Introduction to Accelerator Physics.

DESY Summer Student Lecture 2008

Winni Decking
DESY –MPY-

Accelerator: A Definition

Device that provides accelerated particles with defined and controllable properties:

- Particle Species (electron, proton, ion)
- Energy
- Direction
- Time structure
- Intensity
- Density

Accelerators are tailored to applications needs

Employ cutting edge and special developed technology to advance parameter space in any of the above areas

Accelerator Applications

- Direct particle beams on specific targets or collide beams with each other
- Produce thin beams for synchrotron light

- HEP: structure of atom, standard model, …
- Bombard targets to obtain new materials with different properties
- Synchrotron radiation: spectroscopy, X-rat diffraction, X-ray microscopy, crystallography (of proteins), …
- Medicine: use for Positron Emission Tomography (PET, cancer therapy, surgery
- Nuclear waste transmutation (conversion of long lived into short lived nuclides)

Accelerators Worldwide

CATEGORY	NUMBER
Ion implanters	7000
Industry	1500
Radiotherapy	7500
Medical isotopes	200
Hadron therapy	20
non-nuclear research	1000
SR sources	70
Nuclear & Particle physics res.	120
TOTAL	**17390**

courtesy W. Mondelaers, JUAS 2004

Accelerator Physics – What's needed

- Basic knowledge
  - relativistic particle dynamics
  - classical theory of electromagnetism (Maxwell's equations)

- Advanced Studies
  - Hamiltonian mechanics, optical concepts
  - Quantum scattering, radiation of charged particles
  - Statistical mechanics
  - Surface physics
  - Computing, ….

What we will do

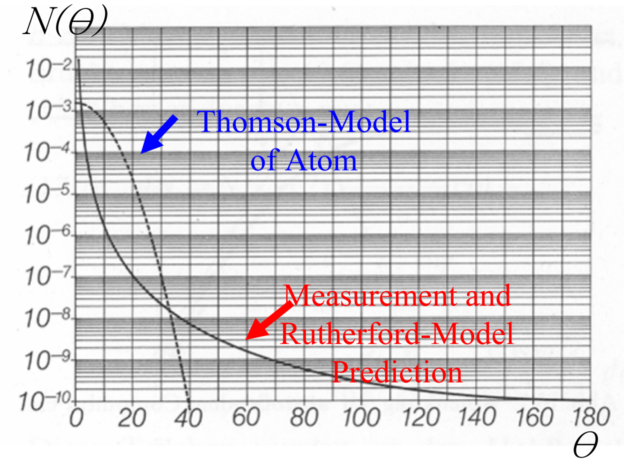
- Historical introduction
- Review of relativity
- Acceleration concepts
- Ring concepts
- Optical Functions
- Luminosity
- Synchrotron Radiation
- PETRA III as an example

- Questions and discussions

Rutherford Scattering

1906 – 1913 using α particles from a radioactive source to bombard a thin gold foil:

Discovery of massive, non point-like structure of the nucleus



1927 E. Rutherford says, addressing the Royal Society : "... *if it were possible in the laboratory to have a supply of electrons and atoms of matter in general, of which the individual energy of motion is greater even than that of the alfa particle, ... this would open up an extraordinary new field of investigation....*"

But bright sources with such energies were not yet available: man-made devices, mainly built to produce X-rays, operated at the time in the hundred KV range.

Villard X-ray Tube 1898-1905

Basic Principles: Relativity

Relativistic Energy

$$E = mc^2 = m_0 \gamma c^2$$

Relativistic Momentum

$$p = mv = m_0 \gamma \beta c$$

$$\beta = \frac{v}{c} \qquad \gamma = 1 \Big/ \sqrt{1 - \frac{v^2}{c^2}} = 1 \Big/ \sqrt{1 - \beta^2}$$

Relationship between Momentum and Energy

$$E^2 = p^2 c^2 + m_0^2 c^4$$

Kinetic Energy

$$T = E - m_0 c^2 = m_0 c^2 (\gamma - 1)$$

Basic Principles: Relativity

Speed of Light	$c = 2.9979 \times 10^8 \, \mathrm{m}/\mathrm{s}$
Electron Rest Mass	$m_{0,electron} = 511 \, \mathrm{keV}/c^2$
Proton Rest Mass	$m_{0,proton} = 938.3 \, \mathrm{MeV}/c^2$
Electron Charge	$e = 1.6021 \times 10^{-19} \, \mathrm{Coulomb}$
Electron Volts	$1 \mathrm{eV} = 1.6021 \times 10^{-19} \, \mathrm{Joule}$
Energy in eV	$E[\mathrm{eV}] = \dfrac{mc^2}{e} = \dfrac{m_0 c^2}{e} \gamma$
Energy and rest mass	$1 \mathrm{eV}/c^2 = 1.78 \times 10^{-36} \, \mathrm{kg}$

Motion in Electric and Magnetic Fields

Equation of motion under Lorentz Force

$$\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{E} + v \times \vec{B}\right)$$

$$E^2 = \vec{p}^{\,2} c^2 + m_0^2 c^4$$

$$\Rightarrow E\frac{dE}{dt} = c^2 \vec{p}\frac{d\vec{p}}{dt} = qc^2 \vec{p}\left(\vec{E} + \vec{v} \times \vec{B}\right) = qc^2 \vec{p}\vec{E}$$

Magnetic Fields do not change the particles energy, only electric fields do!

Technical challenge: provide large enough electric fields

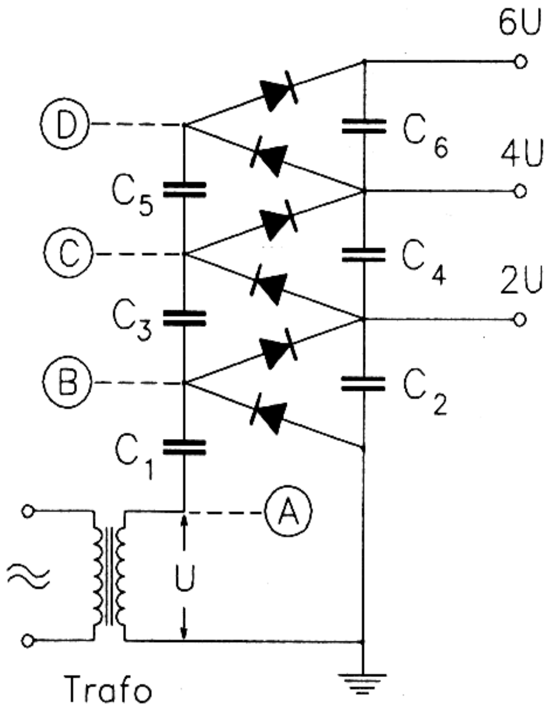
Electrostatic Accelerators

Cockroft & Walton:

$$\vec{F} = q\vec{E} = q\frac{U}{d}$$

$$\Delta E = \int_a^b \vec{F}d\vec{r} = \int_a^b q\frac{U}{d}d\vec{r} = qU$$

useful unit charge×voltage = eV



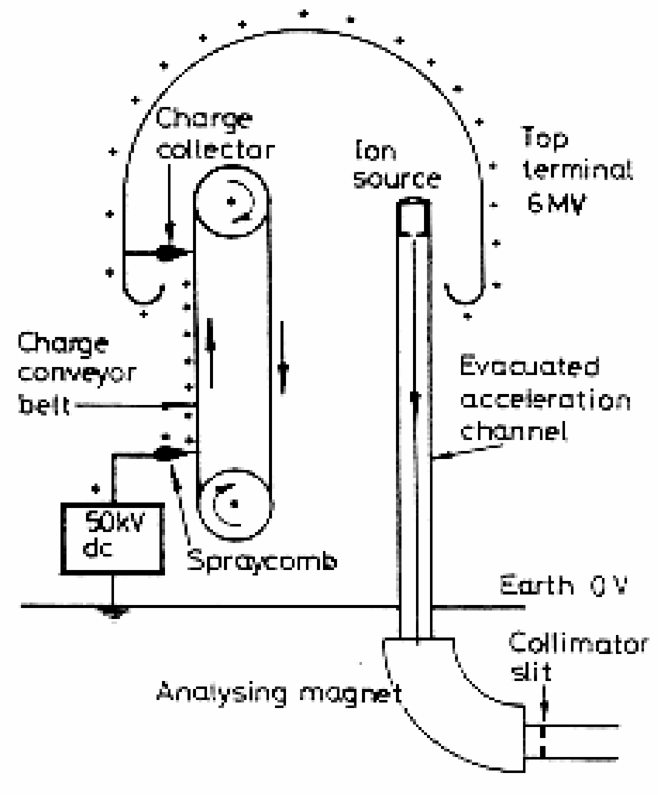
1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV

Today: Proton Pre-accelerator at PSI (Villingen)

Electrostatic Accelerators

Van de Graaff:

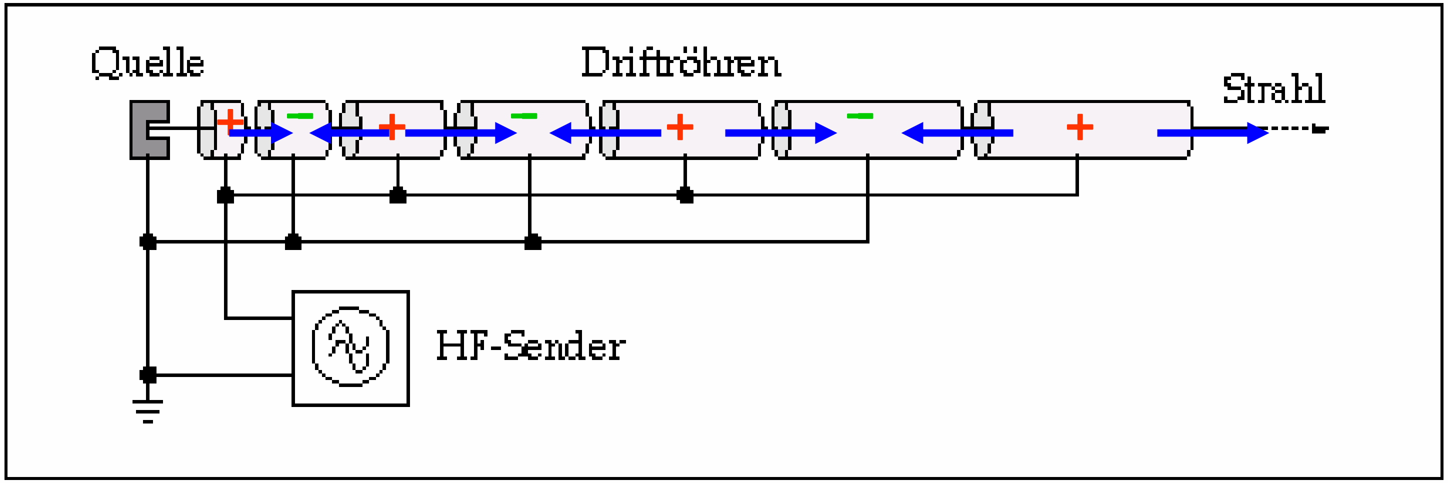
- Mechanical transport of charges creates high voltages
- Maximum potential around 20 MV (needs already SF6 gas surrounding to prevent discharges)



Today: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

Linear Accelerators (Basic Principle)

Wideroe (1928): apply acceleration voltage several times to particle beam



- acceleration of the particle in the gap between the tubes

- voltage has to be „flipped" to get the right sign in the next gap
  → RF voltage
  → shield the particle in drift tubes during the negative half wave of the RF voltage
  → vary the tube length with increasing energy/velocity

Acceleration in the Wideroe Structure

Energy gained after *n* acceleration gaps

$$E_n = n * q * U_0 * \sin\psi_s$$

n number of gaps between the drift tubes
q charge of the particle
U_0 Peak voltage of the RF System
Ψ_S synchronous phase of the particle

Kinetic energy of the particles

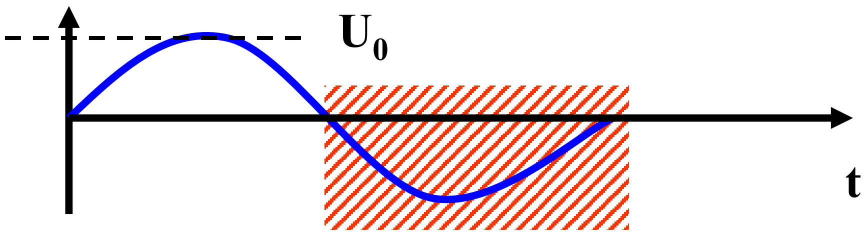
$$E_n = \frac{1}{2} m * v_n{}^2$$

valid for non relativistic particles ...

