Theoretical uncertainty estimates for on-loop corrections of the triple Higgs coupling Real Singlet Extension of the SM with an unbroken $\mathbb{Z}_2$ symmetry

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Abstract

We use the the code `anyBSM` to compute one-loop perturbative corrections of the triple Higgs coupling λ_{hhh} in the Standard Model and in the Real Singlet Extension of SM with an unbroken $\mathbb{Z}_2$ symmetry. We estimate the unknown higher-order contributions to the λ_{hhh} in the Real Singlet Extension and we study this uncertainty in the decoupling and non-decoupling limits.

1 Introduction

The Standard Model (SM) [1] has established itself in the scientific community as one of the most accurate theories in history thanks to its experimental successes. It was developed between 1970 and 1973 and has achieved a large number of successes such as the prediction of the gluon before its measurement in 1979 [2], the prediction of the the W and Z bosons before their measurement [3],[4] in 1983 or the most famous discovery to date, the confirmation of the existence of the Higgs boson at the LHC in 2012 [5]. Apart from direct measurements of particles, many of their properties have also been measured with high precision.

However the SM also has problems such as the lack of a description of gravity, dark matter, dark energy, the asymmetry in the amount of matter and antimatter, the mass of neutrinos or the for point two sigma discrepancy between theory and experiment found n the muon anomalous magnetic moment. These and many other problems have motivated the scientific community to develop Beyond Standard Model (BSM) theories. These theories often start from the SM and add e.g. new fields or symmetries to solve the problems of the SM. The focus of this project is on BSM models that modify the Higgs sector of the SM. There are different ways to check whether these models are in agreement with experimental constraints. One is checking a prediction of a BSM model for an observable property of a SM particle. For this a SM parameter that has not been measured is needed, for example, the triple Higgs coupling λ_{hhh}.

The objective of this project is to calculate one-loop corrections to the triple Higgs coupling in BSM theories, estimate the size of the unknown higher-order corrections the BSM parameter space use the experimental bounds of λ_{hhh}. In order to do so, we will employ a software under development called `anyBSM`. This software calculates one-loop corrections to the triple Higgs coupling λ_{hhh} in BSM theories.

2 Triple Higgs Coupling

The triple Higgs coupling measures the intensity with which three Higgs bosons interact. This coupling can be be extracted from the Lagrangian

$$\mathcal{L} \supset -\frac{1}{2}m_h^2 h^2 - \frac{1}{3!}\lambda_{hhh} h^3 - \frac{1}{4!}\lambda_{hhhh} h^4. \tag{1}$$

If this equation is compared with the corresponding part of the SM Lagrangian, one can obtain the expression for the triple Higgs coupling in the SM

$$\lambda_{hhh} = \frac{3m_h^2}{v}. \tag{2}$$

The spontaneous breaking of electroweak symmetry of the Higgs potential is the mechanism wich gives the SM particles their masses. To understand and characterise this potential is one of the biggest objectives in high energy physics. Up to this moment only the quadratic term of Higgs potential is known due to the measurement of the Higgs boson which corresponds to the curvature around our actual relative minimum of the potential. However the complete shape of the potential is yet unknown. The shape of the Higgs potential is important, for example, because there could be more minima to which the universe may tunnel. Although this is only possible in models beyond the SM. This requires knowledge of the triple Higgs coupling but unfortunately it is not a physical observable. However, it can be extracted from physical observables, for example in the production of two Higgs bosons.

In lepton colliders, like the Large Electron-Positron Collider (LEP), this coupling is important, for example, in the prediction of the $e^+e^- \to Zhh$ cross section. Figure 1 (right) shows the leading Feynman diagram contributing to the $e^+e^- \to Zhh$ cross section where the trilinear self-coupling enters. Thus, the numerical value of λ_{hhh} changes the prediction of the $e^+e^- \to Zhh$ cross-section. Double-Higgs production is also accessible at hadron colliders, like the Large Hadron Collider (LHC), where one has to take into account this coupling in the di-Higgs production as is shown in the left diagram of Fig. 1. The LHC has done huge advantages in the measurement of this coupling, although there is not an experimental value because the cross section of the di-Higgs production in the LHC is too small for the current experimental sensitivity. experiments at the LHC have been able to constraint [5] the coupling. A selection of these results is summarised in Fig. 2.

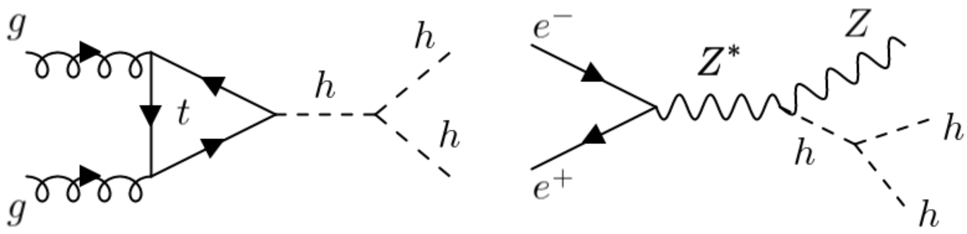


Figure 1: Left: is shown a tree level two Higgs production channel of hadron colliders. Right: is shown a tree level two Higgs production channel of lepton colliders

The lack of an experimental value the triple Higgs coupling leaves still plenty of room for new physics. For example, BSM models that modify the Higgs sector of the SM which are the main objective of this project can significantly modify λ_{hhh}. In most BSM models there may be new parameters which can affect the relation of the coupling to other parameters at tree-level or higher orders. Thus with the experimental constraints of the triple Higgs coupling one can constrain the new BSM parameters that enter the prediction of λ_{hhh}. Some of these BSM parameters do not appear in the tree level prediction but they can have an important role in higher order corrections due to new Feynman diagrams.

