# Introduction to Particle Accelerators

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*based on Lecture by Bernhard Holzer, DESY*
**DESY Summer Student Lectures 2006**

## I) Introduction

*historical development & first principles*
*components of a typical accelerator*

## II) The state of the art in high energy machines:

*The synchrotron:*
*linear beam optics*
*colliding beams, luminosity*
*synchrotron radiation*

# Introduction to Particle Accelerators

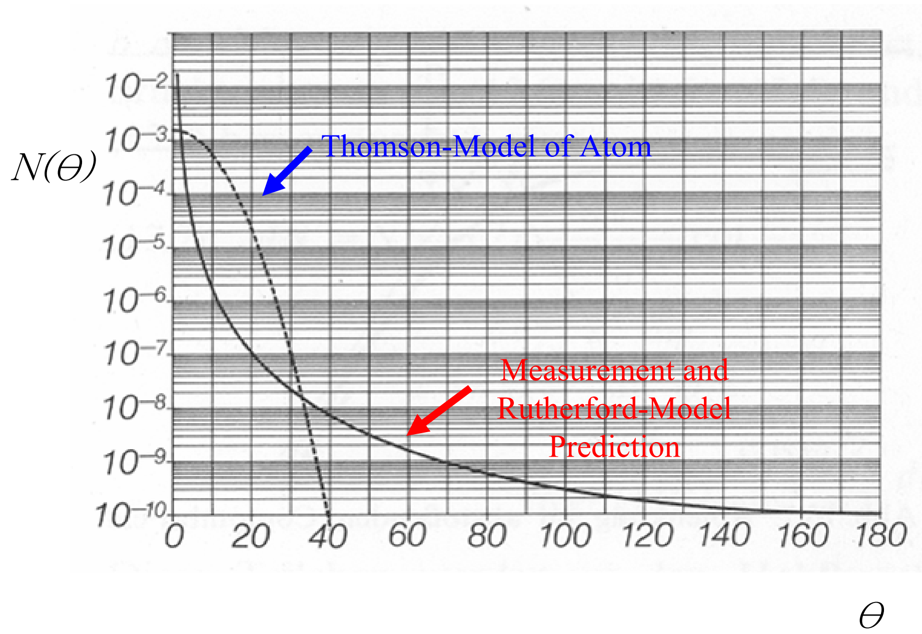
## Historical note:

**... the first steps in particle physics**

***Rutherford Scattering,
1906…1913***

**Using radioactive particle sources:
α-particles of some MeV energy**

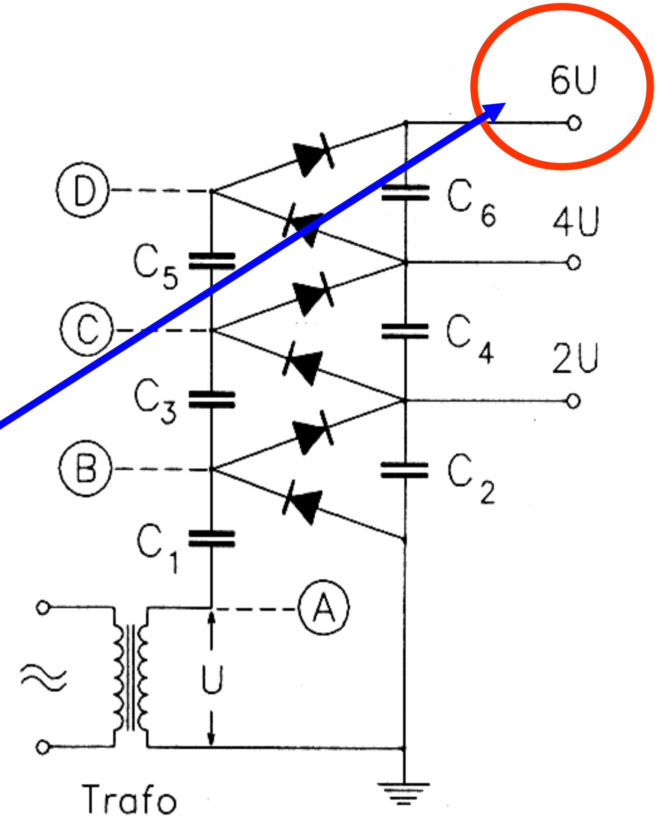
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



# Electrostatic Machines

I.) The Cockroft Walton Generator (1928):

**create a high Voltage DC by rectifying an AC Voltage: „cascade generator“**




Force: $$\left|\vec{F}\right| = q * \left|\vec{E}\right| = q * \frac{U}{d}$$

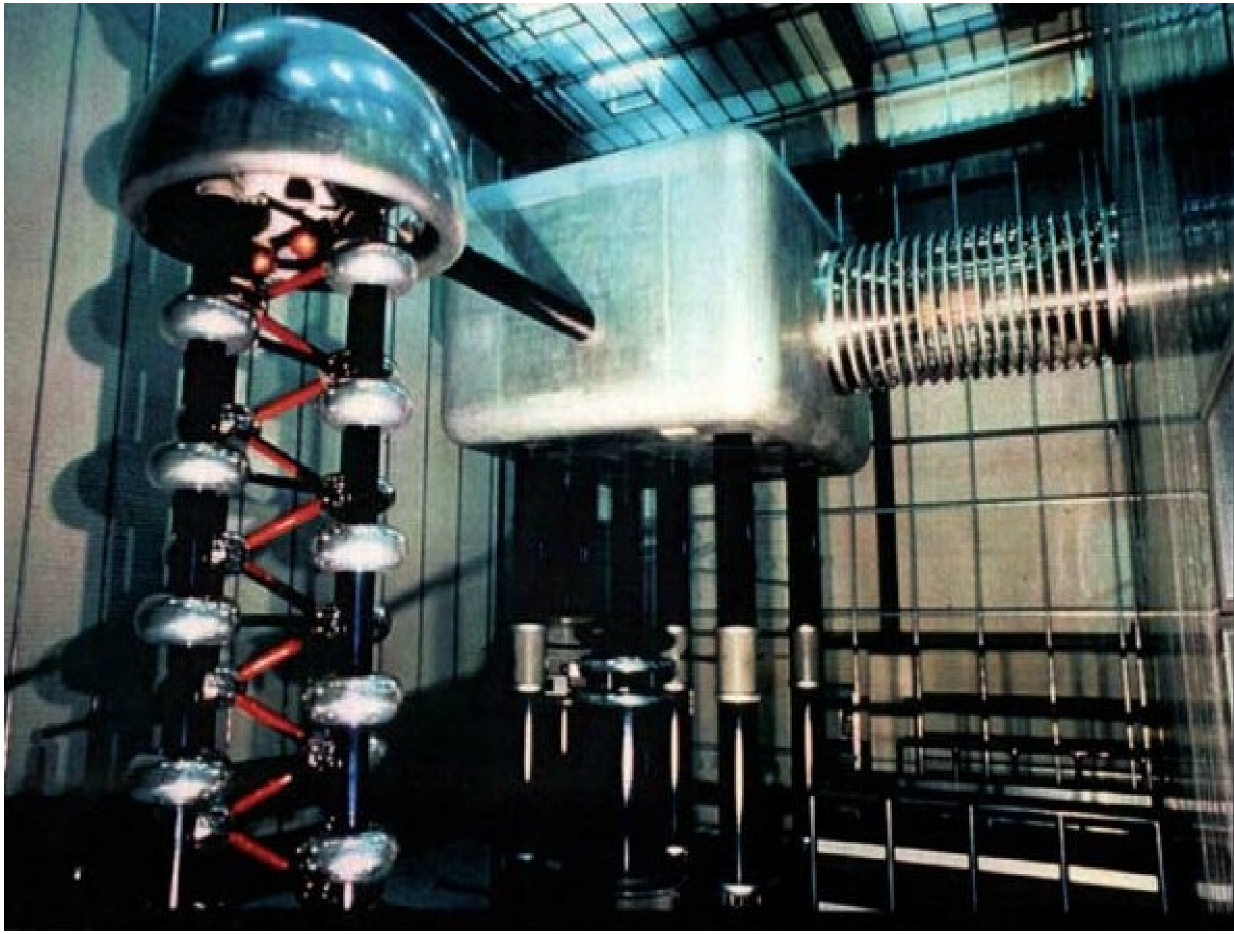
Energy: $$\Delta W = \int_{r1}^{r2} \vec{F}\, d\vec{r} = \int_{r1}^{r2} q * \vec{E}\, d\vec{r} = q * U$$

***usefull energy unit: „eV“ ...***

***...1 eV = 1.6*10^-19 J*** ($1\ eV = 1.6 \cdot 10^{-19}\ J$)

**1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV**

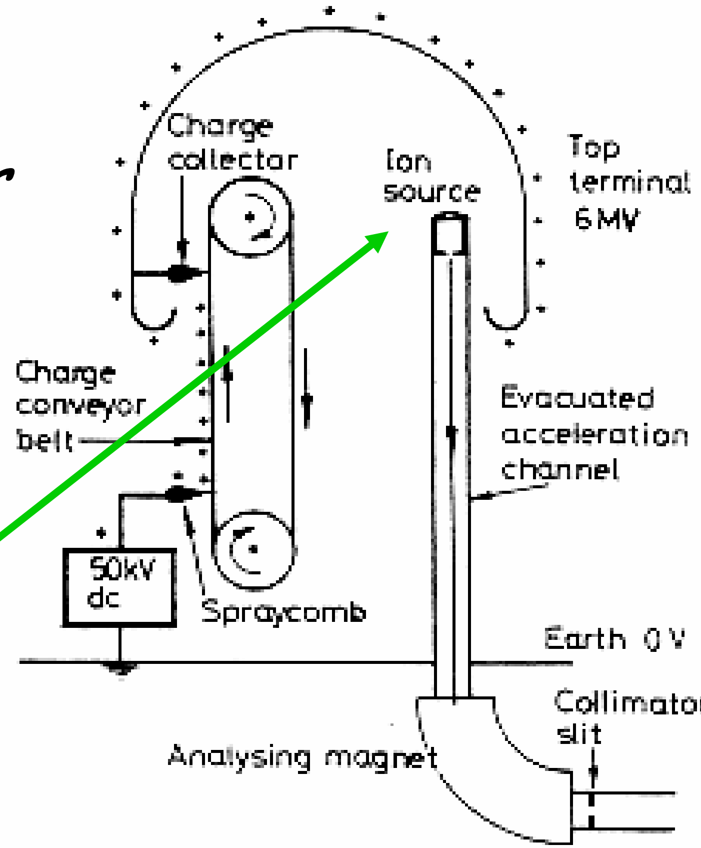
**Particle source:** Hydrogen discharge tube on a 400 kV level
**Accelerator:** evacuated glas tube
**Target:** Li-Foil on earth potential



**Example:**
Pre-accelerator for Protons at PSI (Villingen)

# Electrostatic Machines

## *II.) (Tandem -) van de Graaff Accelerator*




**creating high voltages by mechanical transport of charges**

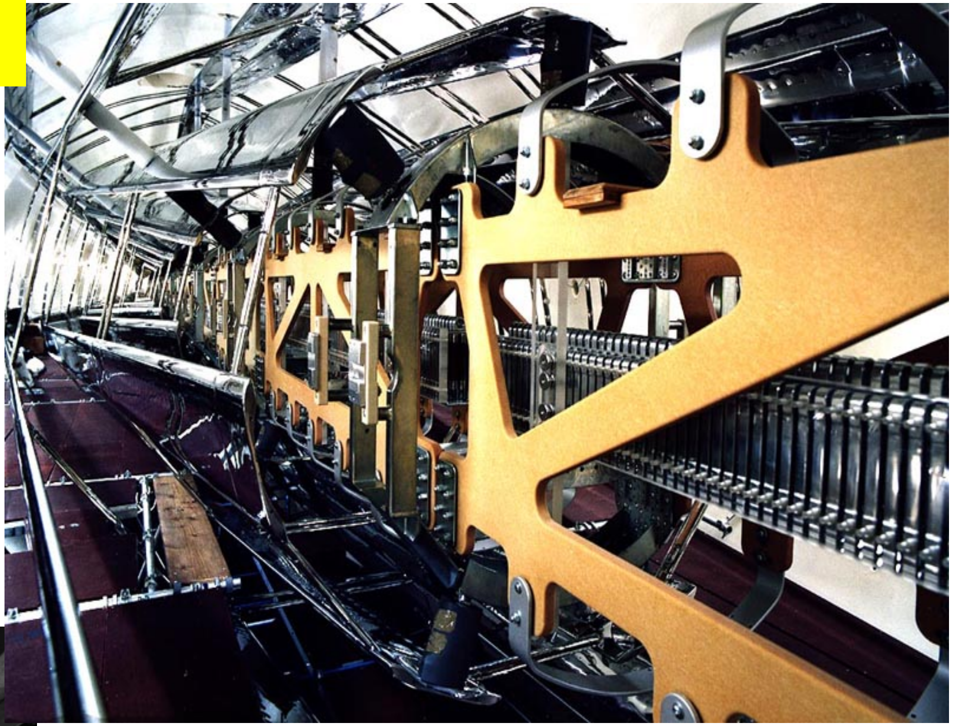
*** Terminal Potential: U ≈ 12 ...28 MV**
**using high pressure gas to suppress discharge ( $SF_6$ )**

**Problems: * Particle energy limited by high voltage discharges**
*** high voltage can only be applied once per particle ...**
**... or twice ?**

* The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. $H^-$) and
stripping the electrons in the centre of the structure

***Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg***

# And the inside of such an „steam engine"

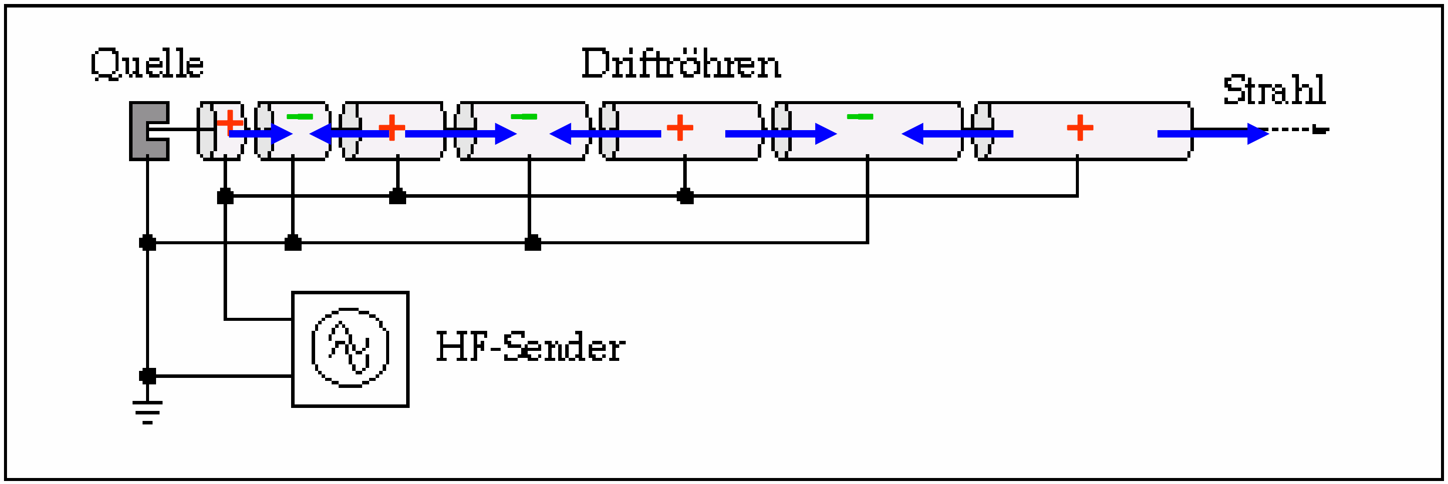


mechanical transport of charge using glas fiber belt

# Linear Accelerators

**1928, Wideroe:** **how can the acceleration voltage be applied several times to the particle beam**

**schematic Layout:**




*** acceleration of the proton in the first gap**
*** voltage has to be „flipped" to get the right sign in the second gap → RF voltage**
**→ shield the particle in drift tubes during the negative half wave of the RF voltage**

## Beam Energy I: Acceleration in the Wideroe Structure

### 1.) Energy gained after *n* acceleration gaps

$$E_n = n * q * U_0 * \sin\psi_s$$

- ***n*** number of gaps between the drift tubes
- ***q*** charge of the particle
- $U_0$ Peak voltage of the RF System
- $\Psi_S$ synchronous phase of the particle

### 2.) kinetic energy of the particles

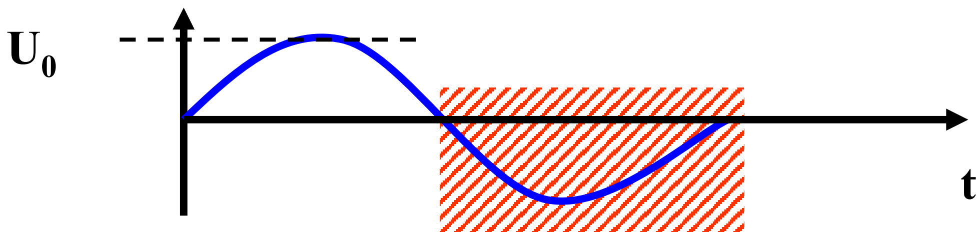
$$E_n = \frac{1}{2} m * {v_n}^2$$

valid for non relativistic particles ...

### velocity of the particle (from (1) and (2))

$$v_n = \sqrt{\frac{2E_n}{m}} = \sqrt{\frac{2 * n * q * U_0 * \sin\psi_S}{m}}$$

## 3.) shielding of the particles during the negative half wave of the RF




**Time span of the negative half wave: $\tau_{RF}/2$**

### Length of the n-th drift tube:

$$l_n = v_n * \frac{\tau_{RF}}{2} = v_n * \frac{1}{2\nu_{RF}}$$

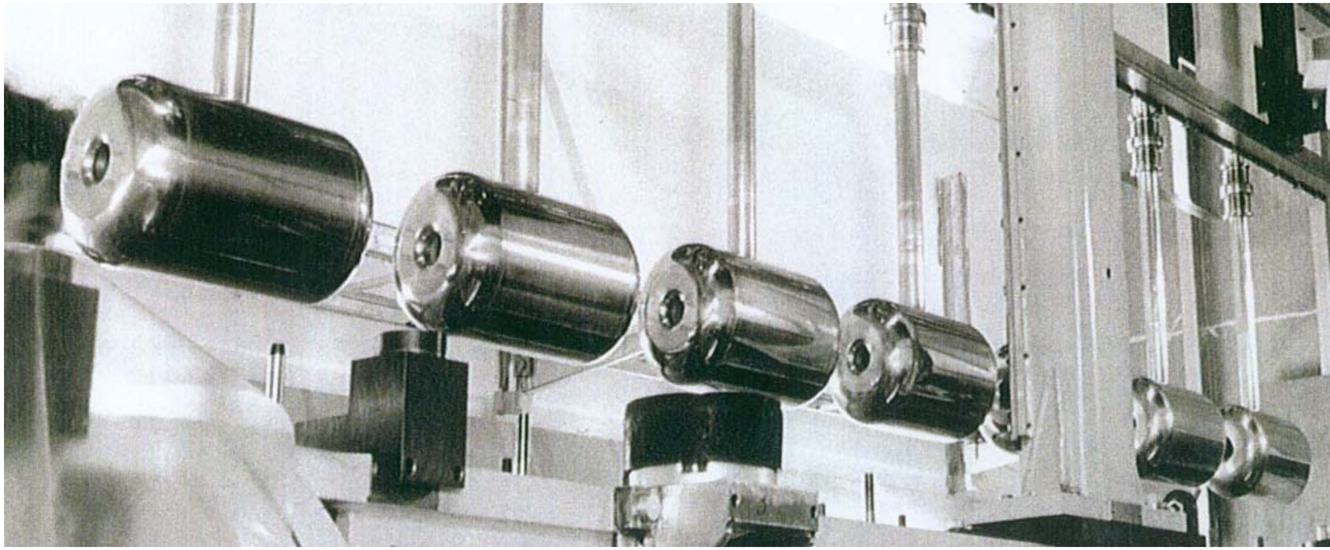
! high RF frequencies make small accelerators

**length of the *n-th* drift tube ... or ... distance between two accelerating gaps:**

$$l_n = \frac{1}{\nu_{RF}} \sqrt{\frac{n * q * U_0 * \sin\psi_S}{2m}}$$

# Example: DESY Accelerating structure of the Proton Linac

$$E_{total} = 988\,MeV$$



*reminder of some relativistic formula*

**rest energy** $E_0 = m_0 c^2 = 938\,MeV$

**total energy** $E = \gamma * E_0 = \gamma * m_0 c^2$

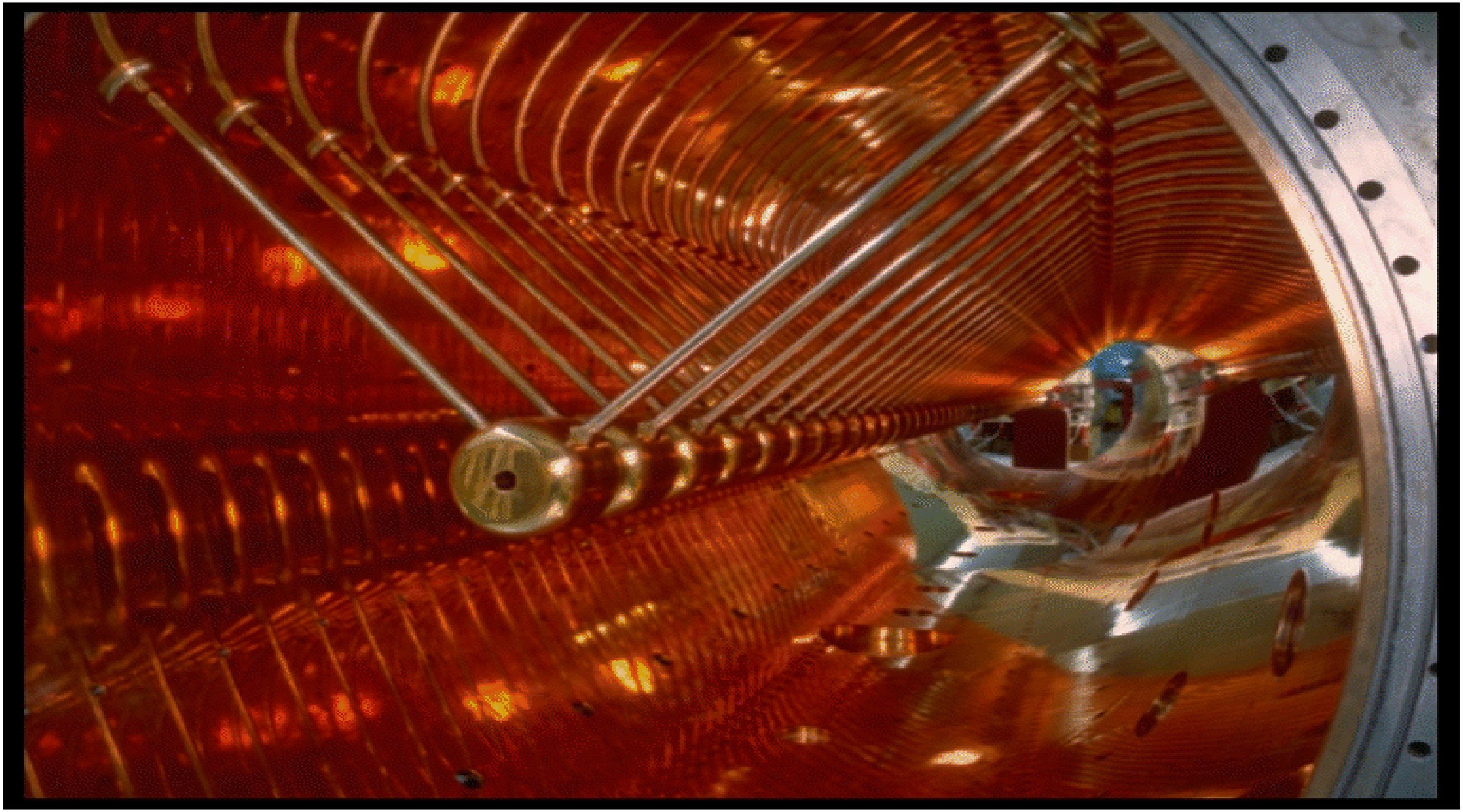
$$E_{kin} = E_{total} - m_0 c^2$$

$$E_{kin} = 50\,MeV$$

**momentum** $E^2 = c^2 p^2 + m_0{}^2 c^4$

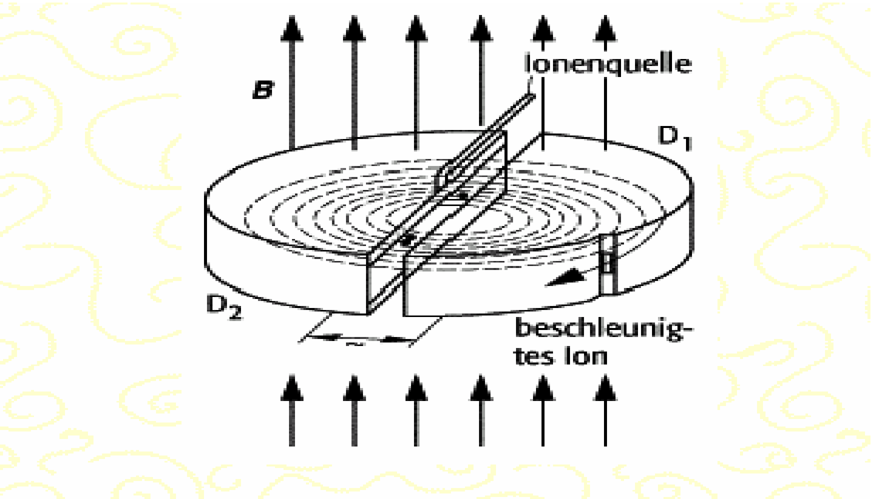
$$p = 310\,MeV/c$$

***GSI:*** ***Unilac, typical Energie ≈ 20 MeV per Nukleon, $\beta \approx 0.04 \ldots 0.6$, Protons/Ions, $\nu = 110$ MHz***



# The Cyclotron: (~1930)

**circular accelerator with a constant magnetic field**
**B = const**




Lorentz force:

$$\vec{F} = q * (\vec{v} \times \vec{B})$$

centrifugal force:

$$F = \frac{m * v^2}{\rho}$$

condition for a circular particle orbit:

$$q * v * B = \frac{m * v^2}{\rho} \quad \rightarrow \quad \boxed{B * \rho = p / q}$$

$$\rightarrow \quad \frac{\rho}{v} = \frac{m}{q * B}$$

time for one revolution:

$$T = 2\pi \frac{\rho}{v} = 2\pi \frac{m}{q * B_z}$$

revolution frequency

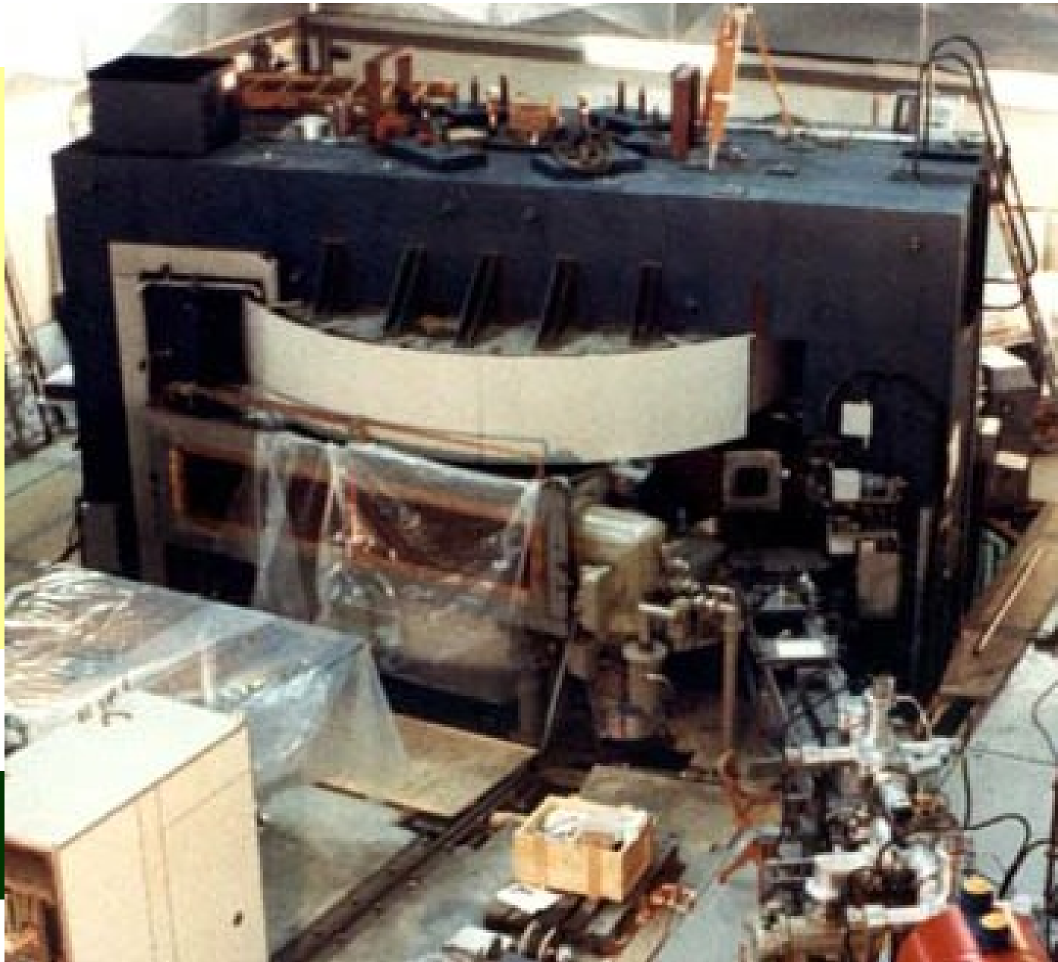
$$\omega_z = 2\pi \frac{1}{T} = \frac{q}{m} * B_z \quad \rightarrow \quad \omega_z = const.$$

! ω is constant for a given q & B

!! B*ρ = p/q
large momentum → huge magnets

!!!! **ω ~ 1/m ≠ const.**
works properly only for non relativistic particles

**Example: cyclotron at PSI**



I) are there any questions until now ???