=> velocity of the particle

$$v_n = \sqrt{\frac{2E_n}{m}} = \sqrt{\frac{2 * n * q * U_0 * \sin\psi_S}{m}}$$

shielding of the particles during the negative half wave of the RF



Length of the n-th drift tube:

$$l_n = v_n * \frac{\tau_{RF}}{2} = v_n * \frac{1}{2\nu_{RF}}$$

Examples

DESY proton linac (LINAC III)

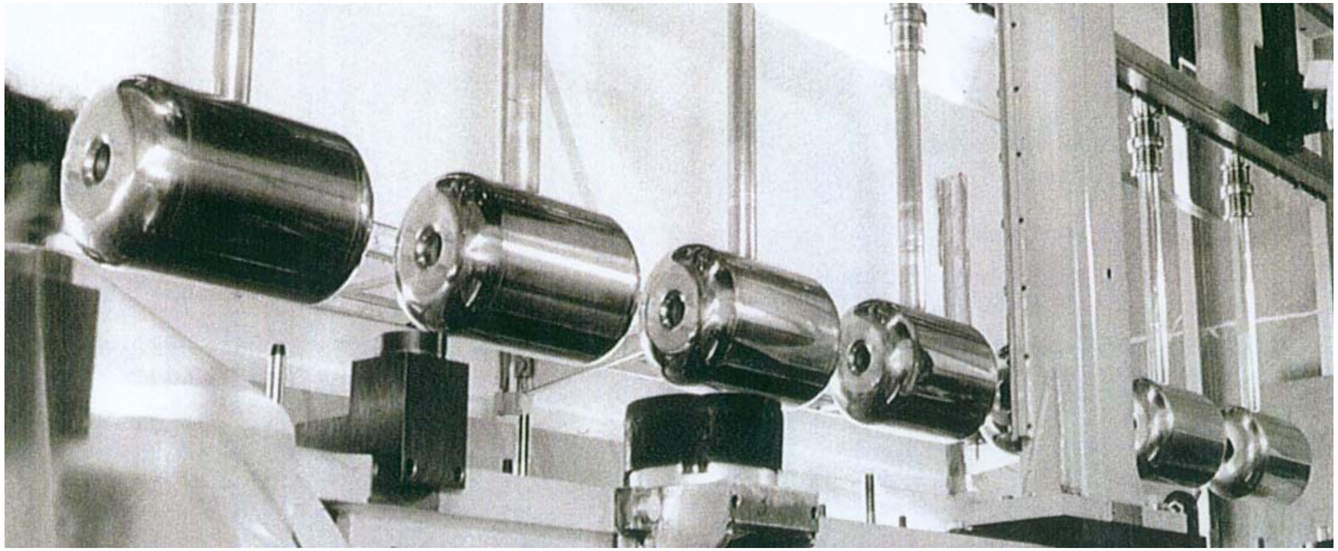
$$E_{total} = 988\,MeV$$

$$E_{kin} = E_{total} - m_0 c^2$$

$$E_{kin} = 50\,MeV$$

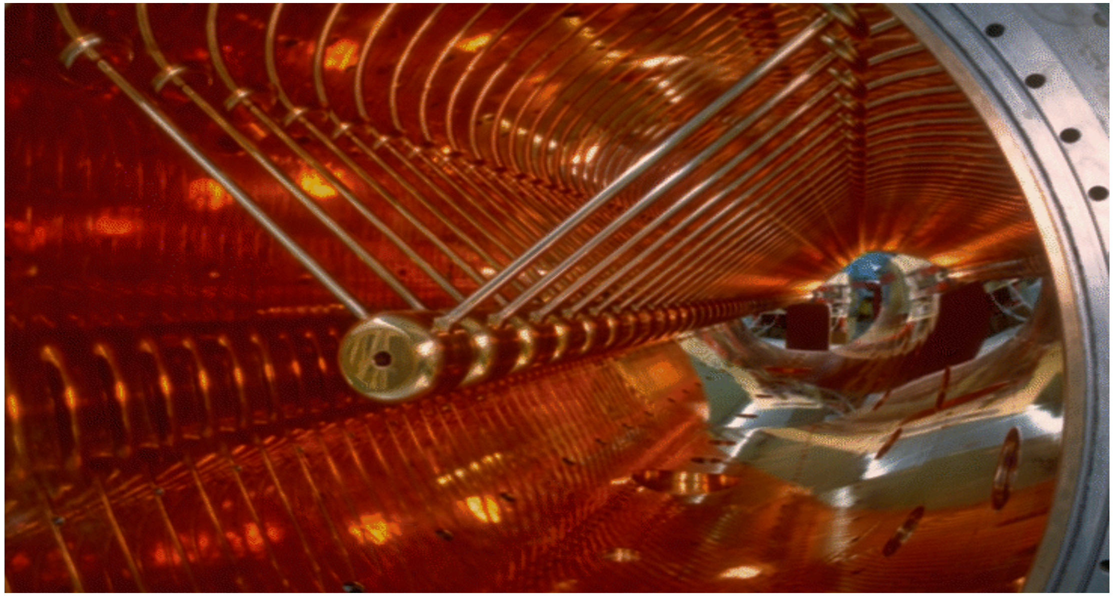
$$E^2 = c^2 p^2 + m_0^2 c^4$$

$$p = 310\,MeV/c$$

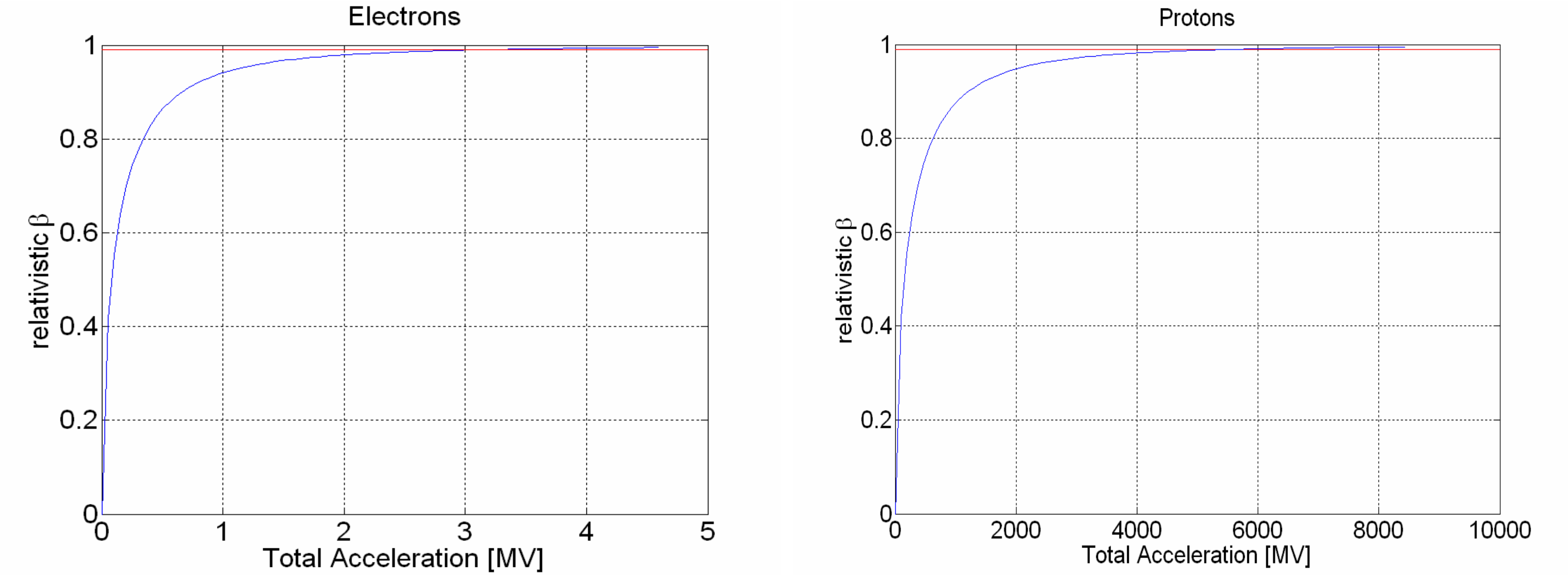


GSI Unilac

E ≈ 20 MeV per Nukleon
$\beta \approx 0.04 \ldots 0.6$
Protons/Ions
ν = 110 MHz



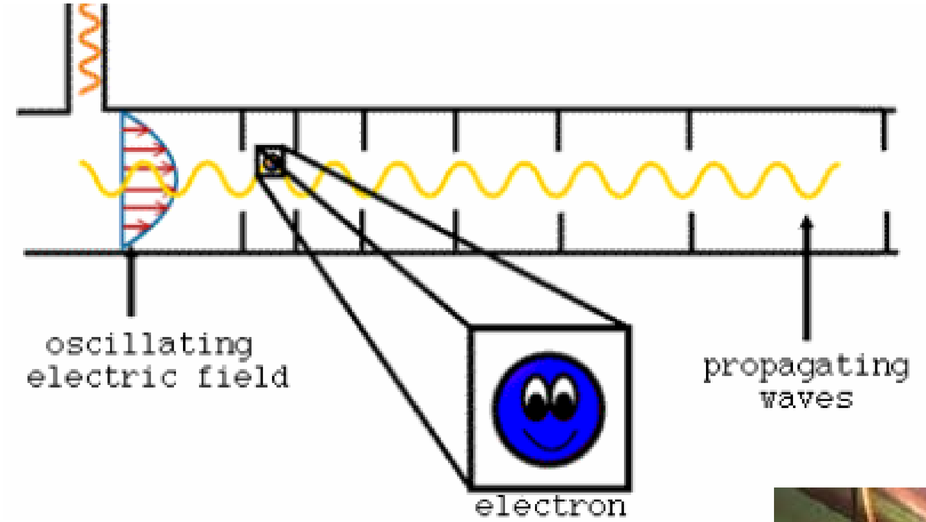
What is relativistic?



