Deutsches Elektronen-Synchrotron DESY

DESY summer student program

Study of $tW\gamma$ generation at NLO in QCD

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Contents

Abstract

In this summer student project the Monte-Carlo based event generation of the $tW\gamma$ process has been extended to Next-To-Leading Order (NLO) in QCD. For this purpose a sample generation strategy is presented in order to produce a set of events for a given full final state particle configuration in a computationally efficient way.

The overlap of the $tW\gamma$ process at NLO with LO contributions to $tt\gamma$ is demonstrated and the core concepts of diagram removal techniques are introduced which are then used in order to prevent double countings caused by this overlap.
A $tW\gamma$ NLO sample has been successfully generated by directly implementing the DR1 removal scheme within the madgraph framework. Additionally, major contributions to the implementation of the DR2 scheme have been made but were not finalized over the course of this project.

1 Introduction

Top quark analyses are a crucial part of modern High Energy Physics. Precise measurements of its production production and decay properties allow to probe the predictions of the Standard Model and its possible extensions. In this summer school project a study on the $tW\gamma$ process has been conducted. Among other things this one grants access to the electroweak top-photon coupling (1), which is modified in certain BSM models.

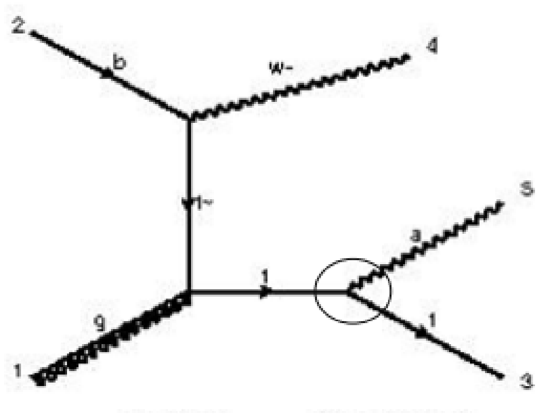

Figure 1: electroweak top-photon coupling in $tW\gamma$

Extending the analysis of this process to NLO in QCD leads to serious complications. Real emission Feynman diagrams which then contribute to the cross section calculation overlap with contributions to $tt\gamma$ at LO. This poses an ill-defined cross section for $tW\gamma$ beyond the LO. Handling the overlap between NLO contributions of $tW\gamma$ and $tt\gamma$ at LO is crucial for a precise data analysis of $tW\gamma$. It requires the use of diagram removal techniques to avoid a double counting. The implementation of these methods has been the focal point of this summer student project.

2 Strategy of Monte-Carlo event generation

For the production of the required event samples, madgraph [1] has been used which is a well established framework for MC event generation at NLO in High Energy Physics. Principally, one would generate a sample for the complete set of final state particles which is of interest, i.e. in the case of $tW\gamma$ an interesting final state combination is the case of both W bosons decaying leptonically [2].
However, due to the high dimensionality of the phase space volume in the final state in the case of many particles, their generation typically is computationally expensive for large samples. Instead, a generation strategy is developed in order to simulate the process as efficient as possible. In this work it was tested if the doubly leptonic final state mentioned above could be covered only by the intermediate process assuming a stable top and W boson.

For this purpose the differential distributions with respect to the transverse momentum and the pseudorapidityof the photon and the top quark have been compared in order to see if the underlying kinematics of the processes are matching eacht other. Since Monte-Carlo data is used, one still had direct access to the kinematics of the intermediate top quark in the process considering the actual final state particles which made a comparison possible in the first place.