II) The state of the art in high energy machines:

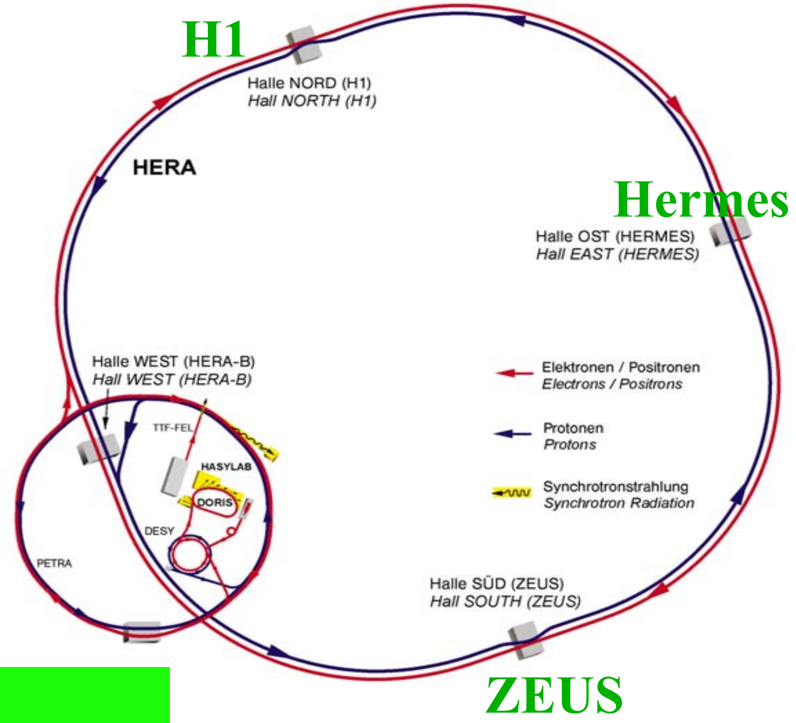
The synchrotron:

- linear beam optics
- colliding beams, luminosity
- synchrotron radiation

# The state of the art in high energy acceleration: The Synchrotron

# Design of the Machine

| | |
|---|---|
| 422 | s.c. dipole magnets |
| 224 | s.c. main quads, |
| 400 | s.c. correction quads |
| 200 | s.c. correction dipoles |
| > 1000 | n.c. electron magnets |




| | | |
|---|---|---|
| **Circumference:** | 6.3 km | |
| **Proton Beam:** | Injection Energy | 40 GeV |
| | Lumi-Energy | 920 GeV |
| **Electron Beam:** | Injection Energy | 12 GeV |
| | Lumi Energy | 27.5 GeV |
| **Magnetic field p-ring:** | 5.1 Tesla | |
| | at I=5500 A for 920 GeV | |

# Design Principles of a Synchrotron
## I.) the bending magnets

**„ ... in the end and after all it should be a kind of circular machine“**
**→ need transverse deflecting force**

**1.) ... again ... the Lorentz force**

$$\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines: $v \approx c \approx 3 * 10^8 \, m/s$

***old greek dictum of wisdom:***
***if you are clever, you use magnetic fields in an accelerator where ever it is possible.***

**But remember: magn. fields act allways perpendicular to the velocity of the particle**
**→ only bending forces, → no „beam acceleration“**

# Lattice Design: Prerequisites

Lorentz force $\vec{F} = q * (\vec{v} \times \mathbf{B})$

**High energy accelerators → circular machines**
**somewhere in the lattice we need a number of dipole magnets, that are bending the design orbit to a closed ring**

**In a constant external magnetic field the particle trajectory will be a part of a circle and ... the centrifugal force will be equal to the Lorentz force**

$$e * v * B = \frac{m v^2}{\rho} \qquad \rightarrow e * B = \frac{mv}{\rho} = p / \rho$$

$$\rightarrow B * \rho = p / e$$

p = momentum of the particle,
ρ = curvature radius

B*ρ is called the "beam rigidity"

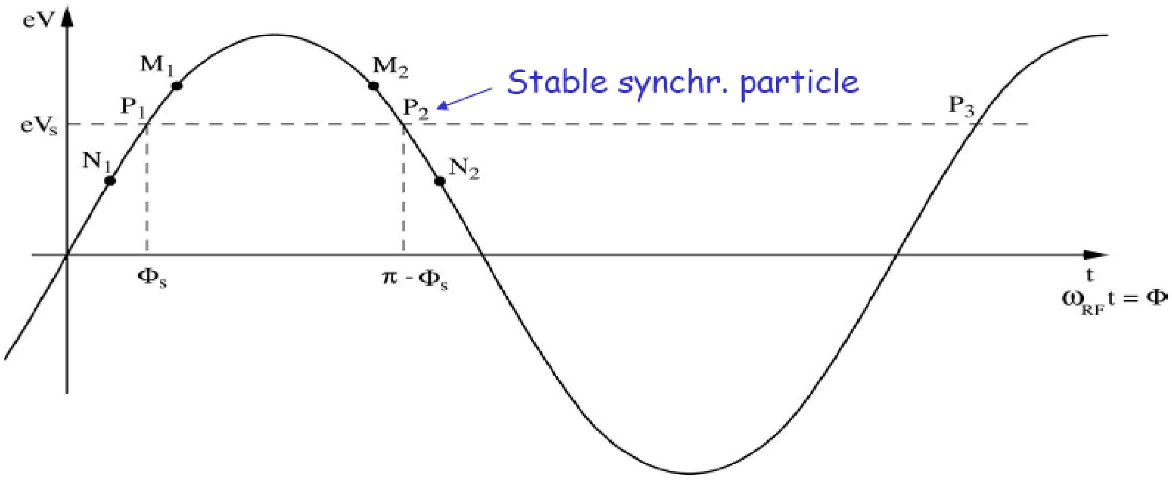
*Example:*
*heavy ion storage ring: TSR*
*8 dipole magnets of equal bending strength*

# The Synchrotron

$$\rho = \frac{p/q}{B}$$

increase magnet field **B** synchronous to momentum **p**

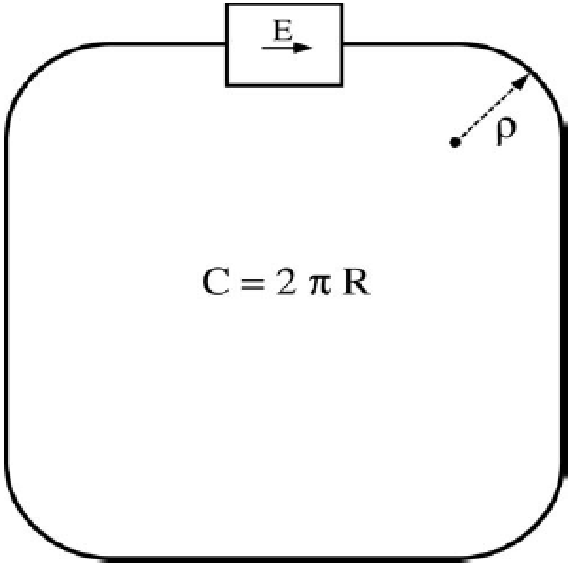
Where is the acceleration?
Install an RF accelerating structure in the ring:




$$\Delta p / q = eU_0 \sin(\phi)$$

# Synchrotron

The synchrotron is a synchronous accelerator since there is a synchronous RF phase for which the energy gain fits the increase of the magnetic field at each turn. That implies the following operating conditions:




$e\hat{V}\sin\Phi \longrightarrow$ Energy gain per turn

$\Phi = \Phi_s = cte \longrightarrow$ Synchronous particle

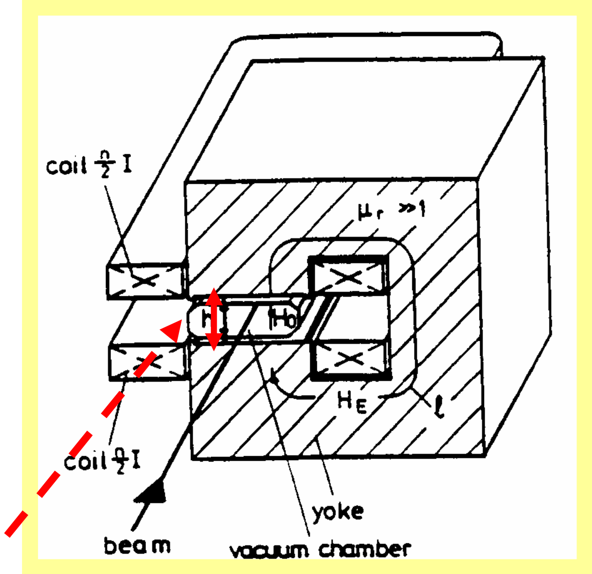
$\omega_{RF} = h\omega_r \longrightarrow$ RF synchronism

$\rho = cte \quad R = cte \longrightarrow$ Constant orbit

$B\rho = P/e \Rightarrow B \longrightarrow$ Variable magnetic field

If v = c, $\omega_r$ hence $\omega_{RF}$ remain constant (ultra-relativistic $e^-$)

# Magnetic Dipole Fields:




### Technical design of a dipole magnet

a magnet with two flat, parallel pole shoes creates a homogeneous dipole field

**Maxwells equations:**

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\delta \vec{D}}{\delta t}$$

$$\int_A \left( \vec{\nabla} \times \vec{H} \right) \vec{n}\, da = \oint \vec{H} d\vec{s} = h * H_0 + l_{Fe} * H_{Fe} = nI$$

$$H_0 = \frac{B_0}{\mu_0}, \; H_{Fe} = \frac{B_0}{\mu_0 * \mu_{Fe}}$$

**n*I = number of coil windings, each carrying the current I**

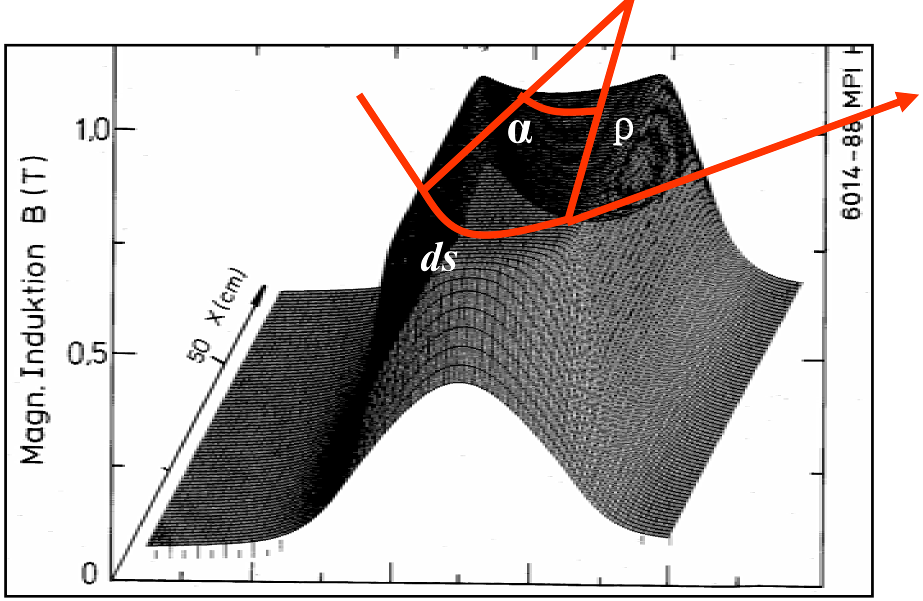
**$\mu_r$ = rel. permeability of the material, $\mu_r$ (Fe) ≈ 3000**

$$B_0 = \frac{\mu_0 * nI}{h}$$

**the magnetic field B depends on**
- **the current,**
- **the number of windings**
- **the gap height**

# Circular Orbit:

$$\alpha = \frac{s}{\rho} \approx \frac{l}{\rho} \qquad \alpha = \frac{B*l}{B*\rho}$$