Electrons: β > 0.99 at 3.7 MeV

Protons: β > 0.99 at 6.7 GeV

Acceleration in RF fields



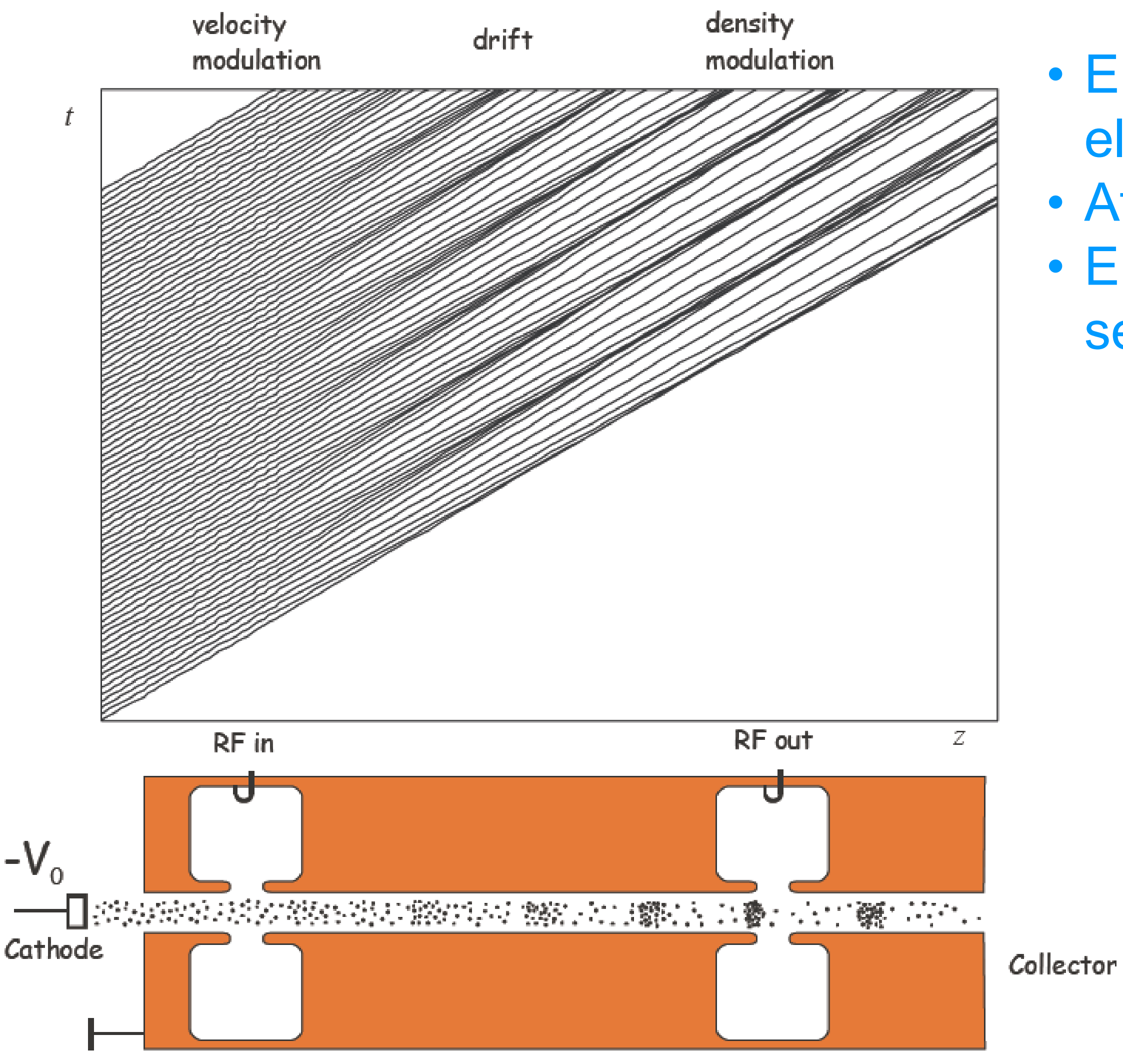
Traveling wave structure:

- Particles in phase with waveform
- Reduce phase velocity in waveguide with irises

Example: SLAC 2 mile LINAC

E up to 50 GeV
in operation since mid 60's
Electrons/Positrons
ν = 2.8 GHz, 35 MV/m

Klystron



courtesy E. Jensen, RF for LINACS, CAS 2005

- Energy/velocity modulated electron beam in first resonator
- After drift density modulation
- Electric field extracted in second resonator

Motion in a constant B-field (E=0)

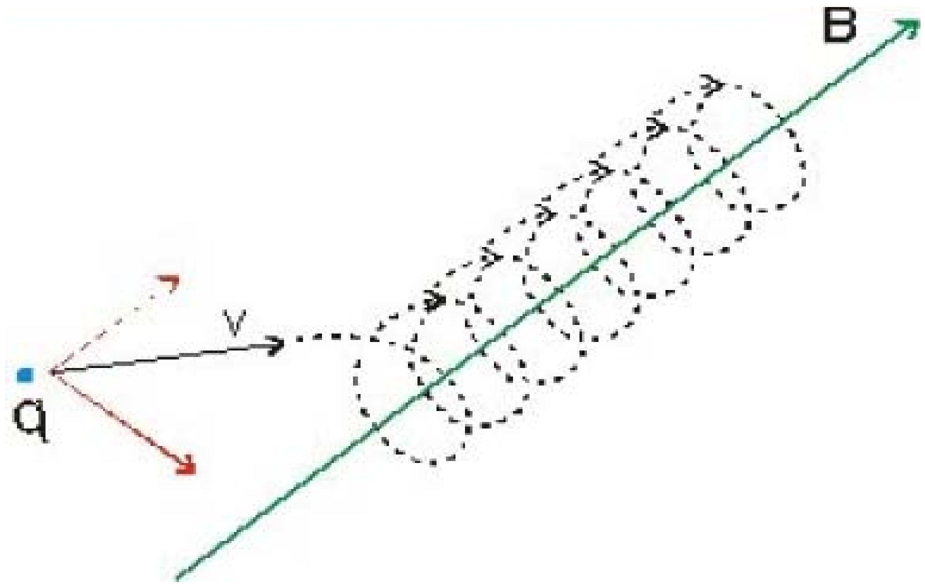
- Energy stays constant
- Spiraling trajectory along the uniform magnetic field

Lorentz Force

$$\vec{F} = q * (\vec{v} \times \vec{B})$$

Zentrifugal Force

$$F = \frac{m * v^2}{\rho}$$



$$q * v * B = \frac{m * v^2}{\rho} \quad \rightarrow \quad B * \rho = p / q$$

Cyclotron

Uses magnetic field to force particles to pass through accelerating fields at regular intervals

Cyclotron:

- **Constant B-field**
- Time for one revolution

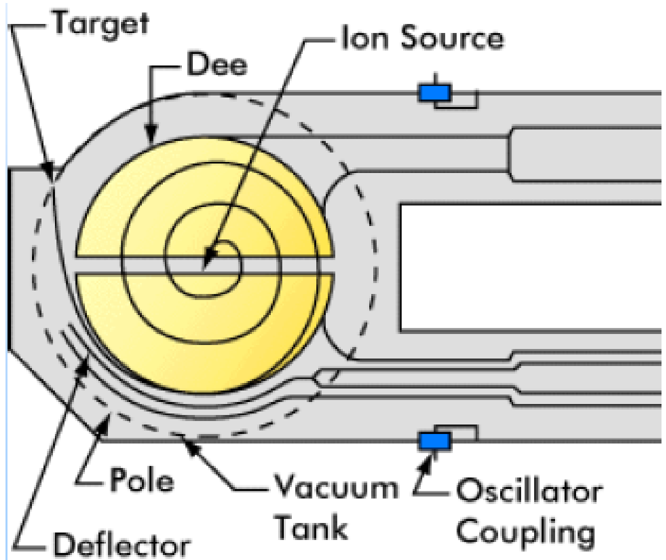
$$T = 2\pi \frac{\rho}{v} = 2\pi \frac{m}{q * B_z}$$

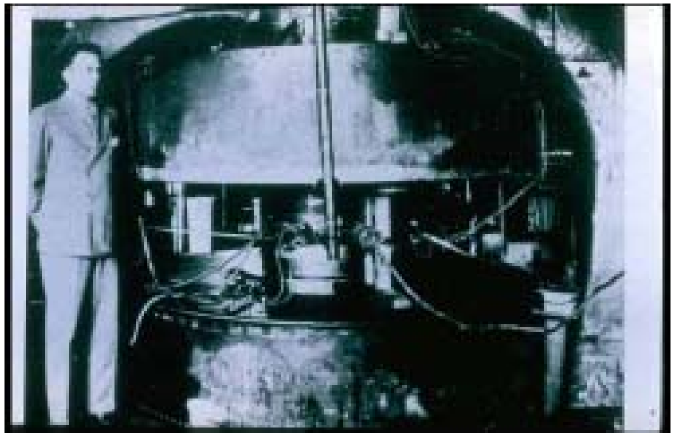
- Constant revolution frequency = constant accelerating frequency

$$\omega_z = 2\pi \frac{1}{T} = \frac{q}{m} * B_z \quad \rightarrow \quad \omega_z = const.$$

- Works only if *m=const* => non-relativistic particles
- Large momentum => large magnets

$$B * \rho = p/q$$





E. Lawrence and his first cyclotron

Synchrotron

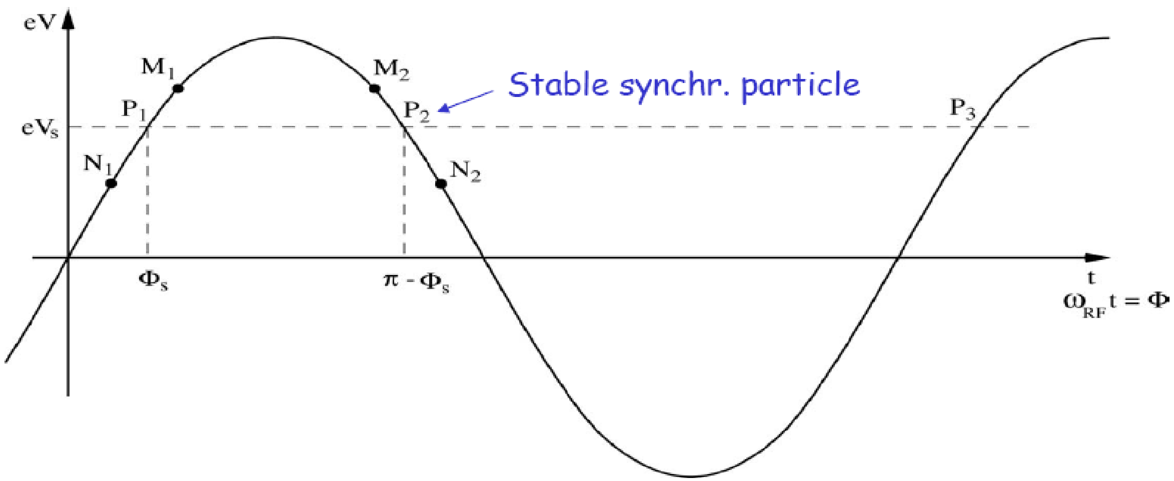
- **Varying B-field**
- **Constant ρ**

$$B * \rho = p/q$$

- Increase B-field synchronous to momentum p

Where is the acceleration?

- RF field in accelerating cavity with the right (synchronous) phase

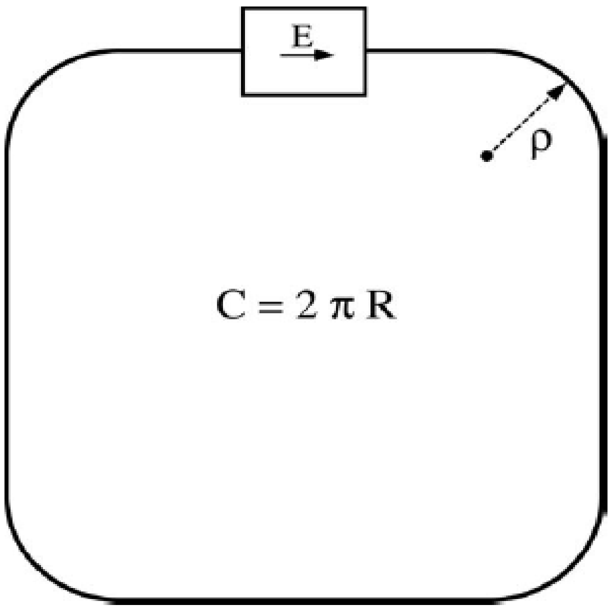


$$\Delta p/q = eU_0 \sin(\phi)$$

Increase $B \rightarrow$ decreases $\rho \rightarrow$ particle comes early $\rightarrow$ gains more energy

Synchrotron

Synchronous accelerator where there is a synchronous RF phase for which the energy gain fits the increase of B-field at each turn.



