x}-\vec{y}|}$$

falls off exponentially outside light cone, but is non-zero

- Measurement is determined through the commutator:

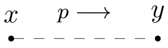
$$[\phi(x), \phi(y)] = D(x-y) - D(y-x) = 0 \qquad (\text{for } (x-y)^2 < 0)$$

- Cancellation of causality-violating effects by Feynman prescription:
  - Particles propagated into the future (retarded)
  - Antiparticles propagated into the past (advanced)

$$D_F(x-y) = \left\{ \begin{array}{l} D(x-y) \text{ for } x^0 > y^0 \\ D(y-x) \text{ for } x^0 < y^0 \end{array} \right\} = \theta(x^0 - y^0)\,\langle 0|\phi(x)\phi(y)|0\rangle$$
$$+\,\theta(y^0 - x^0)\,\langle 0|\phi(y)\phi(x)|0\rangle \equiv \langle 0|\mathrm{T}\left[\phi(x)\phi(y)\right]|0\rangle$$

- **Feynman propagator** (time-ordered product, causal Green's function)

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)} =$$



$i\epsilon$ prescription

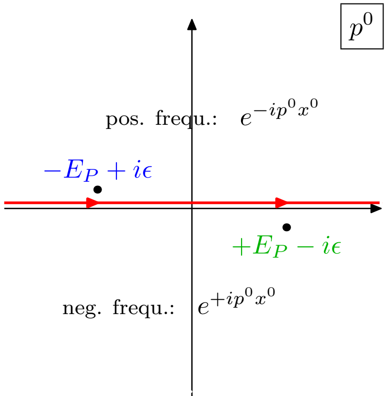
- Solution of wave equation by Fourier transform:

$$(\Box + m^2)\phi(x) = i\delta^4(x-y) \quad \xrightarrow{F.T.}$$

$$\tilde{\phi}(p) = \frac{i}{p^2 - m^2} = \frac{i}{(p^0 - E_p \quad)(p^0 + E_p \quad)}$$

with $E_p = +\sqrt{\vec{p}^2 + m^2}$

- Prescription tells you where to propagate in time:



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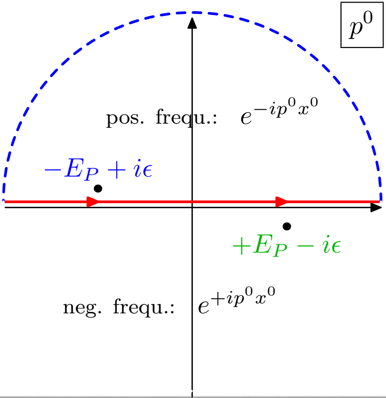

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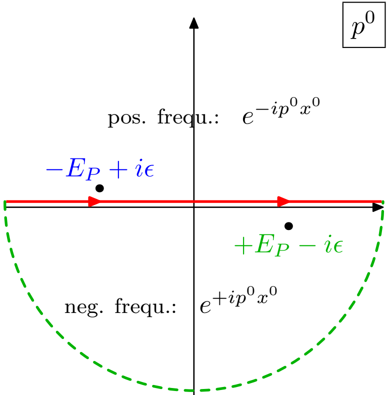


Quantization of the Dirac field

- Spin-statistics theorem (Fierz/Lüders/Pauli, 1939/40):
  - Spin $0, 1, 2, \ldots$: bosons $\Rightarrow$ commutators
  - Spin $\frac{1}{2}, \frac{3}{2}, \ldots$: fermions $\Rightarrow$ anticommutators
- equal-time anticommutators: $\left\{\psi(x)_\alpha, \psi^\dagger(y)_\beta\right\} = \delta^{(}\vec{x}-\vec{y})\delta_{\alpha\beta}$
- Field operators:

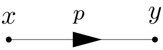
$$\psi(x) = \int \widetilde{dp} \sum_s \left(a_p^s u^s(p) e^{-ipx} + b_p^{s\,\dagger} v^s(p) e^{+ipx} \right)$$

$$\overline{\psi}(x) = \int \widetilde{dp} \sum_s \left(a_p^{s\,\dagger} \overline{u}^s(p) e^{+ipx} + b_p^s \overline{v}^s(p) e^{-ipx} \right)$$

- **Feynman propagator** (time-ordered product, causal Green's function)

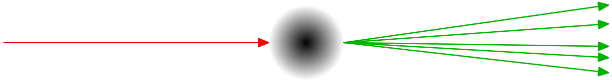
$$S_F(x-y) = \left\{ \begin{array}{rl} \langle 0 | \psi(x)\overline{\psi}(y) | 0 \rangle & \text{for } x^0 > y^0 \\ - \langle 0 | \overline{\psi}(y)\psi(x) | 0 \rangle & \text{for } x^0 < y^0 \end{array} \right\} = \langle 0 | \mathrm{T}\left[\psi(x)\overline{\psi}(y)\right] | 0 \rangle$$

$$S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p}+m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)} =$$



The rocky road from the S matrix to cross sections

- 4D QFTs without interactions exactly solvable, otherwise *not*

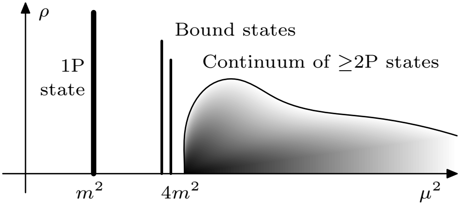


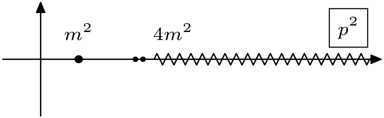
- Idea: For scattering process sharply located in space-time, use:
  - Asymptotically free quantum state for $t \to -\infty$
  - Interaction described completely local in space-time
  - Asymptotically free quantum state for $t \to +\infty$
- General axioms of QFT:
  1. All eigenvalues of P^μ are in the forward lightcone
  2. There is a Poincaré-invariant vacuum state $|0\rangle$
  3. For every particle there is 1-particle state $|p\rangle$
  4. Asymptotic fields are free fields whose creation operators span a Fock space (asymptotic completeness)
- Use the (Källen-Lehmann) spectral representation:

$$\text{F.T.}\ \langle 0 | \mathrm{T}\left[\Phi(x)\Phi(y)\right] | 0 \rangle = \int_0^\infty d\mu^2 \rho(\mu^2) \frac{i}{p^2 - \mu^2 + i\epsilon}$$

$$\longrightarrow \frac{i\,Z}{p^2 - m^2 + i\epsilon} + \int_{4m^2}^{\infty} d\mu^2 \rho(\mu^2) \frac{i}{p^2 - \mu^2 + i\epsilon}$$

Z: Wave function renormalization, m: renormalized mass





- Asymptotic LSZ (Lehmann-Symanzik-Zimmermann) condition:
  Heisenberg fields of full theory are asymptotically free fields (up to wave function and mass renormalization)

$$\Phi(x) \overset{x^0 \to -\infty}{\longrightarrow} \sqrt{Z}\phi_{in}(x) \qquad \Phi(x) \overset{x^0 \to +\infty}{\longrightarrow} \sqrt{Z}\phi_{out}(x)$$

- Asymptotic fields obey free field equations! $\Rightarrow$ Simple Fock spaces

$$\mathcal{V}_{in} = \left\{ a^\dagger_{in,k_1} a^\dagger_{in,k_2} \ldots a^\dagger_{in,k_n} |0\rangle \right\}$$
$$\mathcal{V}_{out} = \left\{ a^\dagger_{in,k_1} a^\dagger_{in,k_2} \ldots a^\dagger_{in,k_n} |0\rangle \right\}$$

Assumption: $\mathcal{V}_{in} \equiv \mathcal{V}_{out} \equiv \mathcal{V}$

The S-Matrix

Wheeler, 1939; Heisenberg 1943

- Scattering probability from initial to final state: Prob. $= |\langle \beta_{out} | \alpha_{in} \rangle|^2$
- S-Matrix transforms in into out states: $\langle \beta_{out} | \alpha_{in} \rangle = \langle \beta_{in} | S | \alpha_{in} \rangle$

  1. S-Matrix is unitary (prob. conserv.): $S^\dagger S = S S^\dagger = \mathbb{1}$
  2. Transforms asymptotic field operators: $\phi_{out}(x) = S^\dagger \phi_{in}(x) S$
  3. S-matrix invariant under symmetries: $[Q, S] = 0$

▸ **Four major steps to calculate scattering cross sections**

1. LSZ formula: S-matrix as n-point Green's functions of full theory
2. Gell-Mann–Low formula: Green's functions of full theory expressed by perturbation series of free field Green's functions
3. Wick's theorem, Feynman rules (and elimination of vacuum bubbles)
4. Phase space integration: scattering cross section from S-matrix elements

Step 1: LSZ formula

Lehmann/Symanzik/Zimmermann, 1955

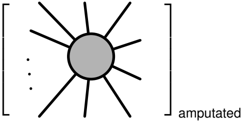
Idea:

- For $t \to \pm\infty$ asymptotic fields are free
- Project out creation operators from free field operators by inverse Fourier transform
- S-Matrix element is (upto normaliz.) Green's function multi-particle pole
- One gets tis pole by amputation of external legs of the Green's function

$$\langle p_1, \ldots p_n, out | q_1, \ldots q_l, in\rangle = \langle p_1, \ldots p_n, in | S | q_1, \ldots q_l, in\rangle =$$

$$\text{(disconn. terms)} + \left(\frac{i}{\sqrt{Z}}\right)^{n+l} \left(\prod_{i=1}^{n} \int d^4 y_i e^{ip_i y_i} (\Box_{y_i} + m_i^2)\right) \cdot$$

$$\left(\prod_{j=1}^{l} \int d^4 x_j e^{-iq_j x_j} (\Box_{x_j} + m_j^2)\right) \langle 0 | \mathrm{T}\left[\Phi(y_1)\Phi(y_2)\ldots\Phi(x_l)\right] | 0\rangle =$$



- Yields Feynman rules for external particles: 1P on-shell wavefunctions

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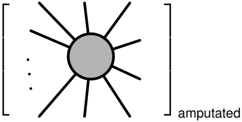
$$\left(\prod_{j=1}^{l} \int d^4 x_j e^{-i q_j x_j} (\Box_{x_j} + m_j^2) \right) \langle 0 | \mathrm{T} \left[\Phi(y_1) \Phi(y_2) \ldots \Phi(x_l) \right] | 0 \rangle =$$



- Yields Feynman rules for external particles: 1P on-shell wavefunctions

Step 2: Gell-Mann–Low formula

Gell-Mann/Low, 1951

- Use: field and time evolution operators in the interaction picture
- Transform fields of the full theory into asymptotically free fields asymptotically free fields
- Solve the Schrödinger equation of IA picture time-evolution operator as a (time-ordered) perturbation series

$$U(t) = \mathrm{T}\left[\exp\left(-i\int_{-\infty}^{t} dt' H_{int}(t')\right)\right]$$

- Time evolution of field operators and also vacuum state (vacuum polarization!) $\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_{int}$

$$\langle 0|\mathrm{T}\left[\Phi(x_1)\ldots\Phi(x_n)\right]|0\rangle = \frac{\langle 0\Big|\mathrm{T}\left[\phi_{in}(x_1)\ldots\phi_{in}(x_n)\exp\left(i\int d^4x\mathcal{L}_{int}[\phi_{in}(x)]\right)\right]\Big|0\rangle}{\langle 0\Big|\mathrm{T}\left[\exp\left(i\int d^4x\mathcal{L}_{int}[\phi_{in}(x)]\right)\right]\Big|0\rangle}$$

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$$U(t) = \mathrm{T}\left[\exp\left(-i\int_{-\infty}^{t} dt' H_{int}(t')\right)\right] \qquad \overset{t\to+\infty}{\longrightarrow} \qquad S$$

- Time evolution of field operators and also vacuum state (vacuum polarization!) $\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_{int}$

$$\langle 0|\mathrm{T}\left[\Phi(x_1)\ldots\Phi(x_n)\right]|0\rangle = \frac{\langle 0|\mathrm{T}\left[\phi_{in}(x_1)\ldots\phi_{in}(x_n)\exp\left(i\int d^4x\mathcal{L}_{int}[\phi_{in}(x)]\right)\right]|0\rangle}{\langle 0|\mathrm{T}\left[\exp\left(i\int d^4x\mathcal{L}_{int}[\phi_{in}(x)]\right)\right]|0\rangle}$$

Step 3: Wick's theorem, Feynman Rules

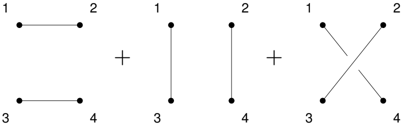
Wick, 1950

- Task is to calculate VEV of time-ordered product of free fields:
  $\langle 0|\mathrm{T}\left[\phi(x_1)\phi(x_2)\ldots\phi(x_n)\right]|0\rangle$ (Note: $\mathcal{L}_{int}\left[\phi(x)\right]$ is a polynomial of free fields!)
- Decompose fields in annihilator (pos. freq.) and creator (neg. freq.) part to show $\mathrm{T}\left[\phi(x)\phi(y)\right] = \,:\phi(x)\phi(y):\, + D_F(x-y)$
- Wick's theorem (proof by induction) $\phi_i := \phi(x_i)$ etc.

$$\mathrm{T}\left[\phi_1\ldots\phi_n\right] = \,:\phi_1\ldots\phi_n:\, + :\phi_1\ldots\phi_{n-2}:\, D_{F,n-1,n} + \text{permut.}$$
$$:\phi_1\ldots\phi_{n-4}:\,\Big[D_{F,n-3,n-2}D_{F,n-1,n} + D_{F,n-3,n-1}D_{F,n-2,n}$$
$$+ D_{F,n-3,n}D_{F,n-2,n-1}\Big] + \text{perm.} + \ldots + \text{product of only Feynman propagators}$$

- $\Rightarrow \langle 0|\mathrm{T}\left[\phi_1\ldots\phi_n\right]|0\rangle = \prod_{\text{all perm.}} D_{F,i,j}$ for example:

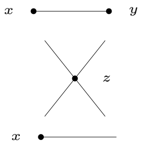
$$\langle 0|\mathrm{T}\left[\phi_1\phi_2\phi_3\phi_4\right]|0\rangle = D_{F,12}D_{F,34}$$
$$+ D_{F,13}D_{F,24} + D_{F,14}D_{F,23} =$$



- Signs from fermion anticommutations arise from Wick's theorem!