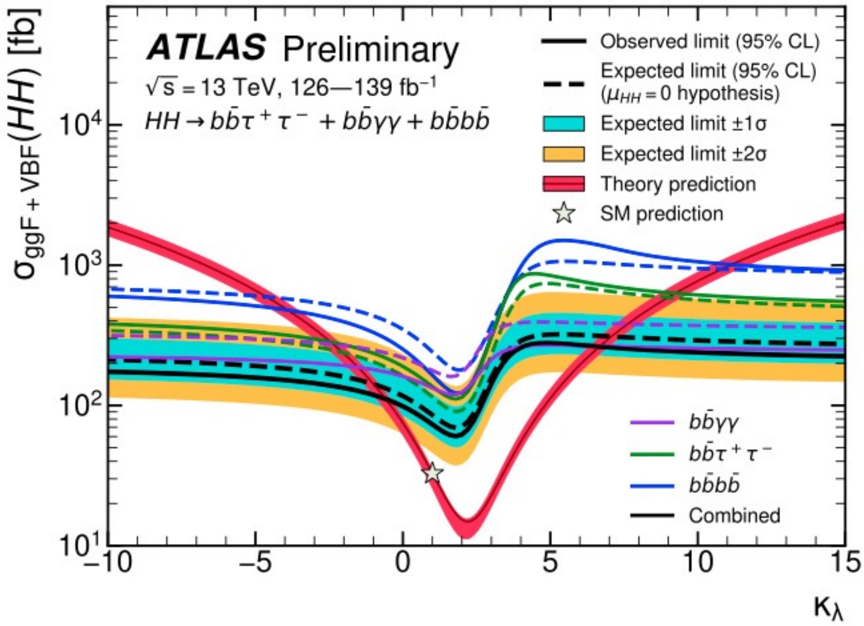


Figure 2: The cross section of two Higgs production in the LHC is shown as a function of κ_λ. The continuous black line is the experimental limit, the red line is the SM prediction varying the value of λ_{hhh} and the star is the Sm result for the SM λ_{hhh}.

3 One loop corrections

Loop corrections are pertubative quantum corrections [6] of tree-level relations predicted by the classical field theory. In this project all one-loop correction to λ_{hhh} are going to be taken into account. The aim of this calculations is to obtain a more precise value of the triple Higgs coupling and to study possible new relations between BSM parameters and this coupling.

For doing this calculation one has to compute the Higgs three point function including all diagrams with at most one-loop. The experimental constraints of [5] are given for the effective triple Higgs coupling, this means that the momenta of the external legs are zero. Therefore, we are going to calculate also the effective triple Higgs coupling because we want to compare the results with the experimental constraints. Its important to note that only the momenta on the external legs are zero, the internal momenta of the loops is integrated from 0 to ∞. In the following it will be shown that this introduces a problem when the integrals appearing in the calculations are solved in 4 dimensional space-time because some of these integrals diverge. To fix this, a series of mathematical techniques called *renormalisation* have been developed in the past.

3.1 Renormalisation

When one encounters divergences in the loop-integration, a first step is to separate the divergent and finite part. This process is called regularization. There are different ways of doing the regularization and in this project the dimensional regularization (DREG) [7] will be employed. This regularization, unlike others, preserves Lorentz and gauge invariance as well as unitarity. Once the calculation has been regularized and the divergent and the finite part have been separated one can continue with the renormalisation. In this second process the divergent part is eliminated using counter-terms which are determined from so called renormalisation conditions. The choices of these conditions are in general not unique since they connect the parameters of the theory to experimental observables.

We begin with an example from [8], the regularization of the function $\mathbf{A}$ which appears in many perturbative calculations,

$$\mathbf{A}(x) = (16\pi^2) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+x} = 2\int_0^\infty \frac{k^3}{k^2+x} dk \to \infty. \tag{3}$$

If this function is integrated in 4 dimensional space-time, it is going to diverge at the upper integration boundary. A crucial observation is that for lower space-time dimensions it dose not diverge. Thus, a new number of dimensions d can be defined as $d = 4 - 2\epsilon$ and the function can be evaluated in this space-time dimension instead. When $\epsilon \to 0$, the 4 space-time dimensions are recovered, $d \to 4$.

$$\begin{aligned}\mathbf{A}(x) = (16\pi^2)\mu^{2\epsilon} \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2+x} = \frac{16\pi^2}{(2\pi)^d}\pi^{d/2}\mu^{2\epsilon}\Gamma(1-\frac{d}{2})x^{d/2-1} = \\ = x\left[-\frac{1}{\epsilon} + \gamma_E - \log 4\pi - \log\mu^2 + \log x - 1\right].\end{aligned} \tag{4}$$

In this new integral there is a new mass dimension parameter μ which is a non-physical parameter, it is called regularization scale. This parameter is used to maintain the dimension of the coupling, the final physical observables can not depend on this parameter. In this new expression there is a term $\frac{1}{\epsilon}$ which is a UV pole: i.e. when the 4 space time dimensions are recovered $d \to 4$ so $\epsilon \to 0$ this pole goes to infinite, although the rest of the function stays finite. In other words, we have separated the divergent part of the function from the finite one. Now one can proceed with the topic of renormalisation [6], which is going to remove the divergence. Two different schemes are going to be used to do so, the On Shell (OS) scheme and the modified Minimal Subtraction ($\overline{\text{MS}}$) scheme.

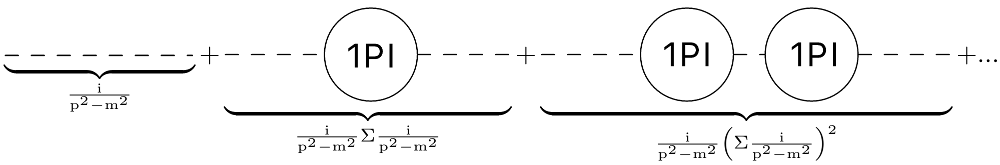
One of the parameters that is necessary for this project, is going to be used as example of renormalisation, the SM like Higgs mass. From now on the following notation is going to be used: Pole mass M_h, renormalised mass $\hat{m}_h$, bare mass m_h, unrenormalised Higgs self-energy Σ_h and renormalised self-energy $\hat{\Sigma}_h$.