$$\tau = \frac{2\pi R}{v} \approx \frac{L}{c}$$

revolution time

$$\frac{\omega}{2\pi} = \frac{1}{\tau} \approx \frac{c}{L}$$

revolution frequency

$$\omega_{rf} = h\omega \approx \frac{hc}{L}$$

RF frequency
h = harmonic number
=number of possible bunches

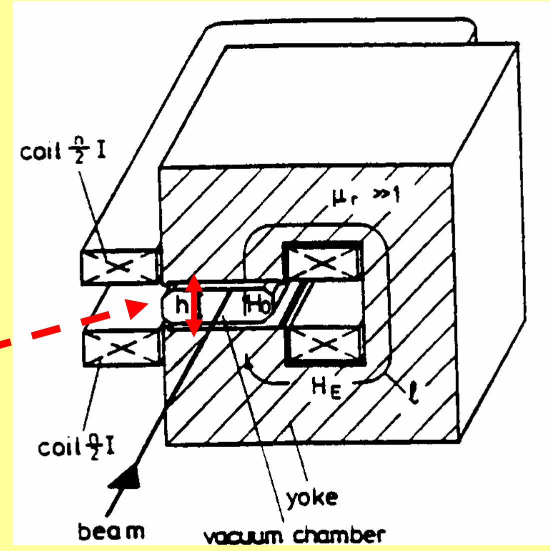
$$\rho = \left|\frac{p}{qB}\right|$$

Magnetic rigidity

$$B\rho = \frac{p}{q}$$

Magnetic Dipole Fields

Technical design of a dipole magnet:
a magnet with two flat, parallel pole shoes
creates a homogeneous dipole field



Maxwells equations: $\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\delta \vec{D}}{\delta t}$

$$\int_A \left(\vec{\nabla} \times \vec{H}\right)\vec{n}\,da = \oint \vec{H}d\vec{s} = h * H_0 + l_{Fe} * H_{Fe} = nI$$

$$H_0 = \frac{B_0}{\mu_0},\ H_{Fe} = \frac{B_0}{\mu_0 * \mu_{Fe}}$$

n*I = number of coil windings, each carrying the current I

μ_r = rel. permeability of the material,

μ_r (Fe) ≈ 3000

$$B_0 = \frac{\mu_0 * nI}{h}$$

the magnetic field B depends on
*the current
* the number of windings
* the gap height

Dipole Fields and Max. Energies

For high energy machines ($\gamma >> 1$)

$$E^2 = p^2c^2 + m_0^2c^4 \Rightarrow E \approx pc$$

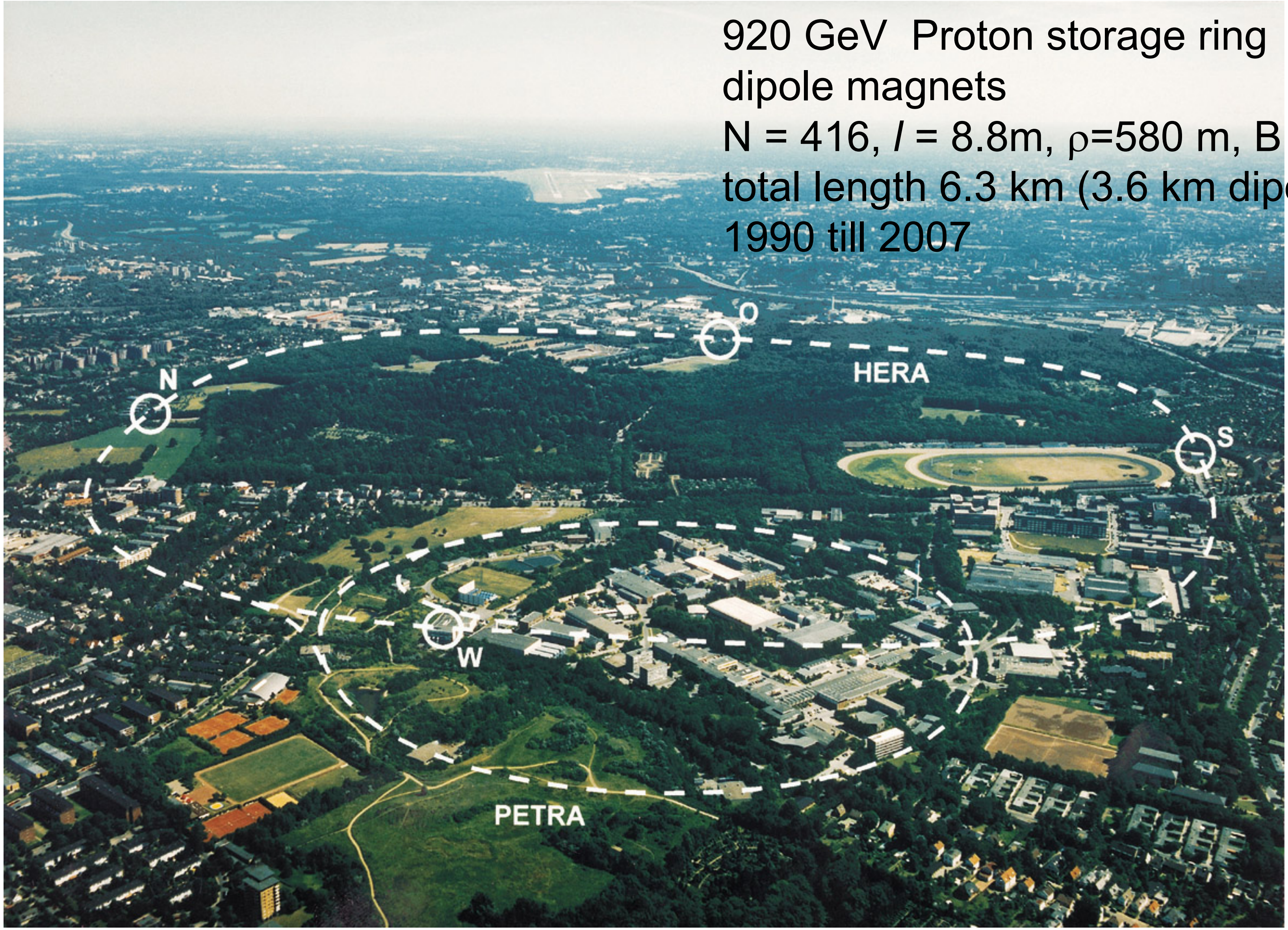
$$B * \rho = p/q \Rightarrow B * \rho = E/ce \Rightarrow E \approx 0.3 \times B[\mathrm{T}] \times \rho[\mathrm{m}]$$

Technology
normal conducting: $B_{max} \approx 2$ T
super conducting : $B_{max} \approx 7$ T

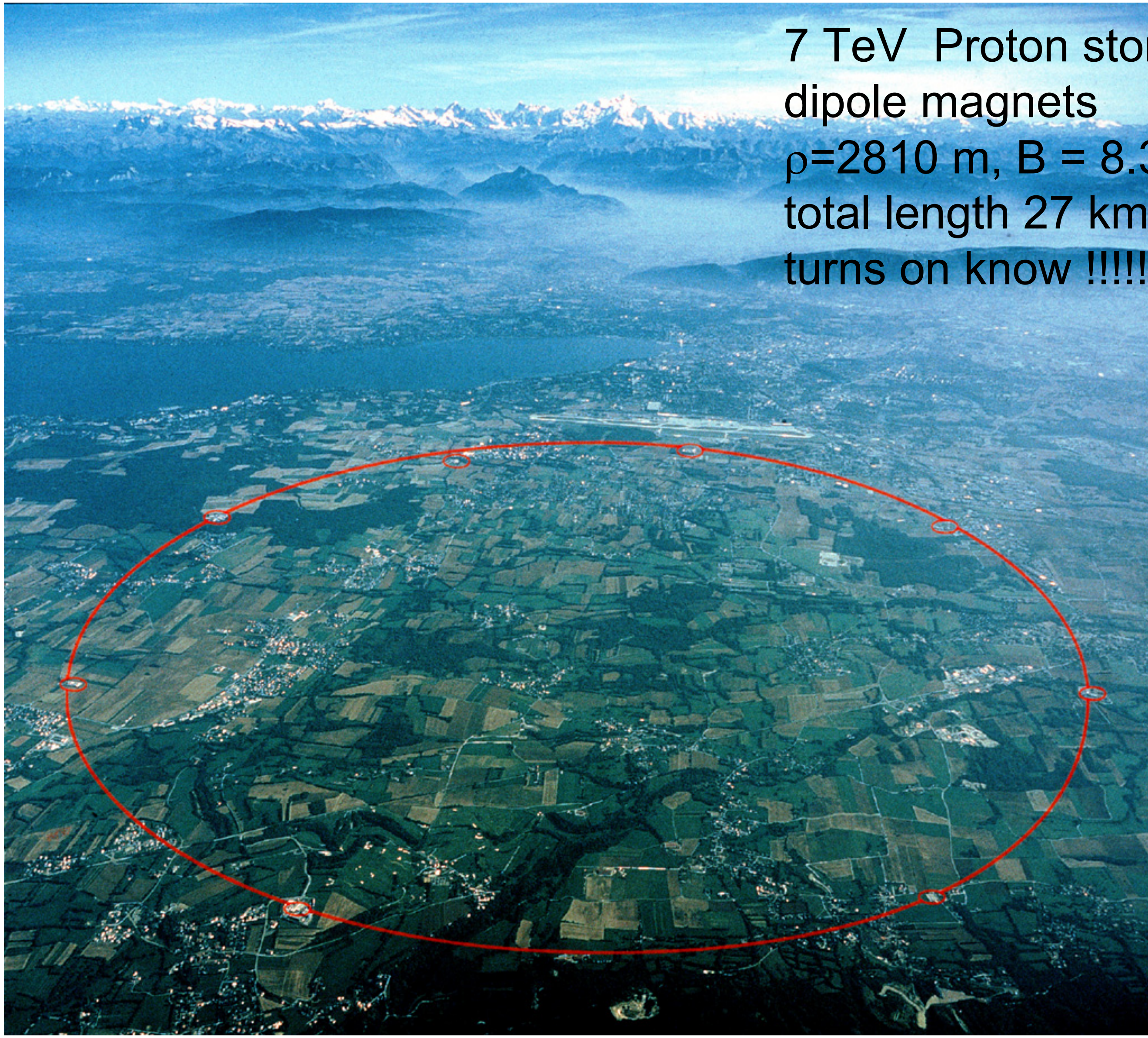
Size (= money)

Example: HERA p

920 GeV Proton storage ring
dipole magnets
N = 416, l = 8.8m, ρ=580 m, B = 5.5 T
total length 6.3 km (3.6 km dipoles)
1990 till 2007