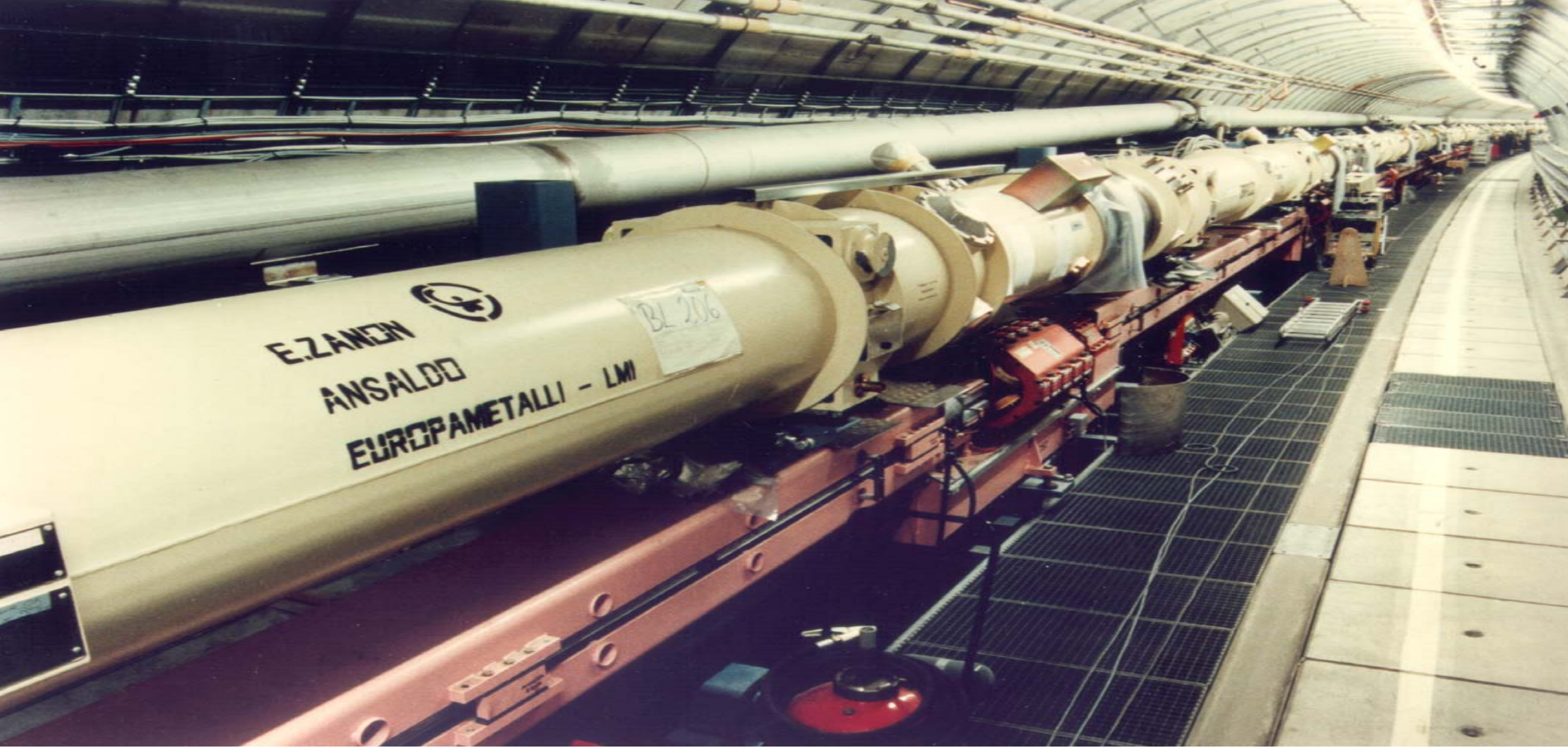
*field map of a storage ring dipole magnet*

The angle swept out in one revolution must be 2π, so

$$\alpha = \frac{\int B\,dl}{B*\rho} = 2\pi \quad \text{... for a full circle}$$

$$\rightarrow B*L = 2\pi * \frac{p}{q}$$

***The overall length of all dipole magnets multiplied by the dipole field corresponds to the momentum (≈ energy) of the beam !***




***Example HERA:***
920 GeV Proton storage ring
number of dipole magnets N = 416
$l$ = 8.8m
q = +1 e

$$\int B dl \approx N * l * B = 2\pi \, p / q$$

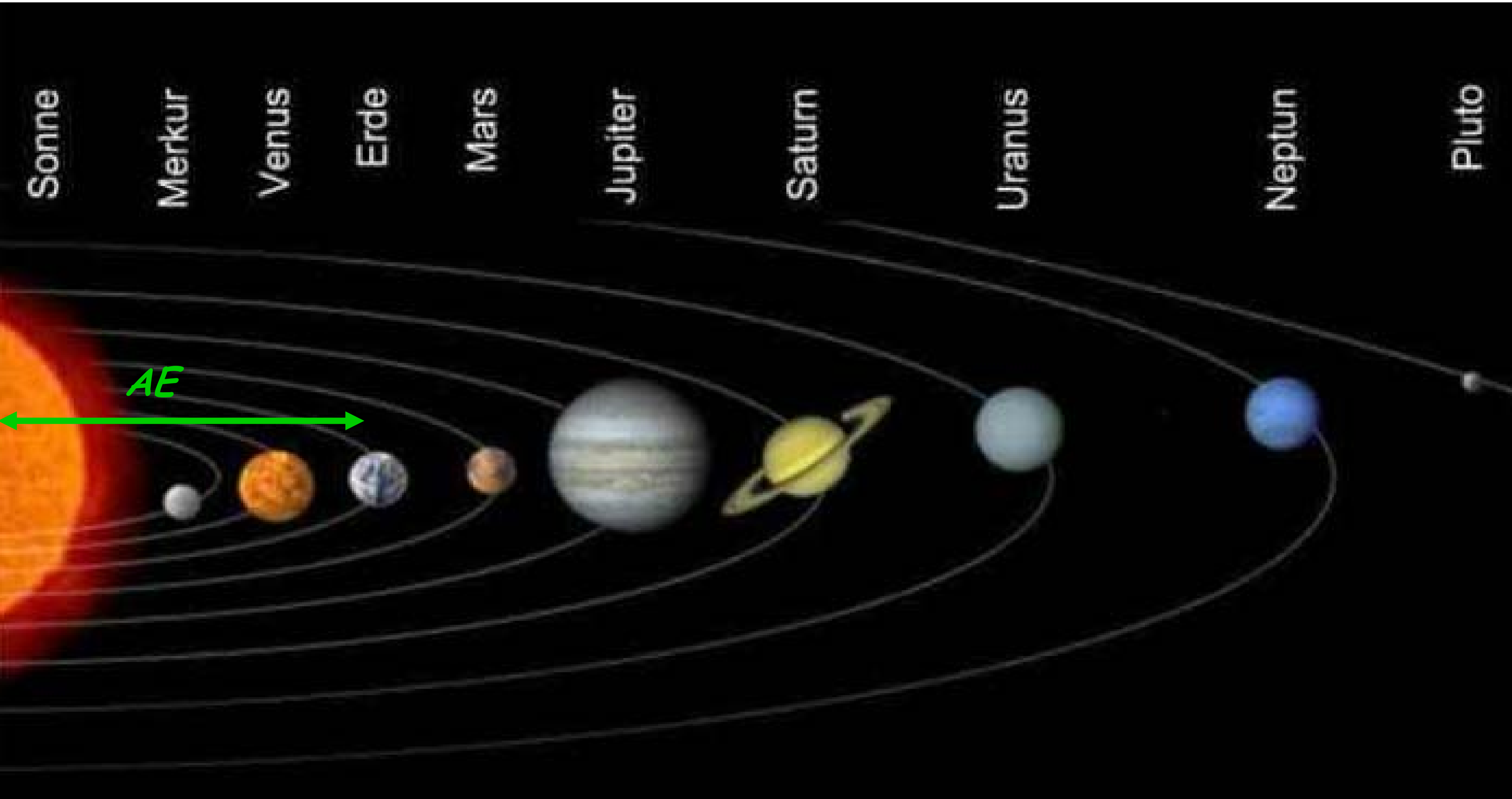
$$B \approx \frac{2\pi * 920 * 10^9 eV}{416 * 3 * 10^8 \frac{m}{s} * 8.8m * e} \approx 5.15 \, Tesla$$