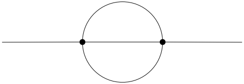
Feynman rules (position space)

Example for $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$



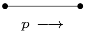
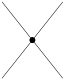
- Propagator: $D_F(x-y)$
- Vertex: $(-i\lambda)\int d^4z$
- external point: $\exp[-ipx \cdot \text{sgn}(in/out)]$
- divide by the symmetry factor

- Note: Position integrals correspond to QM superposition principle
- Example for symmetry factor:

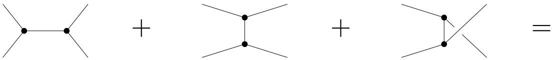


yields a symmetry factor 3!

Feynman rules (momentum space)

– Propagator		$i/(p^2 - m^2 + i\epsilon)$
– Vertex		$-i\lambda$
– external point		1 (or u, v etc.)
– momentum conservation		at each vertex
– integrate over undetermined momenta		$\int d^4p/(2\pi)^4$
– divide by symmetry fac.		

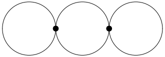
- Note: Momentum integrals correspond to QM superposition principle
  Example in $\lambda\phi^3$ theory: Mandelstam variables: $s \equiv (p_1 + p_2)^2 \quad t \equiv (p1 - p3)^2 \quad u \equiv (p_1 - p_4)^2$



$$(-i\lambda)^2 \left(\frac{i}{s-m^2} + \frac{i}{t-m^2} + \frac{i}{u-m^2} \right)$$

Bubbles, bubbles, vacuum bubbles

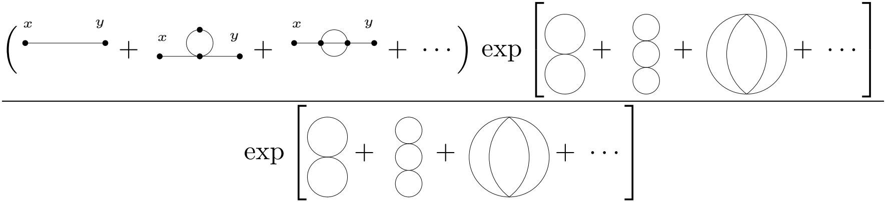
- Vacuum bubbles are infinite diagrams (Fermion loops: (-1))



$$\sim \quad \left(\delta^4(p)\right)^2 \longrightarrow (2\mathrm{T}) \cdot (\mathrm{Vol})$$

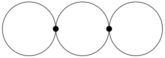
- Vacuum bubbles in the numerator and denominator of Gell-Mann–Low formula
- Symmetry factors $\Rightarrow$ vacuum bubbles exponentiate
- Just a normalization factor: cancels out

$$\langle 0|\mathrm{T}\left[\Phi(x)\Phi(y)\right]|0\rangle = \frac{\langle 0|\mathrm{T}\left[\phi(x)\phi(y)\exp(-i\int dt H_{int}(t))\right]|0\rangle}{\langle 0|\mathrm{T}\left[\exp(-i\int dt H_{int}(t))\right]|0\rangle} =$$



Bubbles, bubbles, vacuum bubbles

- Vacuum bubbles are infinite diagrams (Fermion loops: (-1))



$$\sim \quad \left(\delta^4(p)\right)^2 \longrightarrow (2\mathsf{T}) \cdot (\mathsf{Vol})$$

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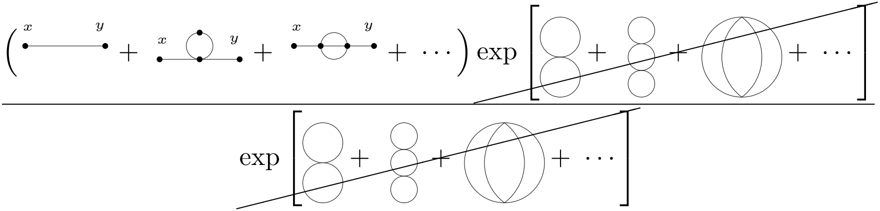
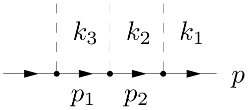
$$\langle 0|\mathrm{T}\left[\Phi(x)\Phi(y)\right]|0\rangle = \frac{\left\langle 0\middle|\mathrm{T}\left[\phi(x)\phi(y)\exp(-i\int dt H_{int}(t))\right]\middle|0\right\rangle}{\left\langle 0\middle|\mathrm{T}\left[\exp(-i\int dt H_{int}(t))\right]\middle|0\right\rangle} =$$



Fermion lines

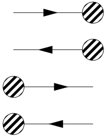
- Signs from disentangling field operator contractions:

$$\langle 0| b \overline{\psi}(x) \psi(x) \overline{\psi}(y) \psi(y) \overline{\psi}(z) \psi(z) b^\dagger |0\rangle$$

$p + k1 + k2 + k3 = q$  $=$

$$\overline{u}(p) \frac{i(\not{p} + \not{k}_1 + m)}{(p+k_1)^2 - m^2 + i\epsilon} \frac{i(\not{p} + \not{k}_1 + \not{k}_2 m)}{(p+k_1+k_2)^2 - m^2 + i\epsilon} u(q)$$

- External fermions:



$u^s(p)$

$\overline{v}^s(p)$

$\overline{u}^s(p)$

$v^s(p)$

Step 4: Phase Space Integration and Cross Sections

- Number N of scattering events for 2 particle beams with particle densities $\rho_{1,2}$, relative velocity v (V scattering volume, T scattering time)

$$N = V \cdot T \cdot \rho_1 \cdot \rho_2 \cdot v \cdot \sigma$$

- Constant of proportionality: cross section
- effective scattering area (e.g. geometric scattering $\sigma = \pi r^2$)
- How to get the cross section?
  1. Probability to get any final state $|n\rangle$ from initial state $|\alpha\rangle$: $\sum_n |\langle n|S|\alpha\rangle|^2$
  2. Project on a specific final state
  3. Use Fermi's Golden Rule
  4. Momentum integral from projection becomes phase space integral over final-state momenta

Flux factor, invariant matrix element, phase space measure

$$d\sigma_{\alpha\to\beta} = \frac{|\mathcal{M}_{\beta\alpha}|^2}{4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2}} \left(\prod_{i=1}^n \widetilde{dp_i}\right) (2\pi)^4 \delta^4(p_1 + p_2 - \sum_{i=1}^n q_i)$$

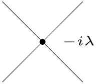
- Analogous decay width Γ of a particle (Life time $\tau = 1/\Gamma$)

$$d\Gamma_{\alpha\to\beta} = \frac{|\mathcal{M}_{\beta\alpha}|^2}{2m_\alpha} \left(\prod_{i=1}^{n} \widetilde{dp_i} \right) (2\pi)^4 \delta^4(p - \sum_{i=1}^{n} q_i)$$

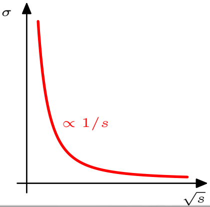
- $2 \to 2$ scattering depends only on $\sqrt{s}$ and $\cos\theta$:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{q}_{out}|}{|\vec{p}_{in}|} |\mathcal{M}_{\beta\alpha}|^2$$

- Simple example, $\lambda\phi^4$ theory:

$\mathcal{M}_{\beta\alpha} =$ 

$\Rightarrow \quad |\mathcal{M}_{\beta\alpha}|^2 = \lambda^2 \quad \Rightarrow \quad \sigma = \frac{\lambda^2}{4\pi} \cdot \frac{1}{s}$



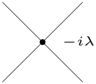
- Analogous decay width Γ of a particle (Life time $\tau = 1/\Gamma$)

$$d\Gamma_{\alpha\to\beta} = \frac{|\mathcal{M}_{\beta\alpha}|^2}{2m_\alpha} \left(\prod_{i=1}^{n} \widetilde{dp_i} \right) (2\pi)^4 \delta^4(p - \sum_{i=1}^{n} q_i)$$

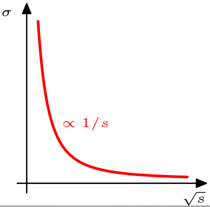
- $2 \to 2$ scattering depends only on $\sqrt{s}$ and $\cos\theta$:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \qquad |\mathcal{M}_{\beta\alpha}|^2$$ all masses equal

- Simple example, $\lambda\phi^4$ theory:

$\mathcal{M}_{\beta\alpha} =$ 

$$\Rightarrow \quad |\mathcal{M}_{\beta\alpha}|^2 = \lambda^2 \quad \Rightarrow \quad \sigma = \frac{\lambda^2}{4\pi} \cdot \frac{1}{s}$$



Quantization of the Electromagnetic Field

- $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu$$
- Euler-Lagrange equation $\Box A^\mu - \partial^\mu (\partial \cdot A) = j^\mu$ invariant under gauge transformation: $A^\mu(x) \to A^\mu(x) + \partial^\mu f(x)$
- Gauge invariance makes life hard (but for the wise easy!)
  - $\dot{A}_0$ absent $\Rightarrow$ no Π^0
  - Kinetic term is singular, hence not invertible
  - $[A_\mu(\vec{x}), A_\nu(\vec{y})] = -i\eta_{\mu\nu}\delta^3(\vec{x} - \vec{y})$ leads to negative and zero norm states on Fock space!

$$A_\mu(x) = \int \widetilde{dk} \sum_{\lambda=0}^{3} \left(a_k^{(\lambda)} \epsilon_\mu^{(\lambda)}(k) e^{-ipx} + a_k^{(\lambda)\,\dagger} \epsilon_\mu^{(\lambda)\,*}(k) e^{+ipx} \right)$$

- Solution: gauge fixing: $\partial \cdot A = 0$
  - Physical states $|\alpha\rangle$: Transversal polarizations ($\epsilon(k) \cdot k = 0$)
  - Unphysical states $|\chi\rangle$: longitudinal (space-like) pol./ scalar (time-like) pol.
  - Physical Fock space $\{|\alpha\rangle\}$ contains only positive-norm states
  - $|\alpha\rangle \to |\alpha\rangle + |\chi\rangle$ just corresponds to a gauge transformation
  - Gupta-Bleuler quantization: $\boxed{\langle\alpha|(\partial \cdot A)|\beta\rangle = 0}$

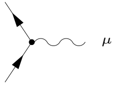
Quantum Electrodynamics (QED)

- Local (gauge) transformations

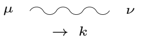
$$\psi(x) \to \exp[ieQ_{el.}\theta(x)]\psi(x) \qquad A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x)$$

- Derivative terms are no longer invariant ⇒ covariant derivatives
  $\partial_\mu\psi_f \to D_{\mu,f}\psi_f = (\partial_\mu - ieQ_f A_\mu)\,\psi_f$

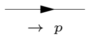
$$\boxed{\mathcal{L}_{QED} = \sum_f \overline{\psi}_f\,(i\not{D}_f - m_f)\,\psi_f \; - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$