The Higgs mass is extracted from the Higgs propagator, so the first thing to do is writing the corrected propagator obtained from [9]. The tree level mass is obtained from the tree level propagator,

$$iS = \frac{i}{p^2 - m_h^2} \tag{5}$$

This propagator receives infinite quantum correction characterized as an infinite series of Feynman diagrams which can be organized using groups of 1PI diagrams,

$iS =$



$$= \frac{i}{p^2 - \hat{m}^2 - \hat{\Sigma}}. \tag{6}$$

The 1PI diagrams are one Particle Irreducible diagrams, this refers to diagrams which can not be separated into two diagrams by cutting only one line. This series corresponds to a Dyson series as we can see in the eq. 6. In the one-loop calculation the 1PI bloops can only have one closed loop. If one wants to maintain the tree level definition of the propagator at higher orders, the definition of the mass has to be changed. The definition of the pole mass is $p^2 = M_h^2$ such that $p^2 - \hat{m}_h^2 - \hat{\Sigma}_h = 0$. The resulting expression of the Higgs pole mass reads:

$$\frac{i}{p^2 - M_h^2} = \frac{i}{p^2 - \hat{m}_h^2 - \hat{\Sigma}_h(p^2)} \;\rightarrow\; M_h^2 = \hat{m}_h^2 + \hat{\Sigma}_h(p^2 = M_h^2). \tag{7}$$

Now we can write the new definition of the pole mass in terms of the unrenormalised quantities and we can split this unrenormalised quantities in the divergent and the finite parts,

$$M_h^2 = \hat{m}^2 + \Sigma_h(M_h^2) + \delta^{CT} m_h^2 = \hat{m}^2 + \Sigma_h^{DIV} + \Sigma_h^{FIN} + \delta^{CT} m_h^2. \tag{8}$$

Until now all the steps are scheme independent but now the divergent part of the mass has to be eliminate. Two different schemes for doing this are going to be discussed: the modified Minimal Subtraction scheme ($\overline{\text{MS}}$) and the On Shell scheme (OS).

OS scheme

In the OS scheme the pole mass is required to be equal to the renormalised mass so the counter-term will cancel both, the divergent and the finite part of the self-energy. As a result of this definition the OS

Higgs mass is equal to the Higgs Pole mass:

$$\begin{aligned}
&\text{OS} : \delta^{CT}_{OS} m_h^2 = -\Sigma_h^{DIV+FIN} \;\rightarrow \\
&M_h^2 = \hat{m}^2_{\text{OS}} + \Sigma_h^{DIV} + \Sigma_h^{FIN} + \delta^{CT} m_h^2 \\
&\quad = \hat{m}^2_{\text{OS}} + \Sigma_h^{DIV} + \Sigma_h^{FIN} - \Sigma_h^{DIV+FIN} \\
&\quad = \hat{m}^2_{\text{OS}}.
\end{aligned} \tag{9}$$

The triple Higgs coupling, as one can see in the eq. 2, is defined in terms of the Higgs mass and the vev, therefore to renormalise this coupling also the vev has to be renormalised. In this project, the vev is defined in terms of the W mass, the Z mass and the electric charge,

$$v = \frac{2M_W}{e}\sqrt{1 - \frac{M_W^2}{M_Z^2}}. \tag{10}$$

Thus to renormalise the vev this parameters have to be renormalised. The renormalisation of the W and Z masses is analogous to the one of the Higgs mas but the electric charge in more difficult and it is explained in [10]. The vev counterterm is calculated as follows:

$$\delta^{CT} v = \frac{\partial v}{\partial \hat{m}_W}\delta^{CT} m_W + \frac{\partial v}{\partial \hat{m}_Z}\delta^{CT} m_Z + \frac{\partial v}{\partial e}\delta^{CT} e \tag{11}$$

Although in the OS scheme all this parameters are equal to the pole ones due to the definition of the counter-term. There are still UV-divergences and scale-dependence terms in the counter-terms that have to be cancelled. This is done with the counter-term of the λ_{hhh},

$$\delta^{CT}\lambda_{hhh} = \sum_x \frac{\partial \lambda_{hhh}}{x}\delta^{CT} x. \tag{12}$$

The terms introduced by this counter-term cancel all the UV-divergences and scale-dependencies.

$\overline{\text{MS}}$ scheme

In the $\overline{\text{MS}}$ scheme the counter-term cancels only the divergent part of the self-energy. As a result of this the $\overline{\text{MS}}$ Higgs mass is not equal to the Pole mass:

$$\begin{aligned}
&\overline{\text{MS}} : \delta^{CT}_{\overline{\text{MS}}} m_h^2 = -\Sigma_h^{DIV} \;\rightarrow \\
&M_h^2 = \hat{m}^2_{\overline{\text{MS}}} + \Sigma_h^{DIV} + \Sigma_h^{FIN} + \delta^{CT} m_h^2 \\
&\quad = \hat{m}^2_{\overline{\text{MS}}} + \Sigma_h^{DIV} + \Sigma_h^{FIN} - \Sigma_h^{DIV} \\
&\quad = \hat{m}^2_{h,\overline{\text{MS}}} + \Sigma_h^{FIN}(M_h^2)
\end{aligned} \tag{13}$$