# Largest storage ring: The Solar System

astronomical unit: average distance earth-sun

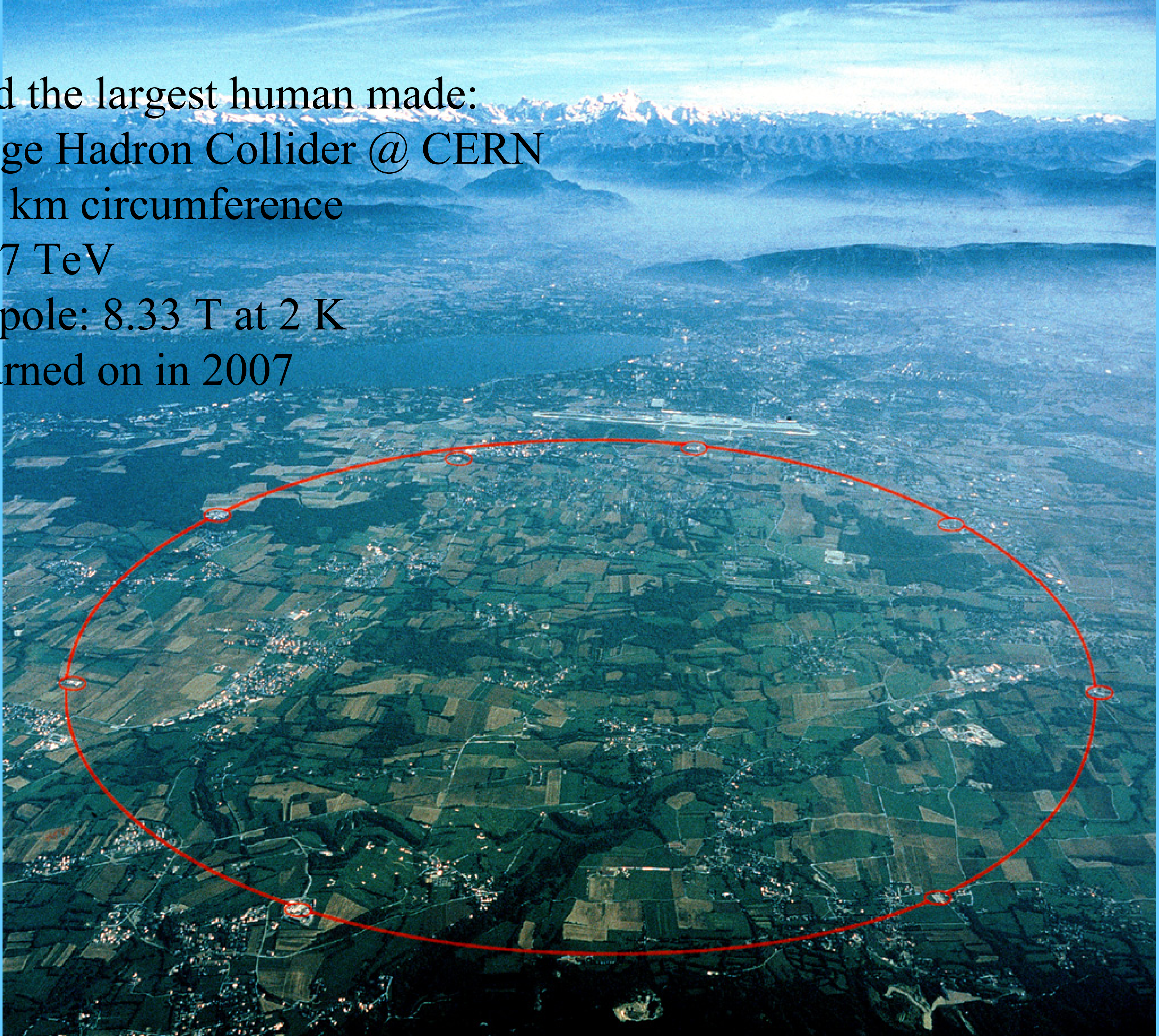
$1AE \approx 150 \cdot 10^6$ km

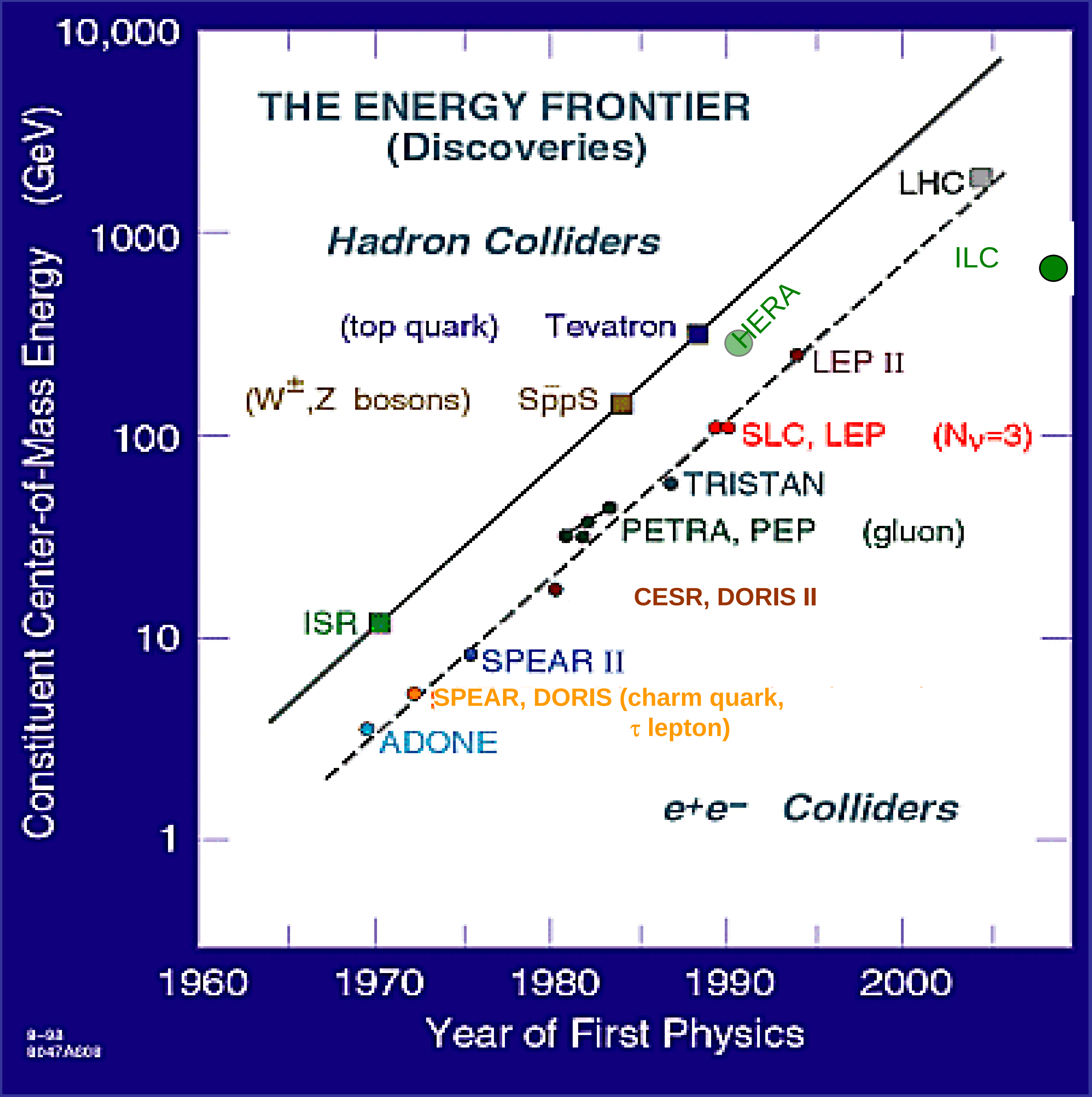
Distance Pluto-Sun $\approx$ 40 AE



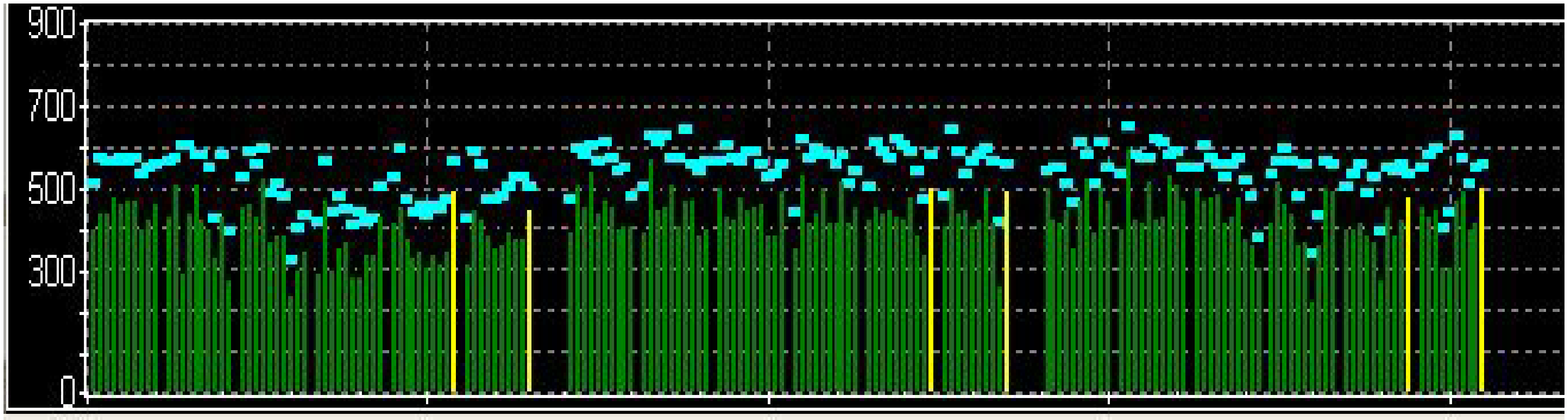
And the largest human made:
Large Hadron Collider @ CERN

- 27 km circumference
- 2*7 TeV
- Dipole: 8.33 T at 2 K
- Turned on in 2007




THE ENERGY FRONTIER
(Discoveries)
Hadron Colliders
(top quark)
Tevatron
LHC
ILC
HERA
LEP II
(W±,Z bosons)
Sp̄pS
SLC, LEP
(Nν=3)
TRISTAN
PETRA, PEP
(gluon)
CESR, DORIS II
ISR
SPEAR II
SPEAR, DORIS (charm quark,
τ lepton)
ADONE
e+e− Colliders
10,000
1000
100
10
1
Constituent Center-of-Mass Energy (GeV)
1960
1970
1980
1990
2000
Year of First Physics

**What about the beam ?** HERA: 100mA protons stored in 180 bunches




**number of particles per bunch**

$$N_b = \frac{100mA}{180} * \frac{\tau_{rev}}{e}$$

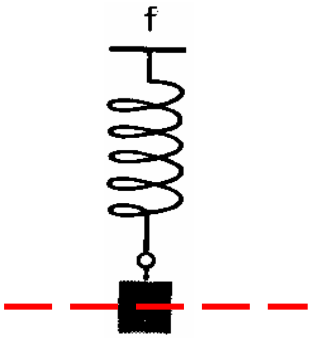
$$N_b = \frac{100*10^{-3}}{180} * \frac{Cb}{s} * \frac{21*10^{-6}}{1.6*10^{-19}} * \frac{s}{Cb}$$

$$N_b = 7.3*10^{10}$$

***A particle beam consits of 180 moscito clouds each containing 7.3*10 $^{10}$ Moscitos, running back and forth at the speed of light between sun and Pluto***

# *the focusing properties – transverse beam optics*

**classical mechanics: pendulum**




there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

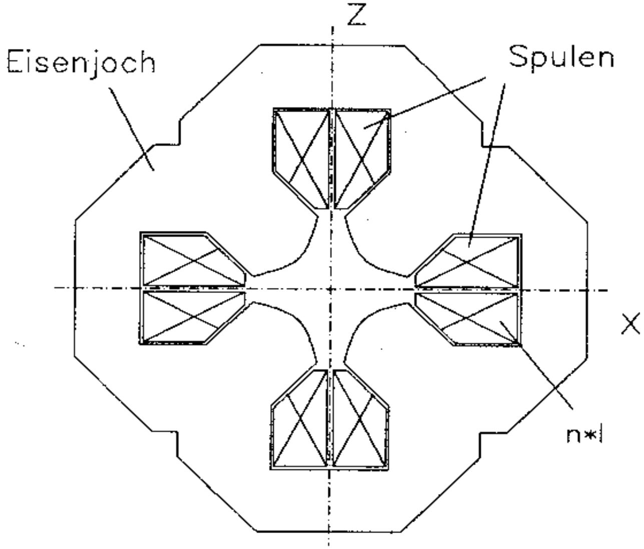
$$x(t) = A * \cos(\omega t + \varphi)$$

**Storage Ring: we need a Lorentz force that rises as a function of the distance to ........ ?**

**................... the design orbit**

$$F(x) = q * v * B(x)$$

# Quadrupole lenses to focus the beam




four iron pole shoes
of hyperbolic contour

linear increasing magnetic field

$$B_z = g * x\,, \quad B_x = g * z$$

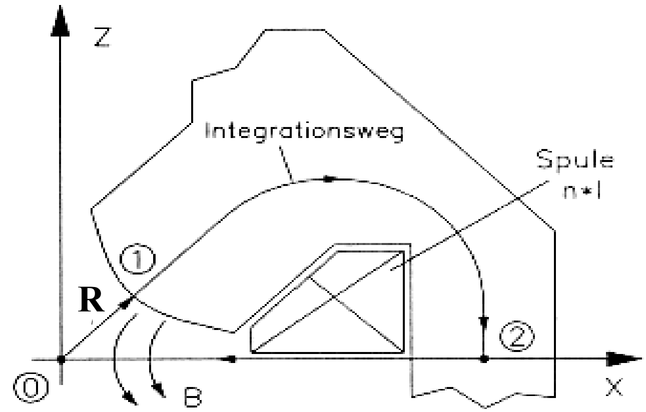
Maxwell‘s equation at the location of the beam
**... no current, no electr. field**

$$\vec{\nabla} \times \vec{B} = 0$$

→ the B field can be expressed as gradient of a scalar potential ***V:***

$$\vec{B} = -\vec{\nabla} V\,, \quad V(x,z) = g\,x\,z$$

equipotential lines $V = gxz$ correspond to hyperbolic curves. Surface of the iron pole shoes (lines of const. potential) has to be hyperbolic.




Quadrupole Field:

$$\boldsymbol{n * I = \oint H ds}$$

$$= \int_0^R H(r)\,dr + \int_1^2 \vec{H}_E\,d\vec{s} + \int_2^0 \vec{H}\,d\vec{s}$$

$$= \frac{1}{\mu_0}\int_0^R g \cdot r\,dr$$

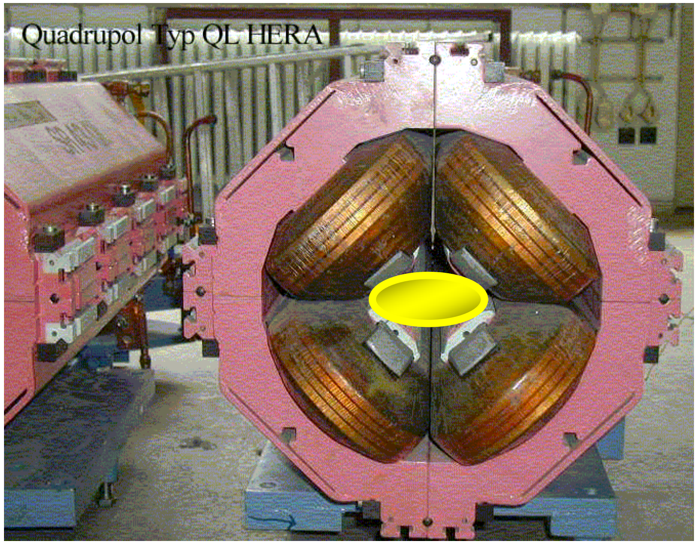
$$g = \frac{2\mu_0 n I}{R^2}$$

Magnetic field of the quadrupole: $B = g * x$

Gradient of the quadrupole field: $g = \frac{dB_z}{dx}$

$$g = \frac{2\mu_0 nI}{R^2}$$

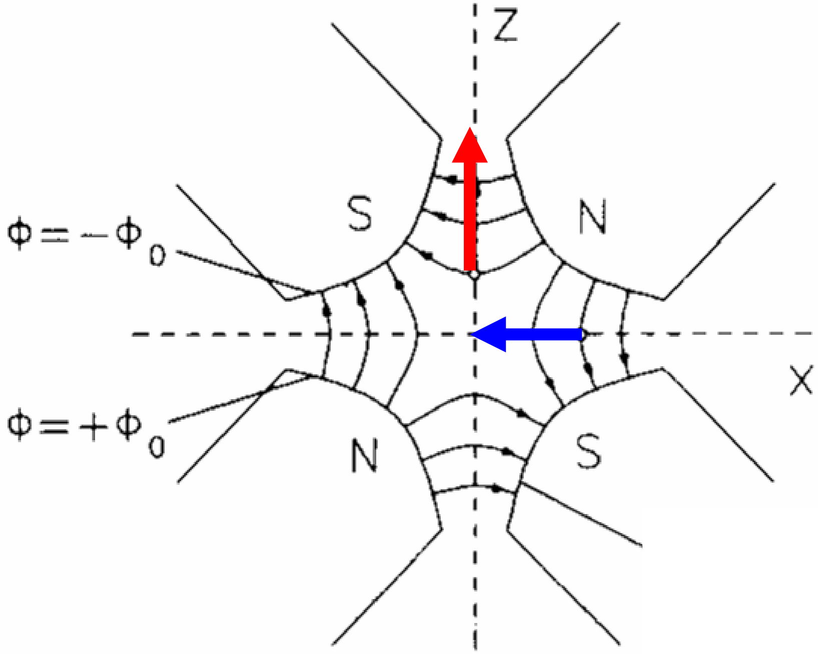
*Example: quadrupole magnet of HERA electron storage ring*




Nota bene:

I) quadrupole lenses that need large apertures have small gradients

II) nothing in life is for free
a horizontal focusing lens
will defocus in the vertical
plane (atowr).



# Focusing forces and particle trajectories:

1.) normalise magnet fields to momentum
(remember: $B*\rho = p / q$ )

*Dipole Magnet*

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

*Quadrupole Magnet*

$$k := \frac{g}{p / q}$$

## Example: HERA Ring

**Momentum:**

**p = 920 GeV/c**

**Bending field:**

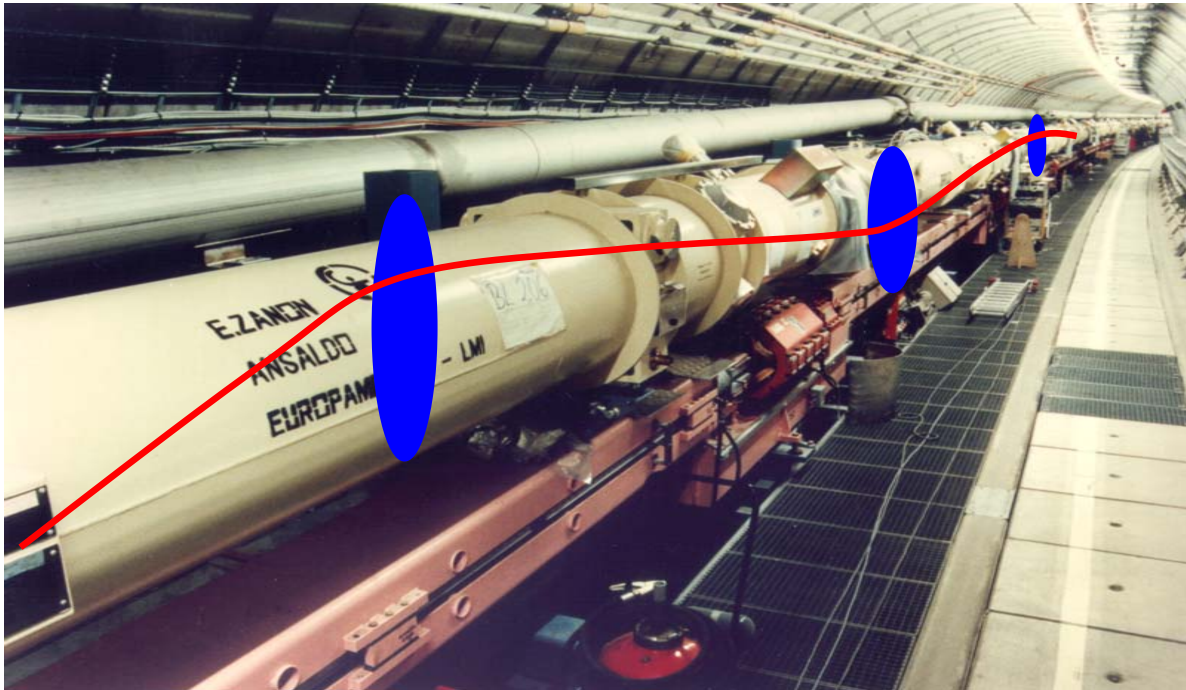
**B = 5.5 Tesla**

**Quadrupol Gradient**

**G= 110 T/m**

→ **k = $33.64 \cdot 10^{-3}$ /m²**

→ **$1/\rho$ = $1.7 \cdot 10^{-3}$ /m**



# the focusing properties - Equation of motion

**Under the influence of the focusing and defocusing forces the differential equation of the particles trajectory can be developed:**

$$x'' + k * x = 0$$ **horizontal plane**

***x = distance of a single particle to the center of the beam***

$$x' := \frac{dx}{ds}$$

***vert. plane:*** $k \Rightarrow -k$

if we assume ....

