Physics at $e^+e^-$ Colliders

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• Introduction
• Achievements with LEP, SLC
• Physics beyond the Standard Model: supersymmetry
• Techniques at the high-energy $e^+e^-$ collider
• ILC physics potential in view of LHC expectations
• Summary and some literature for further studies
Few words before ...

- You heard already a lot about
  - how e+e- colliders work
  - how they are limited
  - how the physics is detected
  - how we describe the physics theoretically
  - summary on physics issues

I do not want to repeat the things, therefore I will focus on only a few physics topics (top, Higgs, SUSY, ED) and a few technical tools (threshold scans, continuums measurements, beam polarization)
- Discussion: calculate problems together + all your questions....
**Introduction**

Characteristics of pp collider composite particles collide
\( E(\text{CM}) < 2 \ E(\text{beam}) \)
strong interaction in initial state
superposition with spectator jets
LHC: \( \sqrt{s} = 14\text{TeV} \),
used \( \hat{s} = x_1 x_2 s \) few TeV
small fraction of events analyzed
multiple triggers
`no' polarization applicable

and of the \( e^+e^- (\gamma e, \gamma \gamma) \) collider
Pointlike particles collide
\( E(\text{CM}) = 2 \ E(\text{beam}) \)
well defined initial state
clean final state
ILC: \( \sqrt{s} = 500 \text{GeV} -- 1 \text{ TeV} \)
most events in detector analyzed
no triggers required
polarized initial beams possible

Large potential for direct discovery

Large potential for discovery via high precision

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Discoveries at e+e- colliders

• Of course, also direct discoveries happened at e+e- colliders:

J/ \Psi at SPEAR at SLAC (1974)

Gluons at PETRA at DESY (1979)

• famous ‘3 jet events’
The unique advantage of $e^+e^-$

- Their clean signatures allow precision measurements
  - Sensitive to the theory at quantum level (i.e. contributions of virtual particles, ‘higher orders’)

- Such measurements allow predictions for further, still undiscovered particles, but whose properties are defined by theory at quantum level
Predictions of top mass

• Predicted discovery of the top quark at the Tevatron 1995:
• The history of physics is full of predicted discoveries:
• $e^+$, $\pi$, $\eta$, $q$, $g$, $W$, $Z$, $c$, $b$, $t$
• Future examples: Higgs, SUSY ??? -- see later
**Interplay: hadron and e+e- colliders**

- The interplay between electron and hadron machines has a long and fruitful tradition
  - $J/\psi$ at SPEAR (e+e−) and AGS (proton fixed target)
  - $\Upsilon$ discovery at E288 (p fixed target), precision B studies at the e+e− B factories
  - top quark at LEP and Tevatron

- To be continued in the form of LHC and ILC -- see examples later
History from LEP: results, techniques

‘LEP Tunnel’ = now ‘LHC tunnel’
Some LEP data

- Circumference 27 km
- $\sqrt{s}$ 91.2 GeV (LEP1) to 209 GeV (LEP2)
- Accelerating Gradient Up to 7MV/m (Superconducting cavities)
- Number of Bunches $4 \times 4$
- Current per Bunch $\approx 750$ $\mu$A
- Luminosity at LEP1 $24 \times 10^{30}$ cm$^{-2}$ s$^{-1}$ ($\approx 1$ $Z^0$ s$^{-1}$)
- Luminosity at LEP2 $50 \times 10^{30}$ cm$^{-2}$ s$^{-1}$
  ($\approx 3$ W+W$^-$ h$^{-1}$)
- Interaction regions 4 (ALEPH, DELPHI, L3, OPAL)
- Energy calibration $< 1$ MeV (at $Z^0$)
1990 – $\approx 91$ GeV
1995 5 Million $Z^0$/exp.
1995 Test phase for LEP2 130GeV
1996 161 – 172 GeV
WW-Threshold
1997 183 – 209 GeV
2000 10 000 WW-pairs/exp.

Searches for new physics
0 (?) Higgs bosons

LEP was shut down and dismantled to make room for LHC in Nov. 2000

Integrated Luminosities

LEP was dismantled to make room for LHC in November 2000 … now first injection
The Basic Process at LEP1

- **$Z^0$ lineshape**: $Z^0$ mass, $Z^0/\gamma$-interference
- **Number of neutrinos**, etc.
- **Precision tests of the QFD**: forward-backward asymmetries
- **Precision tests of QCD**: Confirmation of SU(3)
- **Together with $m_W$**: Prediction of the top quark mass
- Many other precision tests of the SM
- **Very successful**: more than 2400 publications from 4 collaborations!
First Z - event

• $e^+e^- \rightarrow Z \rightarrow q\bar{q}$  (13.8.89 !)
  - Tracking chambers not yet fully operational, therefore only ECAL
• $Z^0$ gives a dramatic resonance

• cross section well described (at quantum level, not only at tree level!)
Differential cross section (tree level)

\[
\frac{d\sigma}{d\Omega} = N_c \frac{\alpha_{em}^2}{4s} \left\{ (1 + \cos^2 \theta) \left[ Q_f^2 - 2\chi_1 v_e v_f Q_f - \chi_2 (a_e^2 + v_e^2)(a_f^2 + v_f^2) \right] + 2 \cos \theta \left[ -2\chi_1 a_e a_f Q_f + 4\chi_2 a_e a_f v_e v_f \right] \right\}
\]

\[
\chi_1 = \frac{s(s - M_Z^2)}{16 \sin^2 \theta_W \cos^2 \theta_W \left( (s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \right)}
\]

\[
\chi_2 = \frac{s^2}{256 \sin^4 \theta_W \cos^4 \theta_W \left( (s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \right)}
\]

\[
a_e = -1; \quad v_e = -1 + 4 \sin^2 \theta_W; \quad a_f = 2l_f; \quad v_f = 2l_f - 4Q_f \sin^2 \theta_W
\]
Z^0 Mass Measurement

- Very important input to SM fits!
- Uncertainty is only $\Delta m_Z \sim 2.1$ MeV
- Important to understand systematics of the beam energy measurement!
Systematics: Beam Energy Measurement

- Uncertainty is only 1MeV!
- Further systematics have been: water level, tides, TGV
- Remark: polarization not used for physics, but for calibration!
\( Z^0 \) branching ratios: neutrinos

- SM makes precise predictions for the branching ratios of the \( Z^0 \)

\[
\Gamma_{\nu\nu} = \frac{G_F M_Z^3}{12\pi\sqrt{2}} \approx 162 \text{ MeV}
\]

\[
\Gamma_{ee} = \Gamma_{\mu\mu} = \Gamma_{\tau\tau} = 4 \sin^4 \theta_W \Gamma_{\nu\nu} \approx 84 \text{ MeV}
\]

\[
\Gamma_{uu} = \Gamma_{cc} = 3 \left( \frac{32}{9} \sin^4 \theta_W - \frac{8}{3} \sin^2 \theta_W + 1 \right) \Gamma_{\nu\nu} \approx 287 \text{ MeV}
\]

\[
\Gamma_{dd} = \Gamma_{ss} = \Gamma_{bb} = 3 \left( \frac{8}{9} \sin^4 \theta_W - \frac{4}{3} \sin^2 \theta_W + 1 \right) \Gamma_{\nu\nu} \approx 370 \text{ MeV}
\]

(here: neglecting the quark masses)

- How can we measure the \( \Gamma \), especially \( \Gamma_{\nu\nu} \)?

  \( \text{measure ‘invisible’ events!} \) (also important for SUSY, see later)
Counting neutrinos via photons!

- Using radiative neutrino production:
  - leads to signal only in ECAL
Fitting the cross section:

- Fit prefers 3 families
  - but rather large error

- Some theory assumptions
  - but better than nothing…
Other method for counting neutrinos

• Measuring the total width of the Z-boson

\[ \Gamma_{\text{tot}} = \Gamma_{\ell\ell} + \Gamma_{qq} + N_{\text{fam}} \Gamma_{\nu\nu} \]

- Total width depends on the number of neutrino families!
- Result:
  \[ N_{\text{fam}} = 2.9841 \pm 0.0083 \]
  - Result before LEP: \( N_{\text{fam}} < 5.9 \)
Exploiting further observables: angular distributions!