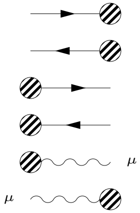
$ieQ_f\gamma^\mu$



$\frac{-i\eta_{\mu\nu}}{k^2+i\epsilon}$



$\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}$



$u^s(p)$

$\overline{v}^s(p)$

$\overline{u}^s(p)$

$v^s(p)$

$\epsilon^*_\mu(k)$

$\epsilon_\mu(k)$

Ward identities: gauge invariance for the wise

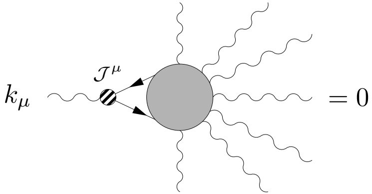
- Noether theorem: continuos symmetry implies a conserved current where the conserved charge is the symmetry generator

$$\partial_\mu \mathcal{J}^\mu = 0 \quad \Rightarrow \quad Q := \int d^3x\, \mathcal{J}^0(\vec{x}) \qquad \text{with} \quad \frac{d}{dt} Q = 0$$

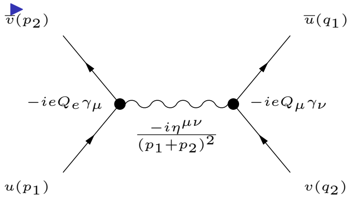
$$[iQ, \phi(x)] = \delta\phi(x) \quad \text{symmetry of the Lagrangian}$$

- Current conservation implies Ward identities: (contact terms vanish on-shell)
$0 = \partial_x^\mu \langle 0 | \mathrm{T} \left[\mathcal{J}(x) \phi_1(x_1) \ldots \phi_n(x_n) \right] | 0 \rangle$

- Generalization for off-shell amplitudes (and also non-Abelian gauge theories): Slavnov-Taylor identities



$e^+e^- \to \mu^+\mu-$: A sample calculation



$$\mathcal{M} = ie^2 Q_e Q_\mu \left(\overline{u}(q_1)\gamma^\mu v(q_2)\right) \frac{1}{s} \left(\overline{v}(p_2)\gamma_\mu u(p_1)\right)$$

- Square the matrix element, sum over final state spins, average over initial spins

Spin sums: $\sum_{s=\pm} u^s_\alpha(p)\overline{u}^s_\beta(p) = (\not{p} + m\mathbb{1})_{\alpha\beta} \qquad \sum_{s=\pm} v^s_\alpha(p)\overline{v}^s_\beta(p) = (\not{p} - m\mathbb{1})_{\alpha\beta}$

Use $\sum_{r,s}(\overline{u}^r(p_1)\gamma_\mu v(p_2)^s)(\overline{u}^r(p_1)\gamma_\nu v^s(p_2))^* = \sum_{r,s}(\overline{u}^r(p_1)\gamma_\mu v^s(p_2))(\overline{v}^r(p_2)\gamma_\nu u^s(p_1))$

$$= \sum_{r,s} \text{tr}\left[(\overline{u}^r(p_1)\gamma_\mu v^s(p_2))(\overline{v}^r(p_2)\gamma_\nu u^s(p_1))\right] = \text{tr}\left[(\not{p}_1 + m)\gamma_\mu(\not{p}_2 - m)\gamma_\nu\right]$$

$$= 4(p_{1,\mu}p_{2,\nu} + p_{1,\nu}p_{2,\mu}) - 2s\eta_{\mu\nu}$$

- Neglect all masses:

$$[4(p_{1,\mu}p_{2,\nu} + p_{1,\nu}p_{2,\mu}) - 2s\eta_{\mu\nu}]\,[4(q_{1,\mu}q_{2,\nu} + q_{1,\nu}q_{2,\mu}) - 2s\eta_{\mu\nu}]$$
$$= 32(q_1p_1)(q_2p_2) + 32(q_1p_2)(q_2p_1) = 8(t^2 + u^2) = 4s^2(1 + cos^2\theta)$$

-

$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{1}{2}\sum_{s,r}|\mathcal{M}|^2$$
$$= \frac{e^4}{32\pi^2 s}\frac{1}{2}(2\pi)\int_{-1}^{1} d(\cos\theta)(1+\cos^2\theta) = \frac{4\pi\alpha}{3s}$$

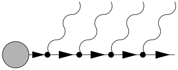
- Sommerfeld's fine structure constant $\alpha = e^2/(4\pi) \sim 1/137$
- Result:

$$\boxed{\sigma(e^+e^- \to \mu^+\mu^-) \;=\; \frac{4\pi\alpha^2}{3s}}$$

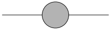
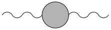
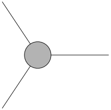
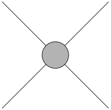
Part II (1. Abend)

The Mighty Valkyrie: Non-Abelian Gauge Theories and Renormalization

Quantum (a.k.a. radiative) corrections

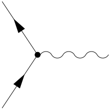
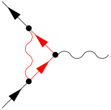


- **Real corrections:** radiation of photons etc.
- **Virtual (loop) corrections**

Self energies		Vacuum polarization	
Vertex corrections		Box diagrams	

- Example: e^- anomalous magnetic moment $\quad g\,\overline{\psi}\frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right]F_{\mu\nu}\psi$

Dirac theory tree level: $g = 2 \qquad \alpha := e^2/4\pi$

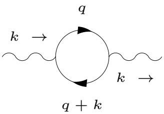
 $+$  $\Rightarrow$

$$g = 2\left(1 + \frac{\alpha}{2\pi}\right) = 2.00232282$$

Experimentally: $\quad g = (2.00231930436222 \pm 0.00000000000148)$

Ultraviolet and infrared (mass) singularities

- Ultraviolet singularities (in loops)

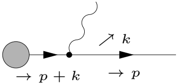


$$:= -i\Sigma_{\mu\nu}(k) = -e^2 \int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{tr}[\gamma_\mu(\not{q}+m)\gamma_\nu(\not{q}+\not{k}+m)]}{(q^2-m^2)((q+k)^2-m^2)}$$

$$\longrightarrow \int \frac{d^4q}{q^4}\{1, q, q^2\} \sim \int dq\,\{q^{-1}, 1, q\}$$

$\Rightarrow$ Logarithmic, linear, quadratic divergencies

- Collinear and soft (mass or infrared) singularities



$$\longrightarrow \frac{1}{(p+k)^2 - m^2} = \frac{1}{2p\cdot k}$$

$$= \frac{1}{2E_e E_\gamma (1 - \beta\cos\theta)} \qquad \text{with} \qquad \beta = p_e/E_e \sim 1$$

Collinear singularity: $\cos\theta \to 1$ Soft singularity: $E_\gamma \to 0$

Dimensional Regularization

- Divergent integrals need to be cast in a finite form to be treated
- Analytical continuation of

loop integrals	to $D = 4 - 2\epsilon < 4$	for ultraviolet singularities
phase space integrals	to $D = 4 + 2\epsilon > 4$	for soft/collinear singularities

- Keep dimensionality of integrals $\Rightarrow$ unphysical scale μ

$$\int \frac{d^4 q}{(2\pi)^4} \quad \longrightarrow \mu^{4-D} \int \frac{d^D q}{(2\pi)^D}$$

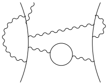
- E.g. QED vacuum polarization:

$$\Sigma_{\mu\nu}(k) = \left(\frac{2\alpha}{\pi}\right) \left(k^2 \eta_{\mu\nu} - k_\mu k_\nu\right) \int_0^1 dx x(1-x) \cdot \left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) - \ln \frac{-x(1-x)k^2 + m^2}{\mu^2}\right]$$

($\gamma_E = 0.577\ldots$ Euler constant) $\qquad \Delta := 1/\epsilon - \gamma_E + \ln(4\pi)$

Power Counting

- Question: Which Feynman diagrams do contain UV divergencies?



$$\sim \int \frac{d^4k_1 d^4k_2 \ldots dk_L}{(\not{k}_i - m)\ldots(k_j^2)\ldots(k_n^2)} \sim \frac{k^{\#\text{ Numerator}}}{k^{\#\text{ Denominator}}}$$

- Define superficial degree of divergence $\mathcal{D}$ (in D dimensions)

$$\boxed{\mathcal{D} = (\#\text{ Numerator}) - (\#\text{ Denominator}) = D\cdot L - I_e - 2I_\gamma}$$

E_e = # external $e^\pm$, E_γ = # extern. γ, I_e = # internal $e^\pm$, I_γ = # intern. γ, V = # vertices, L = # loops

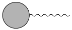
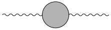
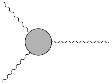
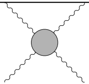
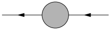
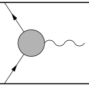
- Fermion propagators contribute k^{-1}, bosons k^{-2}
- Vertices with n derivatives contribute k^n
- Euler identity: $\boxed{L = I_e + I_\gamma - V + 1}$ (by induction)
- Simple line count: $\boxed{V = 2I_\gamma + E_\gamma = \frac{1}{2}(2I_e + E_e)}$

- **Power counting master formula:**

$$\boxed{\mathcal{D} = D(I_e + I_\gamma - V + 1) - I_e - 2I_\gamma = D - E_\gamma - \frac{3}{2}E_e}$$

Depends only on number of external lines

QED Singularities – Classification of QFTs

	$\mathcal{D} = 4$ (irrelevant, normalization)		$\mathcal{D} = 3$ $= 0$ (Furry's theorem)
	$\mathcal{D} = 2$ $\mathcal{D} = 0$, Ward id., $k^2\eta_{\mu\nu} - k_\mu k_\nu$		$\mathcal{D} = 1$ $= 0$ (Furry's theorem)
	$\mathcal{D} = 0$ finite, Ward identity		$\mathcal{D} = 1$ $D = 0$ (chirality)
	$\mathcal{D} = 0$	$\mathcal{D} = 4 - E_\gamma - \frac{3}{2}E_e$	

Superrenormalizable Theory	$\mathcal{D} \sim -I$	Only finite # of (sup.) divergent graphs
Renormalizable Theory	$\mathcal{D} \sim 0 \cdot I$	Finite # of (sup.) div. ampl., but at all orders of pert. series
Non-Renormalizable Theory	$\mathcal{D} \sim +I$	all amplitudes of sufficiently high order diverge

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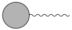
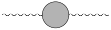
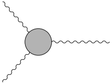
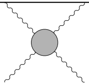
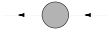
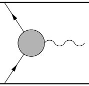
	$\mathcal{D} = 4$ (irrelevant, normalization)		$\mathcal{D} = 3$ $= 0$ (Furry's theorem)
	$\mathcal{D} = 2$ $\mathcal{D} = 0$, Ward id., $k^2\eta_{\mu\nu} - k_\mu k_\nu$		$\mathcal{D} = 1$ $= 0$ (Furry's theorem)
	$\mathcal{D} = 0$ finite, Ward identity		$\mathcal{D} = 1$ $D = 0$ (chirality)
	$\mathcal{D} = 0$	$\mathcal{D} = 4 - E_\gamma - \frac{3}{2}E_e$	

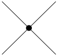
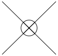
Superrenormalizable Theory	$\mathcal{D} \sim -I$	Coupling constants have positive mass dimension
Renormalizable Theory	$\mathcal{D} \sim 0 \cdot I$	Coupling constants are dimensionless
Non-Renormalizable Theory	$\mathcal{D} \sim +I$	Coupling constants have negative mass dimension

Renormalization

- In a renormalizable theory infinities cancel in the calculation of physical observables!
- Example: UV divergence of vertex correction and Z_e cancel
- Infinities cancel: there are finite shifts in physical parameters ($g - 2$!)
- Easier calculational prescription: Renormalized perturbation theory
  - Express Lagrangian of bare fields and parameters $\mathcal{L}(\phi_0, m_0, \lambda_0) = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$ by renormalized ones:
  - $\phi^0 = \sqrt{Z}\phi_{ren.}$ eliminates wave function factor
  - Expand parameters/fields $\delta_Z = Z - 1$, $\delta m = m_0^2 Z - m^2$, $\delta_\lambda = \lambda_0 Z^2 - \lambda$

$$\mathcal{L} = \frac{1}{2}(\partial\phi_{ren})^2 - \frac{1}{2}m^2\phi_{ren}^2 - \frac{\lambda}{4!}\phi_{ren}^4 + \frac{1}{2}\delta_Z(\partial\phi_{ren})^2 - \frac{1}{2}\delta_m\phi_{ren^2} - \frac{\delta_\lambda}{4!}\phi_{ren^4}$$

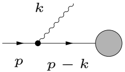
  - This generates counterterms, e.g.