* linear retrieving force
* constant magnetic field
* first oder terms of displacement ***x***

... we get the general solution (hor. focusing magnet):

$$x(s) = x_0 * \cos(\sqrt{k}s) + \frac{x_0'}{\sqrt{k}} * \sin(\sqrt{k}s)$$

$$x'(s) = -x_0\sqrt{k} * \sin(\sqrt{k}s) + x_0' * \cos(\sqrt{k}s)$$

More elegant description: Matrix formalism

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M * \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

## Matrices of lattice elements

**Hor. focusing Quadrupole Magnet**

$$M_{QF} = \begin{pmatrix} \cos(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sin(\sqrt{K} * l) \\ -\sqrt{K} \sin(\sqrt{K} * l) & \cos(\sqrt{K} * l) \end{pmatrix}$$

**Hor. defocusing Quadrupole Magnet**

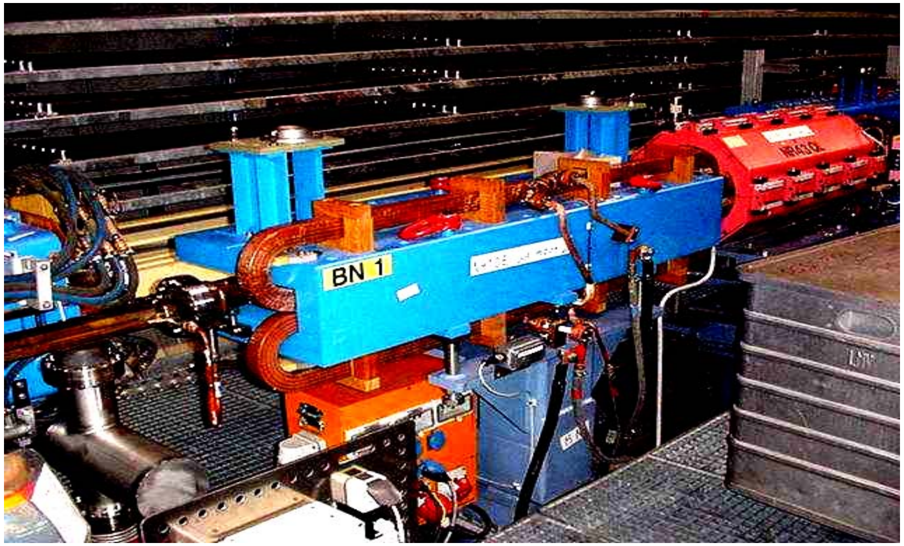
$$M_{QD} = \begin{pmatrix} \cosh(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sinh(\sqrt{K} * l) \\ \sqrt{K} \sinh(\sqrt{K} * l) & \cosh(\sqrt{K} * l) \end{pmatrix}$$

**Drift space**

$$M_{Drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

Nota bene:
formalism is only valid within one lattice element where ***k = const***

in reality: ***k = k(s)***



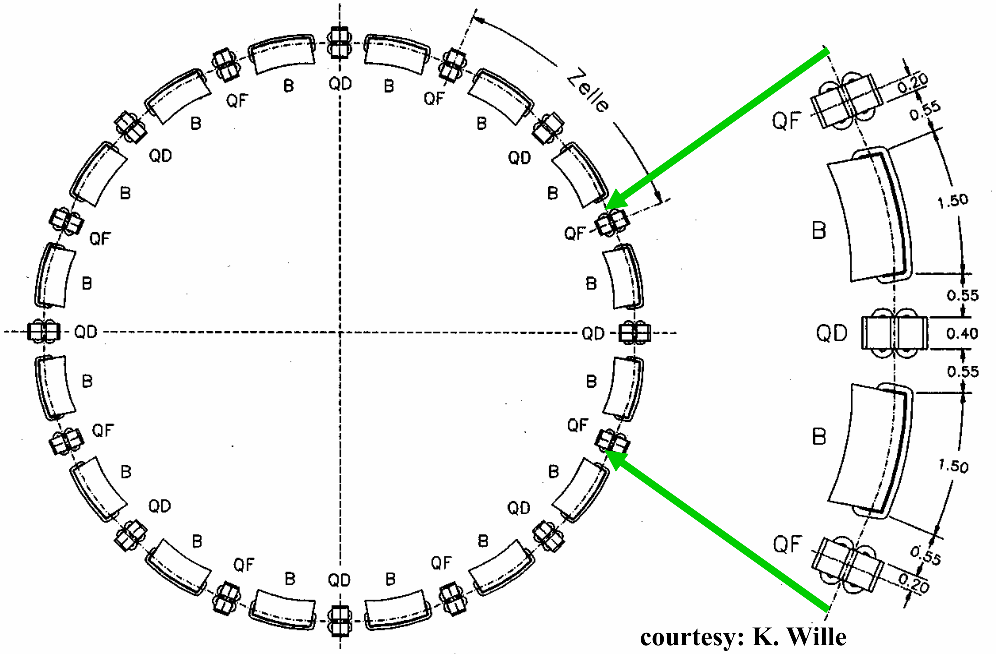
## „veni vidi vici ...“ .... or in english .... „we got it !“

* we can calculate the trajectory of a single particle within a single lattice element
* for any starting conditions $x_{0}$, $x'_{0}$
* we can combine these piecewise solutions together and get the trajectory for the complete storage ring.

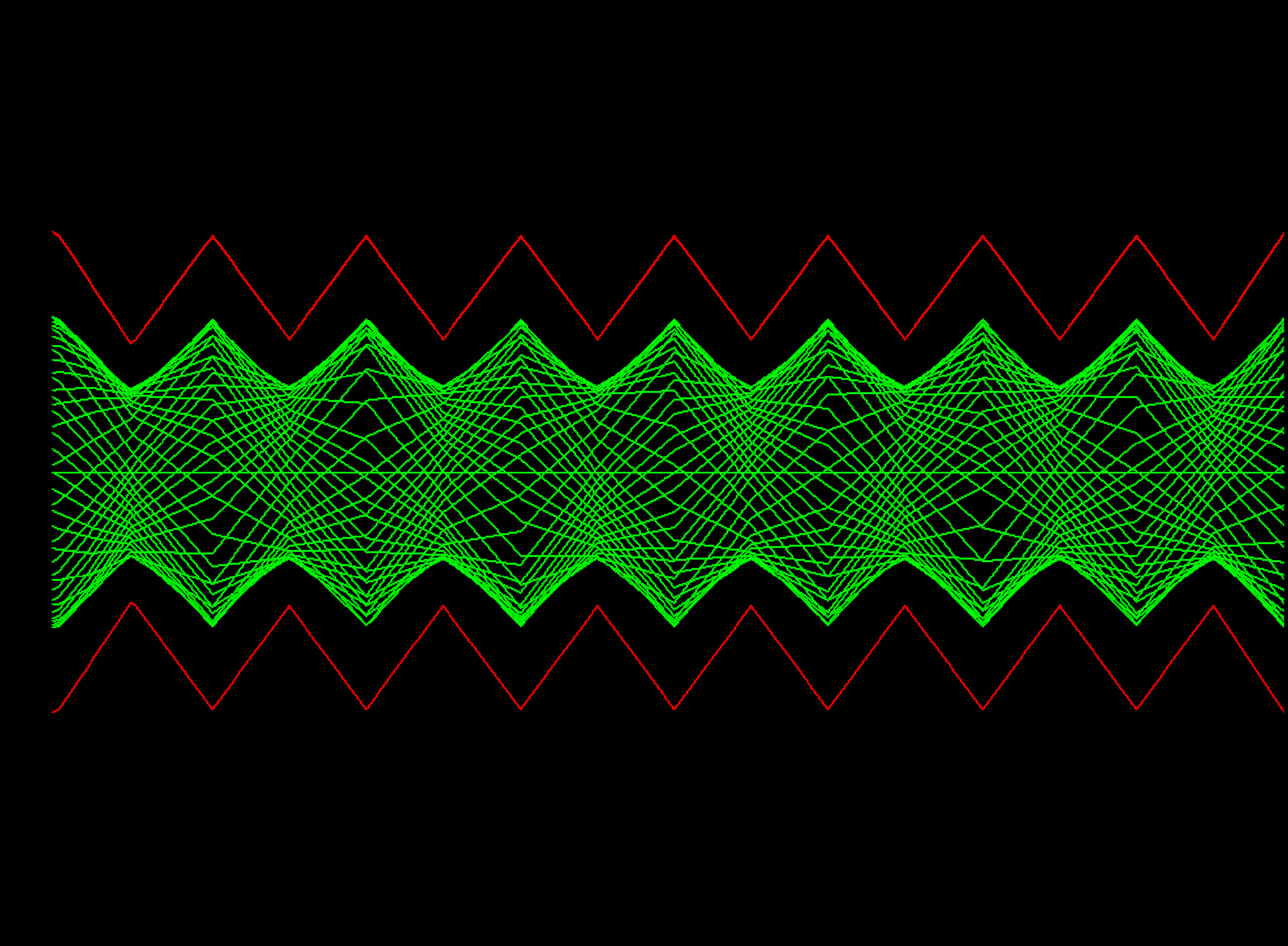
$$M_{lattice} = M_{QF1} * M_{D1} * M_{QD} * M_{D1} * M_{QF2} \ldots\ldots$$

Example:
storage ring for beginners

Dipole magnets and QF & QD quadrupole lenses




courtesy: K. Wille

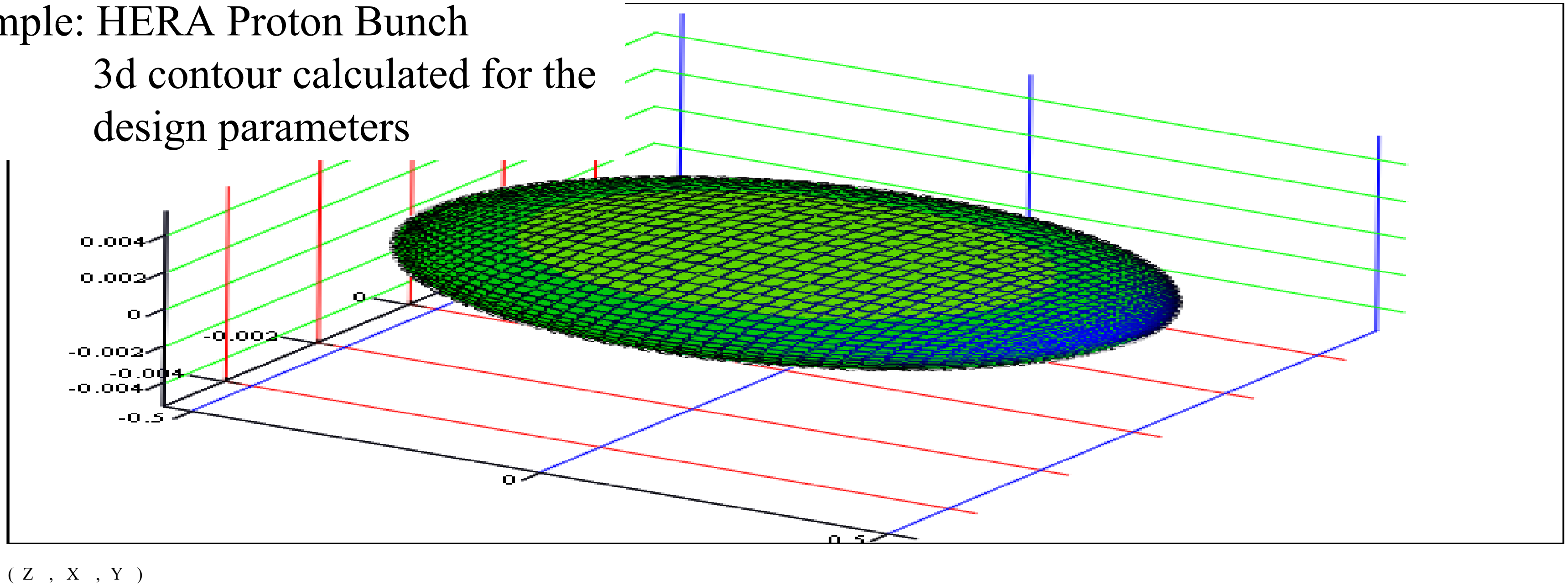

Teilchenbahnen und Envelope

Contour of a particle bunch given by the external focusing fields (arc values)

| | $e$ | $p$ |
|---|---|---|
| $\sigma_x$ | *1.0 mm* | *0.75 mm* |
| $\sigma_z$ | *0.2 mm* | *0.46 mm* |
| $\sigma_s$ | *10.3 mm* | *190 mm* |
| $N_p$ | $3.5*10^{10}$ | $7.3*10^{10}$ |

Example: HERA Proton Bunch

3d contour calculated for the design parameters



# Twiss Parameters

***Astronomer Hill:***

***differential equation for motions with periodic focusing properties: „Hill's equation"***

*Example: particle motion with periodic coefficient*

***equation of motion:*** $$x''(s) - k(s)x(s) = 0$$

***restoring force $\neq$ const,***
***k(s) = depending on the position s***
***k(s+L) = k(s), periodic function***

***we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.***

# *The Beta Function*

*in this case the solution can be written in the form:*

*Ansatz:*

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

*ε, Φ = integration constants determined by initial conditions*

*β(s) given by focusing properties of the lattice ↔ quadrupoles*

*ε beam emittance = woozilycity of the particle ensemble,*
*intrinsic beam parameter,*
*cannot be changed by the foc. properties.*