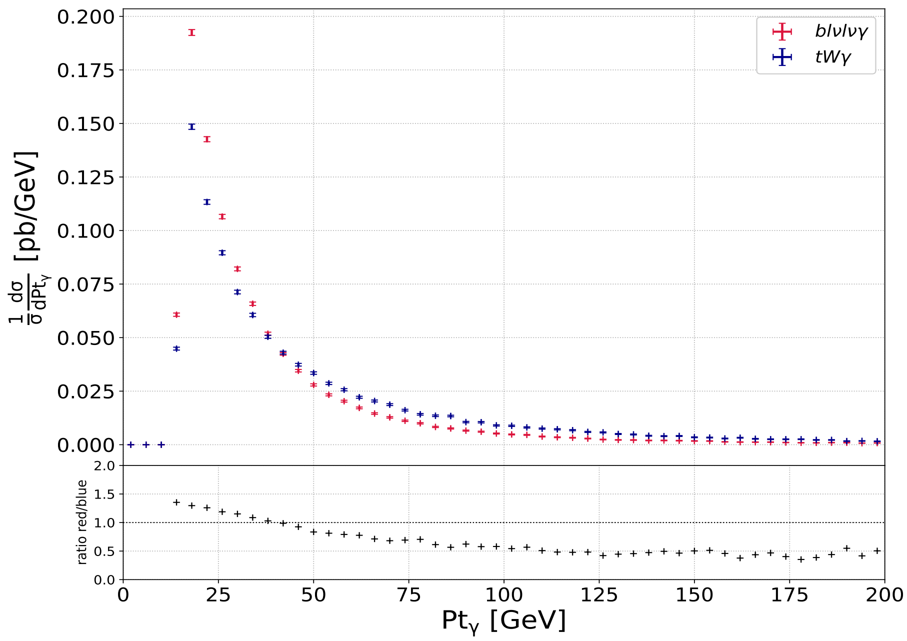

Figure 2: photon Pt for the full and intermediate final state

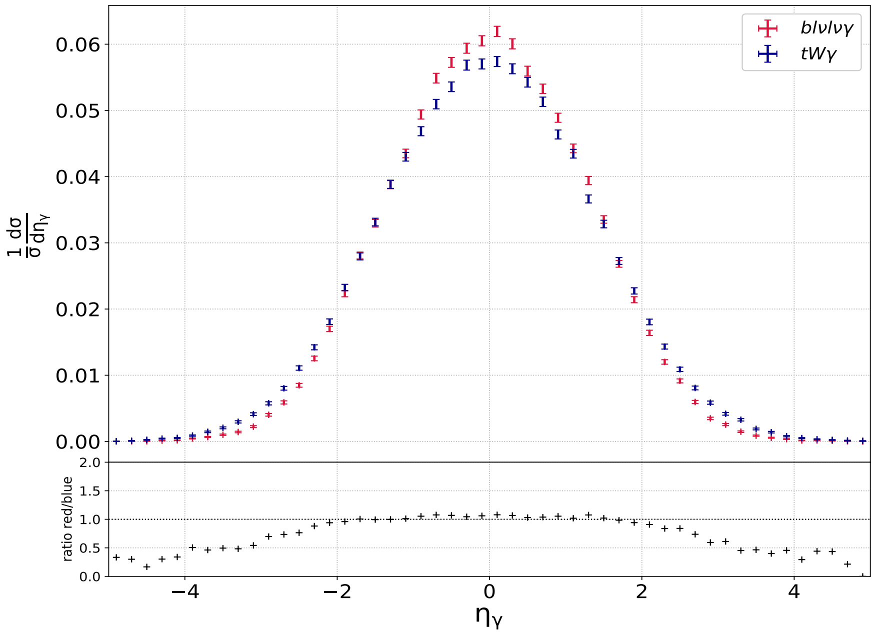

Figure 3: photon η for the full and intermediate final state

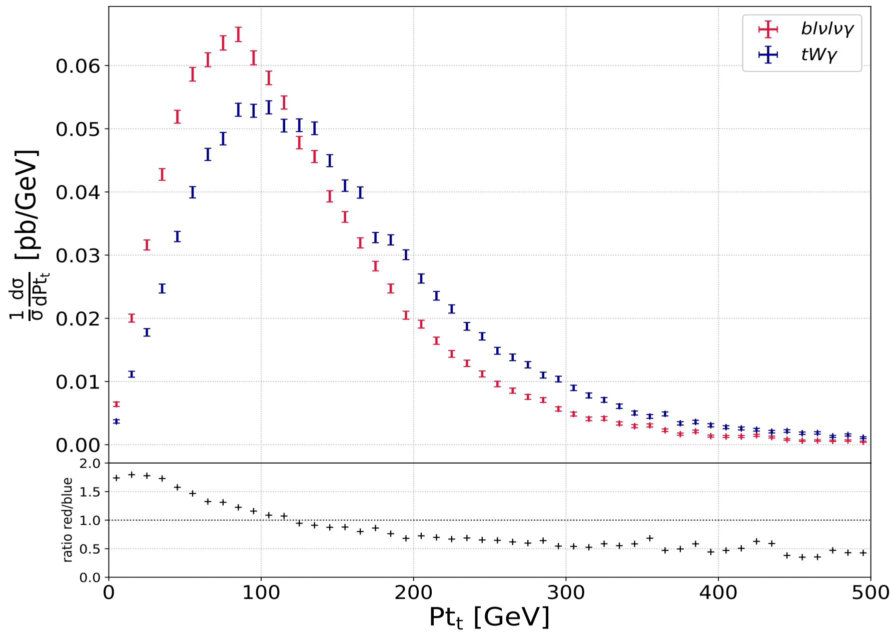

Figure 4: top Pt for the full and intermediate final state

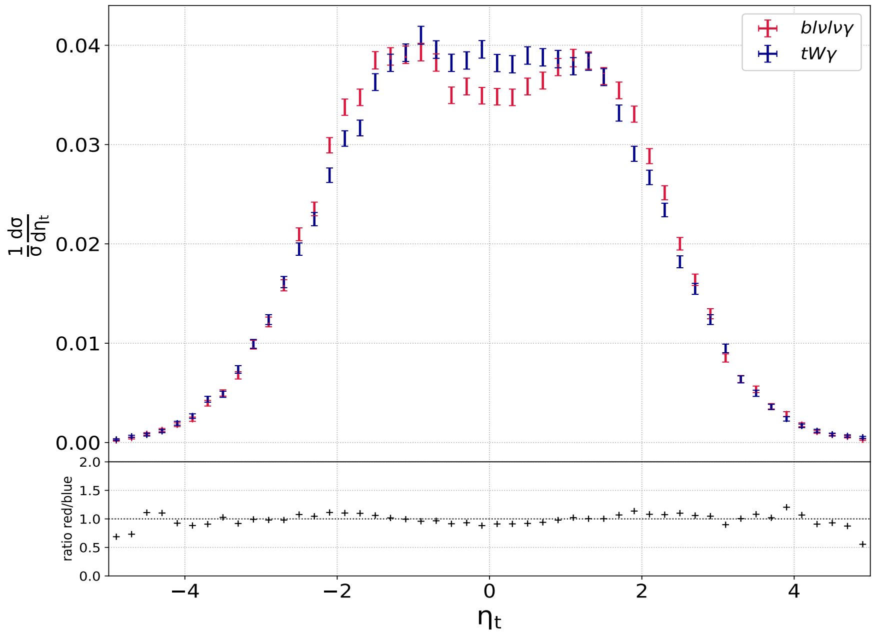

Figure 5: top η for the full and intermediate final state

The four plots above illustrate the comparison between the underlying kinematics of both processes, the one with all final state particles included and the intermediate state one assuming a stable top and W boson. Especially in the plots of the transverse momenta one can see in both cases a softer spectrum with a higher peak for the complete final state. This results from final state photon radiation which simply cannot be accounted for in the intermediate final state sample.

Thus, it was concluded that one has to generate the intermediate state process plus the whole final state but only considering photon emission from one of the final state particles in order to properly simulate the whole process.

3 Overlap of $tW\gamma$ and $tt\gamma$ at NLO

Extending the $tW\gamma$ analysis to NLO in QCD comes along with a whole lot of new Feynman diagrams contributing to the cross section. Besides the Born-level diagrams now also virtual and real corrections have to be taken into account. Generally, these diagrams can contain either none, one or two resonant topquarks. The class of real emission diagrams from the third class do cause significant problems to the perturbative definition of the NLO cross section.