 $= -i\lambda \qquad \longrightarrow$  $= -i\delta_\lambda$

- Loop diagrams and counterterms added up are finite!
- Counterterms are not unique: on-shell scheme, $\overline{\text{MS}}$ scheme

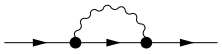
KLN: Discarding mass singularities

- Soft/collinear singularities only appear for (quasi) massless particles
- They appear both in real diagrams $(Q^2 = \vec{k}^2_{\perp,max})$

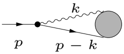


$$d\sigma^{e\to\gamma R}(p, p_R) \sim \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m_e^2}\right) \int_0^1 dx \frac{1+x^2}{1-x} d\sigma^{e\to R}(xp, p_R) + \cdots$$

and virtual diagrams



$$\delta Z_e \sim -\frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m_e^2}\right) \int_0^1 dx(1-x) + \text{non-log terms}$$



$$d\sigma^{e\to\gamma R}_{virt.}(p, p_R) \sim -\frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m_e^2}\right) d\sigma^{e\to R}(xp, p_R) \int_0^1 dx \frac{x}{1-x} + \cdots$$

soft singularity: $m_e \to 0$ collinear singularity: $x \to 1$
Soft-coll. singularities cancel between the three contributions

- Splitting function: $P_{ee} = (1+x^2)/(1-x)$ (cf. later)

KLN Theorem Bloch/Nordsieck, 1937; Kinoshita 1962, Lee/Nauenberg, 1964

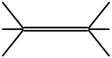
Unitarity guarantees that transition amplitudes are finite when summing over all degenerate states in initial and final state, order by order in perturbation theory (or for renormalization schemes free of mass sing.)

Renormalization group and running couplings

- High-momentum modes ≡ short-distance quantum fluctuations
- Fourier integrating over these $\lambda\Lambda < |k| < \Lambda$ modes (path integral)
  Consider $S = \int d^Dx \left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - B\phi^4\right)$ Rescale:

$$x' = \lambda x \qquad k' = k/\lambda \sim \mu \qquad \phi' = \sqrt{\lambda^{2-D}(1+\delta Z)}\phi \qquad (0 < \lambda < 1)$$

  Integrating out generates higher-dimensional operators, e.g.



$$\sim D \cdot \phi^6$$

- Effect is a shift in the masses, parameters, and field normalization (hence a renormalization)

$$\int d^Dx\mathcal{L}\Big|_{\mu<\lambda\Lambda} = \int d^Dx' \left[\frac{1}{2}(\partial\phi')^2 - \frac{1}{2}m'^2\phi'^2 - B'\phi'^4 - C'(\partial\phi)'^4 - D'\phi'^6 + \ldots\right]$$

$$\begin{aligned} m'^2 &= (m^2+\delta m^2)(1+\delta Z)^{-1}\lambda^{-2} \\ B' &= (B+\delta B)(1+\delta Z)^{-2}\lambda^{D-4} \\ C' &= (C+\delta C)(1+\delta Z)^{-2}\lambda^{D} \\ D' &= (D+\delta D)(1+\delta Z)^{-3}\lambda^{2D-6} \end{aligned}$$

  - Close to a fix point: $m^2, B, C, D, \ldots = 0$
  - Keep only linear terms
  - Relevant operators
  - Marginal operators
  - Irrelevant operators

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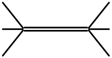

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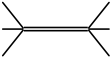

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Renormalization Group Equation

Wilson, 1971; Callan, Symanzik, 1970

- Bare Green's function do not depend on renormalization scale μ

$$0 = \mu \frac{d}{d\mu} G^{(n)}(g_0, m_0, \text{reg.}) = \mu \frac{d}{d\mu} \left\{ \left[Z^{-n/2} \cdot G^{(n)}_{ren} \right] (g(g_0, m_0, \mu), m(g_0, m_0, \mu), \mu, \text{reg.}) \right\}$$

$$\boxed{\left[\mu \frac{\partial}{\partial \mu} + \mu \frac{\partial g}{\partial \mu} \frac{\partial}{\partial g} - \left(-\frac{1}{m} \mu \frac{\partial m}{\partial \mu} \right) m \frac{\partial}{\partial m} - n \left(-\frac{1}{2Z} \mu \frac{dZ}{d\mu} \right) \right] G^{(n)}_{ren} = 0}$$

- $\beta(g, m, \mu) = \partial g / \partial (\ln \mu)$ β (RG) function
- $\beta(g, m, \mu) = -\partial (\ln m) / \partial (\ln \mu)$ anomalous mass dimension
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Typical one-loop values: $\beta(g) = \beta_0 \frac{g^3}{16\pi^2}$ $\gamma_m(g) = \gamma_{m,0} \frac{g^2}{16\pi^2}$ $\gamma(g) = \gamma_0 \frac{g^2}{16\pi^2}$

- Solution of RG equation:

$$\frac{dg}{d \ln \mu} = \frac{\beta_0 g^3}{16\pi^2} \quad \Rightarrow$$

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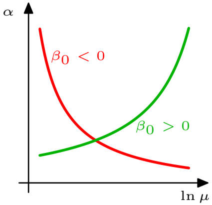

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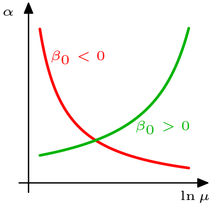

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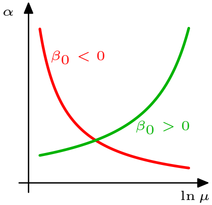

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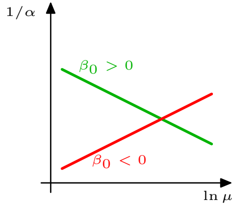


Non-Abelian Gauge (Yang-Mills) theories

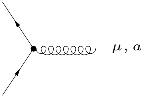
Yang/Mills,1954

- Local symmetry based on a (semi-)simple Lie group or direct product:

$$\phi_i(x) \to \exp\left[ig\theta^a(x)T^a\right]_{ij}\psi_j(x) \qquad \left[T^a, T^b\right] = if_{abc}T^c \qquad D_{\mu,ij} = \partial_\mu\delta_{ij} + igT^a_{ij}A^a_\mu$$

- Matter-gauge boson vertex contains a non-Abelian generator:

$$-g\overline{\psi}T^a_{ij}\not{A}^a\psi \qquad \Rightarrow$$

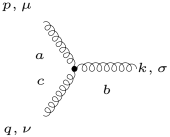


$$-ig\gamma_\mu T^a_{ij}$$

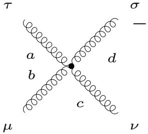
- Non-Abelian field strength tensor:

$$[D_\mu, D_\nu] = igF^a_{\mu\nu}T^a = -ig\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf_{abc}A^b_\mu A^c_\nu\right)T^a$$

does contain gauge boson self interactions:



$$-gf^{abc}\left[(p-k)^\nu\eta^{\mu\sigma} + (q-p)^\sigma\eta^{\mu\nu} + (k-q)^\mu\eta^{\nu\sigma}\right]$$



$$-\mathrm{i}g^2\left[f^{abc}f^{ade}(\eta^{\mu\sigma}\eta^{\nu\tau} - \eta^{\mu\tau}\eta^{\nu\sigma}) + f^{abd}f^{ace}(\eta^{\mu\nu}\eta^{\sigma\tau} - \eta^{\mu\tau}\eta^{\nu\sigma}) + f^{abe}f^{acd}(\eta^{\mu\nu}\eta^{\sigma\tau} - \eta^{\mu\sigma}\eta^{\nu\tau})\right]$$

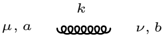
Dynkin index T_R : $\quad \mathrm{tr}\left[T^a_R T^b_R\right] = T_R\delta_{ab} \to T_F = \frac{1}{2}$ Quadratic

Casimir: $\sum_a T^a_R T^a_R = C_R \mathbb{1} \qquad C_{adj.} := C_A \to 3 \qquad C_{fund.} := C_F \to \frac{4}{3}$

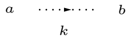
Quantization of Yang-Mills theories

- Use path integral representation of Green's functions
- Summing over physically equivalent field configurations (gauge orbits)
- Gauge fixing: Choose one configuration per space-time point
- Can be written as a functional determinant Faddeev/Popov, 1967
- Leads to gauge-fixing/ghost Lagrangian:

$$\mathcal{L}_{GF+FP} = -\frac{1}{2\xi}(\partial \cdot A)^2 - \bar{c}^a \partial^\mu D_\mu^{ab} c^b$$

- First term leads to invertible gluon propagator
- Faddeev-Popov ghosts $c, \bar{c}$: fermionic scalars!!! cancel unphysical longitudinal and scalar gluon modes, preserve S-Matrix unitarity
- 

$$\frac{-i}{k^2+i\epsilon}\left(\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\delta_{ab}$$



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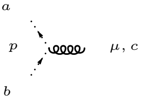


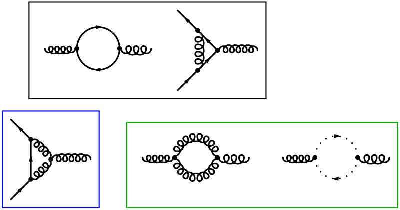
$$-g f_{abc} p_\mu$$

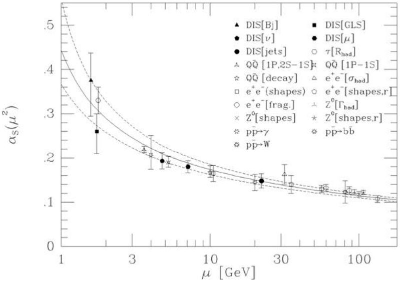
- Ghosts decouple in QED
- After gauge fixing: gauge invariance lost, remainder global, non-linear BRST symmetry Becchi/Rouet/Stora,1976, Tyutin, 1975, Batalin/Vilkoviskiy, 1976

QCD: Asymptotic freedom, Confinement

- Quantum Chromodynamics (QCD) is $SU(3)$ Yang-Mills theory of strong interactions
- QCD β function is negative!

Gross,Politzer,Wilczek, 1973



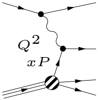


$$\beta_0 = C_F N_f T_R - \frac{11}{3} C_A \to \frac{2}{3} N_f - 11 < 0$$

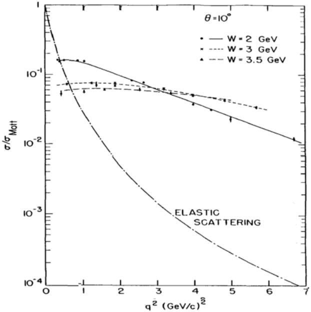
- **Asymptotic Freedom:** $\alpha_s \to 0$ for $\mu \to \infty$
  Quarks quasi-free particles (Antiscreening of YM field)
- **Confinement/Infrared Slavery:**
  Landau pole: $\alpha_s \to \infty$ for $\mu \sim \Lambda_{QCD} \sim 0.2$ GeV
- QCD forms bound states at scale $(1-3) \times \Lambda_{QCD}$: mesons ($q\bar{q}$) and baryons ($qqq$)

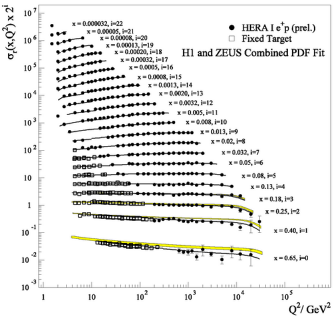
Discovery of QCD: Deep Inelastic Scattering (DIS)

- 1969 SLAC electron beam to hadronic fixed target
- Cross section const., no $1/s$ drop-off



$$\frac{d\sigma}{dQ^2}(P) = \int_0^1 dx \sum_f F_f(x, Q^2) \times \frac{d\sigma}{dQ^2}(e^- f \to e^- f, xP)$$



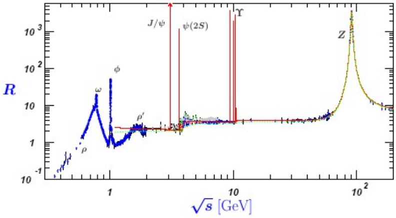


- Bjorken scaling $F_f(x) \sim const.$: scattering at quasi-free partons

  Bjorken, Feynman, 1969

- Scaling violations: $F_f(x) := F_f(x, Q^2) \sim \ln Q^2$ described by logarithmically enhanced higher order QCD radiative corrections (Altarelli-Parisi equations)

Proofs: R Ratio, Jet Events, and all that...



e^+e^- annihilation into hadrons

$$\sigma(e^+e^- \to \mu^+\mu^-) := \sigma_0 = \frac{4\pi\alpha^2}{3s}$$
$$\sigma(e^+e^- \to \text{hadrons}) = \sigma_0 \cdot N_c \cdot \sum_f Q_f^2$$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{5}{3} \qquad \text{below } 2m_s$$

- Gluon discovery:
  3 jets @ PETRA   Wolf (DESY), 1979
- $x_1 = E_q/E_e$, $x_2 = E_{\bar{q}}/E_e$, $x_3 = E_g/E_e$

$$\frac{d\sigma}{dx_1 dx_2}(e^+e^- \to q\bar{q}g) = \sigma_0 \times$$
$$(3\sum_f Q_f^2)\,\frac{2\alpha_s}{3\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