- As shown in tree-level formulae before: linear dependence on scattering angle $\cos \theta$
  - a forward-backward Asymmetry $A_{FB}$:
    $$A_{FB} = \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma(\cos \theta > 0) + \sigma(\cos \theta < 0)}$$
  - Pure $A_{FB}$ is better than a fit to the whole distribution, since detector systematics cancels
Measuring $Z^0$ couplings

- Vector- and axial-vector couplings:
  - $g_{Vl} = T_{3l} - 2e\sin^2\theta_W$
  - $g_{Al} = T_{3l}$

$T_{3l} =$ weak isospin
  - $=-1/2$ for $e$
Measuring the ew mixing angle

- Measuring the AFB can be interpreted as measuring $\sin^2 \theta_W$

- Result (only LEP):
  $\sin^2 \theta_W = 0.23221 \pm 0.00029$
  - Result improved by inclusion of other experiments, e.g. SLD (see later)


### Top mass prediction

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Measurement</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{WW}^{(Z)}$ [m$_W$]</td>
<td>0.02758 ± 0.00035</td>
<td>0.02766</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>91.1875</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>2.4957</td>
</tr>
<tr>
<td>$\sigma_{had}$ [nb]</td>
<td>41.540 ± 0.037</td>
<td>41.477</td>
</tr>
<tr>
<td>$R_0$</td>
<td>20.767 ± 0.025</td>
<td>20.744</td>
</tr>
<tr>
<td>$A_{T}^{O/P}$</td>
<td>0.01714 ± 0.00095</td>
<td>0.01645</td>
</tr>
<tr>
<td>$A_{T}(P_c)$</td>
<td>0.1485 ± 0.0032</td>
<td>0.1481</td>
</tr>
<tr>
<td>$R_0$</td>
<td>0.21629 ± 0.00066</td>
<td>0.21586</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.1721 ± 0.0030</td>
<td>0.1722</td>
</tr>
<tr>
<td>$A_{T}^{O,b}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1038</td>
</tr>
<tr>
<td>$A_{T}^{O,c}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0742</td>
</tr>
<tr>
<td>$A_T$</td>
<td>0.923 ± 0.020</td>
<td>0.935</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.670 ± 0.027</td>
<td>0.668</td>
</tr>
<tr>
<td>$A_{T}(SLD)$</td>
<td>0.1513 ± 0.0021</td>
<td>0.1481</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{(T)}$</td>
<td>0.2324 ± 0.0012</td>
<td>0.2314</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>80.396 ± 0.025</td>
<td>80.374</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>2.140 ± 0.060</td>
<td>2.091</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>170.9 ± 1.8</td>
<td>171.3</td>
</tr>
</tbody>
</table>
So far we have done …

• Discussion of LEP1 results, only as an example

• Because of time: not mentioned details from other e^+e^- experiments
  – **SLD**: very important also for $\sin^2\theta_W$ (used polarized beams, see tomorrow)
  – **LEP2**: but also very rich program, as e.g. precision W mass measurement, searches for the Higgs boson, but also for new physics ….negative, so far

• **But why do we need physics beyond the SM and what are the experimental challenges?**
Shortcomings of the Standard Model

- doesn't contain gravity
- doesn't explain neutrino masses
- doesn't have candidate for dark matter
  23% of universe is cold dark matter!
- no unification of gauge couplings possible
- further problem: 'hierachy problem'
  Higgs mass unstable w.r.t. large quantum corrections:

\[ \delta M_H^2 \sim \Lambda^2 \]
The Hierarchy Problem

Consider loop corrections to propagators $\longleftrightarrow$ corrections to masses

$$\Delta(p^2) \sim \frac{1}{p^2 - m^2 + \Sigma(p^2)}$$

Photon self-energy in QED:

$$\Sigma_{\gamma\gamma}(0) = 0$$

Consequence of $U(1)$ gauge invariance of QED $\longrightarrow$ photon stays massless

$$\Delta_{\gamma\gamma}^{-1}(p^2) \rightarrow 0 \text{ for } p^2 \rightarrow 0$$
Hierarchy Problem 2

Electron self-energy in QED:

\[ \gamma \]
\[ e^- \hspace{1cm} e^- \]

for \( \Lambda \to \infty \):

\[ \sum^{ee} \sim m_e \int^{\Lambda} \frac{dk}{k} \to \ln \Lambda \]

\[ \rightarrow \text{logarithmically divergent correction to electron mass } \delta m_e \]

Within QED: divergence can be removed via renormalization
\[ \Rightarrow k \to \infty \text{ possible} \]

QED as effective theory, underlying more fundamental theory at scale \( \Lambda \Rightarrow \text{cutoff scale} \)

For \( \Lambda = M_{PL} \):

\[ \delta m_e \approx 2\frac{\alpha}{\pi} m_e \log(M_{PL}/m_e) \approx 0.2 m_e \]

\[ \rightarrow \text{modest correction, proportional to } m_e \]

reason: chiral symmetry in limit \( m_e \to 0, \psi_e \to \exp(i\gamma_5 \theta)\psi_e \)

\[ \rightarrow \text{breaking proportional to } m_e \text{ symmetry protects } m_e \]
Hierarchy Problem 3

Contribution of heavy fermions to Higgs self-energy:

\[ \Sigma_f^{\phi\phi} \sim -2N(f)\lambda_f^2 \int d^4k \left( \frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right) \]

for \( \Lambda \to \infty \):

\[ \Sigma_f^{\phi\phi} \sim -2N(f)\lambda_f^2 \left( \int \frac{d^4k}{k^2} + 2m_f^2 \int \frac{dk}{k} \right) \sim \Lambda^2 \ln \Lambda \]

\( \text{quadratically divergent!} \)

For \( \Lambda = M_P \):

\( \delta M_\phi^2 \sim M_P^2 \Rightarrow \delta M_\phi^2 \approx 10^{30} M_\phi^2 \) (\( M_\phi \lesssim 1 \text{ TeV} \))

no additional symmetry for \( M_\phi = 0 \), no protection against large corrections

\( \text{in general: scalar masses tend to be near highest theory mass scale} \)

\( \text{hierarchy problem, extreme fine-tuning necessary to get small } M_\phi \)
Hierarchy Problem 4

Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale!

E.g.: Grand unified Theory (GUT): $\delta M_{\Phi}^2 \approx \lambda < v_{GUT}>^2$

Hierarchy problem is not just a problem of the Higgs mass; problem: why is $M_W \ll M_{GUT}, M_{PL}$ why is $V_{Coulomb} \gg V_{Newton}$?

Note however: there is another fine-tuning problem in nature, for which we have no clue so far ---- cosmological constant
**Supersymmetry – intro 1**

- Symmetry between fermions and bosons
  
  \[ Q | \text{boson} > = | \text{fermion} > \quad \text{and} \quad Q | \text{fermion} > = | \text{boson} > \]

- In other words: SM particles have SUSY partners (e.g. \( f_{L,R} \rightarrow \bar{f}_{L,R} \))

**SUSY:**

\[
\sum_{\frac{\phi}{\varphi}} \sim N(\bar{f}) \bar{\lambda}_f \int d^4k \left( \frac{1}{k^2 - m_{f_L}^2} + \frac{1}{k^2 - m_{f_R}^2} \right) + \text{terms without quadratic divergencies}
\]

For \( \Lambda \rightarrow \infty \):
\[
\sum_{\frac{\phi}{\varphi}} \sim 2N(\bar{f}) \bar{\lambda}_f \Lambda^2
\]
Supersymmetry – intro 2

Quadratic divergencies cancel for:

\[ N(\tilde{f}_L) = N(\tilde{f}_R) = N(f) \]

\[ \tilde{\lambda}_f = \lambda_f^2 \]

Complete correction vanishes if furthermore:

\[ m_{\tilde{f}} = m_f \]