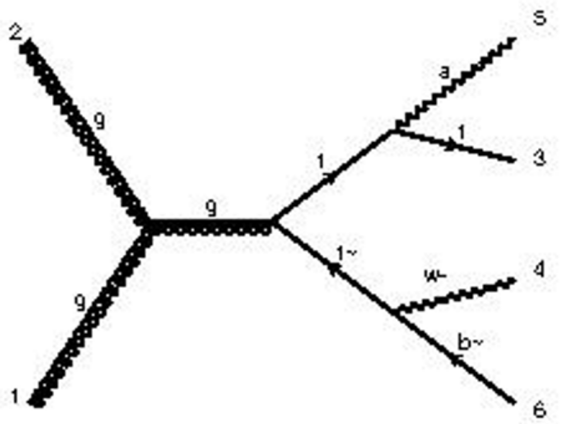

Figure 6: exemplary Feynman diagram contributing to both $tW\gamma$ and $tt\gamma$

In the figure 6 above is one example of this class of diagrams illustrated. The depicted process meets all requirements in order to be identified as a $tW\gamma$ contribution with a real emission manifested by the bottom quark. However, it also does clearly represent a $tt\gamma$ contribution at LO. This poses a direct partial overlap between the contributions of these two different processes. This overlap has to be treated in order to avoid double counting in data analyses. How to remove this overlap will be the focal part of the rest of the report.

4 Diagram removal techniques

In the previous section the occuring overlap between contributing Feynman diagrams between $tW\gamma$ at NLO and LO $tt\gamma$ has been demonstrated. It is only one example of this phenomenon which generally poses a problem to perturbative computations beyond the LO. In this section a brief introduction to diagram removal techniques will be presented which are widely used to treat these presented overlaps.

The general problem is the following: the cross section of a given process is ill-defined in perturbation theory beyond the LO. Extending a process to NLO adds real emission diagrams. However, these interfere with contibutions to an underlying resonant process which contains a particle which goes on to decay into a particle present in the first process and an additional jet so that the same situation as in the NLO real emission case is recreated.
The concept of diagram removal techniques is based on splitting the complete transition amplitude of the process of interest into a resonant and a non-resonant part. The latter contains all resonant contributions which originally stem from the resonant process which is in our special case $tt\gamma$:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{non-res}}|^2 + 2\Re(M^*_{\text{non-res}} M_{\text{res}}) + |\mathcal{M}_{\text{res}}|^2. \tag{1}$$

The exact way to treat the resonant part is scheme-dependent. Most prominent are the DR1 and DR2 scheme. Within the first one the resonant part is completely removed. The resonant amplitudes are simply not taken into account anymore for the calculation:

$$|\mathcal{M}|^2_{\text{DR1}} = |\mathcal{M}_{\text{non-res}}|^2. \tag{2}$$

As for the DR2 scheme one only discards the squared matrix element of the resonant part, however the interference term is kept:

$$|\mathcal{M}|^2_{\text{DR2}} = |\mathcal{M}_{\text{non-res}}|^2 + 2\Re(M^*_{\text{non-res}} M_{\text{res}}). \tag{3}$$

In practice both schemes are used parallelly and the difference between the respective results is then used to put an estimate on the uncertainty of the MC data.

5 Implementation of the diagram removal

5.1 Problems with state-of-the-art plugins

The madgraph framework which has been used to produce the Monte-Carlo event samples is supported by a number of extra plugins provided by its community to extend the possibilities of the tool. There do also exist established plugins which can take care of resonant diagrams and remove overlapping contributuons beyond the LO. The prime example is the MadSTR plugin [3]. However, it is not able to remove a certain type of diagrams, namely those which contain a 1 to 3 particles decay. An example Feynman diagram 7 of such a process is depicted below:

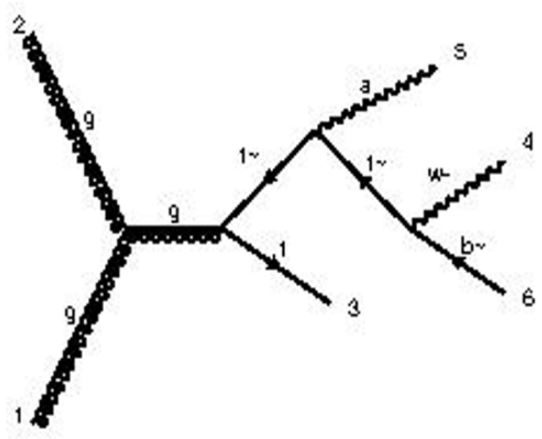

Figure 7: exemplary Feynman diagram which can not be removed by MadSTR

For this specific reason it was not possible to simply use the plugin but the overlap had to be removed by hand within the corresponding madgraph routines. Many thanks to Hesham El Faham for his important contributions to achieve this goal! At this point it is noteworthy that it has been used before in a tWZ analysis. The reason that it does not work in the present case is not, as first suspected, due to the photon being massless, but rather because it can act as a final state particle which is the origin of diagrams of this type which require this proceeding.

5.2 Madgraph implementation of DR1

In this chapter a conceptual tutorial on how to implement the DR1 scheme in madgraph is presented. The first step marks the generation of the process within madgraph. Afterwards the resonant amplitudes need to be identified. The contributing diagrams of the different sub processes can be found in the 'SubProcesses' folder of the generated process. For each sub process every diagram has an assigned number. One now has to identify all the amplitudes that shall be removed. Afterwards, go into the fortran matrix files generated by madgraph within the 'Subprocesses' folder. The implementation of the DR1 scheme consists of one step only at this point. After all the different amplitudes have been called within the fortran file and before the 'Jamp' variable is specified, the resonant amplitudes have to be set to 0. See below 8 for how to achieve this:

```
C       DR1 procedure

        AMP(4) = DCMPLX(0D0,0D0)
        AMP(5) = DCMPLX(0D0,0D0)

        AMP(7) = DCMPLX(0D0,0D0)
        AMP(8) = DCMPLX(0D0,0D0)

        AMP(16) = DCMPLX(0D0,0D0)
        AMP(19) = DCMPLX(0D0,0D0)

```

Figure 8: removal of resonant amplitudes within the DR1 scheme

This already completes the implementation. If one tries to launch the event generation of the $tW\gamma$ process without the correct DR1 implementation one will cause an error which results in the calculations not converging. This holds true until all the resonant diagrams are removed.
It is a neat cross check, if the event generation works ones can be sure that all the necessary diagrams have been removed. However, one still has to make sure that one did not remove any other non-necessary amplitude by accident.

5.3 Madgraph implementation of DR2

During the timespan of the summer student project a fully successful implementation of the DR2 scheme has not been achieved and is still pending final corrections. Nevertheless, the necessary steps will be discussed in the following:

```
      COMPLEX*16 ZTEMP, AMP(NGRAPHS), JAMP(NCOLOR,NAMPSO), W(8
     $ ,NWAVEFUNCS)
      COMPLEX*16 TMP_JAMP(7)
      COMPLEX*16 ZTEMP_str, AMP_keep(NGRAPHS), JAMP_str(NCOLOR,NAMPSO)
      COMPLEX*16 TMP_JAMP_str(7)
      double PRECISION MDL_WT_keep
      PARAMETER(MDL_WT_keep=1.48d0)
```

Figure 9: extra definition of variables for DR2

As can be seen in the code fragment 9 above, the first step is to make a copy of the depicted variables in the fortran matrix files of madgraph. These will act as auxiliary variables in order to calculate the resonant only parts of the amplitude which we wish to subtract from the final result produced by madgraph. Additionally a finite width for the top quark is defined at this stage.