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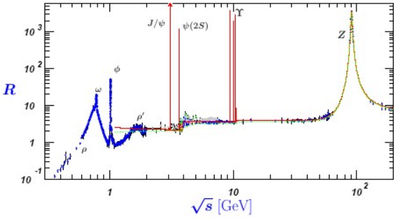

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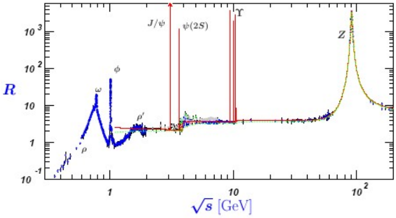

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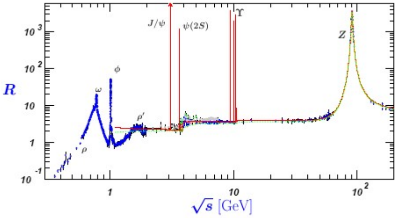

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$$\begin{aligned} \sigma(e^+e^- \to \mu^+\mu^-) &:= \sigma_0 = \frac{4\pi\alpha^2}{3s} \\ \sigma(e^+e^- \to \text{hadrons}) &= \sigma_0 \cdot N_c \cdot \sum_f Q_f^2 \end{aligned}$$

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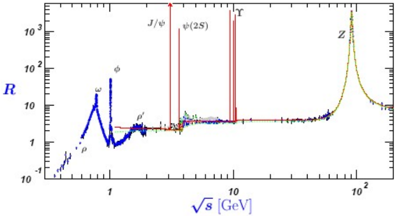

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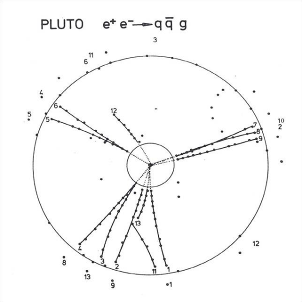

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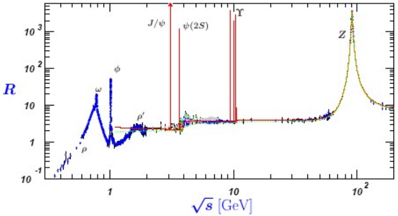

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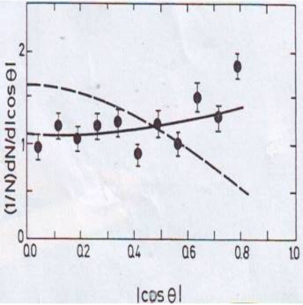
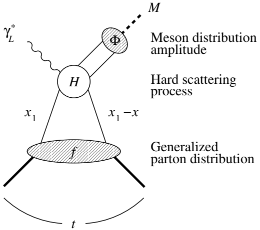


The Factorization Theorem



- Cornerstone of perturbative QCD:
  Factorization theorem Sterman, 1979; Collins/Soper, 1981
- Hadronic cross sections can be split into
  - Perturbative part: hard scattering process
  - Non-perturbative part:
    parton distribution functions
  - Non-perturbative part:
    jet or fragmentation functions
- Hard scattering cross sections perturbatively calculable
- Parton distribution and fragmentation functions from experimental fits
- Perturbative evolution of PDFs/fragmentation func. to different scales:

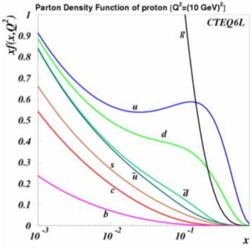
$$\frac{d}{d\ln Q}F_g(x,Q) = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\Big\{P_{g\leftarrow q}(z)\sum_f\Big[F_q(\frac{x}{z},Q)+F_{\bar q}(\frac{x}{z},Q)\Big] + P_{g\leftarrow g}(\frac{x}{z},Q)F_g(\frac{x}{z},Q)\Big\}$$

$$\frac{d}{d\ln Q}F_q(x,Q) = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\Big\{P_{q\leftarrow q}(z)F_q(\frac{x}{z},Q)+P_{q\leftarrow g}(\frac{x}{z},Q)F_g(\frac{x}{z},Q)\Big\}$$

$$\frac{d}{d\ln Q}F_q(x,Q) = \frac{dz}{z}\Big\{P_{\bar q\leftarrow \bar q}(z)F_{\bar q}(\frac{x}{z},Q)+P_{\bar q\leftarrow g}(\frac{x}{z},Q)F_g(\frac{x}{z},Q)\Big\}$$

Hadronic Cross Sections, PDFs

- Parton Distribution Functions (PDFs): $F_f(x)dx =$ prob. of finding constituent f with longitudinal momentum fraction x
- Momentum sum rule: $\int_0^1 dx x \left[\sum_q F_q(x) + \sum_{\bar{q}} F_{\bar{q}}(x) + F_g(x)\right] = 1$
- Charge sum rules: $\int_0^1 dx \left[F_u(x) - F_{\bar{u}}(x)\right] = 2, \quad \int_0^1 dx \left[F_d(x) - F_{\bar{d}}(x)\right] = 1$
  - PDFs Have to be fitted from experiments
  - Hadronic cross section are calculated according to the factorization theorem:



$$\sigma(p(P_1) + p(P_2) \to Y + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_f F_f(x_1) F_{\bar{f}}(x_2) \cdot \sigma_{showered}(f(x_1 P_1) + \bar{f}(x_2 P_2) \to Y) \cdot \prod_Y \tilde{F}(Y \to M, B)$$

  - Parton shower describes QCD radiation

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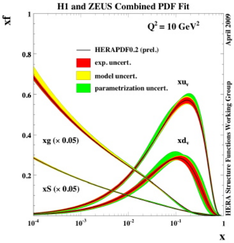
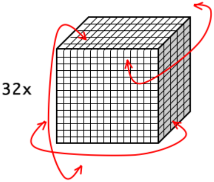


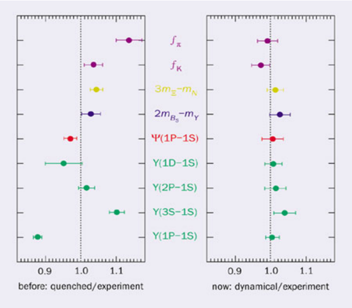
    - Parton shower describes QCD radiation

Lattice QCD

- Discretize space and time
- Solve Yang-Mills equation of motion numerically
- Gluon fields are links between lattice points
- Fermions sit on the sites of the lattice
- Problem: artefacts from continuum limit



- Hadron spectra are ”measured” on the lattice
- m_π is the usual input
- Old days: ”quenched” (no fermions)
- One sees $V(r) \sim r$ confinement potential



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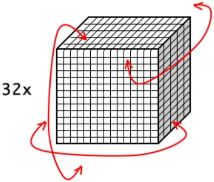

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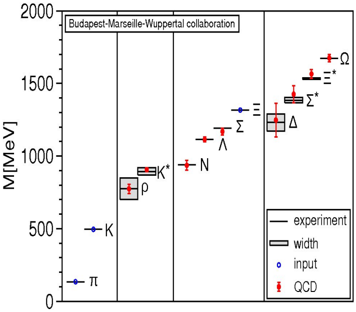


Part III (2. Abend)

The Noble Hero: Hidden Symmetries (formerly known as Spontaneous Symmetry Breaking)

Basics of Hidden Symmetries

- Hidden symmetry is obeyed by the Lagrangian (and the E.O.M.)
- It is not respected by the spectrum, especially the ground state
- In principle only possible in a system of infinite volume
- **Nambu-Goldstone Theorem** Goldstone, 1961; Nambu, 1960; Goldstone/Salam/Weinberg, 1962

  For any broken symmetry generator of a global symmetry there is a massless boson (Nambu-Goldstone boson) in the theory.

  Two cases:

  i) $Q^a \left|0\right\rangle = 0 \forall a$ unbroken or Wigner-Weyl phase

  ii) $Q^a \left|0\right\rangle \neq 0$ for at least one a $\Rightarrow$ Nambu-Goldstone phase

- Simple proof:

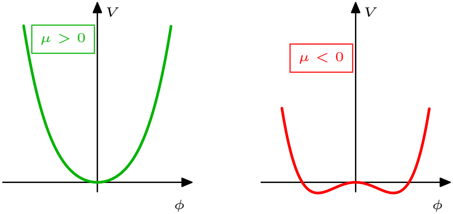
$$\phi_i \to i\theta^a T^a_{ik}\phi_k \quad \Rightarrow \quad \frac{\partial \mathcal{V}}{\partial \phi_i} T^a_{ij}\phi_j = 0 \quad \Rightarrow$$

$$\underbrace{\frac{\partial^2 \mathcal{V}}{\partial \phi_i \partial \phi_j}\bigg|_{\langle 0|\phi|0\rangle}}_{=(m^2)_{ij}} T^a_{jk} \left\langle 0|\phi_k|0\right\rangle + \underbrace{\frac{\partial \mathcal{V}}{\partial \phi_j}\bigg|_{\langle 0|\phi|0\rangle}}_{=0} T^a_{ji} = 0$$

The Nambu-Goldstone Theorem

- N-component real scalar field, possesses $O(N)$ symmetry

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^T)(\partial^\mu \phi) - \frac{\mu^2}{2}\phi^T\phi - \frac{g}{4}(\phi^T\phi)^2 \quad \text{with} \quad \phi = (\phi_1, \ldots, \phi_N)$$



Minimizing the potential:
$\langle\phi\rangle = 0$ (metastable) or
$\langle\phi^T\phi\rangle \sim \langle\phi\rangle^T \langle\phi\rangle = -\mu^2/g > 0$

- Without loss of generality: $\langle\phi_i\rangle = (0, 0, \ldots, 0, \langle\phi_N\rangle)$ VEV in n-th comp.
- Mass squared matrix:

$$(M^2)_{ij} = \frac{\partial^2 V(\phi)}{\partial\phi_i \partial\phi_j}\bigg|_{\phi=\langle\phi\rangle} = 2g\langle\phi_i\rangle\langle\phi_j\rangle = \left(\begin{array}{c|c} 0_{(N-1)\times(N-1)} & 0_{1\times(N-1)} \\ \hline 0_{(N-1)\times 1} & 2g\langle\phi\rangle^2 \end{array}\right)$$

- $O(N)$ symmetry group broken down to $O(N-1)$ symmetry group
- # broken symmetry generators = # Goldstone bosons = $\frac{1}{2}N(N-1) - \frac{1}{2}(N-1)(N-2) = N-1$

Chiral Symmetry Breaking in QCD

- Light quarks (almost) massless $\Rightarrow$ $SU(2)_L \times SU(2)_R$ global symmetry

$$Q := \begin{pmatrix} u \\ d \end{pmatrix} \qquad \mathcal{L} = \overline{Q} \not{D} Q = \overline{Q}_L \not{D} Q_L + \overline{Q}_R \not{D} Q_R$$

(mass term: $-m\overline{Q}Q = -m\overline{Q}_L Q_R + \overline{Q}_R Q_L$

- Rewrite left- and right-handed rotations into vector and axial transformations:

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \exp\left[i\frac{\vec{\sigma}}{2}\vec{\theta}_L \mathcal{P}_L\right] \exp\left[i\frac{\vec{\sigma}}{2}\vec{\theta}_R \mathcal{P}_R\right] \begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow \exp\left[i\frac{\vec{\sigma}}{2}\vec{\theta}_V\right] \exp\left[i\frac{\vec{\sigma}}{2}\vec{\theta}_A \gamma^5\right] \begin{pmatrix} u \\ d \end{pmatrix}$$

- For massive quarks, the axial Noether current is not conserved:

$$\partial_\mu \vec{J}^\mu_V = 0 \qquad \partial_\mu \vec{J}^\mu_A = -2m\overline{Q}\frac{\vec{\sigma}}{2}\gamma^5 Q \overset{m\to 0}{\longrightarrow} 0$$

- If $SU(2)_A$ were exact: $|h\rangle \Rightarrow T_A |h\rangle$ degenrate opposite parity pairs of hadrons, not seen in Nature
- $SU(2)_A$ hidden symmetry $\vec{T}_A |h\rangle = |h + \vec{\pi}\rangle$
- Pions are the Nambu-Goldstone bosons (NGB) of spontaneously broken $SU(2)_A$

What breaks chiral symmetry?