For \[ m_{\tilde{f}}^2 = m_f^2 + \Delta^2 \]

\[ \Rightarrow \sum_{f+\tilde{f}} \sim N(f) \lambda_f^2 \Delta^2 + \ldots \]

\[ \text{correction acceptable small if mass splitting is of weak scale} \]

\[ \text{realized if mass scale of SUSY partners} \]

\[ M_{\text{SUSY}} \lesssim 1 \text{ TeV} \]

SUSY at TeV scale provides attractive solution of hierarchy problem
Supersymmetry – intro 3

Symmetry: group of transformations that leave Lagrangian invariant
- generators of the group fulfill certain algebra
- Noether's theorem: symmetries ↔ conservation laws

How to get unification of fundamental interactions?
- electroweak and strong interactions:
  - described by gauge theories: internal symmetries
    - $\gamma, Z, W^{\pm}$: spin 1
- gravity:
  - described by general relativity: invariance under space-time transformations
  - graviton $G$: spin 2
Supersymmetry – intro 4

Haag, Lopuszanski, Sohnius theorem '75:

'no direct symmetry transformations between fields with different integer spins'

particles with different spin in the same multiplet only possible for SUSY theories, Q | boson >= | fermion > and Q | fermion >= | boson >

- symmetry generator Q: fermionic operator, needs to have spin ½
  spin 2 ---> spin 3/2 ---> spin 1
  graviton  gravitino  photon

Q changes spin (behaviour under spatial rotations) by ½

SUSY transformation influences in general both space-time and internal quantum numbers!
Consequences of the SUSY algebra:

- Global SUSY transformation: \( \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2(\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu \)

  constant translation in space-time

- If SUSY transformations are made local:

  space-time transformation differing from point to point

Invariance under local SUSY transformations:

- invariance under local coordinate change

- general relativity

- local SUSY includes gravity, called `supergravity'!
Supersymmetry – intro 6

$Q_\alpha$ changes spin of particle by $\frac{1}{2}$

$Q_\alpha |\text{boson}\rangle = |\text{fermion}\rangle$ and $Q_\alpha |\text{fermion}\rangle = |\text{boson}\rangle$

Consider fermionic state $|f\rangle$ with mass $m$:

$P^2 |f\rangle = m^2 |f\rangle$

Bosonic state: $|b\rangle = Q_\alpha |f\rangle$

$P^2 |b\rangle = P^2 Q_\alpha |f\rangle = Q_\alpha P^2 |f\rangle = Q_\alpha m^2 |f\rangle = m^2 |b\rangle$

For each fermionic state there is a bosonic state with the same mass.

states are paired bosonic $\leftrightarrow$ fermionic

Experimentally excluded, so SUSY must be broken symmetry!
**Soft SUSY Breaking**

**Exact SUSY:** \( m_f = m_f \) \( \rightarrow \) SUSY must be broken in nature!

**Only way for model of SUSY breaking:**
- spontaneous SUSY breaking

Specific SUSY-breaking schemes yield effective Lagrangian at low energies, which is supersymmetric except for explicit soft breaking terms.

**Soft SUSY-breaking terms:** do not alter dimensionless couplings (i.e. dimension of coupling constants of soft SUSY-breaking terms \( > 0 \))

\( \rightarrow \) no quadratic divergences (in all orders of perturbation theory)

Scale of SUSY-breaking terms:

\[ M_{\text{SUSY}} \leq 1 \text{ TeV} \]
Free parameters in the MSSM

- mass matrices are 3 x 3 hermitian
  \[ m_Q^2, m_U^2, m_d^2, m_L^2, m_e^2 \] : 45 parameters

- gaugino masses \( M_1, M_2, M_3 \) are complex numbers: 6

- trilinear couplings \( a_u, a_d, a_e \) are 3 x 3 complex matrices: 54

- bilinear coupling \( b \) is 2 x 2 matrix: 4

- Higgs masses \( m_{Hu}^2, m_{Hd}^2 \) : 2

altogether 111 parameter ???

Symmetries (lepton + baryon number, Peccei-Quinn, R symmetry) lead to 'rotations':

- 4 non-trivial field redefinitions

- 2 in the Higgs sector (since minimal model only 2 parameters in the Higgs sector)

  remain 105 free new parameters in the MSSM!
Unconstrained MSSM

- no particular SUSY breaking mechanism assumed, parametrization of possible soft SUSY-breaking terms
  - relations between dimensionless couplings unchanged
  - no quadratic divergencies

- most general case:
  - 105 new parameters: masses, mixing angles, phases
    - Good phenomenological description for universal breaking terms (FCNC, etc.)

- Constrained models (e.g. CMSSM, mSUGRA, etc.):
  - assumption on the scheme of SUSY breaking
  - unification assumptions
  - prediction of soft SUSY-breaking terms in terms of small set of parameters

Experimental determination of SUSY parameters: patterns of breaking
Particle content in the MSSM

**Superpartners for Standard Model particles**

\[
\begin{align*}
[u, d, c, s, t, b]_{L,R} & \quad [e, \mu, \tau]_{L,R} & \quad [\nu_e, \mu, \tau]_L & \quad \text{Spin } \frac{1}{2} \\
[\bar{u}, \bar{d}, \bar{c}, \bar{s}, \bar{t}, \bar{b}]_{L,R} & \quad [\bar{e}, \bar{\mu}, \bar{\tau}]_{L,R} & \quad [\bar{\nu_e}, \mu, \tau]_L & \quad \text{Spin 0}
\end{align*}
\]

\[
\begin{align*}
\gamma, Z, H_1^0, H_2^0 & \quad \text{Spin 1 / Spin 0} \\
g, W^\pm, H^\pm & \\
\tilde{g}, \tilde{\chi}_1^{\pm}, \tilde{\chi}_1^0 & \quad \text{Spin } \frac{1}{2}
\end{align*}
\]

**Enlarged Higgs sector:**

- Two Higgs doublets, physical states: \( h^0, H^0, A^0, H^\pm \)

**Breaking of \( SU(2) \times U(1)_Y \) (electroweak symmetry breaking)**

- Fields with different \( SU(2) \times U(1)_Y \) quantum numbers can mix if they have the same \( SU(3)_c, U(1)_{em} \) quantum numbers
**SUSY breaking schemes**

- `Hidden sector`: SUSY breaking
  - `Gravity-mediated`: mSUGRA
  - `Gauge-mediated`: GMSB
  - `Anomaly-mediated`: AMSB

- Visible sector: MSSM

- **SUGRA**: mediating interactions are gravitational
- **GMSB**: mediating interactions are ordinary electroweak and QCD gauge interactions
- **AMSB**: SUSY breaking happens on a different brane in a higher-dimensional theory

**Feature of schemes: lead to 'characteristic' mass spectra**

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Gravity-mediated SUSY breaking

Quantum field theory of supergravity:

- QFT with spin 2 (and spin 3/2) field is not renormalizable
- cannot be extended to arbitrarily high energies
- QFT of supergravity has to be interpreted as effective theory
- contains non-renormalizable terms prop. to inverse powers of $M_{Pl}$

Best candidate for fundamental theory: string theory

SUSY breaking in hidden sector:

- supergravity Lagragian contains non-renormalizable terms that communicate between hidden and visible sector $\sim 1/M_{Pl}$
**Gravity-mediated SUSY**

- **CMSSM -- five independent parameters:**
  - $m_0$: universal scalar mass parameter
  - $m_{1/2}$: universal gaugino mass parameter
  - $A_0$: universal trilinear coupling
  - $\tan \beta$: ratio of Higgs vacuum expectation values
  - $\text{sign}(\mu)$: sign of supersymmetric Higgs parameter

- **mSUGRA, if:** $m_3 = m_0$ and $b = A_0 - 1$

- **Typical CMSSM or mSUGRA features:**
  - lightest stable SUSY particle is the neutralino and is bino-like
  - almost mass degenerated $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ and are mainly wino-like
    - phenomenologically interesting: couple only 'left-handed'
  - light gaugino/higgsino and slepton spectrum but rather heavy coloured particles

**Most studies done in this class of scenarios!**

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Properties of SUSY - Unification

Gauge coupling unification:

- Running of gauge couplings:

\[
\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln \left(\frac{\mu^2}{\mu_0^2}\right)
\]

- Coupling constant unification in MSSM for

Unification of couplings at high scale ↔ ‘Grand unified theories’ (GUT)

- E.g. SO(10) GUTs, can naturally accommodate right-handed neutrinos

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**New quantum number R-parity**

Most general gauge-invariant and renormalizable superpotential with chiral superfields in the MSSM:

\[
\nu = \nu_{\text{MSSM}} + \frac{1}{2} \chi^{ijk} L_i L_j E_k + \chi^{njk} L_i Q_j D_k + \mu^n L_i H_u + \frac{1}{2} \chi^{mjk} U_i D_j D_k
\]

- violates baryon number
- violates lepton number

If **both** lepton+baryon number violated: rapid proton decay!