For the next step copy the code segment in which all of the different amplitudes are called, paste it right after again and exchange every Amp with the previously defined Ampkeep and also replace the top width MDWT to the finite one MDWTkeep. By this the same procedure as before is repeated but with a finite width of the top quark.

```
C      Amplitude(s) for diagram number 1
       CALL FFV1_0(W(1,9),W(1,8),W(1,5),GC_1,AMP_keep(1))
       CALL FFV1_2(W(1,6),W(1,5),GC_1,ZERO,ZERO,W(1,10))
C      Amplitude(s) for diagram number 2
       CALL FFV1_0(W(1,10),W(1,8),W(1,7),GC_5,AMP_keep(2))
       CALL FFV1_1(W(1,3),W(1,5),GC_2,MDL_MT,MDL_WT_keep,W(1,11))
C      Amplitude(s) for diagram number 3
       CALL FFV2_0(W(1,9),W(1,11),W(1,4),GC_11,AMP_keep(3))
       CALL FFV2_2(W(1,6),W(1,4),GC_11,MDL_MT,MDL_WT_keep,W(1,12))
C      Amplitude(s) for diagram number 4
       CALL FFV1_0(W(1,12),W(1,11),W(1,7),GC_5,AMP_keep(4))
       CALL VVV1_3(W(1,5),W(1,4),GC_25,MDL_MW,MDL_WW,W(1,13))
       CALL FFV1_1(W(1,3),W(1,7),GC_5,MDL_MT,MDL_WT_keep,W(1,14))
C      Amplitude(s) for diagram number 5
       CALL FFV2_0(W(1,6),W(1,14),W(1,13),GC_11,AMP_keep(5))
C      Amplitude(s) for diagram number 6
       CALL FFV2_0(W(1,9),W(1,3),W(1,13),GC_11,AMP_keep(6))
C      Amplitude(s) for diagram number 7
       CALL FFV1_0(W(1,12),W(1,14),W(1,5),GC_2,AMP_keep(7))
```

Figure 10: calling the amplitudes with the finite top width

Principally, the same procedure now needs to be repeated for the Jamp variable. It is copied, properly renamed and then delete all the non-resonant amplitudes. This part will then compute the joint amplitude of only the resonant part.

```
      JAMP_str(1,1) = (-1.000000000000000D+00)*AMP(12)+(-1.000000000000000D+00)*AMP(13)
     $ +(-1.000000000000000D+00)*AMP(14)+(-1.000000000000000D+00)
     $ *AMP(15)+(-1.000000000000000D+00)*AMP(37)+((0.000000000000000D
     $ +00,1.000000000000000D+00))*TMP_JAMP_str(7)
      JAMP_str(2,1) = (-1.000000000000000D+00)*AMP(25)+(-1.000000000000000D+00)*AMP(26)
     $ +(-1.000000000000000D+00)*AMP(27)+(-1.000000000000000D+00)
     $ *AMP(28)+(-1.000000000000000D+00)*AMP(38)
     $ +((0.000000000000000D+00,-1.000000000000000D+00))*TMP_JAMP_str(7)
```

Figure 11: calculating the joint amplitude for the resonant part

The following step already marks the last one but is also the one which requires additional corrections. In this section of the matrix files the complete transition amplitude is calculated. As before, one principally copies the variables and adds them again with the subscript in order to repeat the same calculation for the resonant part only. In the following loop the basic idea is to always subtract the resonant part. Right here lies the problem which has not been solved up until this point.

```
      DO M = 1, NAMPSO
        DO I = 1, NCOLOR
          ZTEMP = (0.D0,0.D0)
          DO J = 1, NCOLOR
            ZTEMP = ZTEMP + CF(J,I)*JAMP(J,M)
            ZTEMP_str = ZTEMP_str + CF(J,I)*JAMP_str(J,M)
          ENDDO
          DO N = 1, NAMPSO
            RES(SQSOINDEX2(M,N)) = RES(SQSOINDEX2(M,N)) + ZTEMP
     $        *DCONJG(JAMP(I,N)) - ZTEMP_str*DCONJG(JAMP_str(I,N))
          ENDDO
        ENDDO
      ENDDO

      DO I=1,NSQAMPSO
        RES(0)=RES(0)+RES(I)
      ENDDO
```

Figure 12: subtraction of resonant term from complete amplitude

6 Results

Only after implementing a suitable diagram removal procedure a problem-free generation of a $tW\gamma$ sample at NLO in QCD is possible in madgraph. Using the DR1 scheme such a sample has been generated. In order to test this set of events, differential distributions of the photon and top variables previously used are compared between this and the another sample generated at LO.