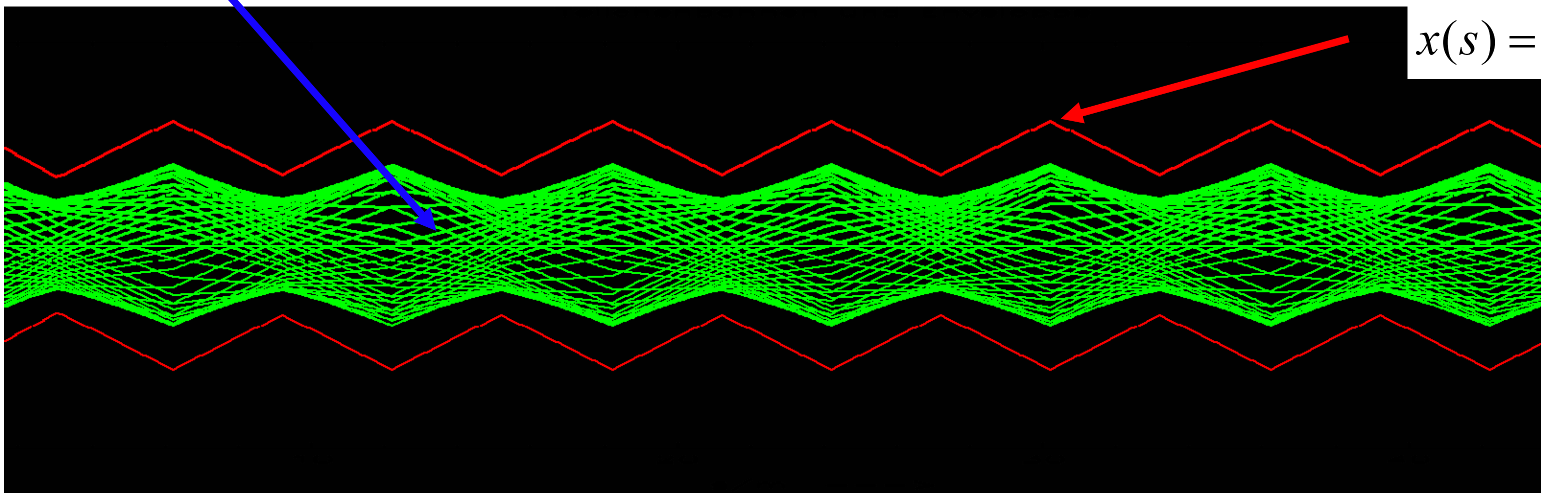
*Scientifiquely spoken: area covered in transverse x, x´ phase space*
*… and it is constant !!!*

# Ensemble of many (...all) possible particle trajectories

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

max. amplitude of all particle trajectories

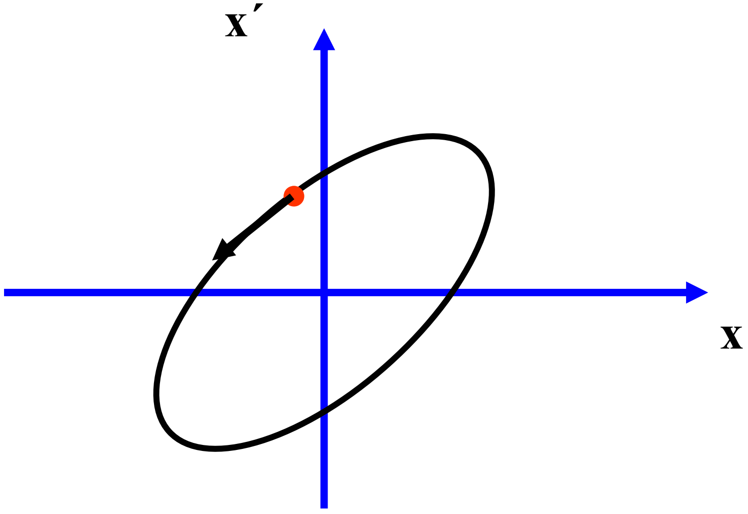
$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)}$$



## Beam Dimension:

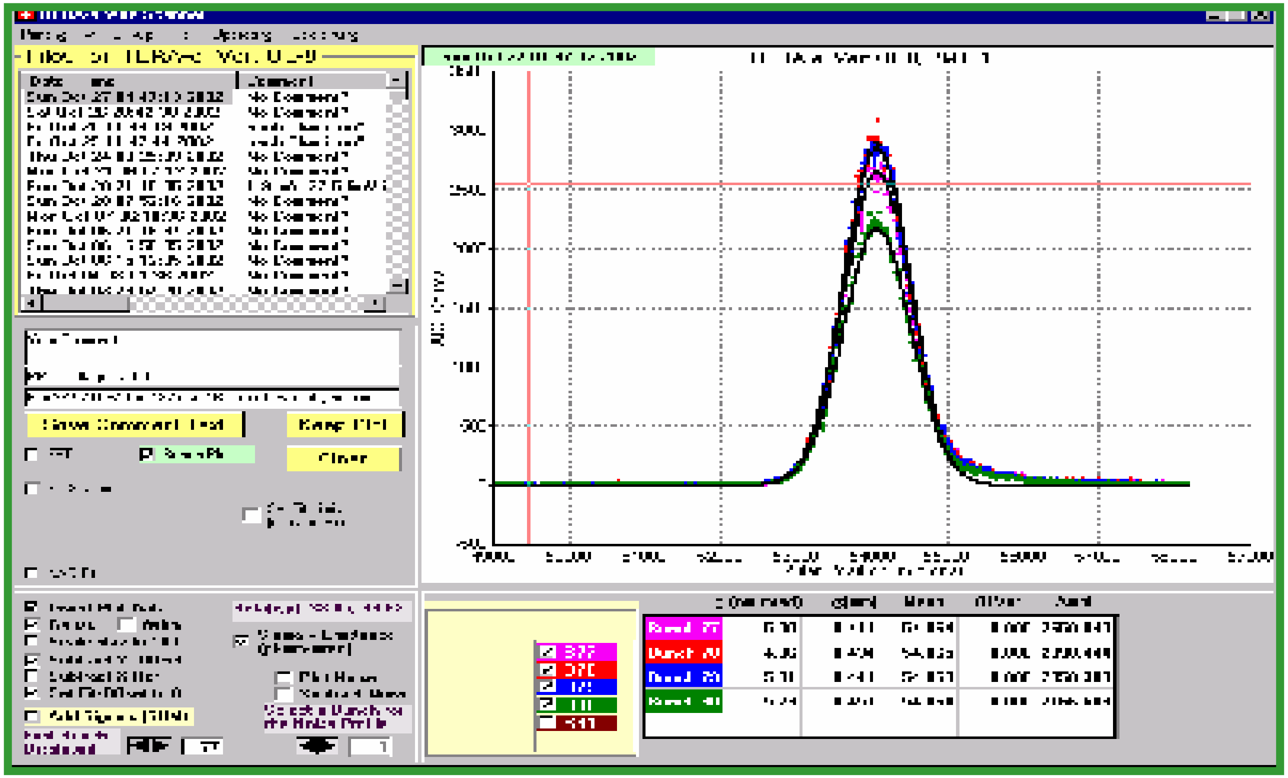
determined by two parameters

$$\sigma = \sqrt{\varepsilon * \beta}$$




*ε = area in phase space*

# Beam Dimension:



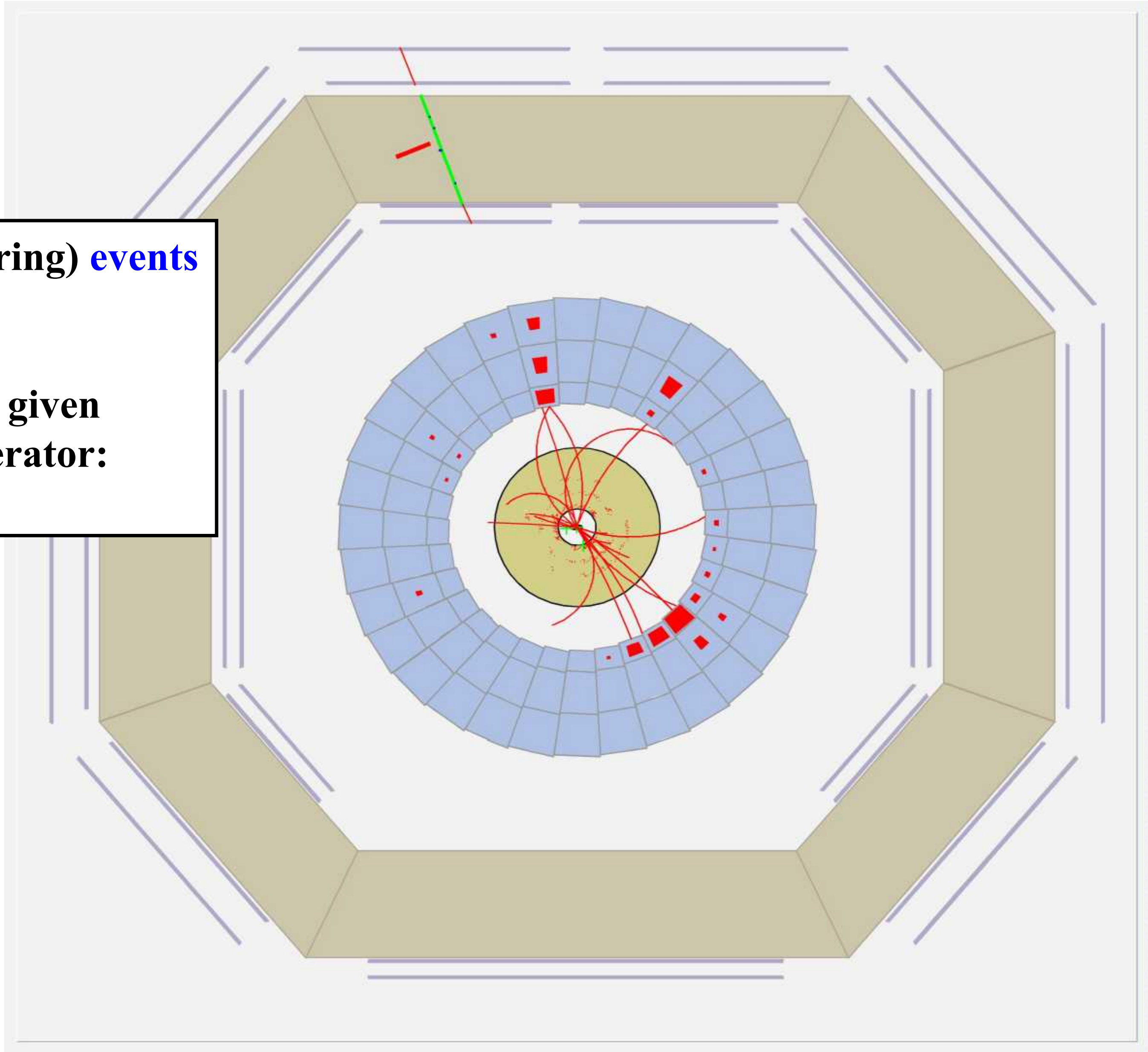
$$\sigma = \sqrt{\varepsilon * \beta}$$

Example: Measurement of the beam dimension in HERA
Wirescanner
particle density ≈ gaussian distributed

# Luminosity

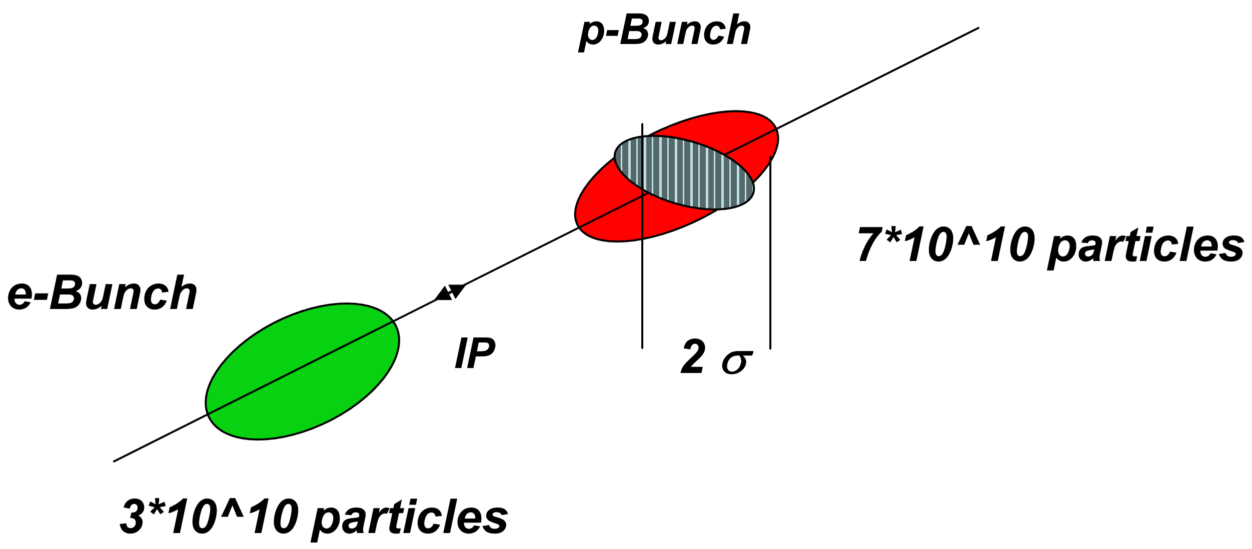
$$R = L * \Sigma_{react.}$$

**production rate of (scattering) events is determined by the cross section $\Sigma_{react}$ and a parameter L that is given by the design of the accelerator: … the luminosity**