**Minimal choice contains only terms with even number of SUSY particles → new quantum number R-parity**

(SM=+1, SUSY=-1)

If **conserved**: -- SUSY only in pairs produced

- decays in SM particles + odd number of SUSY particles
- lightest SUSY particle (LSP) has to be stable (cold dark matter?)
- LSP: neutral, uncoloured → signatures with missing energy
Relations between SUSY parameters

Symmetry properties of MSSM Lagragian (SUSY, gauge invariance) give conditions to couplings and mass relations

- z.B. gauge-boson-fermion coupling = gaugino-fermion-sfermion coupling for U(1), SU(2) and SU(3) gauge groups

- In SM: all masses are free input parameters (except $m_w$--$m_z$ interdependence)

- MSSM:

  relations between chargino and neutralino masses (soft breaking+ew breaking)

  sfermion mass relations (gauge invariance): $m_{\tilde{l}}^2 = m_{\tilde{l}}^2 - M_W^2 \cos(2\beta)$

  upper bound on mass of lightest CP-even Higgs boson

- All relations receive contributions from loop effects

  experimental verification of relations is crucial test of SUSY
Prospects of SUSY at future colliders

- **Tevatron**: slightly increased 1.8 -> 2 TeV, but 100 x higher lumi
  - best prospects for trilepton signal: $\tilde{\chi}_2^0\tilde{\chi}_1^+ \rightarrow e^+e^-\tilde{\chi}_1^0\nu\tilde{\chi}_1^0$
  - $\tilde{\tau}$, $\tilde{b}$ searches, light SUSY Higgs in large tan$\beta$ region

- **LHC**: direct production of `couloured' particles $\tilde{q}$, $\tilde{g}$
  - Very large mass range in searches for jets+missing energy up to 2-3 TeV
  - Electroweak-interacting particles as neutralinos/charginos mainly in decays!
  - e.g. at the LHC in cascades: $\tilde{g} \rightarrow \tilde{q}\tilde{q} \rightarrow \tilde{q}q\tilde{\chi}_2^0 \rightarrow \tilde{q}q\tilde{\tau}\tilde{\tau} \rightarrow \tilde{q}q\tau\tau\tilde{\chi}_1^0$
  - Assumption about particle identities in chains
  - Problem: main background of SUSY is SUSY itself!

- **Test of SUSY relations not easy!**

- **ILC**: direct production of all particles up to kinematical limit
  - Clean signatures: precise tests of all SUSY assumptions!
Goals of physics at the Linear Collider

• **Discovery of New Physics (NP)**
  – complementary to the LHC
  – large potential for **direct searches**
  – impressive potential also for **indirect searches via precision**

• **Unraveling the structure of NP**
  – precise determination of **underlying dynamics and parameters**
  – **model distinction** through model independent searches

• **High precision measurements**
  – **test of the Standard Model (SM)** with unprecedented precision
  – even smallest hints of NP could be observed

• **Discovery of new phenomena via high energy and high precision!**
Needed ILC tools for all searches

- **High statistics** needed
  - $L = 2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

- Clean experimental environment
  - *low beamstrahlung* ($\gamma_{ave}=0.048$)
  - precise *luminosity* ($\Delta L < 10^{-3}$) and
    - *energy* ($\Delta \sqrt{s} < 200 \text{ ppm}$) *measurement*

- Excellent detector resolution
  - *b-, c-tagging*, even the charge!
  - *τ-polarization*
  - $4\pi$ – $\epsilon$ angle coverage
  - exploitation of angular distributions
ILC features, cont.

- **Threshold scans**
  - *Tuneable energy* allows to vary energy around the mass threshold of new particles
  - Cost luminosity
  - Optimization of required energy steps a priori possible via rather accurate continuums measurements

- **Beam polarization**
  - Polarized e- with P(e-)~90% expected
  - Polarized e+ with P(e+)~60% (even in baseline ~40% expected !)
  - Enable to reveal underlying structure of new physics
  - Enhance statistics
**Beam polarization at colliders**

- **Polarization** = ensemble of particles with definite helicity $\lambda = \pm \frac{1}{2}$ left- or right-handed:

  \[ P = \frac{\#N_R - \#N_L}{\#N_R + \#N_L} \]

  → beam polarization gives access to the couplings and unravels the structure of interactions

- **Polarized beams at circular e-e+ colliders:**
  - Polarization of both beams via Sokolov-Ternov effect
    (= spin-flip effect due to synchrotron radiation)
  - At LEP (e+e-): massive depolarization effects; low polarization; not used for physics
  - At HERA (ep): excellent e- / e+ polarization reached, ~50%-70%; spin rotators used to produce longitudinally polarized beams for physics studies

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Beam polarization at linear colliders

🌟 Polarized beams at linear e⁻e⁺ colliders:

← synchrotron radiation due to longitudinal acceleration negligible
← beams have to be polarized at the source!

🌟 Polarized e⁻ source:

← at the SLAC Linear Collider (SLC): excellent e⁻ polarization of about 78%
← led to precision measurement of the weak mixing angle:
\[ \sin\theta_{\text{eff}} = 0.23098 \pm 0.00026 \text{ (SLD)} \]
\[ \text{(LEP: } 0.23221 \pm 0.00029\text{)} \]

🌟 Polarized sources at the ILC:

← expected e⁻ polarization between 80% and 90%

\[ e^+ \text{ polarization is an absolute novelty! Expected } P(e^+) \sim 60\% \]
Electron polarization

Remember again: First polarised $e^-$ beam at a LC at SLAC (1992-98)
with $P(e^-) = [60\%, 78\%]$

How did they polarise the $e^-$?
→ circ. polarised light ($I_z = +1$ or $-1$)
on GaAs cathode
⇒ $P^{-1} = \frac{N_+ - N_-}{N_+ + N_-} = \frac{3 - 1}{3 + 1} = +0.5$

How to get higher polarisation?
→ use strained lattice: grow GaAs on substrate with diff. crystal spacing
⇒ removes degeneracy in lower level
If $h\nu = [E_g, (E_g + \delta)]$:
→ in principle $P^{-1} = 100\%$ possible...
⇒ $P^{-1} = 80 - 90\%$ expected at LC
Polarized positrons

- Conventional source: e- scattering in target → pair production → e+
- Undulator-based scheme: polarized e+ via circularly polarized photons

- deviation of e- beam via helical magnetic field in undulator
- radiated circularly polarized photons onto thin target, pair production
- e+ polarization depends on undulator length

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How to describe the spin?

- **Definition:** Basis of Spinvektors \( s^a, a = 1, 2, 3 \) with \((s^a p) = 0\):
  - build ‘right-hand-system’ in the CMS of \( e^-(p_1)e^+(p_2) \rightarrow X(p_3)Y(p_4) \)
  - Longitudinal Spinvektors: \( s^{3\mu}(p_{1,2}) := \frac{1}{m_{1,2}}(|p_{1,2}|, E\hat{p}_{1,2}) \)
  - Transverse Spinvektors: \( s^{2\mu}(p_1) := (0, \vec{p}_1 \times \vec{p}_3), \quad s^{2\mu}(p_2) = s^{2\mu}(p_1) \)
    \( s^{1\mu}(p_1) := (0, \vec{p}_1 \times \vec{s}^{2}(p_1)), \quad s^{1\mu}(p_2) = -s^{1\mu}(p_1) \)

- **Definition:** ‘left-handed’ and ‘right-handed’ \( \equiv \) with respect to \( \hat{p} \)
  - If Spinvektor \( \vec{s}^3 = \begin{cases} \text{parallel } \vec{p} & \text{('right-handed': } P > 0) \\ \text{antiparallel } \vec{p} & \text{('left-handed': } P < 0) \end{cases} \)
Remarks about couplings structure

Definition: Helicity \( \lambda = \frac{s \cdot \vec{p}}{|\vec{p}|} \) ‘projection of spin’

Chirality = handedness is equal to helicity only of \( m=0 \! \)!