- Strong interactions (QCD) make quark-antiquark pairs condens (like Cooper pairs in a BCS superconductor)
- Quark condensate: $(300 \text{ MeV})^3 \sim \Lambda_{QCD}^3 \sim \langle \overline{q} q \rangle$ is invariant under $SU(2)_V$, but breaks $SU(2)_A$
- Axial current generates pion states:

$$\langle 0 | J_A^{\mu,a} | \pi_b \rangle = i F_\pi \delta_{ab} p_\pi^\mu e^{i p_\pi x} \qquad F_\pi = 184 \text{ MeV from pion decay}$$

- Explicit breaking of $SU(2)_A$ by finite quark masses
- Pions only approximate NGBs, i.e. pseudo NGBs (pNGBs):

$$m_\pi^2 = \frac{4(m_u + m_d) \langle \frac{1}{2}(\overline{u}u + \overline{d}d) \rangle}{F_\pi^2}$$

- Difference $m(\pi^\pm) - m(\pi^0) \approx 5$ MeV from electromagnetic quantum corrections
- Include strange quark: $SU(3) \times SU(3)$ stronger broken $m_s \sim 95$ MeV vs. $m_{u,d} \sim 2 - 6$ MeV

Hidden Local Symmetries

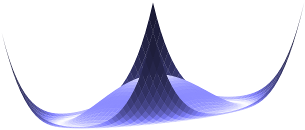
Anderson, 1961; Higgs, 1964; Brout/Englert, 1964; Kibble 1964

- Consider scalar electrodynamics:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$$

$$V(\phi) = -\mu^2 |\phi|^2 + \frac{\lambda}{2} (|\phi|^2)^2$$

- Remember the gauge trafos: $A_\mu \to A_\mu + \partial_\mu \theta(x), \quad \phi(x) \to \exp[-ie\theta(x)]\phi(x)$



  - Minimize the potential $\Rightarrow \langle \phi \rangle = v/\sqrt{2} e^{i\alpha}$ where $v/\sqrt{2} = \mu/\sqrt{\lambda}$
  - Radial excitation: "Higgs field"
  - Phase is the NGB

$$\phi(x) = \frac{1}{\sqrt{2}} (v + h(x)) e^{\frac{i}{v}\pi(x)}$$

- Evaluating the kinetic term

$$|D_\mu \phi|^2 = \frac{1}{2} (\partial h)^2 + \frac{e^2}{2} (v+h)^2 \left(A_\mu - \frac{1}{ev} \partial_\mu \pi \right)^2 A_\mu$$

- Mixture between gauge boson and NGB. Define $B_\mu := A_\mu - \frac{1}{ev} \partial_\mu \pi$
- Field strength term does not change under this redefinition

Hidden Local Symmetries

Anderson, 1961; Higgs, 1964; Brout/Englert, 1964; Kibble 1964

- Consider scalar electrodynamics:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$$

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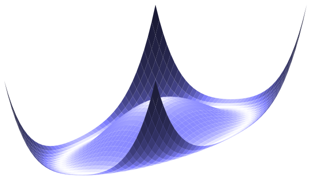

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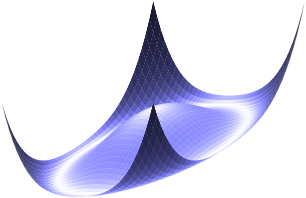

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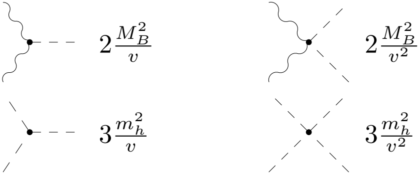
The Higgs Mechanism

- VEV generates mass term for the gauge boson
- Gauge boson mass: only consistent (renormalizable) way 't Hooft/Veltman, 1971
- $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_B^2 B_\mu B^m u + \frac{1}{2}(\partial h)^2 - \frac{1}{2}m_h h^2 - g_{h,3}h^3 - g_{h,4}h^4$$

  with $\boxed{m_h^2 = \lambda v^2}$ $\boxed{M_B = ev}$ $\boxed{g_{h,3} = \frac{m_h^2}{2v}}$ $\boxed{m_h^2 = \frac{m_h^2}{8v^2}}$

- Higgs field generates particle masses proportional to its VEV and its coupling to that particle

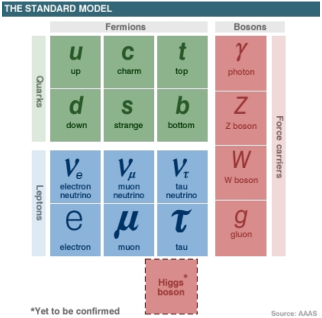
Feynman rules



- Hey, what happened to the Nambu-Goldstone theorem??

  Longitudinal polarisation now becomes physical, Goldstone boson takes over its place in cancelling unphysical degrees of freedom.

The Electroweak Standard Model



- Standard Model (SM) is $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory
- Nuclear forces known since 1930s
- QCD ($SU(3)_c$) proven to be the correc theory in 1968-1980 (DIS, $e^+e^- \to$ jets at SLAC/DESY)
- Weak interactions known since 1895 (beta decay)
- Charged current weak processes, e.g. muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ Fermi, 1934
- Weak interactions couple only to left-handed particles Wu, 1957; Goldhaber, 1958
- Discovery of neutral currents in ν-nucleus scattering 1973, discrepancy in strength to charged current $\Rightarrow$ weak mixing angle
- Production of W, Z bosons (CERN, 1983)

The Lagrangian and its particles *in totaliter*

- Building blocks $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ quantum numbers:

Q_L	u_R	d_R	L_L	e_R	H	ν_R
$(\mathbf{2},\mathbf{3})_{\frac{1}{3}}$	$(\mathbf{1},\mathbf{3})_{\frac{4}{3}}$	$(\mathbf{1},\mathbf{3})_{-\frac{2}{3}}$	$(\mathbf{2},\mathbf{1})_1$	$(\mathbf{1},\mathbf{1})_{-2}$	$(\mathbf{2},\mathbf{1})_1$	$(\mathbf{1},\mathbf{1})_0$

- All renormalizable interactions possible with these fields:

$$\mathcal{L}_{SM} = \sum_{\psi=Q,u,d,L,e,H,\nu} \overline{\psi} \not{D} \psi - \frac{1}{2} \operatorname{tr}\left[G_{\mu\nu} G^{\mu\nu}\right] - \frac{1}{2} \operatorname{tr}\left[W_{\mu\nu} W^{\mu\nu}\right] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + Y^u \overline{Q}_L \epsilon H^\dagger u_R + Y^d \overline{Q}_L H d_R + Y^e \overline{L}_L H e_R \quad \left[+ Y^n \overline{L}_L \epsilon H^\dagger \nu_R\right] + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

Electroweak Symmetry Breaking

- Higgs vev $\langle H \rangle = (0, v/\sqrt{2})$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{em.}$
  $D_\mu \phi = (\partial_\mu + ig\vec{W}_\mu \frac{\vec{\sigma}}{2} + ig' Y B_\mu)\phi$
- Electroweak gauge boson mass term:

$$\Delta\mathcal{L} = \frac{1}{2}(0, v)\left(g\vec{W}_\mu \frac{\vec{\sigma}}{2} + \frac{g'}{2}B_\mu\right)\left(g\vec{W}^\mu \frac{\vec{\sigma}}{2} + \frac{g'}{2}B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}$$

- Three massive vector bosons $W^\pm$, Z

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \qquad m_W = \frac{1}{2}gv$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right) \qquad m_Z = \frac{v}{2}\sqrt{g^2+g'^2}$$

- Orthogonal combination remains massless photon

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'W_\mu^3 + gB_\mu\right) \qquad m_A = 0$$

- Rewrite the covariant derivative: $\sigma^\pm = \frac{1}{2}(\sigma^1 \pm \sigma^2)$

$$D_\mu = \partial_\mu + i\frac{g}{\sqrt{2}}(W_\mu^+\sigma^+ + W_\mu^-\sigma^-) + i\frac{1}{\sqrt{g^2+g'^2}}Z_\mu(g^2T^3 - g'^2Y) + i\frac{gg'}{\sqrt{g^2+g'^2}}A_\mu(T^3+Y)$$

Weak mixing angle: $\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$, $\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

Gell-Mann–Nishijima relation: $Q = T^3 + Y$

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$$D_\mu = \partial_\mu + i \frac{g}{\sqrt{2}} (W^+_\mu \sigma^+ + W^-_\mu \sigma^-) + i \frac{1}{\cos\theta_W} Z_\mu (T^3 - \sin^2\theta_W Q) + i e A_\mu Q$$

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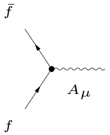
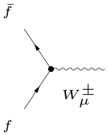
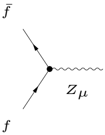
Electroweak Feynman Rules (all momenta outgoing)



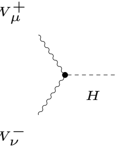
$-ie\gamma_\mu Q_f$



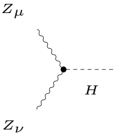
$-i\frac{g}{2\sqrt{2}}\gamma_\mu(1-\gamma^5)$



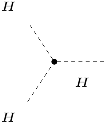
$-i\frac{g}{2\cos\theta_W}\gamma_\mu\left[(T^3_f-2\sin^2\theta_W Q_f)-\gamma^5 T^3_f\right]$



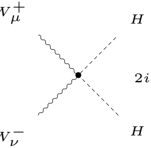
$2i\frac{m_W^2}{v}\eta_{\mu\nu}$



$2i\frac{m_Z^2}{v}\eta_{\mu\nu}$



$-3i\frac{m_H^2}{v}$



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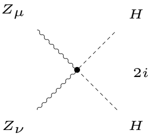

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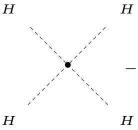
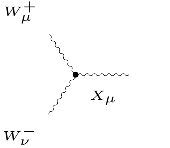


$-3i\frac{m_H^2}{v^2}$

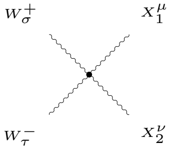
Easily derivable: $\left(m_W^2 W^+_\mu W^{-\mu}+\frac{1}{2}m_Z^2 Z^2\right)\left(1+\frac{H}{v}\right)^2$

Electroweak Feynman Rules (all momenta outgoing)



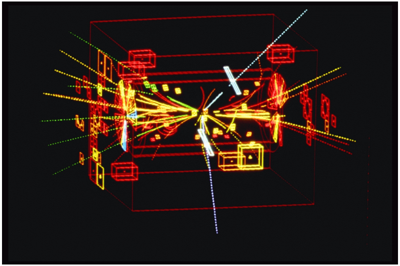
$$-i g_{WWX} \left[(k_- - k_+)^\rho \eta^{\mu\nu} + (q - k_-)^\mu \eta^{\nu\rho} + (k_+ - q)^\nu \eta^{\mu\rho} \right]$$

$$g_{WWZ} = g \cos\theta_W \qquad g_{WW\gamma} = e$$



$$-i g_{WWX_1X_2} \left[2\eta^{\mu\nu}\eta^{\sigma\tau} - \eta^{\mu\sigma}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\tau} \right]$$

$$g_{WW\gamma\gamma} = e^2$$
$$g_{WWZZ} = g^2 \cos^2\theta_W$$
$$g_{WW\gamma Z} = g^2 \cos\theta_W \sin\theta_W$$
$$g_{WWWW} = -g^2$$



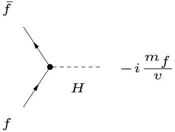
Fermion masses: Yukawa terms

- Fermion mass terms $-m_f(\overline{f}_L f_R + \overline{f}_R f_L)$ forbidden by $U(2)_L \times U(1)_Y$ gauge invariance
- Yukawa coupling is gauge invariant dimension-4 operator:

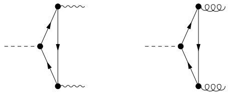
$$\Delta\mathcal{L}_{Yuk.} = -Y_e(\overline{L}_L \cdot \phi)e_R \to -\frac{vY_e}{\sqrt{2}}\overline{e}_L e_R \left(1 + \frac{H}{v}\right)$$

- Again, Higgs boson couples proportional to mass:

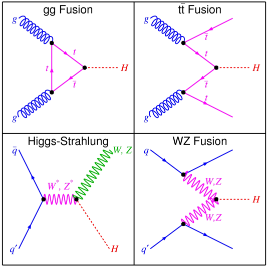
$$\boxed{m_f = \frac{1}{\sqrt{2}} Y_f v}$$



- Hierarchy of Yukawa couplings according to fermion masses: $Y_t \approx 1$, $Y_{c,\tau,b} \approx 10^{-2}$, $Y_{\mu,s} \approx 10^{-3}$, $Y_{e,\nu,d} \approx 10^{-5}$
- $Y_\nu \lesssim 10^{-10}$, but Majorana mass term $\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} m_\nu \overline{\nu^c}_R \nu_R$ possible
- $H\gamma\gamma, Hgg$ couplings: $\frac{1}{v}\frac{Yg^2}{16\pi^2} \cdot c \cdot HF_{\mu\nu}F^{\mu\nu}$



Higgs: Properties and Search



Production: gluon/vector boson fusion

decays predominantly into the heaviest particles

$b\bar{b}$ hopeless: background!