Finite part of the self-energy also enters into the calculation. In the $\overline{\text{MS}}$ scheme also the vev (the W, Z masses and the electric charge) have to be renormalised in the new scheme, therefore all the renormalised parameters are going to be different to the physical ones. The bare parameter is the same in both schemes so using the eqs. 9 and 13 one can define the translation of the parameters from one scheme to the other. The translation of all the parameters that enter in the calculation of the triple Higgs coupling are as follows:

$$
\begin{aligned}
\hat{m}^2_{h,\overline{\text{MS}}} &= \hat{m}^2_{h,OS} - \Sigma_h^{FIN}(M_h^2),\\
\hat{m}^2_{W,\overline{\text{MS}}} &= \hat{m}^2_{W,OS} + \Sigma_W^{FIN}(M_W^2),\\
\hat{m}^2_{Z,\overline{\text{MS}}} &= \hat{m}^2_{Z,OS} + \Sigma_Z^{FIN}(M_Z^2) \text{ and}\\
\hat{v}_{\overline{\text{MS}}} &= \hat{v}_{OS} + \delta^{CT}_{\overline{\text{MS}}} v
\end{aligned}
\tag{14}
$$

In the OS scheme the input parameters are the same that the physical ones so these are tree level parameters. But as one can see in the eq. 14 when this parameters are translated to the $\overline{\text{MS}}$ scheme the self-energy, which is a one-loop quantity, enters in the calculation. Taking this into account the $\overline{\text{MS}}$ parameters are one-loop parameters not tree ones. Therefore when one introduces these one-loop parameters in the one-loop expressions to calculate the one-loop corrections as a result there are higher-order contributions because of this mixing of one-loop quantities in one-loop expressions.

The tadpoles can be also renormalised in the OS or in the $\overline{\text{MS}}$ scheme irrespective of the renormalisation of other parameters. If they are renormalised in the OS scheme the counterterm will absorb the hole tadpole contribution and there will not appear in the genuine calculations, however they will appear in the counterterm on λ_{hhh} shown in the eq. 11. If the $\overline{\text{MS}}$ scheme is used for the tadpoles the counterterm will only abosrve cancel the UV-divergence and there eill be a tadpole contribution in the genuine calculations.

For convenience, the regularization scale μ is going to be used as the renormalisation scale Q defined as $Q = 4\pi e^{-\gamma_E}\mu^2$. Its important to remember that although during the calculations some parameters depend on the renormalisation scale Q the final physical value should not depend on this scale.

3.2 Higher order corrections

One of the objectives of this project is estimating the effect of unknown higher order corrections in the triple Higgs coupling. In orther to do so, one needs to introduce a dependence on higher-order corrections in the coupling and compare the results with a proper one-loop calculation. Another check that one can do is see how much the results depend on the renormalisation scale Q because the results should not depend on this scale.

When one calculates the individual loop diagrams that appear in quantum corrections the expression of each diagram depend on the renormalisation scale. In the translation of the parameters from the OS scheme to the $\overline{\text{MS}}$ e.g. self-energies are used which depend on the renormalisation scale. Therefore the $\overline{\text{MS}}$ parameters are one-loop quantities that depend on the renormalisation scale Q.

In the quantum theory the individual diagrams do not have physical meaning, all the diagrams have to be taken into account to have a physical result. When one does a proper loop calculation taking into account all the diagrams to a given order in perturbation theory all the Q dependencies are cancelled and the final result is scale independent. This is the case of the OS scheme because there are used tree level parameter in one-loop expressions thus there is no order mixing.

However in the $\overline{\text{MS}}$ scheme the input parameters are one-loop parameters and when they are inserted intro the one-loop expression, they give rise to higher-order contributions. However, this way not all higher order diagrams are being taken into account, because only one-loop expressions are being used. Thus final result depends on the renormalisation scale. This renormalisation scale dependency is due to the unknown higher order contributions, therefore this dependency can be used to estimate the unknown higher order contributions.

4 Analysis

The calculations in this project have been performed with the tool `anyBSM`. `anyBSM` is a Python program in development that calculates one-loop corrections to the triple Higgs coupling in a large class of BSM models. In this project two models will be analyzed, the SM and then the Singlet Extension of the Standard Model with an unbroken $\mathbb{Z}_2$ symmetry (SSMZ2).

For the calculations in the SM it has been used the $\overline{\text{MS}}$ scheme with good results. In the SSMZ2 we have found some instabilities in the calculations for high values of the Higgs singlet boson mass using the $\overline{\text{MS}}$ scheme for the tadpoles. We have seen that the higher order Higgs singlet boson tadpole contributions break the calculation so for the SSMZ2 we have used the OS scheme for the tadpoles and with this the problem has been fixed.

4.1 Standard Model

First, we are going to study the SM parameters that enter in the calculation of λ_{hhh} (m_h, vev) in the $\overline{\text{MS}}$ scheme and their dependence on the renormalisation scale Q. The results are presented in Fig. 3.

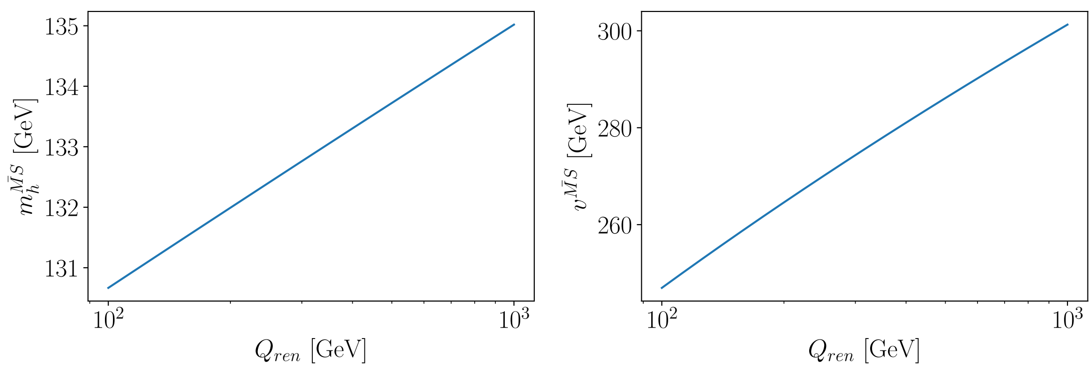


Figure 3: SM $\overline{\text{MS}}$ parameters as a function of the renormalisation scale Q. Left: the Higgs boson mass. Right: the vev.