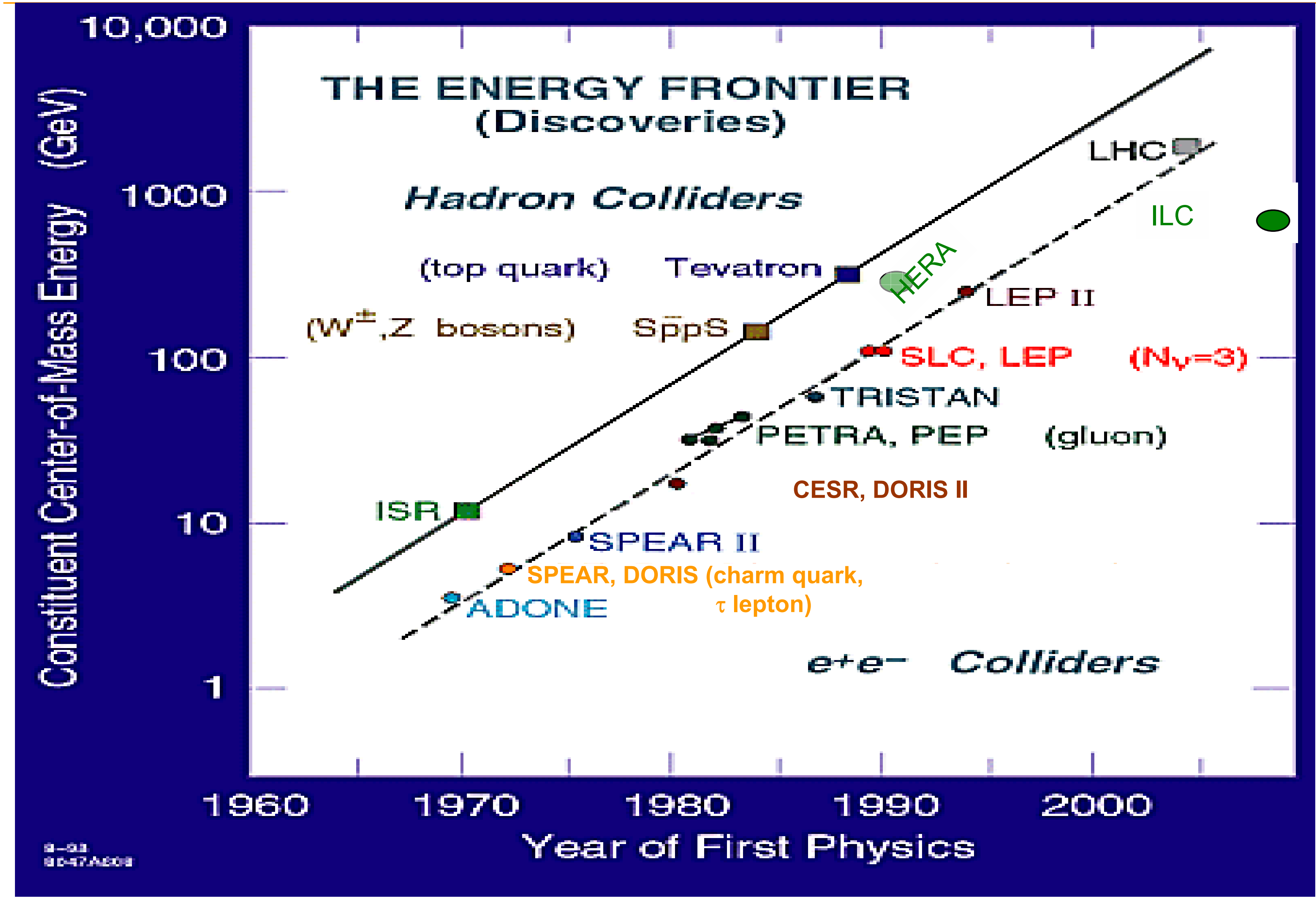


Example LHC



7 TeV Proton storage ring
dipole magnets
ρ=2810 m, B = 8.3 T
total length 27 km (17.6 km dipoles)
turns on know !!!!!!!!!

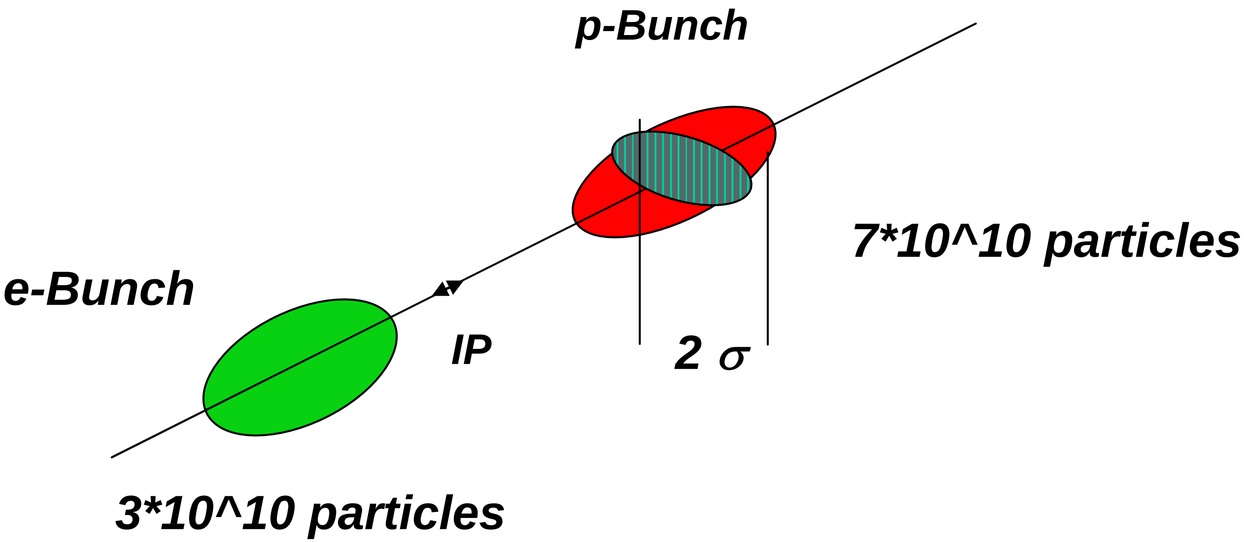
HEP: The energy frontier



Luminosity

$$R = L * \Sigma_{react.}$$

production rate of (scattering) events is determined by the cross section Σ_{react} and a parameter L that is given by the design of the accelerator: … the luminosity

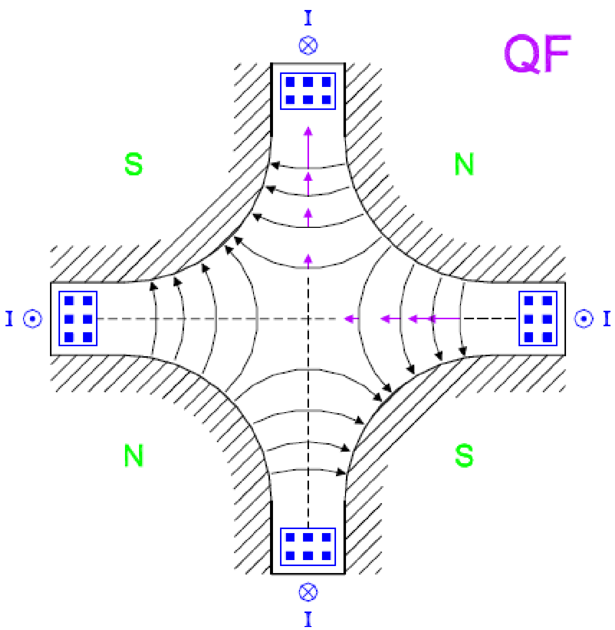


$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_e I_p}{\sigma_x^* \sigma_y^*}$$

Focusing

Magnet imperfections, misalignments, gravitation, earth magnetic field, ….

Particles will not stay on a stable circular orbit (or follow a long straight pass) => need a magnet with increasing B-field away from the centre axis



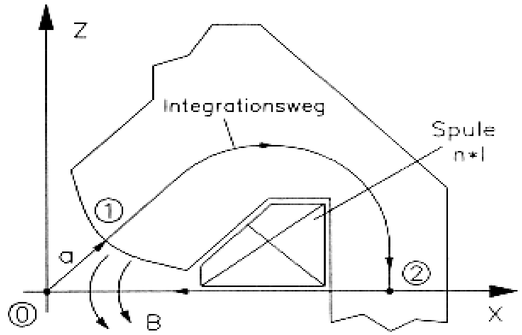
four iron pole shoes of hyperbolic contour

$$B_y = -g * x, \quad B_x = g * y$$

$$F_x = -g * x, \quad F_y = g * y$$

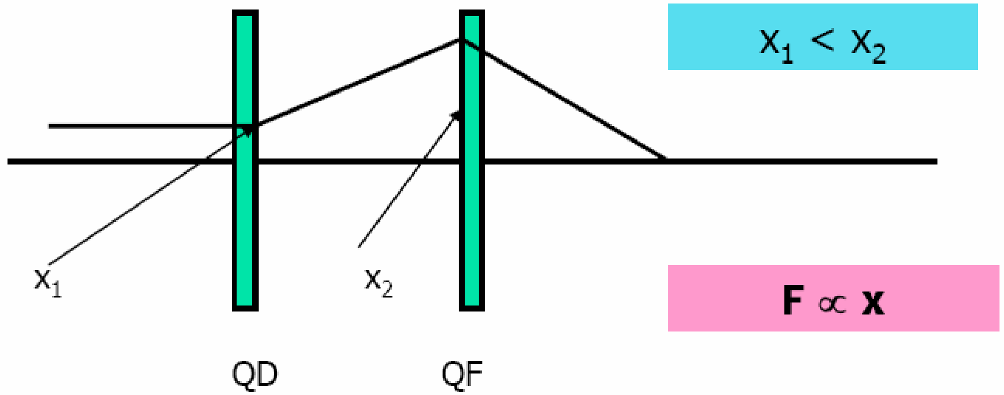
focusing in one plane, defocusing in the other

$$g = \frac{2\mu_0 n I}{R^2}$$

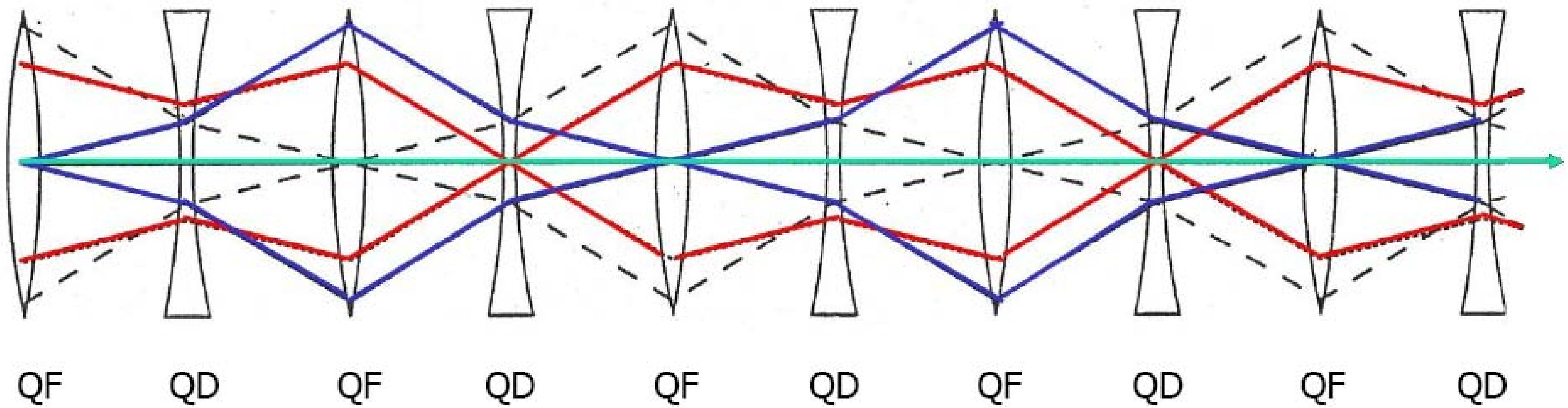


Focusing

Net focusing effect



Put many quadrupoles in a row – motion is confined
'FODO' structure



Focusing forces and particle trajectories

normalise magnet fields to momentum (remember: $B*\rho = p / q$)

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

$$k := \frac{g}{p / q}$$

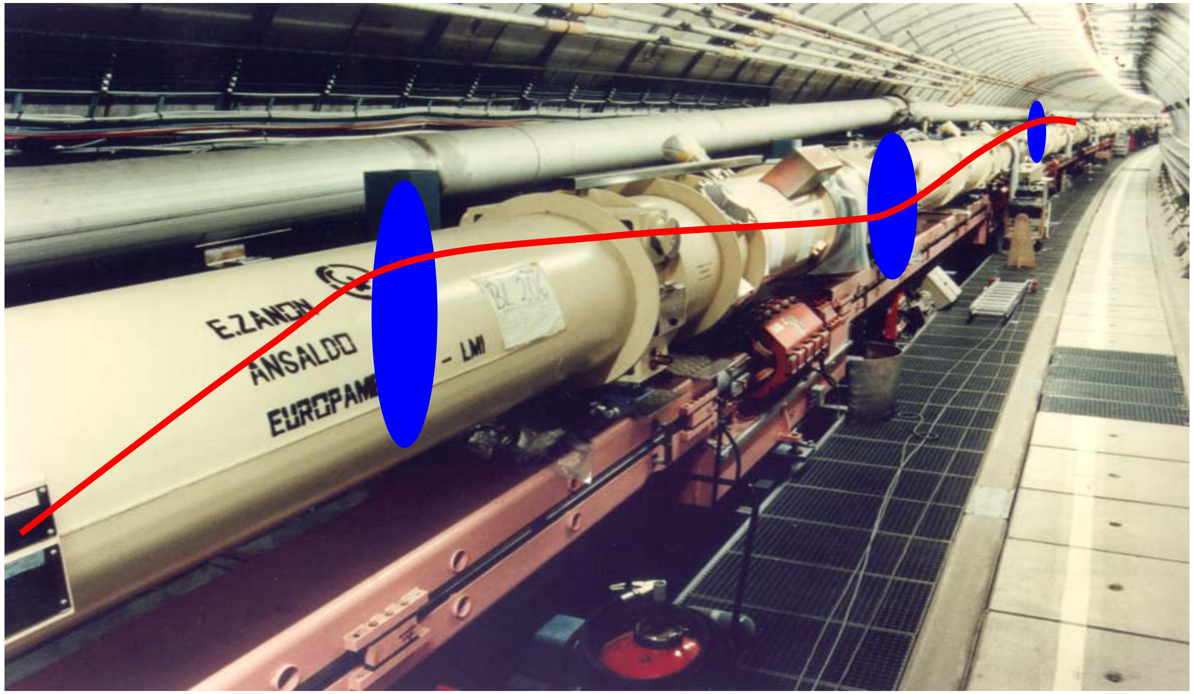
Example: HERA Ring

Momentum: p = 920 GeV/c

Bending field: B = 5.5 Tesla

Quadrupol Gradient
G= 110 T/m

→ $k = 33.64 * 10^{-3} /m^2$
→ $1/\rho = 1.7 * 10^{-3} /m$



Equation of Motion

Under the influence of the focusing and defocusing forces the differential equation of the particles trajectory can be developed:

$$x'' + k * x = 0$$ horizontal plane

x = distance of a single particle to the center of the beam

$$x' := \frac{dx}{ds}$$

vert. plane: $k \Rightarrow -k$

if we assume ....