Def.: left-handed \( \equiv P(e^\pm) < 0 \) right-handed \( \equiv P(e^\pm) > 0 \)

Which configurations are possible in principle?

s–channel:

\[
\begin{array}{c}
\begin{array}{c}
\text{\( e^+ \)} \\
J=1 \\
\text{\( e^- \)} \\
J=0
\end{array}
\end{array}
\]

\( \leftarrow \text{only from RL,LR: SM (}\gamma, Z) \)

\( \leftarrow \text{only from LL,RR: NP!} \)

\( \Rightarrow \) In principle: \( P(e^-) \) fixes also helicity of \( e^+ \! \)!
General remarks, cont.

Which configurations are possible in the crossed channels?

**t–channel:**

\[ e^+ \rightarrow \text{depends on } P(e^+)! \]

\[ e^- \rightarrow \text{depends on } P(e^-)! \]

\[ a \quad b \quad c \]

⇒ helicity of \( e^- \) not coupled with helicity of \( e^+ \)!

**Two examples:**

a) Single \( W \) production

\[ e^+ \rightarrow \text{only influenced by } P(e^+)! \]

\[ e^- \]

\[ W^+ \]

b) Bhabha scattering

\[ \Rightarrow \gamma, \ Z \text{ exchange in } s\text–channel: \]

selects LR, RL

\[ \Rightarrow \gamma, \ Z \text{ exchange in } t\text–channel: \]

LL, RR possible!

<table>
<thead>
<tr>
<th>unpolarised</th>
<th>4.50 pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{e^-} = -80% )</td>
<td>4.63 pb</td>
</tr>
<tr>
<td>( P_{e^-} = -80%, \ P_{e^+} = -60% )</td>
<td>4.69 pb</td>
</tr>
<tr>
<td>( P_{e^-} = -80%, \ P_{e^+} = +60% )</td>
<td>4.58 pb</td>
</tr>
</tbody>
</table>
Statistical arguments for $P(e^+)$

Polarized cross sections can be subdivided in:

$$
\sigma_{P_e^-P_e^+} = \frac{1}{4} \left\{ (1 + P_e^-)(1 + P_e^+) \sigma_{RR} + (1 - P_e^-)(1 - P_e^+) \sigma_{LL} + (1 + P_e^-)(1 - P_e^+) \sigma_{RL} + (1 - P_e^-)(1 + P_e^+) \sigma_{LR} \right\}.
$$

$\sigma_{RR}$, $\sigma_{LL}$, $\sigma_{RL}$, $\sigma_{LR}$ are contributions with fully polarized L, R beams.

In case of a vector particle only (LR) and (RL) configurations contribute:

$$
\sigma_{P_e^-P_e^+} = \frac{1 + P_e^-}{2} \frac{1 - P_e^+}{2} \sigma_{RL} + \frac{1 - P_e^-}{2} \frac{1 + P_e^+}{2} \sigma_{LR} = (1 - P_e^- P_e^+) \frac{\sigma_{RL} + \sigma_{LR}}{4} \left[ 1 - \frac{P_e^- - P_e^+}{1 - P_e^+ P_e^-} \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}} \right] = \left( 1 - P_e^+ P_e^- \right) \sigma_0 \left[ 1 - P_{\text{eff}} A_{LR} \right],
$$
• Polarized cross section reads: \[ \sigma_{P_-P_+} = (1 - P_+P_-) \sigma_0 [1 - P_{\text{eff}} A_{LR}] \]

the unpolarized cross section: \[ \sigma_0 = \frac{\sigma_{RL} + \sigma_{LR}}{4} \]

the left-right asymmetry: \[ A_{LR} = \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}} \]

and the effect \[ \mathcal{L}_{\text{eff}} = \frac{1}{2} (1 - P_+P_-) \mathcal{L} \]

\[ P_{\text{eff}} = \frac{P_- - P_+}{1 - P_+P_-} \]

• With effective luminosity \[ \mathcal{L}_{\text{eff}} = \frac{1}{2} (1 - P_+P_-) \mathcal{L} \]

\[ \sigma_{P_-P_+} = 2 \sigma_0 \left( \mathcal{L}_{\text{eff}} / \mathcal{L} \right) [1 - P_{\text{eff}} A_{LR}] \]
Effective polarization:

\[ P_{\text{eff}} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^+}P_{e^-}} \]

\[ |P_{\text{eff}}| [\%] \]

\[ P_{e^-} = -90\% \]

\[ P_{e^-} = -80\% \]

\[ P_{e^-} = 70\% \]

\( (80\%, 60\%): P_{\text{eff}} = 95\% \) \( (90\%, 60\%): P_{\text{eff}} = 97\% \) \( (90\%, 30\%): P_{\text{eff}} = 94\% \)
Effective polarization

\[ P_{\text{eff}} := (P_{e^-} - P_{e^+})/(1 - P_{e^-}P_{e^+}) \]
\[ = (#LR - #RL)/(#LR + #RL) \]

Fraction of colliding particles

\[ \mathcal{L}_{\text{eff}}/\mathcal{L} := \frac{1}{2}(1 - P_{e^-}P_{e^+}) = (#LR + #RL)/(#all) \]

Colliding particles:

<table>
<thead>
<tr>
<th>( P(e^-) = 0, ) ( P(e^+) = 0 )</th>
<th>RL</th>
<th>LR</th>
<th>RR</th>
<th>LL</th>
<th>( P_{\text{eff}} )</th>
<th>( \mathcal{L}_{\text{eff}}/\mathcal{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.</td>
<td>0.5</td>
</tr>
<tr>
<td>( P(e^-) = -1, ) ( P(e^+) = 0 )</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>( P(e^-) = -0.8, ) ( P(e^+) = 0 )</td>
<td>0.05</td>
<td>0.45</td>
<td>0.05</td>
<td>0.45</td>
<td>-0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>( P(e^-) = -0.8, ) ( P(e^+) = +0.6 )</td>
<td>0.02</td>
<td>0.72</td>
<td>0.08</td>
<td>0.18</td>
<td>-0.95</td>
<td>0.74</td>
</tr>
</tbody>
</table>

⇒ Enhancing of \( \mathcal{L}_{\text{eff}} \) with \( P(e^-) \) and \( P(e^+) \)!

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How are $P_{\text{eff}}$ and $A_{LR}$ related?

$$A_{LR} = \frac{1}{P_{\text{eff}}} A_{LR}^{\text{obs}} = \frac{1}{P_{\text{eff}}} \frac{\sigma_{zz} - \sigma_{zz}}{\sigma_{zz} + \sigma_{zz}},$$

That means:

$$\left| \frac{\Delta A_{LR}}{A_{LR}} \right| = \left| \frac{\Delta P_{\text{eff}}}{P_{\text{eff}}} \right|$$

With pure error propagation (and errors uncorrelated), one obtains:

$$\frac{\Delta P_{\text{eff}}}{P_{\text{eff}}} = \frac{x}{(|P_{e+}| + |P_{e-}|)(1 + |P_{e+}||P_{e-}|)} \sqrt{(1 - |P_{e-}|^2)^2 P_{e+}^2 + (1 - |P_{e+}|^2)^2 P_{e-}^2}$$

With

$$x \equiv \Delta P_{e-}/P_{e-} = \Delta P_{e+}/P_{e+}$$
(80%, 60): $P_{\text{eff}} = 95\%$

$\frac{\Delta A_{LR}}{A_{LR}} = 0.3$

gain: factor~3

(90%, 60%): $P_{\text{eff}} = 97\%$

$\Delta A_{LR}/A_{LR} = 0.27$

factor>3

(90%, 30%): $P_{\text{eff}} = 94\%$

$\Delta A_{LR}/A_{LR} = 0.5$

factor~2

NO gain with only polarized $e^-$!
**Background suppression**

$WW$, $ZZ$ production = large background for NP searches!