Both parameters grow with the renormalisation scale. With the parameters that have been obtained in the $\overline{\text{MS}}$ scheme, we can proceed with the calculation of $\lambda_{hhh}^{\overline{\text{MS}}}$. There are different normalisations for λ_{hhh} in the literature. Thus, in order to be able to better compare the results of different works, we use a quantity known as κ_λ, to present the results. It is defined as the new triple Higgs coupling λ_{hhh} normalized to its tree level SM prediction $\lambda_{hhh}^{(0),SM}$:

$$\kappa_\lambda = \frac{\lambda_{hhh}}{\lambda_{hhh}^{(0),SM}}. \tag{15}$$

Figure 4 shows the prediction for κ_λ at the one-loop order in the OS and $\overline{\text{MS}}$ schemes.

As can be seen in fig. 4 the OS result does not depend on the renormalisation scale because a proper fixed-order one-loop calculation has been done. On the other hand, the $\overline{\text{MS}}$ result depends on the renormalisation scale as expected. Although the $\overline{\text{MS}}$ result is higher than the OS one, both values are lower than the tree level value.

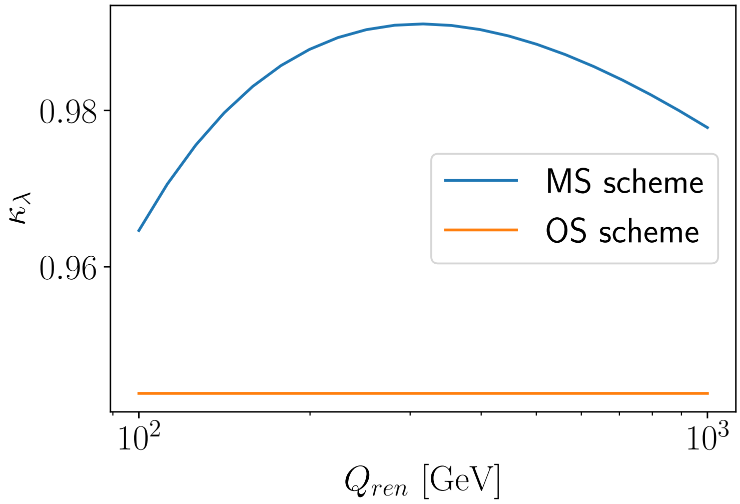


Figure 4: κ_λ for the SM as a function of the renormalisation scale Q, in the $\overline{\text{MS}}$ and OS schemes.

When one switches to the $\overline{\text{MS}}$ scheme a dependence on the renormalisation scale due to the higher-order terms such as $\ln(\frac{m_h^2}{Q^2})$ can appear. These logarithmic terms can become longer for high renormalisation scale values and they can break the perturbative behaviour of the calculations. To prevent this, the Renormalisation Group Equations (RGE) can be used. The RGEs are differential equations that give the dependence of the parameters on the renormalisation scale. For this project SARAH [?] has been used to solve the RGEs for the SM parameters and to check if the Q range that is being used is secure from this high logarithmic terms. The results are shown in Fig. 5.

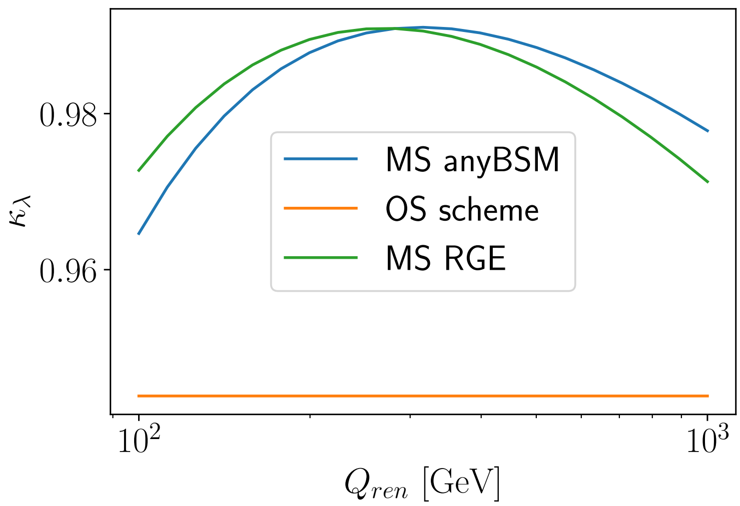


Figure 5: κ_λ for the SM as a function of the renormalisation scale Q, in the $\overline{\text{MS}}$ and OS schemes calculated with the program `anyBSM` and solving the RGEs.

We observe very similar behaviour between the results obtained with and without the RGEs. This means that in the range $Q_{ren} \in [100, 1000]$ GeV the logarithmic terms are not too large and the calculations are safe from large logarithmic enhancements.

4.2 SSMZ2

Once the calculations have been checked for the SM with good results one can go on to analyse BSM models. In this project we investigate the Real Singlet extension of the SM with an unbroken Z2 symmetry. During the calculations of this model we have found some instabilities due to the use of the $\overline{\text{MS}}$ scheme in the renormalisation of the tadpoles, hence for this model we have used the OS scheme for the tadpole renormalisation.