* linear retrieving force
* constant magnetic field
* first oder terms of displacement ***x***

... we get the general solution (hor. focusing magnet):

$$x(s) = x_0 * \cos(\sqrt{k}s) + \frac{x_0'}{\sqrt{k}} * \sin(\sqrt{k}s)$$

$$x'(s) = -x_0\sqrt{k} * \sin(\sqrt{k}s) + x_0' * \cos(\sqrt{k}s)$$

Matrix Formalism

Matrices of lattice elements

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M * \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Hor. focusing Quadrupole

$$M_{QF} = \begin{pmatrix} \cos(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sin(\sqrt{K} * l) \\ -\sqrt{K} \sin(\sqrt{K} * l) & \cos(\sqrt{K} * l) \end{pmatrix}$$

Hor. defocusing Quadrupole

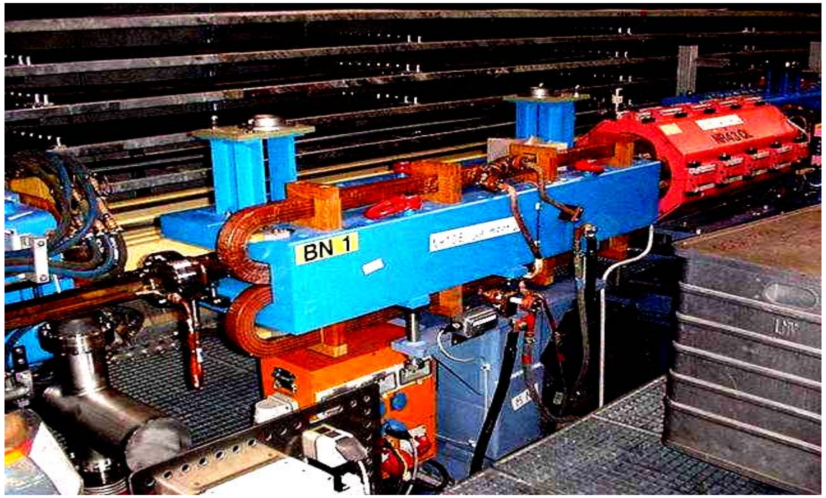
$$M_{QD} = \begin{pmatrix} \cosh(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sinh(\sqrt{K} * l) \\ \sqrt{K} \sinh(\sqrt{K} * l) & \cosh(\sqrt{K} * l) \end{pmatrix}$$

Drift spare

$$M_{Drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

formalism is only valid within one lattice element where ***k = const***

in reality: ***k = k(s)***



Combination of Elements

* we can calculate the trajectory of a single particle within a single lattice element

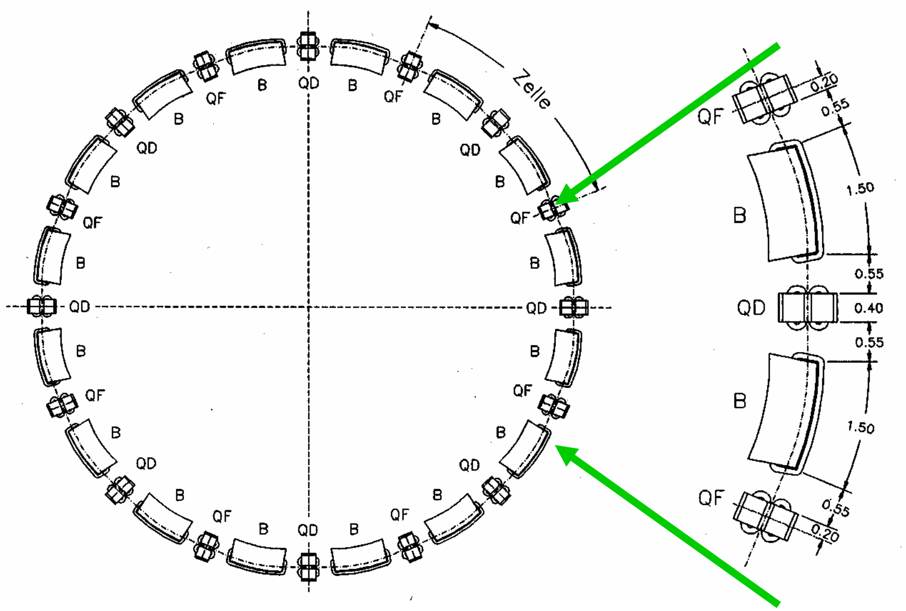
* for any starting conditions x_0, x'_0

* we can combine these piecewise solutions together and get the trajectory for the complete storage ring

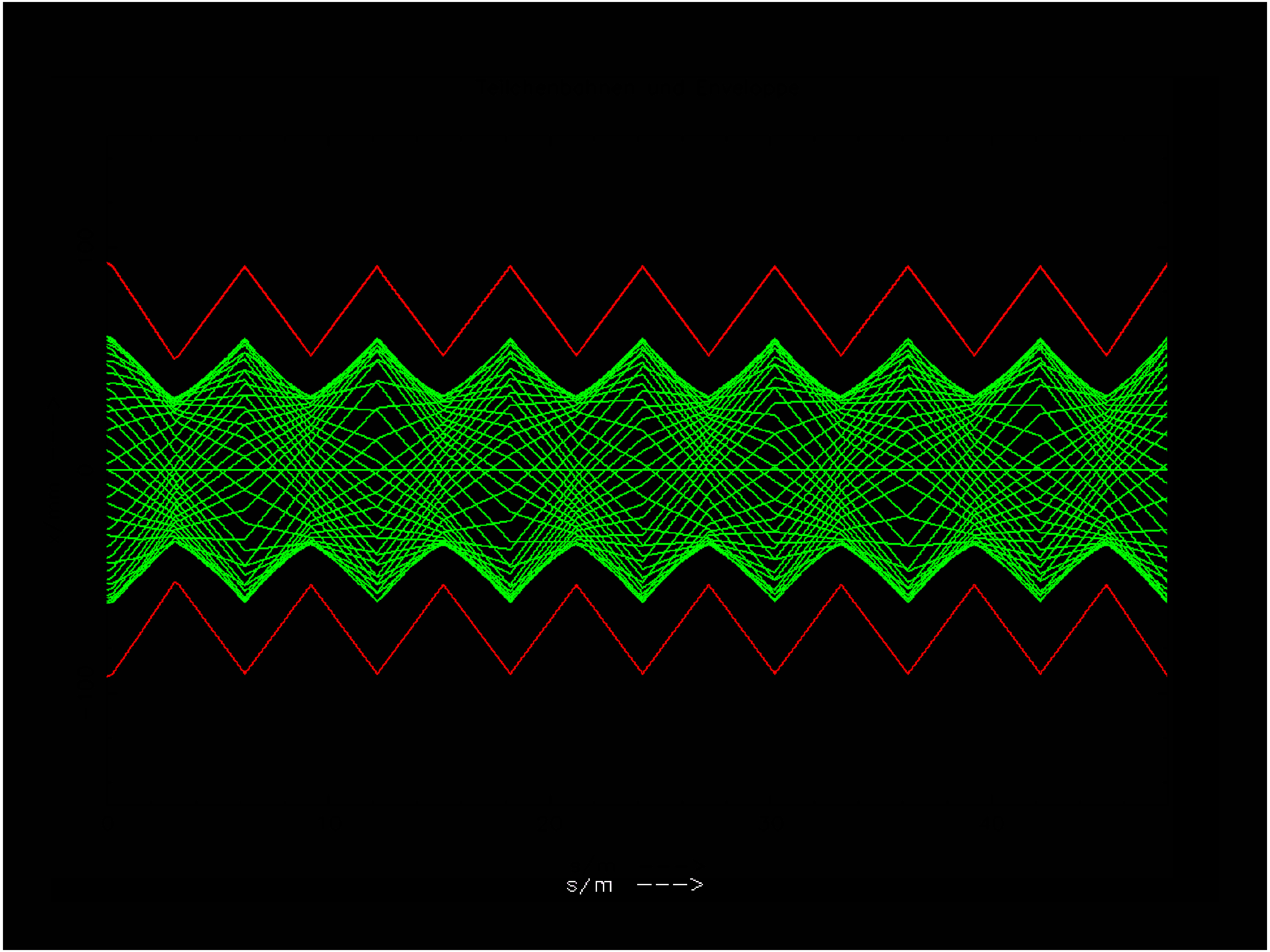
$$M_{lattice} = M_{QF1} * M_{D1} * M_{QD} * M_{D1} * M_{QF2} \ldots\ldots$$

Example:
storage ring for beginners

Dipole magnets and QF & QD quadrupole lenses



courtesy: K. Wille


s/m --->

The 'Beta' Function

equation of motion

$$x''(s) - k(s)x(s) = 0$$

restoring force $\neq$ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring

Solution in the form

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

ε, Φ = integration constants
determined by initial conditions

***β*(s)** given by focusing properties of the lattice ↔ quadrupoles

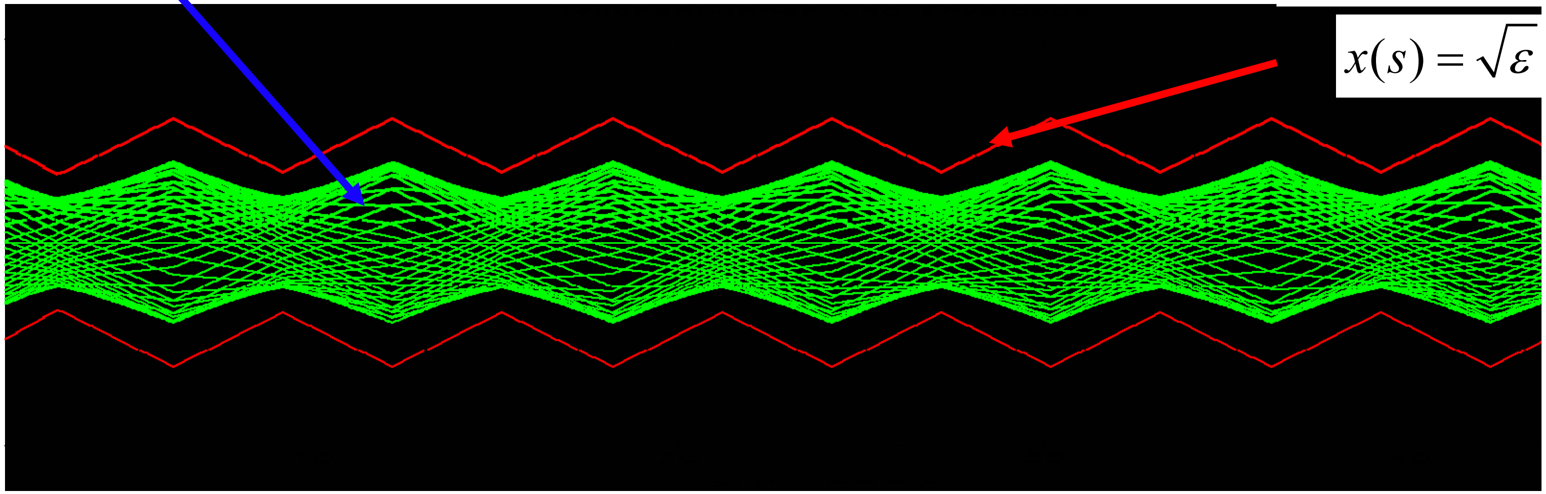
ε beam emittance = intrinsic beam parameter,
cannot be changed by the foc. properties.