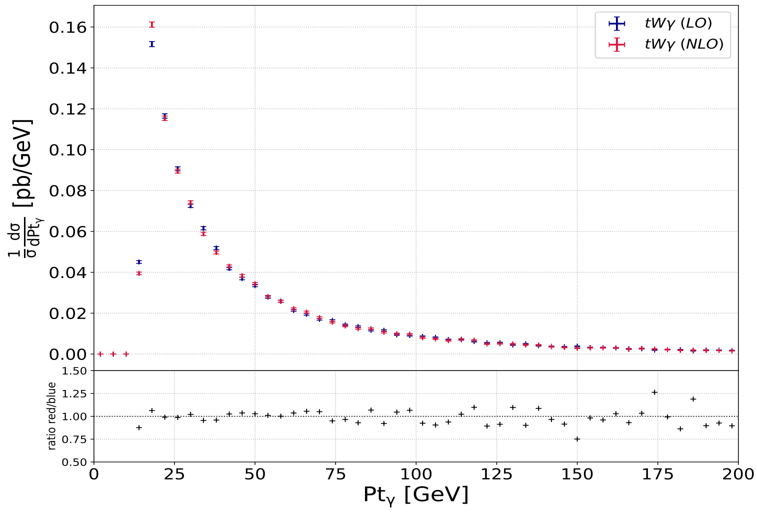

Figure 13: photon Pt in the LO and NLO event sample

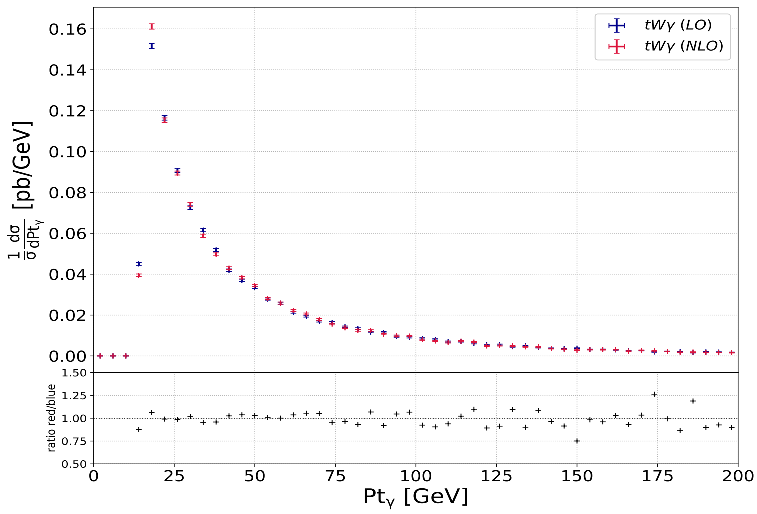

Figure 14: photon η in the LO and NLO event sample

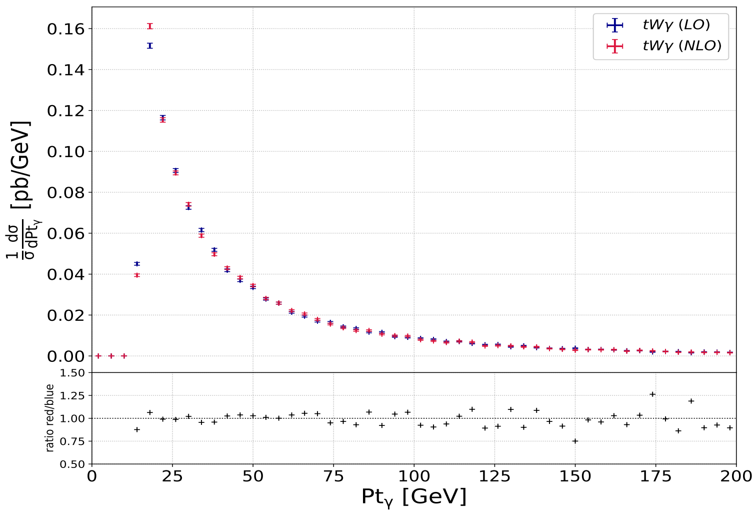

Figure 15: top Pt in the LO and NLO event sample

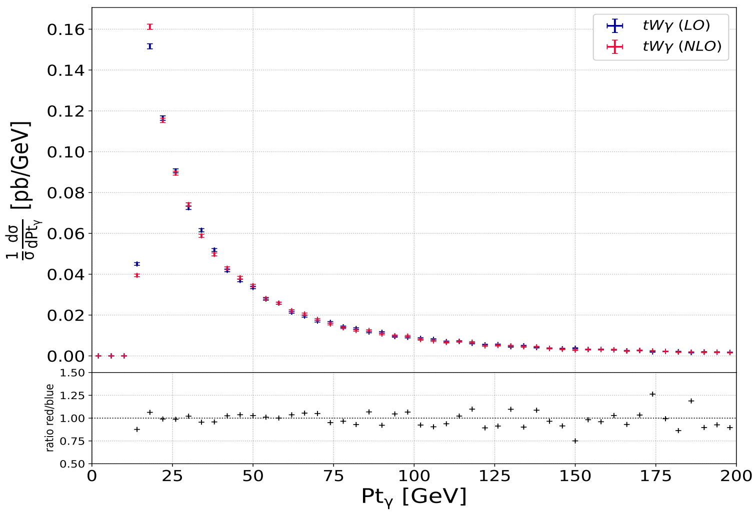

Figure 16: top η in the LO and NLO event sample

Índeed, the underlying kinematics are supposed to differ from each other due to the NLO corrections. However, the general structure of the respective distributions was anticipated to be similar. The observed corrections between both samples seen in the above plots are within the expected range. The initial assumption has thus been confirmed. This illustrates a small cross-check which hints the NLO event generation did not run into obvious problems.

A similar cross-check poses the comparison of the cross sections. To underline the benefit of this simple check it is noteworthy that in a prior analysis on the tWZ process at NLO this test did not meet the expectations and thus hinted to the existence of a hidden problem. In the case of $tW\gamma$ the cross sections are written down below. The NLO corrections are significant which is expected in QCD, however they do match with the expectations.

$$\sigma_{\mathrm{LO}}(pp > tW\gamma) = (0.260 \pm 0.003)\mathrm{pb} \tag{4}$$

$$\sigma_{\mathrm{NLO}}(pp > tW\gamma) = (0.346 \pm 0.005)\mathrm{pb} \tag{5}$$

7 Summary

Over the course of this summer student project the Monte-Carlo event generation of a $tW\gamma$ sample at NLO in QCD has been successfully conducted using the DR1 diagram removal scheme. This sample has then been tested by comparing the underlying kinematics as well as the calculated total cross section to the LO case. Both tests did not give rise to any noticeable problems. This hints to a successful generation of the NLO sample. Additionally, major contributions to the implementation of the DR2 scheme have been made but were not finalized.