4.2.1 Model

The Real Singlet extension of SM adds a new real singlet S (with one new degree of freedom) which translates into a new Higgs boson. This singlet is defined in the eq. 16 together with the Higgs doublet ϕ. The $\mathbb{Z}_2$ symmetry makes the Lagrangian invariant under changes on the sign of the field: $S \to -S$.

$$\phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix} \qquad < S >= 0 \tag{16}$$

This model also modifies the Higgs potential as shown in eq. 17. There are not linear or trilinear terms in the potential due to the $\mathbb{Z}_2$ symmetry that is imposed. Due to this symmetry the Higgs fields do not mix between them.

$$V(\phi, S) = \mu^2 \phi\phi^\dagger + \frac{\lambda}{2}|\phi\phi^\dagger|^2 + \frac{m_S^2}{2}S^2 + \frac{\lambda_S}{2}S^4 + \frac{\lambda_{SH}}{2}S^2\phi\phi^\dagger \tag{17}$$

In the new potential there are three new terms, with three new coupling and this means three new free parameters:m_S, λ_S and λ_{SH}. From the expansion of the potential the new Higgs boson mass M_S can be defined in terms of the Lagrangian parameters as

$$M_S^2 = m_S^2 + \frac{\lambda_{SH}}{2}v^2. \tag{18}$$

Therefore for the calculations of the SSMZ2 M_S, λ_S and λ_{SH} are going to be used as free parameters. If one expands the potential of the eq. 17 and compares the result with the eq. 1 obtains that the tree level expression for λ_{hhh} is the same in both models. But the one-loop quantum corrections are not the same because there is a new particle to take into account in the loops. It is also important to notice that our one-loop calculations do not depend on the quartic coupling of the singlet λ_S because there are not one-loop diagrams with three SM-Higgs like external legs where this coupling enters.

4.2.2 Renormalisation scale dependence

First, we are going to study the SM parameters in the $\overline{\text{MS}}$ scheme and their dependence on the renormalisation scale Q. The results are presented in Fig. 6.

The SM-like Higgs boson mass grows with the renormalisation scale as in the SM but the vev decreases. This behaviour is due to the different treatment of the tadpoles in both models. These tadpoles contribution to the self-energies of the W and Z bosons are significant compared with the other contributions and depend on the renormalisation scale. This difference in the input parameter is compensated at the end with the counterterm of the triple Higgs coupling. With the parameters that have been obtained in the $\overline{\text{MS}}$ scheme, we can proceed with the calculation of $\lambda_{hhh}^{\overline{\text{MS}}}$. Figure 7 shows the prediction for κ_λ at the one-loop order in the OS and $\overline{\text{MS}}$ schemes.

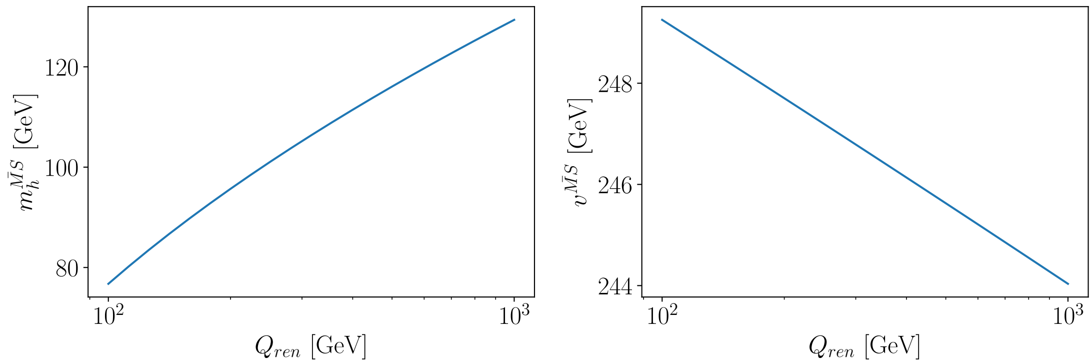


Figure 6: SSMZ2 SM-like $\overline{\text{MS}}$ parameters as a function of the renormalisation scale Q. Left: the Higgs boson mass. Right: the vev.

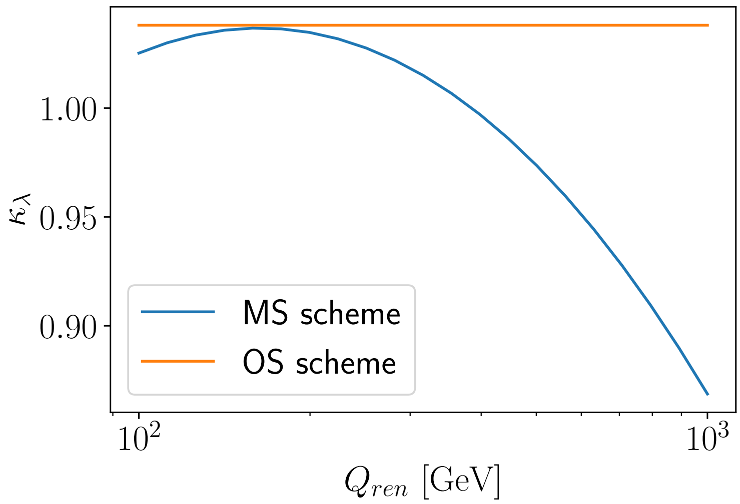


Figure 7: κ_λ for the SSMZ2 as a function of the renormalisation scale Q in the $\overline{\text{MS}}$ and OS schemes. BSM parameter values: $\lambda_{SH} = 3.86$, $M_S = 400$ GeV and $\lambda_S = 0$.

We can see in fig. 7 that for renormalisation scales close to the SM energies, $Q \in [200, 300]\ GeV$, we have the flattest behaviour of the $\overline{\text{MS}}$ result. We also can see how for higher renormalisation scale values κ starts to decrease and this tells us that the SSMZ2 has a bigger dependency on the renormalisation scale than the SM. Taking into account that in this calculations we have only translated to the $\overline{\text{MS}}$ scheme the SM parameters we can conclude that the unknown higher order corrections to the SM-like parameters are higher in the SSMZ2 than in the SM.