The 'Beta' Function

$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$

max. amplitude of all particle trajectories

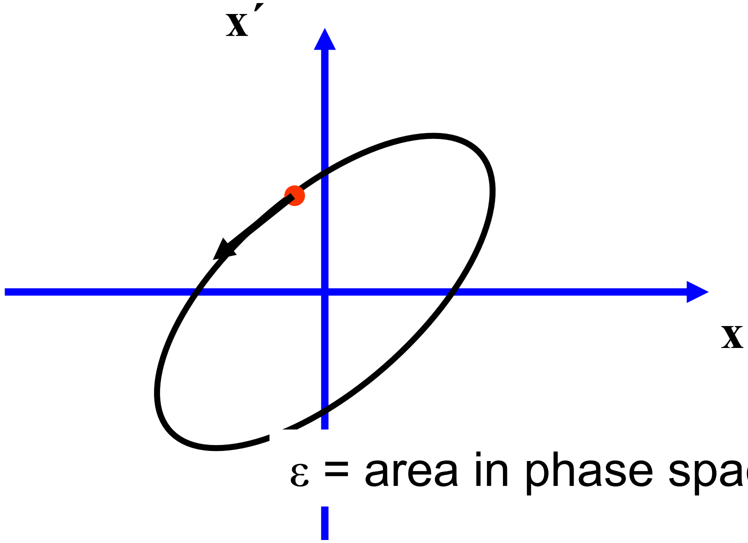
$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)}$



Beam Dimension:

determined by two parameters

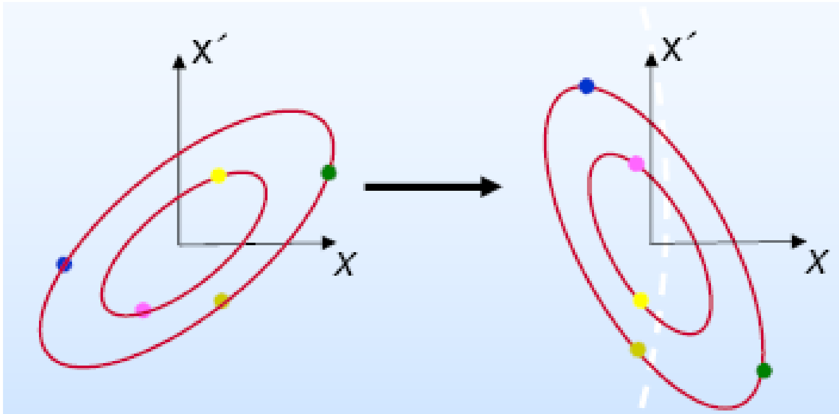
$$\sigma = \sqrt{\varepsilon * \beta}$$



ε = area in phase space

Transverse Phase Space

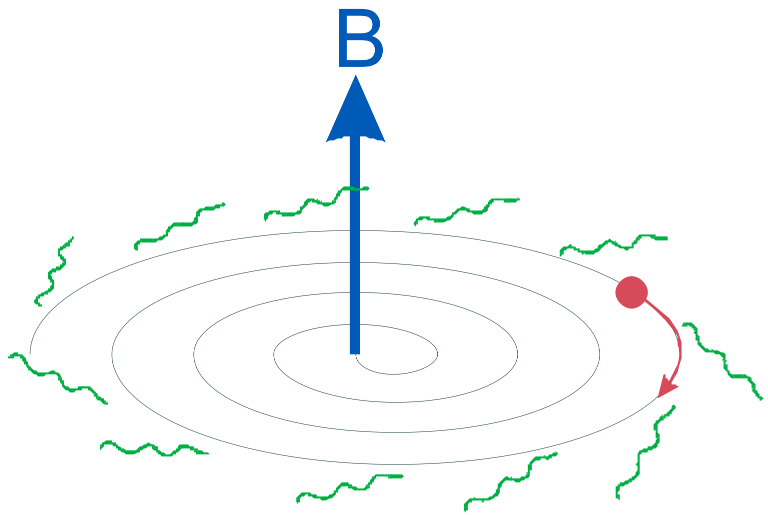
- Under linear forces, any particle moves on an ellipse in phase space
- Ellipses shear in magnets, but their area is preserved



- General equation of ellipse is $\beta x'^2 + 2\alpha x x' + \gamma x^2 = \varepsilon$
- with α,β,γ functions of the distance and ε constant
- Area of ellipse is $\pi\varepsilon$
- Statistical definitions of emittance (for nonlinear beams)
  - area covering 95% of all particles
  - $\varepsilon = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$

Synchrotron Radiation

- Particles radiate when accelerated, particle moving in a dipole is accelerated centrifugally and emits radiation tangential to the trajectory



- Total energy loss after one turn:

$$\Delta E / rev[\mathrm{GeV}] = \frac{6.034 \times 10^{-18}}{\rho[\mathrm{m}]} \left(\frac{E[\mathrm{GeV}]}{m_0 \left[\mathrm{GeV/c^2}\right]} \right)^4$$

- Ratio of proton to electron mass is 1836
- At the same energy and radius: $\Delta E_{electron} : \Delta E_{proton} \approx 10^{13}$

Synchrotron Radiation

- Example HERA electron ring
  - E = 27 GeV
  - B = 0.16 T, ρ=580 m
  - $\Delta E \approx$ 80 MeV
  - Lots of RF stations and power installed in HERA-e

- Example HERA proton ring
  - E = 920 GeV
  - B = 5.5 T, ρ=580 m
  - $\Delta E \approx$ 10 eV
  - RF only needed for longitudinal focusing and acceleration

Synchrotron Radiation

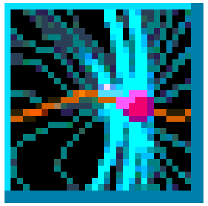
- Radiation is produced in a narrow light cone of angle

$$\theta \approx \frac{1}{\gamma} = \frac{511}{E[\mathrm{keV}]} \quad \text{for electrons and v} \approx \text{c}$$

- Radiation spot size depends thus on electron energy and beam size at source point
- A quality measure for synchrotron radiation is the brilliance (or brightness in US literature): F

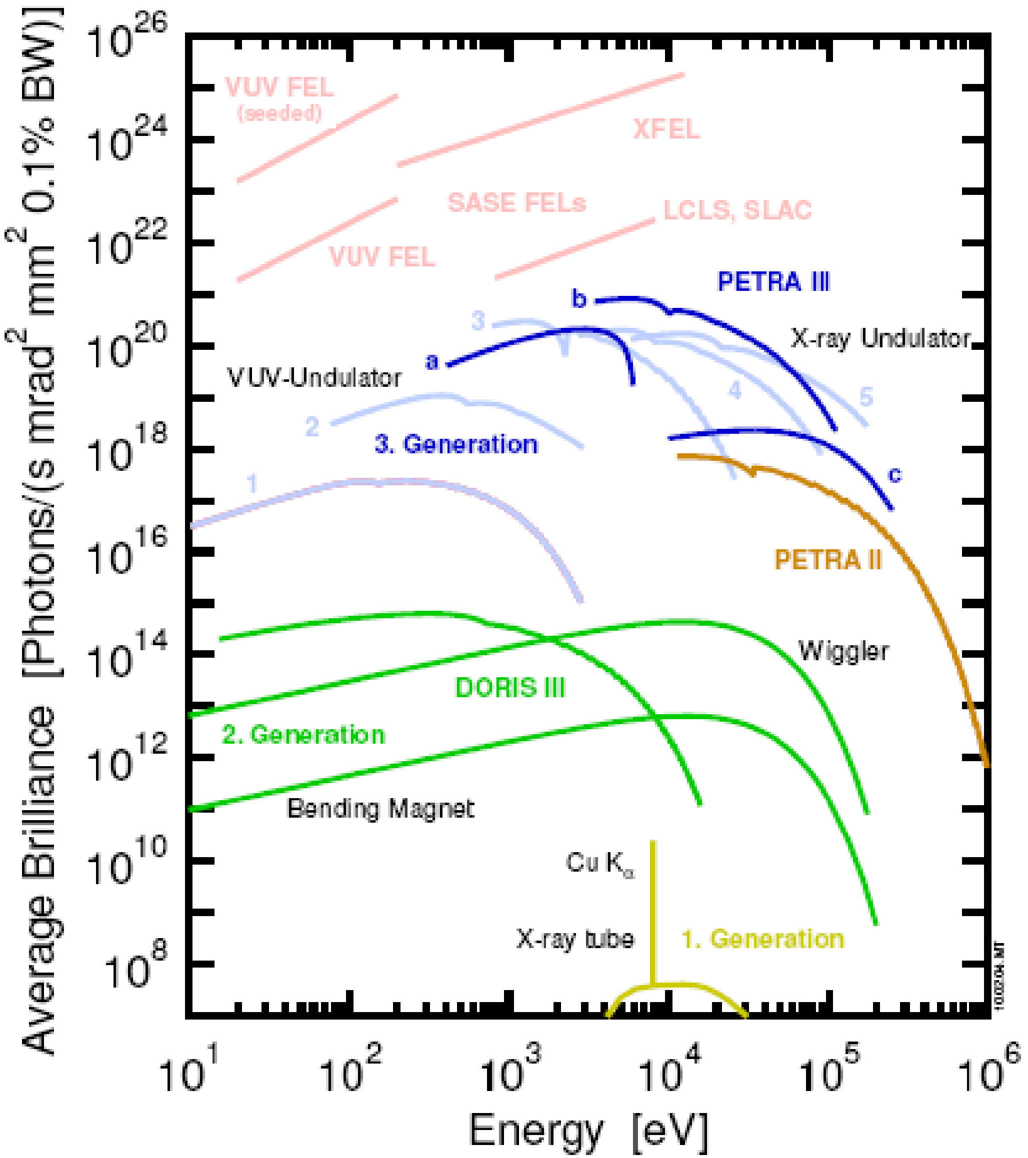
$$B = \frac{F}{4\pi^2 \sigma_{Tx}\sigma_{Ty}\sigma_{Tx'}\sigma_{Ty'}}$$

- Photon flux (per 0.1 % bandwidth) normalized with the total horizontal and vertical photon source size and divergence