$W^-$ couples only **left-handed**:  
$\rightarrow WW$ background strongly suppressed with right polarized beams!

**Scaling factor** $= \sigma^{pol}/\sigma^{unpol}$ for $WW$ and $ZZ$:

\[
\begin{array}{|c|c|c|}
\hline
P_{e^-} = \pm 80\%, P_{e^+} = \pm 60\% & e^+e^- \rightarrow W^+W^- & e^+e^- \rightarrow ZZ \\
\hline
(+0) & 0.2 & 0.76 \\
(-0) & 1.8 & 1.25 \\
(+-) & 0.1 & 1.05 \\
(-+) & 2.85 & 1.91 \\
\hline
\end{array}
\]
Back to the ILC physics case...

• But since the ILC can not start before 2015+, all physics issues have to be seen in view of expected LHC results

• In the following we discuss several physics topics, starting at 500 GeV, 1TeV, multi-TeV

• Applying the mentioned tools, threshold scans, beam polarization, precision measurements

• But only a personal selection of examples ……
Physics up to $\sqrt{s}=500$ GeV: top

Current average: $m_{\text{top}} = 172.4 \pm 1.2$ GeV

Expectations at the LHC:

$\Delta m_{\text{top}} \sim 1$ GeV

Yukawa couplings $\sim 20\%$ (with slight model assumptions)

Expectations at the ILC:

Mass via threshold scans: $m_{\text{top}} \sim 100$ MeV (dominated by theory)

Yukawa couplings via $t\,t\,H$: difficult due to small rates, but $< 20\%$

Unique access to electroweak couplings

Why are top properties so important?

Heaviest detected elementary particle up to now

Opens unique window to physics beyond the SM
Top mass, 2

Additional problem for $m_t$: what is the mass of a coloured object?

Top pole mass is not “IR safe” (affected by large long-distance contributions), cannot be determined to better than $\mathcal{O}(\Lambda_{QCD})$

Current exp. error on $m_t$ from the Tevatron: $\delta m_t^{\text{exp}} = 1.2 \text{ GeV}$

Which mass is actually measured at the Tevatron and the LHC?

Measured mass should be “close to the pole mass”, but how close?

Issue not yet settled, effects of $\mathcal{O}(\Gamma_t)$ are not fully under control
**Top mass 3**

From running at the $t\bar{t}$ threshold:

- Measurement of a “threshold mass parameter” with high precision: $\lesssim 20 \text{ MeV}$
- Transition to suitably defined (short-distance) top-quark mass, e.g. $\overline{\text{MS}}$ mass

We expect at the LC:

- $\delta m_t^{\text{exp}} \lesssim 100 \text{ MeV} \ (\text{dominated by theory uncertainty})$
Importance of ‘top’ mass

Top mass is important input parameter for electroweak precision tests

- SM prediction for $m_W$ and $\sin^2 \theta_{\text{eff}}$: consistency checks, sensitivity to $m_{\text{Higgs}}$
- compare $m_W$ and $\sin^2 \theta_{\text{eff}}$: experimental accuracy with theoretical prediction

Theoretical uncertainties

1. unknown higher orders: $\Delta \sin^2 \theta_{\text{eff}}^{\text{ho}} \sim 5 \times 10^{-5}$, $\Delta m_W^{\text{ho}} \sim 4$ MeV

High precision of top mass mandatory to exploit theory at quantum level!

If $\Delta m_{\text{top}} \sim 1$ GeV (LHC): $\Delta \sin^2 \theta_{\text{eff}}^{\text{input}} \sim 3 \times 10^{-5}$, $\Delta m_W^{\text{input}} \sim 6$ MeV

If $\Delta m_{\text{top}} \sim 0.1$ GeV (ILC): $\Delta \sin^2 \theta_{\text{eff}}^{\text{input}} \sim 0.3 \times 10^{-5}$, $\Delta m_W^{\text{input}} \sim 1$ MeV
**Electroweak symmetry breaking / Higgs**

- Where do we expect the Higgs?

  \[ M_h < 186 \text{ GeV} \]

  (LEP, SLD, CDF, D0 + LEP-2 direct limit)

- Light Higgs expected but heavier

  SM-Higgs not excluded!

- SUSY Higgs < 135 GeV!

- 'Higgs' task for the LC:
  - mass measurement, spin verification, couplings determination

---

Establish the mechanism of electroweak symmetry breaking!
**Determination of Higgs properties**

**Expectations at the LHC:**
- Higgs mass: up to $\Delta m_H = 100-200$ MeV
- Higgs couplings: 15%-40% (with some model assumptions)

**Expectations at the ILC:**
- absolute couplings: 1.5%
- Establishing of ew sym. breaking: triple Higgs couplings at 500 GeV up to 22%
- estimate: further gain of 30%-50% precision if both beams polarized
- process $ttH$: difficult due to small rates (but threshold effects!)
- accuracy about 24% for $mH=120$ GeV (unpolarized beams)
- improvement factor 2.5 when (80%, 0%) -> (80%, 60%)

**LHC input for optimal choices of running scenarios**

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Dominant production mechanisms: Higgsstrahlung and WW-fusion

Higgs mass at ILC

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Higgs mass, 2

• Use Higgsstrahlung: due to well-known initial state and well-observed Z-decays
  – Derive Higgs mass independently from decay

  – Only possible at a LC!
Higgs properties

Spin verification
- threshold scans (i.e. at $\sqrt{s}=205-300$ GeV)
  mainly needed for spin verification
- due to excellent masses from continuum,
  only about 3 energy steps needed

Parity Measurement
- in $H \tau\tau$ decay
- distinguish between CP-odd and even via angular distributions
- independent from production process
Technical requirements – Higgs sector

- Couplings determination: high rates and lumi needed

- measurement of couplings in Higgs-strahlungs process at $\sqrt{s}=350$ GeV

- beam polarization (80%,0) → (80%, 60%): improvement by about 30%

- triple Higgs couplings: e.g. in HHZ at $\sqrt{s}=500$ up to 22% (unpolarized beams)

- estimate: further gain of 30%-50% precision if both beams polarized

Polarized $e^+$ very useful even in Higgs physics (factor 4 in separation, 30% in couplings, etc.), in particular at $\sqrt{s}=350$ GeV and 500 GeV

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New Physics -- Supersymmetry

Remember: free parameters in the MSSM:

- Mass matrices are 3 x 3 Hermitian:
  \[ m_Q^2, m_u^2, m_d^2, m_L^2, m_e^2 : 45 \] parameters

- Gaugino masses \( M_1, M_2, M_3 \) are complex numbers: 6

- Trilinear couplings \( a_u, a_d, a_e \) are 3 x 3 complex matrices: 54

- Bilinear coupling \( b \) is 2 x 2 matrix: 4

- Higgs masses \( m_{H_u}^2, m_{H_d}^2 \) : 2

  Altogether 111 parameter ???

Symmetries (lepton + baryon number, Peccei-Quinn, R symmetry) lead to 'rotations':

- 4 non-trivial field redefinitions

- 2 in the Higgs sector (since minimal model only 2 parameters in the Higgs sector)

  Remain 105 free new parameters in the MSSM!

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**DisneyWorld of SUSY scenarios**

- Often (ab)used: Manhattan plots

- 13 SUSY ‘benchmarks’ scenarios out of millions ... 
  *really a true representative choice?*

- heavy masses often mass degenerated: no resolution 
  (beamstrahlung!) has been taken into account... 
  *really a reliable ‘counting’?*

- experimental verification of properties not studied ... 
  *really a useful basis for future decisions?*

**Physics or just propaganda?**
**SUSY scale expectations**

In which range do we expect SUSY?

- at least **some light particles** should be accessible at 500 GeV
- **best possible tools** needed to get **maximal information** out of only the part of the spectrum

To reveal the structure of the underlying physics, it is important to determine the parameters in a model-independent way and test all model assumptions experimentally

Soon we will have LHC data, but LHC/ILC interplay will be essential and both machines cover a large range of the parameter space!