A Monte-Carlo event generation strategy has been developed in order to efficiently generate a sample for a given full final state configuration. It has been concluded that the whole final state can be covered by the consideration of two different event samples, one being the $tW\gamma$ intermediate state which contains initial state as well as intermediate state photon radiation. Secondly, a event sample of the whole final state is needed in which only photons stemming from a final state particle have to be taken into consideration.

In this report the basic concepts of diagram removal techniques have been covered and it has been emphasized how they are used to treat overlaps of contributing Feynman diagrams in different processes beyond the LO. Moreover, the two most prominent schemes were briefly explained.

8 References

[1] Johan Alwall,Michel Herquet,Fabio Maltoni,Olivier Mattelaer,Tim Stelzer: *MadGraph 5: going beyond*, Journal of High Energy Physics, doi:10.1007/jhep06(2011)128, 2011

[2] Meshreki, John Kamal Rizk: *Measurements of inclusive and differential cross-sections of combined $tt\gamma$ and $tW\gamma$ production in the e channel at $\sqrt{s}$= 13 TeV with the ATLAS experiment*, 40th International Conference on High Energy Physics (ICHEP), report number: ATL-PHYS-SLIDE-2020-316, 2020

[3] Stefano Frixione et al.: *Automated simulations beyond the Standard Model: supersymmetry*, Journal of High Energy Physics, doi:10.1007/JHEP12%282019%29008, 2019