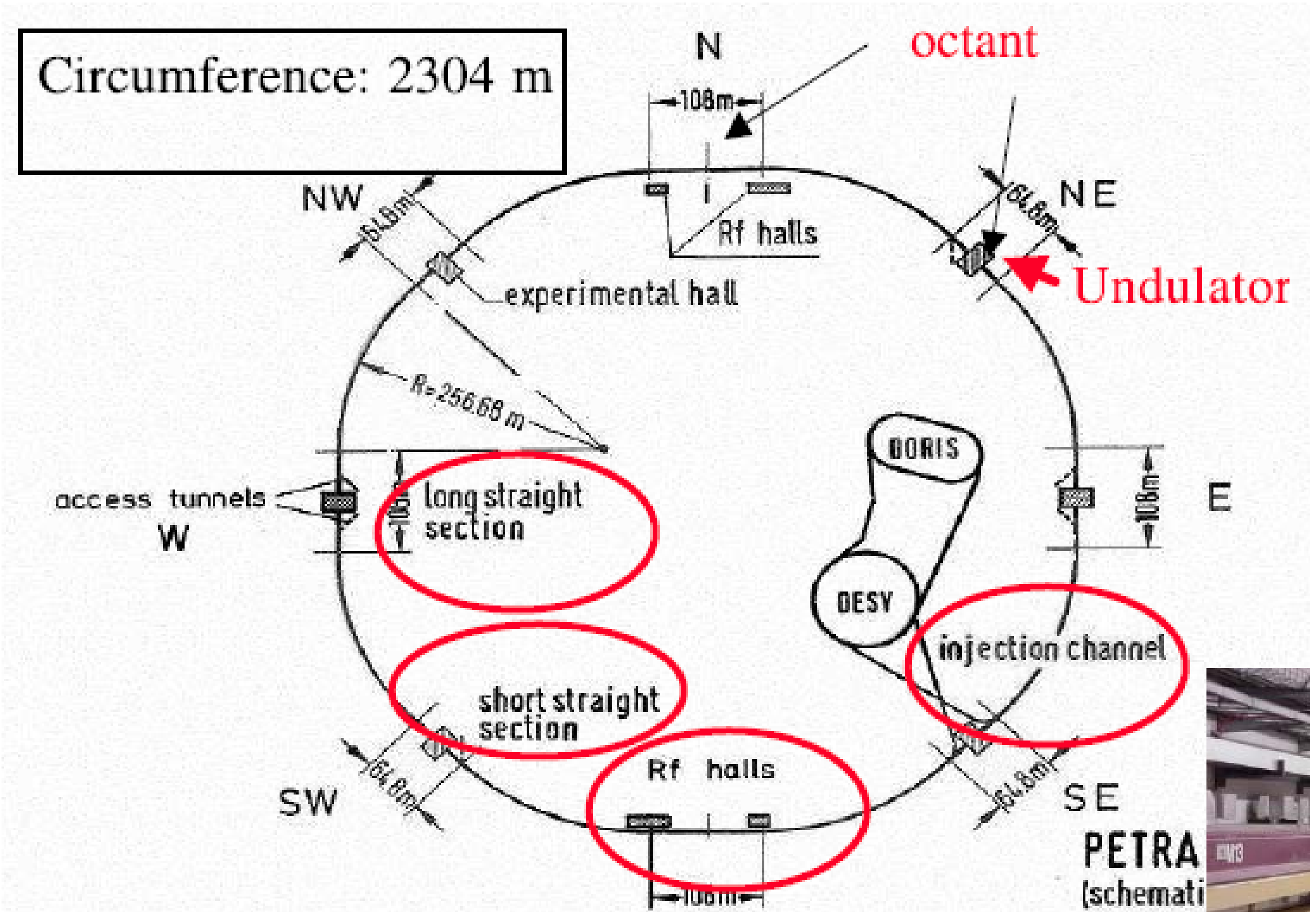


Radiation2D.exe

Brightness Comparison



Synchrotron Light Source PETRA III



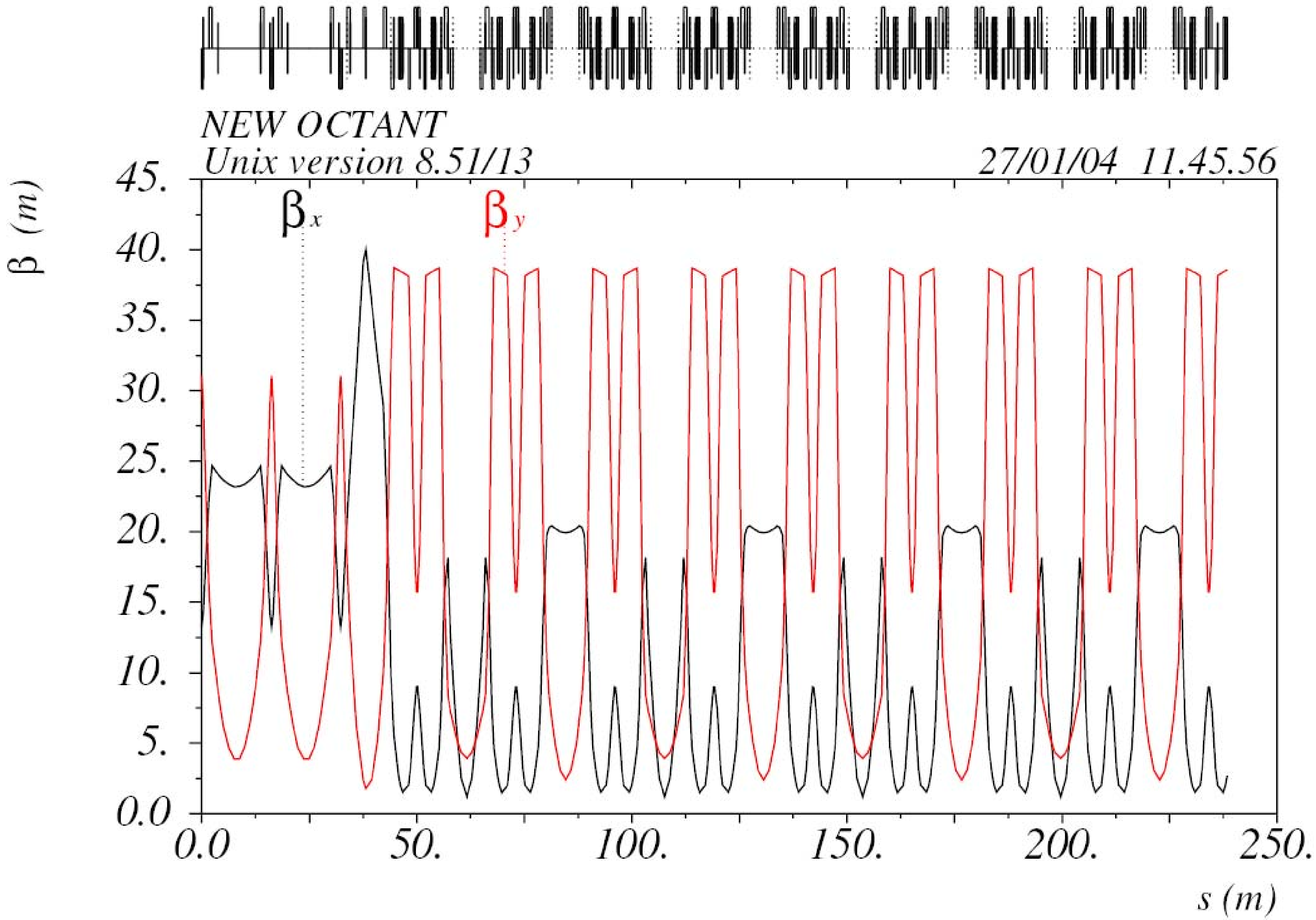
E = 6 GeV

$\varepsilon = 1$ nm rad

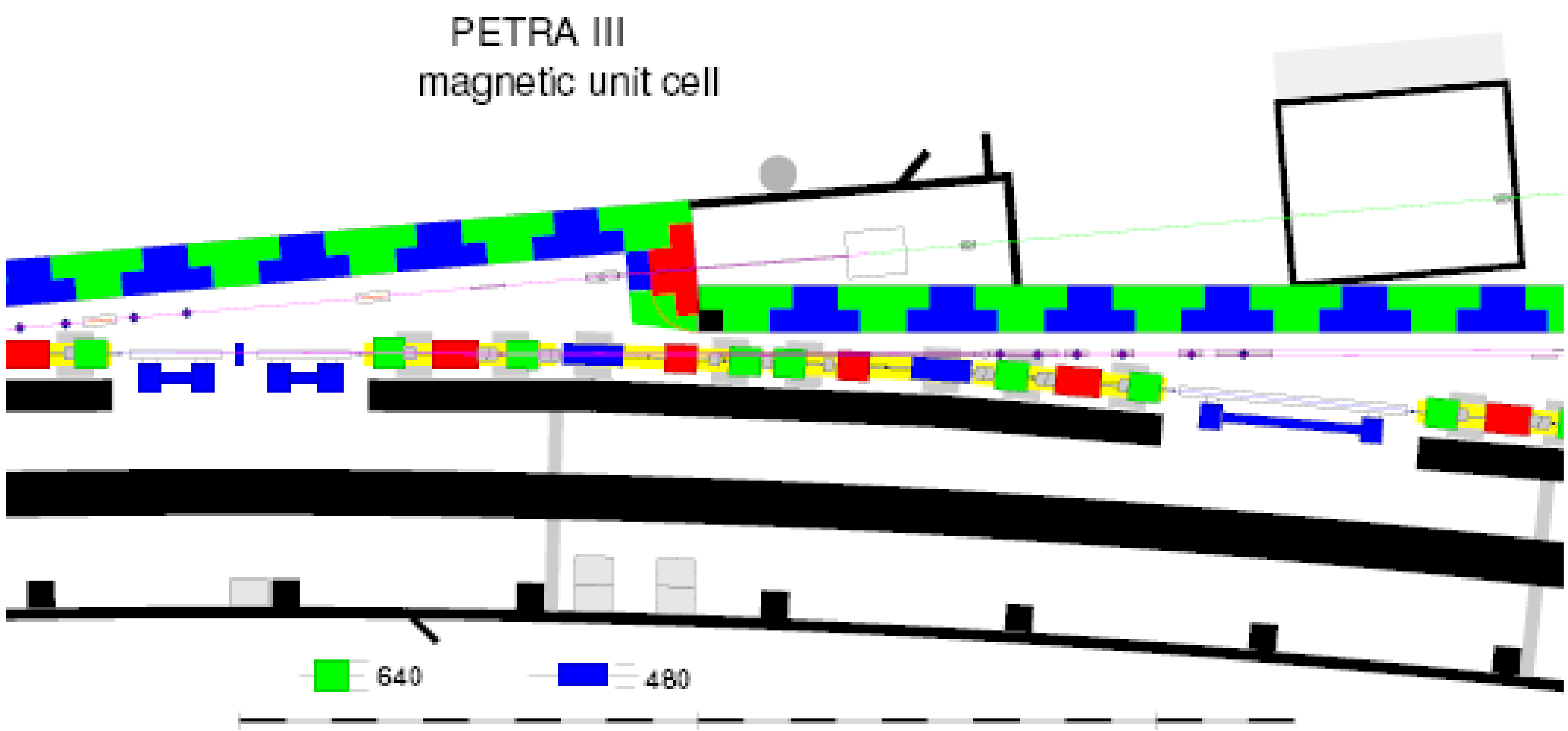
C = 2.3 km

$\rho = 200$ m

PETRA III Optics



Experimental Beamline



Further Issues

- Self Fields (particles interacting with each other)
  - repelling forces of same-charge particles limit particle density
  - higher energy helps
- Interaction with surroundings
  - Fields and image charges of particle beams interact with vacuum chamber walls, creating additional fields
  - Act back on same bunch or next bunch => Instability

Summary

- Accelerator Physics is a very broad and interesting field (which might not become clear at 9:00 on a Monday)
- Upcoming big accelerators are the LHC (this year), the ILC (2015 ?), and many FELs (European XFEL, LCLS, Spring8-XFEL …)
- Future of the field towards more tailored and specialized devices
- Novel acceleration techniques (plasma acceleration) are on the horizon and desperately needed to advance the energy frontier

Acknowledgments

- This talk based on material from
  - Bernhard Holzer (previous summer student lectures)
  - Chris Prior (CAS 2004 and 2006)
  - Daniel Brandt (CAS 2006)
- Further Reading
  - CERN Accelerator School Proceedings http://cas.web.cern.ch/cas/
  - E. Wilson: Introduction to Accelerators
  - S.Y. Lee: Accelerator Physics
  - H. Wiedemann: Accelerator Physics 1&2
  - K. Wille: Beschleunigerphysik
  - ….
- Accelerator Physics Programs
  - MAD-X http://mad.home.cern.ch/mad/
  - elegant http://www.aps.anl.gov/Accelerator_Systems_Division/Operations_Analysis/oagPackages.shtml