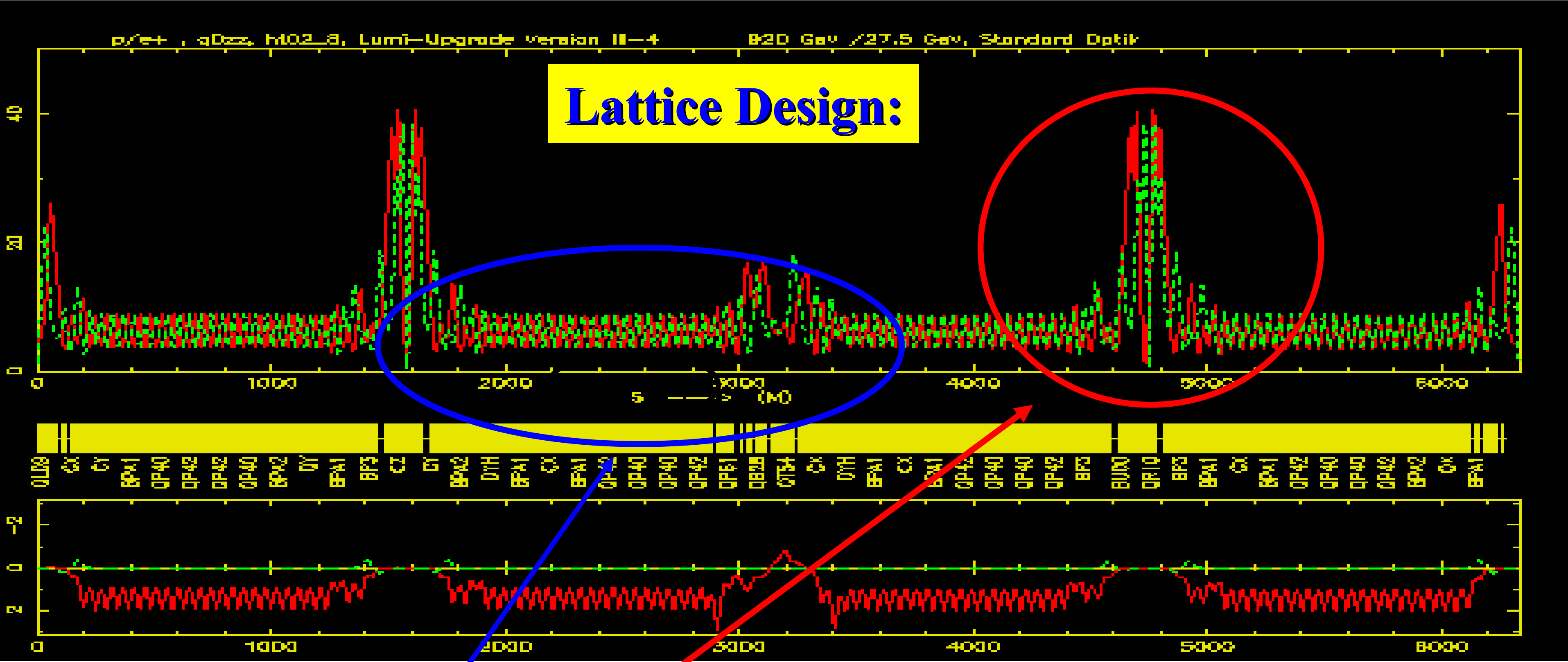
***ZEUS detector: inelastic scattering event of e+/p***

# Luminosity




$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_e I_p}{\sigma_x^* \sigma_y^*}$$

# Lattice Design:




**Arc:** regular **(periodic) magnet structure:**

**bending magnets → define the energy of the ring**

**main focusing & tune control, chromaticity correction,**

**multipoles for higher order corrections**

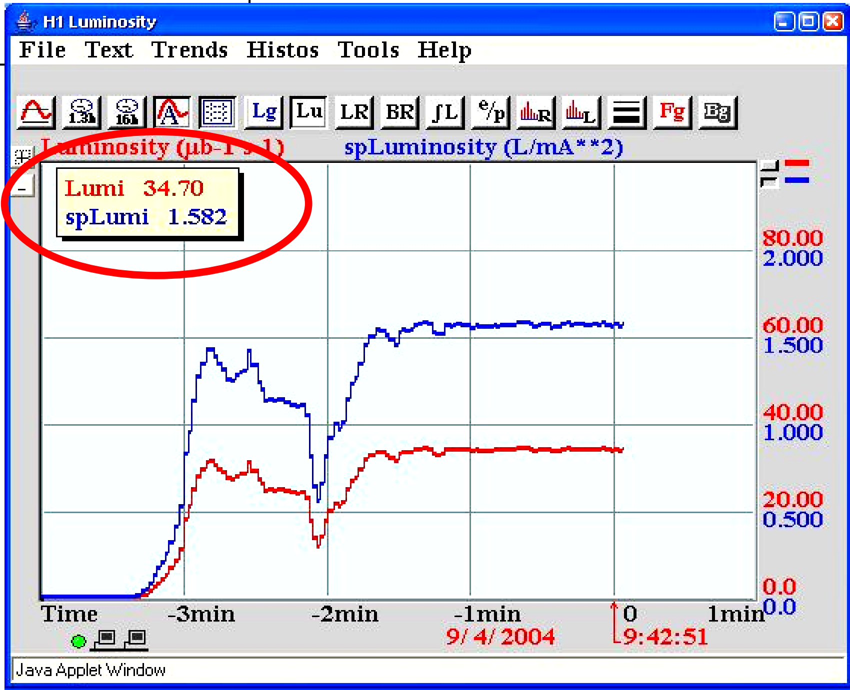
**Straight sections:** **drift spaces for injection, dispersion suppressors,**

**low beta insertions, RF cavities, etc....**

**... and the high energy experiments if they cannot be avoided**

# Luminosity Parameters at a Collider ring (HERA)

| value at IP | horizontal | vertical |
|---|---|---|
| beta function | $\beta_x = 2.45m$ | $\beta_y = 0.18m$ |
| emittance | $\varepsilon_x = 7 * 10^{-9} rad\, m$ | $\varepsilon_y = \varepsilon_x$ |
| beam size | $\sigma_x = 118 \mu m$ | $\sigma_Y = 32 \mu m$ |
| beam currents | $I_e = 43mA$ | $I_p = 84mA$ |
| bunch rev. freq. | $f_0 = 47.3kHz$ | $n_b = 180$ |
| Luminosity | $L = 34.0 * 10^{30} \; {}^{1}\!/_{cm^2 s}$ | |




*HERA luminosity* ***measured at the*** *H1 detector*

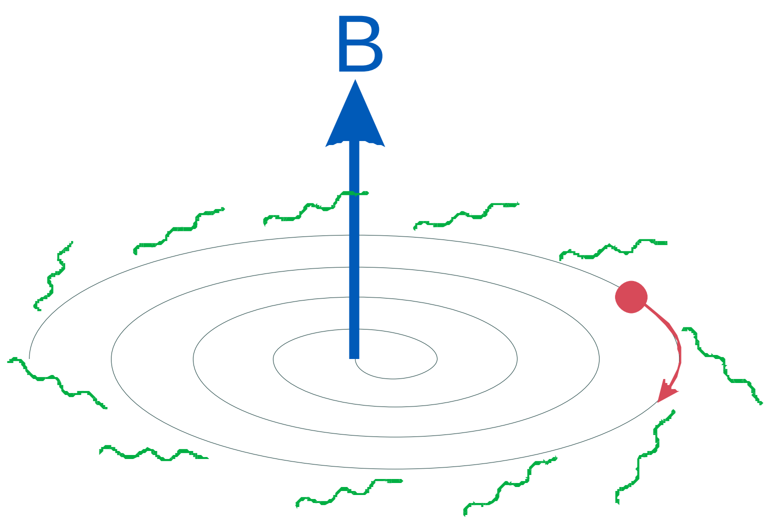
- *injection & acceleration of e+ and protons*
- *E(e+)=27.5 GeV, E(p)=920 GeV*
- *sqeezing both beams (mini beta scheme)*
- *steering the beam position at the IP's*

Des einen Freud – des anderen Leid

One man’s meat is another man's poison

# Synchrotron Radiation

Synchrotron Radiation from an electron in a magnetic field:




$$P_\gamma = \frac{e^2 c^2}{2\pi} C_\gamma E^2 B^2$$

$$C_\gamma \propto \left(m_0 c^2\right)^{-4}$$

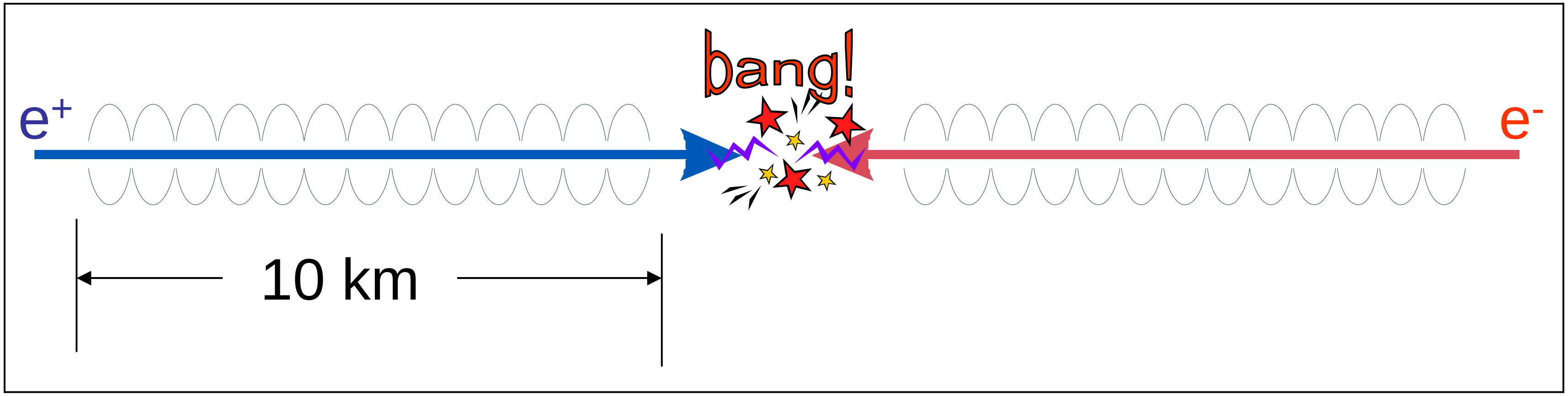
Energy loss per turn of a machine with an average bending radius ρ:

$$\Delta E / rev = \frac{C_\gamma E^4}{\rho}$$

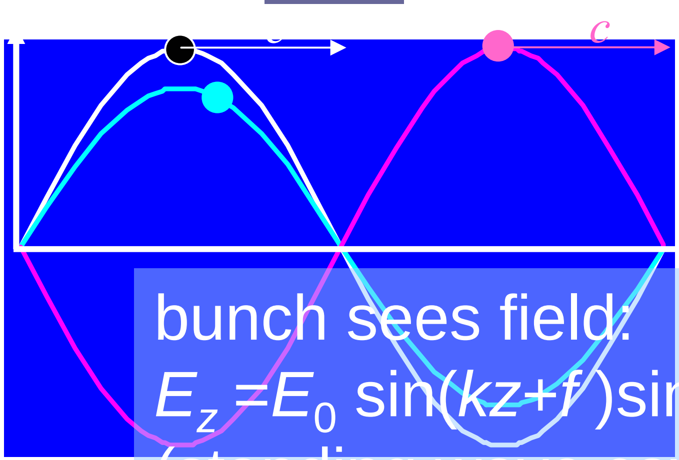
Energy loss must be replaced by RF system

# Linear Collider

## No Bends, but *lots* of RF!




$$ct = \frac{\lambda}{2}$$




bunch sees field:

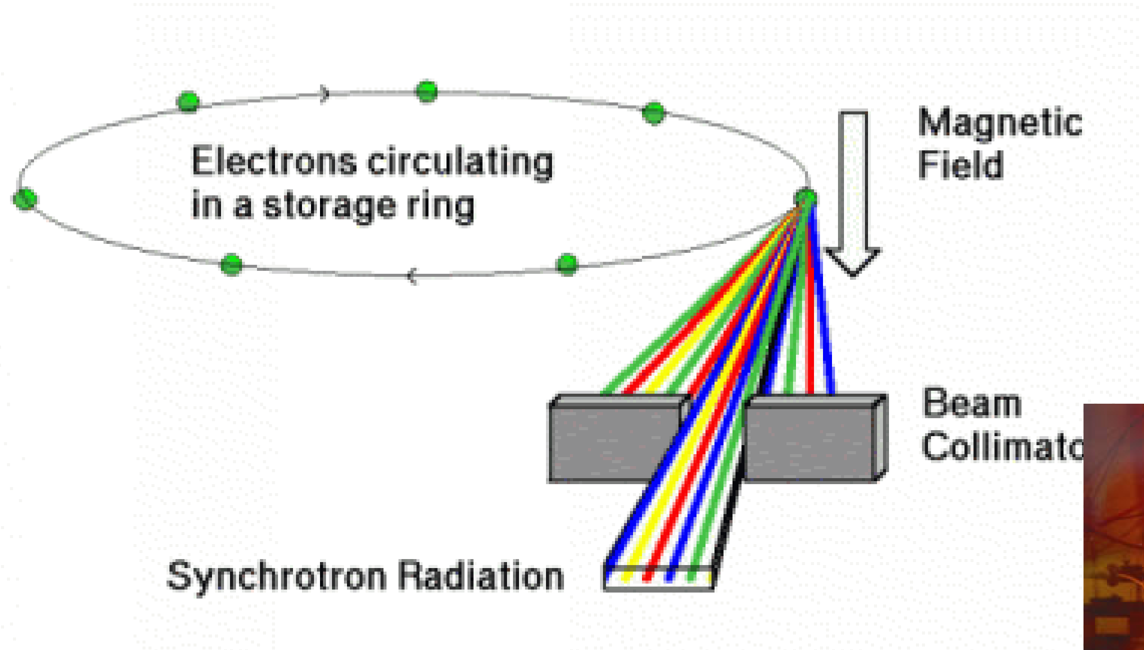
$E_z = E_0 \sin(kz+f)\sin(kz)$

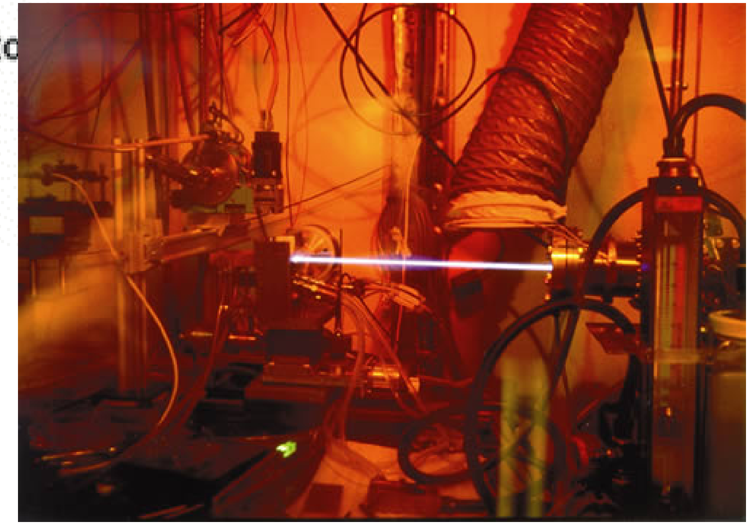
(standing wave cavity)

For a $E_{cm}$ = 500 GeV:

Effective gradient: 25 MV/m

# Synchrotron Radiation Source



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