Ellis, Heinemeyer, Olive, Weber, Weiglein '07

---

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Discovery of SUSY

- What's needed for establishing SUSY?
  - Spin verification: via analysis of angular distributions
  - Couplings measurement: Yukawa couplings = gauge couplings
  - Precise mass measurements
  - Unraveling the SUSY breaking mechanism and test unification
  - 'Model-independent' determination of the parameters (105 already in the MSSM!)

Expectations at the LHC:

- Coloured SUSY partners: discovery reach $m_{q,g} < 2-2.5$ TeV
- Non-coloured partners: a) via Drell-Yan $m_\chi < 250$ GeV
  b) via cascade decay chains
- Parameter determinations: in specific SUSY breaking models

Particularly promising field for LHC/ILC interplay studies!

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SUSY mass determinations at the LHC

Analysis of cascade decays:

\[ \tilde{q}_L \rightarrow q \tilde{\chi}_2^0 \rightarrow \ell_2^\pm \tilde{\ell}^\mp \rightarrow \ell_1^\mp \tilde{\chi}_1^0 \]

\[ \Rightarrow \text{Mass determination from kinematical endpoints} \]

invariant mass distribution:

\[ m_{\ell\ell}^{\text{max}} \sim \left( \frac{m_{\tilde{\chi}_0}^2 - m_{\ell}^2}{m_{\tilde{\chi}_2}^2} \right) \left( m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_0}^2 \right) \]

\[ m_{\tilde{\ell}\ell} \]

\[ m_{\tilde{\chi}_0} \]

\[ m_{\ell} \]

\[ m_{\ell}\tilde{\ell} \]

\[ \Rightarrow \text{Difference of masses are measured} \]

\[ \Rightarrow \text{strong dependence on the LSP mass} \]

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**SUSY mass measurement in continuum**

To optimize threshold scans: precise continuum measurements important!

Worst SM background is WW-pair production

- e.g. \( e^+e^- \rightarrow \tilde{\mu}_{L,R}^+ \tilde{\mu}_{L,R}^- \)

**Strong WW-backgr.:**

- all edges observable only with \( P(e^-) \) and \( P(e^+) \)
- at 65 GeV and 220 GeV

\[
\frac{S}{B} = 0.07 \ (+80\%, 0)
\]

\[
\frac{S}{B} = 0.46 \ (+80\%, -80\%)
\]

\[\Delta (m_{\tilde{\mu}_{L,R}}) \sim \text{few GeV if both beams are polarized}!\]
Mass measurement of the LSP mass

- A promising cold dark matter candidate = lightest SUSY particle (LSP)
  - in many scenarios: \( \tilde{\chi}_1^0 \)
  - excellent mass resolution e.g. in slepton decays \( \tilde{\mu}_R \tilde{\mu}_R \rightarrow \mu \mu \tilde{\chi}_1^0 \tilde{\chi}_1^0 \)

\[ \Delta m_{\tilde{\chi}_1^0} \text{ up to 0.3\%, here 100 MeV!} \]

Further improvement in mass measurements via threshold scans possible!
- costs luminosity, therefore should be optimized via excellent measurements in the continuum

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Test off spin quantum number at ILC

- Clean signatures, known initial state, tunable energy:

\[
\Delta m_{\tilde{\mu}_R} < 1 \times 10^{-3} \text{ GeV}
\]

\[
\Rightarrow \text{ test of } J = 0 \text{ hypothesis}
\]

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One more SUSY Test at the ILC

Test of SUSY assumption: SM ↔ SUSY have same quantum numbers!

⇒ $\tilde{e}_{L,R} \leftrightarrow \tilde{e}_{L,R}$ and $\tilde{e}_{L,R} \leftrightarrow \tilde{e}_{R,L}$

Scalar partners ↔ chiral quantum numbers!

How to test this association?

Strategy: $\sigma(e^+e^- \rightarrow \tilde{e}_{L,R}^+\tilde{e}_{L,R}^-)$ with polarised beams

⇒ 2nd diagram: unique relation between chiral fermion ↔ scalar partner

Use e.g. $\tilde{e}_{L}^+e_{L}^-$

⇒ no annihilation diagram
Chiral quantum numbers, 2

Association of chiral electrons to scalar partners $\ell_{L,R}^-, \ell_{R,L}^-$ and $\ell_{L,R}^+, \ell_{R,L}^+$:

s-channel  t-channel

1. separation of scattering versus annihilation channel
2. test of 'chirality': only $\ell_L^+ \ell_R^-$ may survive at $P(e^-) > 0$ and $P(e^+) > 0$!

Even high $P(e^-)$ not sufficient, $P(e^+)$ is substantial!

DESY Summer Program 2009
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**LHC/ILC interplay**

- If fundamental parameters determined: allows mass predictions for heavier particles

  - significant increase of sensitivity for searches at the LHC and unique identification of particles in decay chain

  - powerful test of the model and distinction between e.g. MSSM vs. NMSSM model!

<table>
<thead>
<tr>
<th></th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$\mu$</th>
<th>$\tan \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>input</td>
<td>99.1</td>
<td>192.7</td>
<td>352.4</td>
<td>10</td>
</tr>
<tr>
<td>LC$_{500}$</td>
<td>99.1 ± 0.2</td>
<td>192.7 ± 0.6</td>
<td>352.8 ± 8.9</td>
<td>10.3 ± 1.5</td>
</tr>
<tr>
<td>LHC+LC$_{500}$</td>
<td>99.1 ± 0.1</td>
<td>192.7 ± 0.3</td>
<td>352.4 ± 2.1</td>
<td>10.2 ± 0.6</td>
</tr>
</tbody>
</table>

- **Strong improvement in parameter determination via LHC/ILC interplay!**

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Dark matter analysis at LC

- High precision in parameter determination required for reliable DM prediction
  - Parameter ranges where abrupt changes of neutralino character happen

- Precise determination of $M_1, M_2$...required
SUSY model distinction

- SUSY scenario in the NMSSM: Higgs and light particle sector (neutralino / chargino ) show no hints for model distinction

- measured at ILC (500 GeV): \( m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_{1,2}^0}, \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \tilde{\chi}_1^0 \tilde{\chi}_2^0) \)

  - Consistent within MSSM-analysis

  - Predictions:

    \[
    m_{\tilde{\chi}_3^0} = [352, 555] \text{ GeV} \rightarrow \text{pure higgsino} \\
    m_{\tilde{\chi}_4^0} = [386, 573] \text{ GeV} \rightarrow \text{larger gaugino comp.} \\
    m_{\tilde{\chi}_2^\pm} = [450, 600] \text{ GeV}
    \]

  \Rightarrow \tilde{\chi}_3^0 \text{ not accessible at LHC}

  - However: \( \tilde{\chi}_3^0 \) in underlying NMSSM scenario has large gaugino component

  \rightarrow visible at LHC \rightarrow inconsistency

Model inconsistency determined via LHC/ILC
Indirect searches: extra dimensions

Hierarchy between $M_{\text{Planck}}$ and $M_{\text{weak}}$ is related to the volume or the geometrical structure of additional dimensions of space

⇒ observable effects at the TeV scale
Extra dimensions

- Models with extra dimension allow also to solve the hierarchy problem.
- **Transversely** polarized beams (only effects detectable with $P(e^-) \times P(e^+)$!)
  - enables to exploit azimuthal asymmetries in fermion production.
- Distinction between SM and different models of extra dimension:
  - asymmetry signals contribution from spin-2 graviton.