4.2.3 Error estimation

We want to estimate the unknown higher order corrections due to the BSM parameters. As we have seen the $\overline{\text{MS}}$ parameters are one-loop quantities. Therefore what we can do is use the OS scheme in the calculation of the triple Higgs coupling but using as input parameter the singlet mass in the $\overline{\text{MS}}$ scheme. The scheme conversion generates 2L (and higher-order) terms of order $\mathcal{O}\left(\frac{M_S^6}{v^5}\right)$ and $\mathcal{O}\left(\lambda_S \frac{M_S^4}{v^3}\right)$, i.e. corresponding to the order of the leading 2L corrections to λ_{hhh}. If one takes the genuine part of the one-loop corrections $\lambda^{(1)}$, introduces the $\overline{\text{MS}}$ singlet mass and expands the result in terms of the OS renormalised parameter one will obtain the OS genuine part of the corrections $\lambda^{(1)}(m^2_{S,\text{OS}})$ plus the higher order contributions,

$$
\begin{aligned}
\lambda^{(1)}(m^2_{S,\overline{\mathrm{MS}}}) =&\lambda^{(1)}(m^2_{S,\mathrm{OS}} + \delta^{CT} m^2_S) \\
=&\lambda^{(1)}(m^2_{S,\mathrm{OS}}) + \delta^{CT} m^2_S \frac{\partial \lambda^{(1)}}{m^2_S}\bigg|_{m^2_{S,\mathrm{OS}}} + ... \\
=&\lambda^{(1)}(m^2_{S,\mathrm{OS}}) + \mathcal{O}(2\mathrm{L}).
\end{aligned} \tag{19}
$$

In the fig. 8 we can see the results for the translation of the Higgs singlet mass as a function of Q.

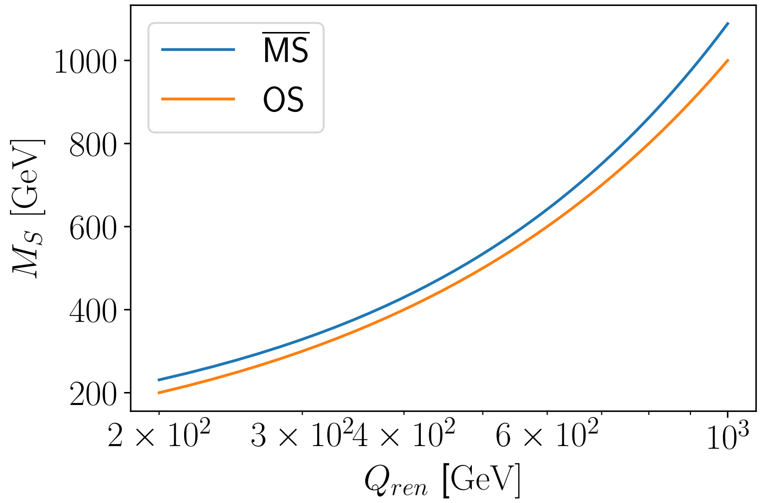


Figure 8: Singlet Higgs boson mass as a function of the renormalisation scale Q in the $\overline{\mathrm{MS}}$ and OS schemes. BSM parameter values: $\lambda_{SH} = 3.86$ and $\lambda_S = 0$.

Now we are going to calculate the triple Higgs coupling in the OS scheme using as an input parameter the singlet mass in the OS scheme and in the $\overline{\mathrm{MS}}$ scheme. The results are shown in Fig. 9.
In the Fig. 9 we can see that the λ_{hhh} tends towards the one-loop SM value and this behaviour is due to the decoupling theorem [11]. The difference that we can see between the two curves is due to higher order corrections, thus, we can use the difference of this two curves to estimate this unknown contributions. The results are shown in the Fig. 10.

Fig. 10 shows that the upper bound of the $\overline{\mathbf{MS}}$ value matches with the OS value and the lower bound of the OS value matches with the $\overline{\mathbf{MS}}$ value. This was expected due to the way we have defined the error. We also can see that the error decreases when we increase M_S, this again was to be expected due to the decoupling theorem. The higher-order terms has to be 0 when the mass goes to infinite to recover the SM value of the coupling, thus in this case the OS value and the $\overline{\mathrm{MS}}$ one when $M_S \to \infty$ are the same.

We can extract information by analysing the results for a renormalisation scale close to the SM energies. In the range of $Q \in [200, 250]$ GeV we have relative errors between 8% and 5%. This uncertainty due to the singlet boson higher-order contributions are reasonable. Now we can compare this uncertainty with the experimental constrains of fig. 2. The constrains set the experimental value in the range $\kappa_\lambda \in [-1, 6]$ and the higher uncertainty that we have found in the decoupling limit for κ_λ is of 0.09. From this we can conclude that when we work in a parameter space near the SM values the one-loop calculations are give a satisfactory level of precision for λ_{hhh}.

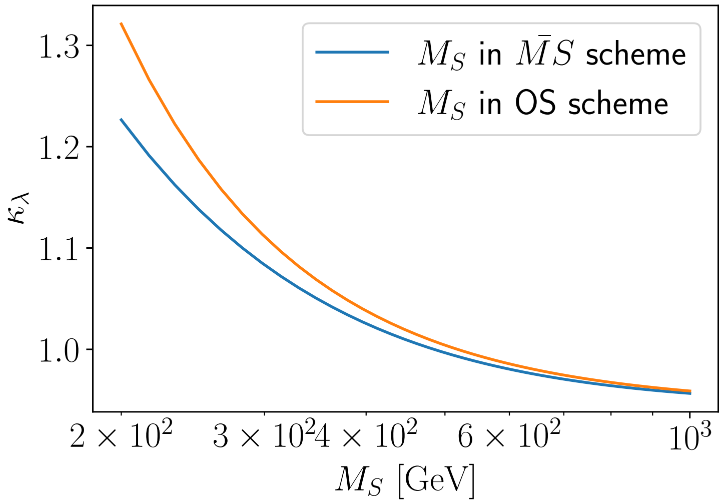


Figure 9: κ_λ for the SSMZ2 as a function of the singlet Higgs boson mass M_S in the OS scheme using as input parameter the singlet mass in the $\overline{\text{MS}}$ and OS schemes. BSM parameter values: $\lambda_{SH} = 3.86$, $M_S = 400$ GeV and $\lambda_S = 0$. The OS curve is represented in function of the OS value of M_S and the $\overline{\text{MS}}$ is represented as a function of the $\overline{\text{MS}}$ singlet mass

However, we have worked in the decoupling limit, this means to fix the λ_{SH} coupling and change the Lagrangian mas parameter m_S to change the the mass M_S. But what one can also fix the Lagrangian parameter and change the coupling λ_{SH}, this is known as the non-decoupling limit and it is shown in fig. 11. We have stopped the calculation in $Q = 700\ GeV$ because the perturbative unitarity is broken beyond this point, the error increases too fast.