Detection of rare decays

Complicated search: many channels

high statistics necessary

$\gamma\gamma$: mass determination

$M_H > 135\,\text{GeV}$: $ZZ^* \to \ell\ell\ell\ell$

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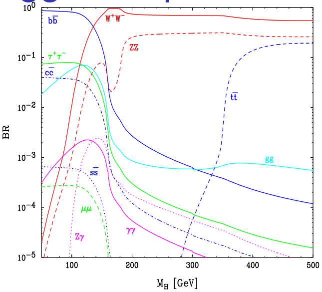

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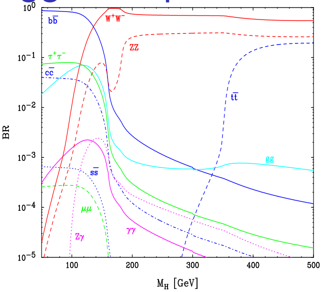

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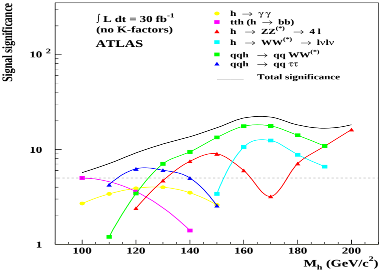

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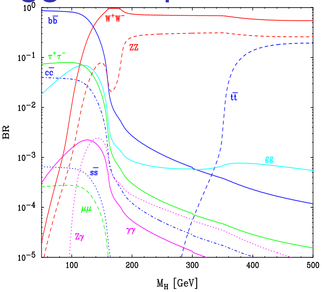

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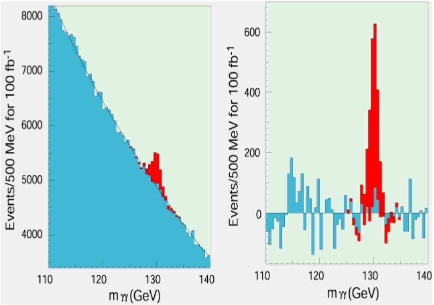


Flavor, the CKM matrix, and CP violation

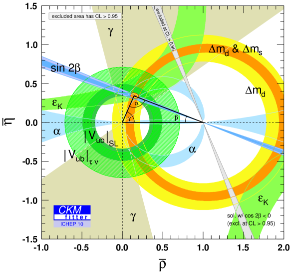
- Three generations of fermions in Nature
- Diagonalization of fermion mass matrices:

$$v^2 Y_u Y_u^\dagger = L_u \text{diag}(m_u^2, m_c^2, m_t^2) L_u^\dagger \qquad v^2 Y_d Y_d^\dagger = L_d \text{diag}(m_d^2, m_s^2, m_b^2) L_d^\dagger$$

- Rotation of quark fields leaves a trace in the charged current:

$$\overline{u}_L \not{W} (L_u^\dagger L_d) d_L = \overline{u}_L \not{W} V_{CKM} d_L$$

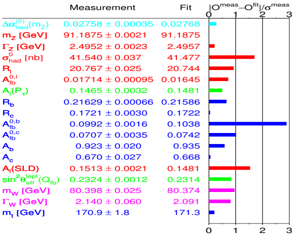
- CKM matrix: unitary, experimentally almost diagonal
- Three angles $\theta_{12}, \theta_{13}, \theta_{23}$, one phase
- Phase violates CP (charge conjugation and parity)
- After discovery of neutrino oscillations: MNS matrix
- CKM describes flavor incredibly well



Part IV (3. Abend)

Götterdämmerung:
Beyond the Standard Model

The Standard Model of Particle Physics – Doubts



– describes microcosm (too well?)

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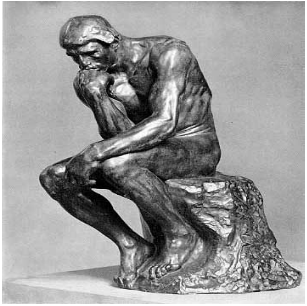
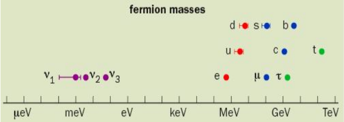
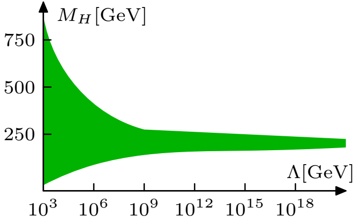


– describes microcosm (too well?)

– 28 free parameters



– form of Higgs potential ?



Hierarchy Problem

chirale Symmetrie: $\delta m_f \propto v \ln(\Lambda^2/v^2)$

no symmetry for quantum corrections to the Higgs mass

$$\delta M_H^2 \propto \Lambda^2 \sim M_{\text{Planck}}^2 = (10^{19})^2\,\text{GeV}^2$$

20000 GeV^2 = (10000000000000000000000000000000020000 –

10000000000000000000000000000000000000) GeV^2

Open questions

– Unification of all interactions (?)

– Baryon asymmetry $\Delta N_B - \Delta N_{\bar{B}} \sim 10^{-9}$
  missing CP violation

– Flavour: three generations

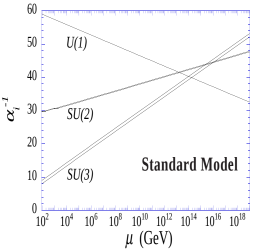
– Tiny neutrino masses: $m_\nu \sim \frac{v^2}{M}$

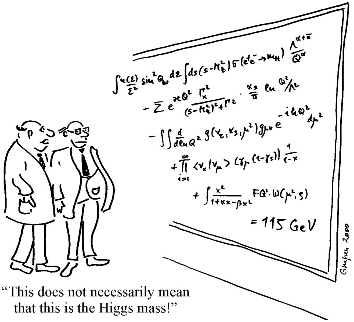
– Dark Matter:
  - stable
  - weakly interacting
  - $m_{DM} \sim 100\,$GeV

– Quantum theory of gravity

– Cosmic inflation

– Cosmological constant





"This does not necessarily mean
that this is the Higgs mass!"


bosonic TC
composite Higgs
TC-TC
hypercolor
supercolor
techni-GIM
extended TC
pseudo NGB
NOT YET THOUGHT OF
effective SUSY
SUSY
NMSSM
nMSSM
MSSM +νR
NMSSM
unnunified SM
axigluon
sterile ν
6th gen
5th gen
4th gen
lepto quark
heavy Majorana ν
Z'LR
Z'ψ
WR
Z'SM
Z'χ
X
Y
Z'η
vector-like family
fractionally charged
milli-charged
mono-pole
S,T,U
NOT YET THOUGHT OF
shadow matter
mirror symmetry
NOT YET THOUGHT OF
doubly charged Higgs
triplet Higgs
Type I
Type II
general 2HDM
spontaneous CP
superweak
milli-weak
Weinberg's 3HD
Majoron
axion
familon
NGB
saxion
axino
worm-hole
Extra dim
quintessence
k-essence
composite W,Z
contact int.
string
N=2
N=4
N=8
heterotic
matrix
M
IIB
IIA
F
I

Ideas for New Physics since 1970

(1) **Symmetry for elimination of quantum corrections**

– Supersymmetry: Spin statistics $\Rightarrow$ corrections from bosons and fermions cancel each other

– Little Higgs Models: Global symmetries $\Rightarrow$ corrections from particles of like statistics cancel each other

(2) **New Building Blocks, Substructure**

– Technicolor/Topcolor: Higgs bound state of strongly interacting particles

(3) **Nontrivial Space-time structure eliminates Hierarchy**

– Extra Space Dimensions: Gravitation appears only weak

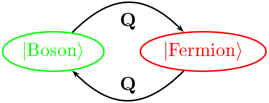
– Noncommutative Space-time: space-time coarse-grained

(4) **Ignoring the Hierarchy**

– Anthropic Principle: Parameters are as we observe them, *since* we observe them

Supersymmetry (SUSY)

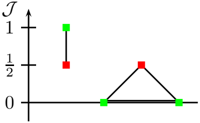
Gelfand/Likhtman, 1971; Akulov/Volkov, 1973; Wess/Zumino, 1974

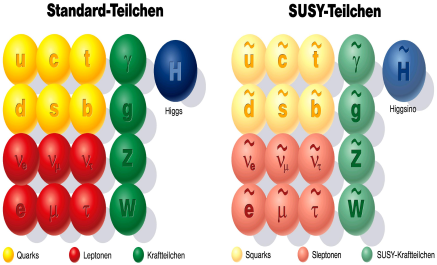


– connects gauge and space-time symmetries

– Multiplets with equal-mass fermions and bosons

⇒ SUSY broken in Nature





– Every particle gets a superpartner

– Minimal Supersymmetric Standard Model (MSSM)

– Mass eigenstates:

Charginos: $\tilde{\chi}^\pm = \tilde{H}^\pm, \tilde{W}^\pm$

Neutralinos: $\tilde{\chi}^0 = \tilde{H}, \tilde{Z}, \tilde{\gamma}$

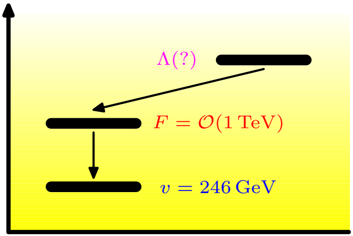
SUSY: Success and Side-Effects

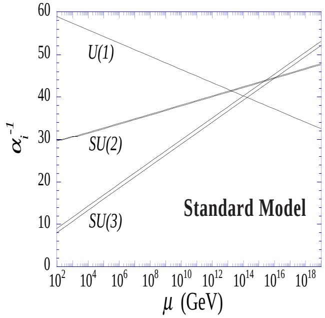
MSSM: spontaneous SUSY breaking ↯ (SUSY partners in MeV range)

Breaking in “hidden sector”

Breaking mechanism induces 100 free parameters

solves hierarchy problem:
$\delta M_H \propto F \log(\Lambda^2)$





- Existence of fundamental scalars
- Form of Higgs potential
- light Higgs ($M_H = 90 \pm 50$ GeV)
- discrete R parity
  - SM particles even, SUSY partners odd
  - prevents a proton decay too rapid
  - lightest SUSY partner (LSP) stable
    Dark Matter $\tilde{\chi}_1^0$
- Unification of coupling constants

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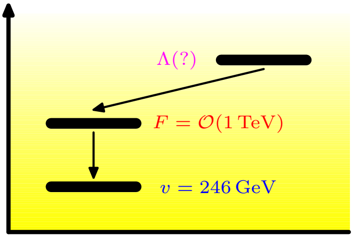

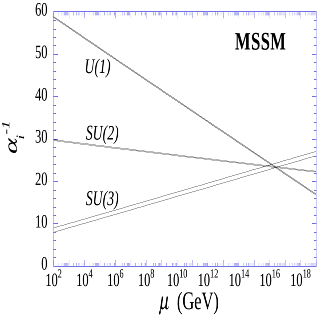

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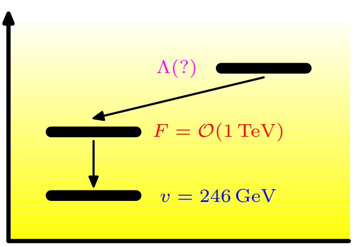

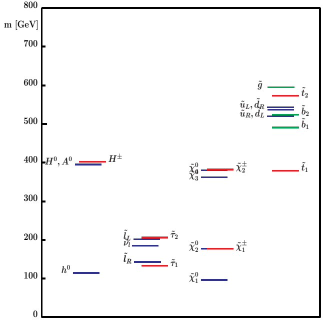

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"SUSY will be discovered, even if non-existent"

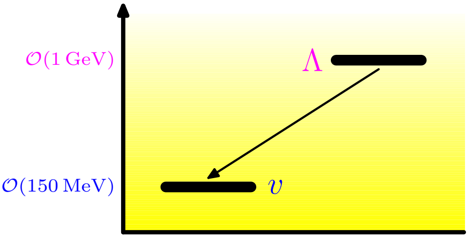
What, if not SUSY?

Higgs as Pseudo-Goldstone boson: Technicolor

Nambu-Goldstone Theorem: Spontaneous breaking of a global symmetrie: spectrum contains massless (Goldstone) bosons 1960/61

Color: Adler/Weisberger, 1965; Weinberg, 1966-69

Light pions as (Pseudo-)Goldstone bosons of spontaneously broken chiral symmetry



Skala Λ: chiral symmetry breaking,
Quarks, $SU(3)_C$

Scale v: pions, kaons, ...