Detect new kind of physics even if new scale is in the multi-TeV range, but transversely polarized beams need polarized $e^-$ and $e^+$. 
**EW precision measurements**

- **GigaZ option at the ILC:**
  - high-lumi running on Z-pole/WW
  - $10^9$ Z in 50-100 days of running
  - Needs machine changes (bypass in the current outline)

- **High precision needs polarized beams**

- **Provides measurement of $\sin^2\theta_W$ with unprecedented precision!**
Electroweak precision tests

Electroweak precision measurements:

\[
\begin{align*}
M_Z [\text{GeV}] & = 91.1875 \pm 0.0021 & 0.002\% \\
G_\mu [\text{GeV}^{-2}] & = 1.16637(1) \times 10^{-5} & 0.0009\% \\
m_t [\text{GeV}] & = 178.0 \pm 4.3 & 2.4\% \\
M_W [\text{GeV}] & = 80.426 \pm 0.034 & 0.04\% \\
\sin^2 \theta^\text{lept}_{\text{eff}} & = 0.23150 \pm 0.00016 & 0.07\% \\
\Gamma_Z [\text{GeV}] & = 2.4952 \pm 0.0023 & 0.09\% \\
\end{align*}
\]

Quantum effects of the theory: loop corrections: \( \sim \mathcal{O}(1\%) \)

\textbf{SM:} \( M_H \) is free parameter
precise measurement of \( M_W, \sin^2 \theta_{\text{eff}}, \ldots \Rightarrow \) constraints on \( M_H \)

\textbf{MSSM:} \( m_H \) is predicted
precise meas. of \( M_W, \sin^2 \theta_{\text{eff}}, m_H, \ldots \Rightarrow \) constr. on \( m_t, \theta_t, m_b, \theta_b, \ldots \)
Electroweak precision test 2

Comparison of ew precision data with theory:

EW precision data:
$M_Z, M_W, \sin^2 \theta_{\text{eff}}^{\text{lept}}, \ldots$

Theory:
$\text{SM, MSSM,} \ldots$

↓

Test of theory at quantum level:

$H$

↓

Improve indirect constraints on unknown parameters: $M_H, m_{\tilde{e}}, \ldots$
Blondel scheme for GigaZ

Measurement of $\sin^2 \theta_{\text{eff}}$ in $e^+e^- \rightarrow Z \rightarrow f\bar{f}$:

usually $\Delta P/ P \sim 0.5\%$ sufficient
(maybe $\Delta P/ P \sim 0.25\%$ reachable !)

$$A_{LR} = \frac{2(1-4 \sin^2 \Theta^\ell_{\text{eff}})}{1+(1-4 \sin^2 \Theta^\ell_{\text{eff}})^2}$$

Blondel

$$\approx \frac{(\sigma^{RR} + \sigma^{RL} - \sigma^{LR} - \sigma^{LL})(-\sigma^{RR} + \sigma^{EL} - \sigma^{LR} + \sigma^{LL})}{(\sigma^{RR} + \sigma^{RL} + \sigma^{LR} + \sigma^{LL})(-\sigma^{RR} + \sigma^{EL} + \sigma^{LR} - \sigma^{LL})}$$

with $\Delta P/ P \sim 0.5\%$ and $P(e^-)=80\%$ only:

$\Rightarrow \Delta \sin^2 \theta^\ell_{\text{eff}} = 9.5 \times 10^{-5}$

(with $\Delta P/ P = 0.25\%$ and $P_{e^-} = 90\%$:

$\Rightarrow \Delta \sin^2 \theta^\ell_{\text{eff}} = 5 \times 10^{-5}$)

with Blondel scheme: [P(e^-), P(e^+)] = [80\%, 60\%]

$\Rightarrow \Delta \sin^2 \theta^\ell_{\text{eff}} = 1.3 \times 10^{-5}$

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**SUSY Constraints from GigaZ**

Gain of about one order of magnitude in $\sin^2 \theta_{\text{eff}}$:

→ Prediction / constraints for $m_h$ and $m_{1/2}$

`Gain' of P(e+): bounds on SM $m_H$ ~ order of magnitude, on $m_{1/2}$ ~ factor 5!
Physics up to 1 TeV

Top couplings
- improvement of top Yukawa couplings
- higher cross section (depends on Higgs mass)
- couplings up to 5%!

Direct search for SUSY particles
- high probability for access to almost the full gaugino/higgsino SUSY spectrum
- powerful consistency tests and model determination

Extrapolation of masses and gauge couplings to high scales
- consistency tests for the underlying SUSY breaking scheme
- consistency check for gauge unification

• Direct search for extra dimensions
Direct search for extra dimensions

- Direct search for gravitons in the process $e^+e^-$
  - measuring the cross sections at two different CMS energies, allows to determine the number of extra dimensions!
  - ILC with polarized beams exceeds/complements discovery region of LHC

- Serious background from $\gamma\gamma\gamma$, similar behaviour
  - Polarized $e^-$ and $e^+$ essential for background suppression
  - $(P_{e^-}, P_{e^+}) = (+80\%, +60\%) / (+80\%, 0)$: suppresses B by factor 2, enhances S by 1.5
Multi-TeV option at CLIC - Higgs

Needed scale and physics case for the multi-TeV option depends on results at LHC and ILC

Improvement in all sectors (direct and indirect searches) if
  - same precision available as at ILC
  - beamstrahlung fully under control

Triple Higgs couplings: improvement by about a factor 2
  - enhancement of cross sections of WW-fusion process
  - uncertainty of triple Higgs couplings up to 13%
  - important for further understanding of the electroweak symmetry mechanism!
Summary

- e+e- physics has been *the core of high precision physics* over the last decade
- Results from LEP, SLD, B-factories provide *tests of the SM at quantum level*
- We expect a fascinating future in the next years: *LHC will shed first light on* the mysteries of *EW symmetry breaking*
- Rich program and high physics potential of the *ILC will unravel the new physics and enter a new precision frontier*

*Stay tuned for the LHC and ILC!*
Some literature

- ILC physics: *TESLA TDR, physics part hep-ph/0106315*
  *ILC RDR, arXiv:0712.1950*

- LHC/ILC interplay:

- Supersymmetry: introduction

- Polarization+Spin:
  *webpage: www.ippp.dur.ac.uk/LCsources*
Ex: Harmonic oscillator in SUSY

Harmonic oscillator in SUSY:

\(\hbar = c = \omega = \cdots = 1\)

We have: \([q, p] = i, \quad a = \frac{1}{\sqrt{2}}(q + ip), \quad a^+ = \frac{1}{\sqrt{2}}(q - ip), \quad [a, a^+] = 1\)

Eigenstates \(\langle n|\): \(a|n\rangle = \sqrt{n}|n - 1\rangle, \quad a^+|n\rangle = \sqrt{n+1}|n+1\rangle\)

Everything bosonic: \(N_B = a^+a, \quad H_B = \frac{1}{2}(p^2 + q^2) = ?\)

What gives \([N_B, a], [N_B, a^+], N_B|n\rangle\) and \(H_B|n\rangle\)?

\(\Rightarrow\) b) Now two-state system (as IS^2, \(S_z\)):

\(|\frac{1}{2}, +\frac{1}{2}\rangle = |+\rangle, \quad |\frac{1}{2}, -\frac{1}{2}\rangle = |\rangle\)

What's the algebra?

Define with \(S_\pm = S_x \pm iS_y\) a fermionic generator+annihilation operators:

\(d^+ := S_+, \quad d := S_-\)

What's the (anti-commuting) algebra of \(d^+\) and \(d\)?

Define: \(N_F = d^+d\), \(H_F = S_z = ?\)

What happens if \(d^+, d, N_F\) act on \(|+\rangle, |\rangle\)?
Harmonic Oscillator II

c) Couple fermionic with bosonic system: \( H := H_B + H_F =? \)

States are: \( |n, +\rangle = |n\rangle \otimes |+\rangle, \quad |n, -\rangle = |n\rangle \otimes |-\rangle \)

How is the spectrum of \( H \)? What's about degeneracy?

d) Derive SUSY generators which fulfill:

\[
\begin{align*}
Q |1, +\rangle &= |2, -\rangle & \text{(allg.: } Q |n, +\rangle &\rightarrow |n + 1, -\rangle) \\
Q^+ |2, -\rangle &= |1, +\rangle & \text{(allg.: } Q^+ |n + 1, -\rangle &\rightarrow |n, +\rangle)
\end{align*}
\]

What's about \( Q, Q^+ \)? Calculate \( [N_{B,F}, Q^{(+)}] \).

What is \( \{Q^{(+)}, Q^{(+)}\} \), \( [H, Q^{(+)}] \)?

And what are the eigenvalues of the energy?