In the limit where the couplings become long, λ_{hhh} starts to increase rapidly and also the uncertainty due to the singlet-Higgs boson higher-order contributions increase. For $Q = 700\ GeV$ we obtain an error in κ_λ of 1 which is huge compared with the experimental constrains. Therefore, we can conclude that if one wants to explore a region parameter space where λ_{hhh} is much higher than the SM values a calculation including higher-order effects is needed.

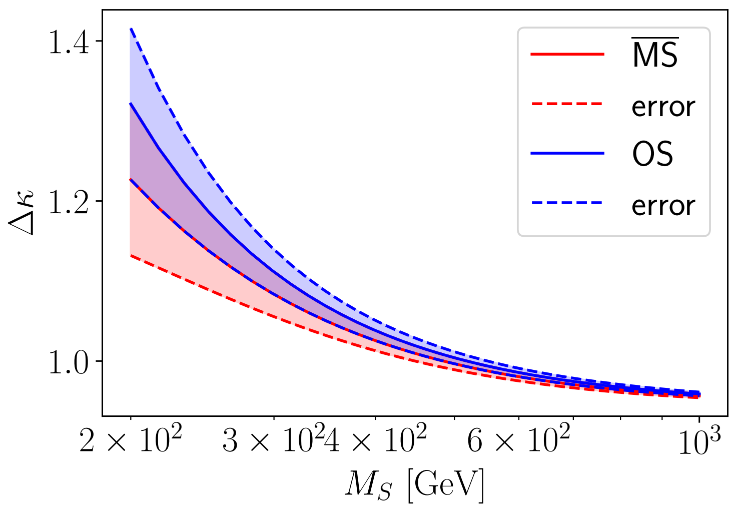


Figure 10: κ_λ for the SSMZ2 as a function of the singlet Higgs boson mass M_S in the OS scheme using as input parameter the singlet mass in the $\overline{\text{MS}}$ and OS schemes, the bands represent the uncertainty due to the higher order contributions of the BSM Higgs mass. BSM parameter values: $\lambda_{SH} = 3.86$ and $\lambda_S = 0$.

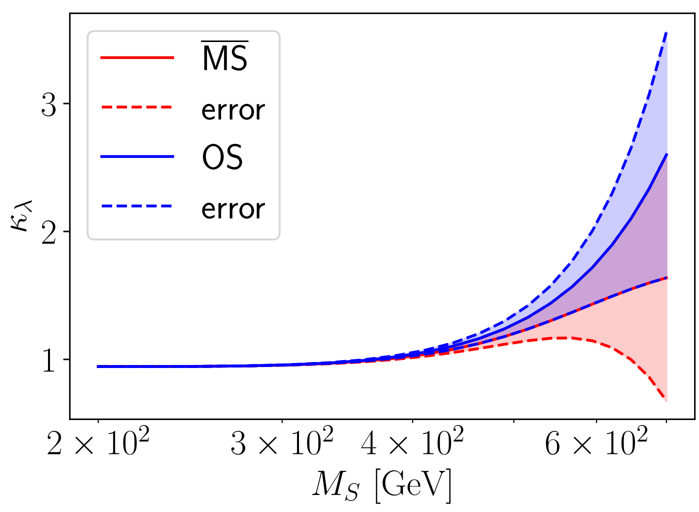


Figure 11: κ_λ for the SSMZ2 as a function of the singlet Higgs boson mass M_S in the OS scheme using as input parameter the singlet mass in the $\overline{\text{MS}}$ and OS schemes with the uncertainty due to the higher order contributions of the BSM Higgs mass with the Lagrangian mass parameter fixed and varying the coupling λ_{SH}. BSM parameter values: $m_S = 200$ GeV and $\lambda_S = 0$.

5 Conclusions and outlook

In this project we have used the anyBSM tool to calculate one-loop perturvative correction to the triple Higgs coupling λ_{hhh} in the SM and in the SSMZ2 and in the SSMZ2 we have also estimated the unknown higher-order contributions to the coupling.

From fig. 5 we can conclude that the one-loop perturbative corrections of the λ_{hhh} in the SM are safe from long logarithmic terms in a range for the renormalisation scale of $Q \in [100, 1000]$.

We can conclude from fig. 7 that the contributions to λ_{hhh} from unknown higher order corrections to the SM-like parameters are higher in the SSMZ2 than in the SM. We can also conclude from fig. 10 that when one wants to calculate perturbative corrections to the λ_{hhh} in the SSMZ2 in a parameter space near the SM value of λ_{hhh} (decoupling limit) the one-loop calculations are enough to obtain a result with a small error compared to the current experimental accuracy. However if one wants to do these calculations in a region of parameter space far from the SM values (non-decoupling limit) we can conclude from fig. 11 that one-loop corrections are not enough and higher-order corrections are needed.

As a continuation of this project the codes HiggsBounds [12] and HiggsSignals [13] can be used to take into account the experimental constrains of the parameter space of the model from direct searches in Tevatron, LEP and LHC and from measurements of the Higgs in the LHC. Taking into account the experimental constraints on the Higgs sector, obtained from HiggsBounds and HiggsSignals, one can constrain the BSM parameters that enter in the calculation of λ_{hhh}.

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