Higgs as Pseudo-Goldstone boson: Technicolor

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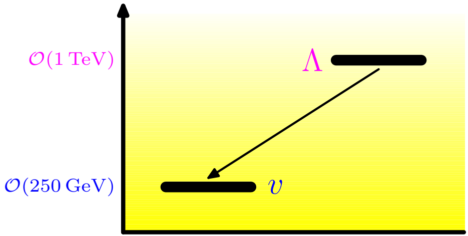
Technicolor: Georgi/Pais, 1974; Georgi/Dimopoulos/Kaplan, 1984

Light Higgs as (Pseudo)-Goldstone boson of a new spontaneously broken chiral symmetry

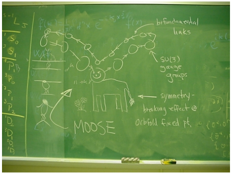


Skala Λ: chiral symmetry breaking, techni-quarks, $SU(N)_{TC}$

Skala v: Higgs, techni-pions

experimentally constrained, but not ruled out

Collective Symmetry Breaking, Moose Models



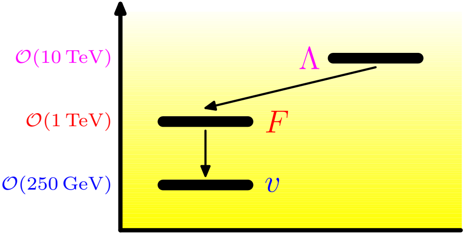
Collective Symmetry Breaking:

Arkani-Hamed/Cohen/Georgi/Nelson/..., 2001

> 2 different global symmetries; if one were unbroken
> $\Rightarrow$ Higgs *exact* Goldstone boson

Higgs mass only by quantum corrections of 2. order:

$$M_H \sim (0.1)^2 \times \Lambda$$

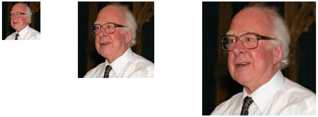


Scale Λ: chiral SB, strong interaction

Scale F: Pseudo-Goldstone bosons, new gauge bosons

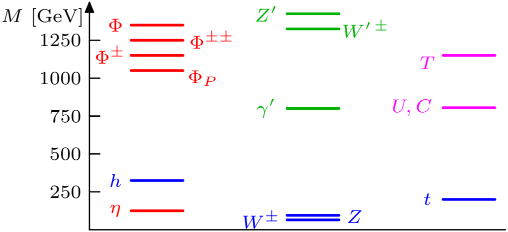
Scale v: Higgs

Little-Higgs Models



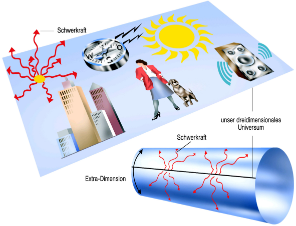
- Economic implementation of collective symmetry breaking
- New Particles:
  - Gauge bosons: $\gamma', Z', W'^{\pm}$
  - Heavy Fermions: $T, U, C, \ldots$
  - Quantum corrections to M_H cancelled by particles of like statistics

Littlest Higgs Arkani-Hamed/Cohen/Katz/Nelson, 2002



- "Little Big Higgs": Higgs heavy $(300 - 500\,\text{GeV})$
- discrete T-(TeV scale) parity:
  - allows for new light particles
  - Dark matter: LTOP (lightest T-odd), often γ'

Extra Dimensions & Higgsless Models



Motivation: String theory

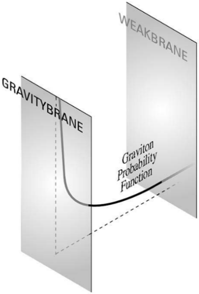
$3+n$ Space dimensions: Radius $R \sim 10^{\frac{30}{n}-17}$ cm Antoniadis, 1990; Arkani-Hamed/Dimopoulos/Dvali, 1998

Gravitation strong in higher dimensions

Particles in quantum well: Kaluza-Klein tower

Production of mini Black Holes at LHC

- "Higgsless Models": Higgs component of higher-dim. gauge field
- "Large Extra Dimensions": continuum of states
- "Warped Extra Dimensions": discrete, resolvable resonances Randall/Sundrum, 1999
- "Universal Extra Dimensions": also fermions/gauge bosons in higher dimensions



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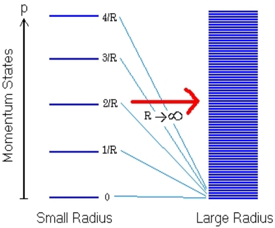

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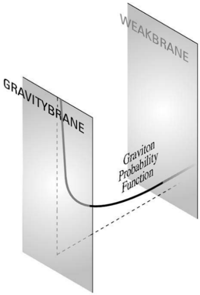
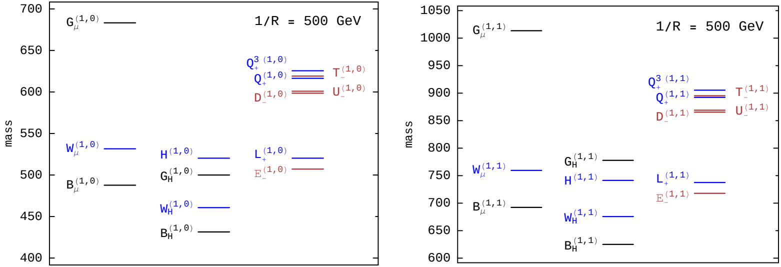


KK parity and Dark Matter

- typical Kaluza-Klein spectra



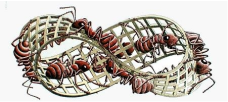
- Spectrum structure similar to SUSY, but shifted in spin

- Dark matter: lightest KK-odd particle (LKP)
  Photon resonance γ' (in 5D vector, in 6D scalar)

- Quote from SUSY orthodoxy:
  "This is a strawman's model invented with the only purpose to be inflamed to shed light on the beauty of supersymmetry!"

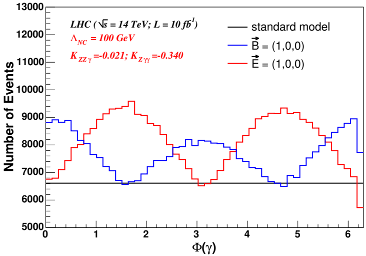
Noncommutative Space-time

Wess et al., 2000



- Assumption: non-commuting Space-time coordinates $[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}$
- Classical analogue: charged particle in lowest Landau level: $\{x_i, x_j\}_P = 2c(B^{-1})_{ij}/e$
- Low energy limit of string theory   Seiberg/Witten, 1999
- Yang-Landau-Theorem violated: $Z \to \gamma\gamma, gg$ possible
- Special direction in the Universe: broken rotational invariance
- Cross sections depend on azimuth

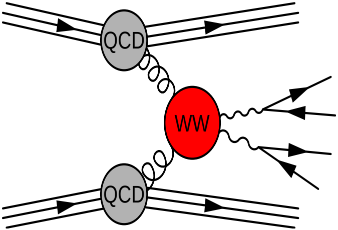
$\Rightarrow$ Varying signals as Earth rotates



- Dark Matter, cosmology, theoretical problems ↯

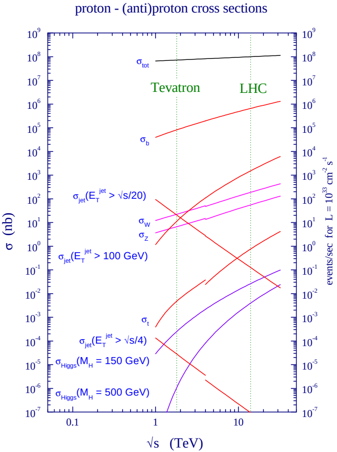
Which model? A Conspiracy Unmasked

The Challenge of the LHC



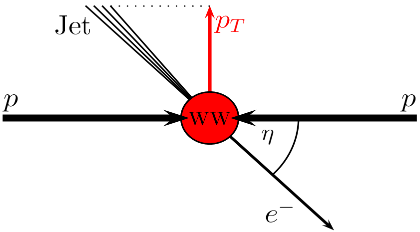
Partonic subprocesses: qq, qg, gg

no fixed partonic energy



$R = \sigma \mathcal{L} \qquad \mathcal{L} = 10^{34}\,\text{cm}^{-1}\text{s}^{-1}$

High rates for t, W/Z, H, $\Rightarrow$ **huge backgrounds**



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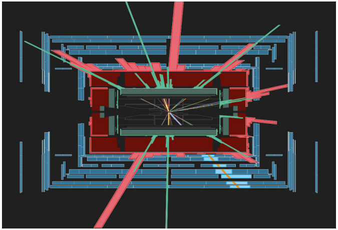

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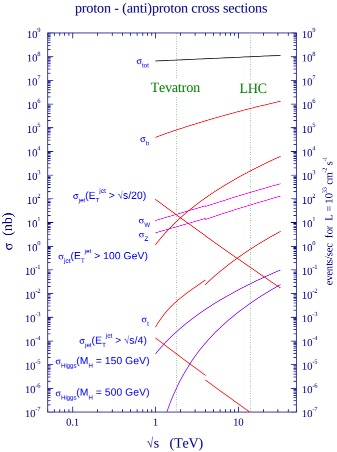

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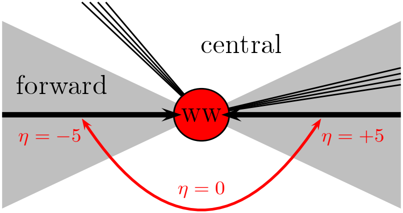
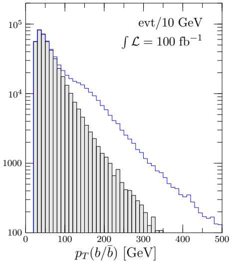


Search for New Particles



Decay products of heavy particles:

- high-p_T Jets
- many hard leptons

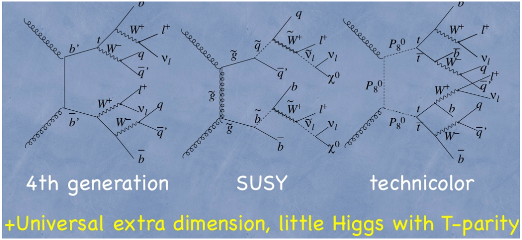
Production of coloured particles

weakly interacting particles only in decays

Dark Matter ⇔ discrete parity (R, T, KK)

- only pairs of new particles ⇒ high energies, long decay chains
- Dark Matter ⇒ large missing energy in detector ($\not{E}_T$)

Different Models/Decay Chains — same signatures



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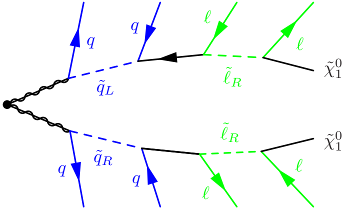

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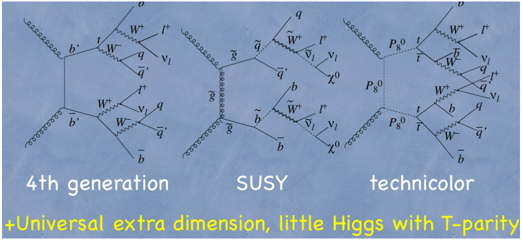
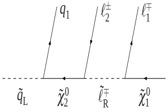
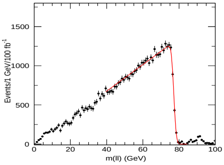
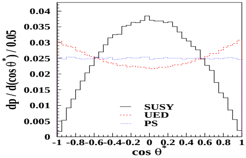


Model Discrimination – A Journey to Cross-Roads

- Mass of new particles: end points of decay spectra







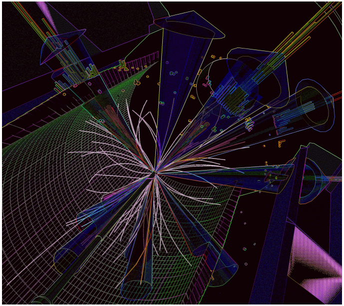
- Spin of new particles: Spin of new particles: angular correlations, . . .
- Model determination: measuring coupling constants

⇒ Precise predictions for signals and backgrounds

- kinematic cuts
- Exclusive multi particle final states $2 \to 4$ up to $2 \to 10$
- Quantum corrections: real and virtual corrections

Outlook

- LHC: new era of physics
- New Particles, new symmetries, new interactions
- Dark Matter
- Interesting times!



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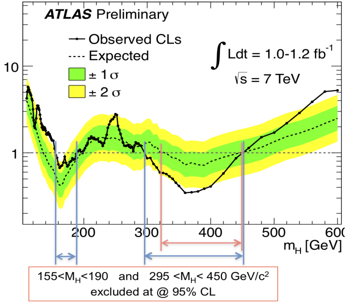

"Will man nun annehmen, dass das abstrakte Denken das Höchste ist, so folgt daraus, dass die Wissenschaft und die Denker stolz die Existenz verlassen und es uns anderen Menschen überlassen, das Schlimmste zu erdulden. Ja es folgt daraus zugleich etwas für den abstrakten Denker selbst, dass er nämlich, da er ja doch selbst auch ein Existierender ist, in irgendeiner Weise distrait sein muss."

Søren Kierkegaard

One Ring to Find Them, One Ring to Rule Them Out?