Investigation of the effects of Mass Hierarchy, Octant Angles and CP violation on Neutrino Oscillation Probabilities using nuSQuIDS.

DESY Summer Student Programme, 2021

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September 8, 2021

Contents

1 Introduction

In recent years, there have been several breakthroughs in neutrino physics and it has emerged as one of the most active fields of research. The Standard Model of particle physics describes neutrinos as massless, chargeless elementary particles that come in three different flavours . However, recent experiments indicate that neutrinos not only have mass, but also have multiple mass eigenstates that are not identical to the flavour states, thereby indicating mixing. As an evidence of mixing, neutrinos have been observed to change from one flavour to another during their propagation – a phenomenon called neutrino oscillation.

Neutrinos are produced in weak interactions which occur in many sources like atmosphere, sun, reactors and accelerators. In the past few decades, several experiments have confirmed the event of flavour change in each of these neutrinos.

In this project, I will investigate how the values of mass hierarchy, octant angles and CP phase ,which are undetermined parameter of the oscillation, affects the probability using nuSQuIDS software.

2 Theory and phenomenology of neutrino oscillations

2.1 Formalism

Neutrinos come in three flavours, electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ). Mixing may be described with the observation that each of the three flavours of neutrinos can be expressed as a superposition of mass eigenstates. Thus, we can write a flavour state as :

$$|v_\alpha\rangle = \sum_i U_{\alpha i} |v_i\rangle \tag{1}$$

Where $U_{\alpha i}$, called leptonic mixing matrix. The mixing matrix assuming $i = 1, 2, 3$ and $\alpha = e, \mu, \tau$ would look like:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{2}$$

The leptonic mixing matrix is unitary $UU^\dagger = U^\dagger U = \mathbf{1}$, and can be parametrized by 3 mixing angles and 6 phases. Out of the six phases only one would be relevant here. So, we can rewrite U as:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

where $c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij}$

2.2 Neutrino Oscillation in Vacuum

Now we consider ν_μ neutrino travels a distance L in vacuum. Using the above equations we can find the oscillations probabilities:

$$
\begin{aligned}
P\left(\nu_\mu \rightarrow \nu_\mu\right) &\simeq 1-\sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \\
P\left(\nu_\mu \rightarrow \nu_e\right) &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \\
P\left(\nu_\mu \rightarrow \nu_\tau\right) &= \cos^4 \theta_{13} \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)
\end{aligned} \tag{4}
$$

In which $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and $L \simeq ct$ is the distance travelled by the neutrino. It is clear that the oscillations in vacuum don't depend on the phase δ_{CP} in the U matrix.

2.3 Neutrino Oscillation in matter

Propagation though matter has an effect on the oscillations probability because of the interaction between neutrinos and matter, which adds an additional term in the Hamiltonian of the particles.

Assuming ultra-relativistic neutrinos, we can get the oscillations probabilities. What would be important for the rest of the report is $P\left(\nu_\mu \rightarrow \nu_e\right)$, give by:

$$
\begin{aligned}
P\left(\nu_\mu \rightarrow \nu_e\right) \approx & \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2\left(\Delta_{31}-aL\right)}{\left(\Delta_{31}-aL\right)^2} \Delta_{31}^2 \\
& +\sin 2\theta_{23} \sin 2\theta_{13} \sin 2\theta_{12} \frac{\sin\left(\Delta_{31}-aL\right)}{\left(\Delta_{31}-aL\right)} \Delta_{31} \frac{\sin(aL)}{(aL)} \Delta_{21} \cos\left(\Delta_{31}+\delta_{CP}\right) \\
& +\cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(aL)}{(aL)^2} \Delta_{21}^2
\end{aligned} \tag{5}
$$

where $\Delta_{ij} = \Delta m_{ij}^2 L / 4E_\nu, a = G_F N_e / \sqrt{2}, G_F$ is the Fermi constant, N_e is the number density of electrons in the Earth, L is the baseline in km, and E_ν is the neutrino energy in GeV.
It is clear that all the four parameter of the leptonic matrix participate in the probability. The probability as well depend on the the differences in the squared masses of the neutrinos, Δm_{21}^2 and Δm_{31}^2, where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and $\Delta m_{31}^2 = \Delta m_{32}^2 + \Delta m_{21}^2$

Five of the parameters governing neutrino oscillations have been measured: all three mixing angles and the magnitude of the two independent mass squared differences. Because the sign of Δm_{31}^2 is not known, there are two possibilities for the ordering of the neutrino masses, called the mass hierarchy: $m_1 < m_2 < m_3$ ("normal hierarchy") or $m_3 < m_1 < m_2$ ("inverted hierarchy"). The value of the CP-violating phase δ_{CP} is unknown. Another remaining question is the octant of θ_{23} : measured values of $\sin^2\left(2\theta_{23}\right)$ are close to, but the data are so far inconclusive as to whether θ_{23} is less than or greater than 45°, the value for maximal mixing between ν_μ and ν_τ.

So, we would investigate how the mass hierarchy, the value of δ_{CP}, and the θ_{23} octant (value of $\sin^2 \theta_{23}$) affect the muon neutrino to electron neutrino oscillation probability over propagation through earth.

2.4 Atmospheric Neutrinos

Atmospheric neutrinos are typically produced around 15 kilometers above Earth's surface. They form when a cosmic ray, an energetic particle from space, crashes into Earth's atmosphere. These particles are typically protons, though they can also be helium or heavier nuclei. When they strike an atomic nucleus in our atmosphere, there is a cascade of particles. Short-lived particles called mesons form, most of them pions. These are unstable particles made of two quarks, and they rapidly decay into muons and muon antineutrinos (or antimuons and muon neutrinos). A muon is also unstable, so it will decay into an electron, electron antineutrino, and muon neutrino. Thus, about two-thirds of atmospheric neutrinos are muon neutrinos and antineutrinos, and the remainder are electron neutrinos and antineutrinos.

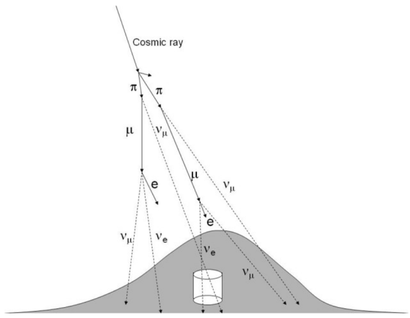

Figure 1: Cosmic rays interacting with an air nucleus in the atmosphere

2.5 Neutrino Detection IceCube

IceCube is a neutrino observatory, consisting of a cubic-kilometer particle detector made of strings with 5160 digital optical modules (DOM) frozen in the Antarctic ice. It is located at South Pole near the Amundsen-Scott South Pole Station. Most of IceCube detector is at the geographic depth of $1,5km$.

There are two additional parts of IceCube - IceTop, surface array, studying extensive air-showers and DeepCore, a denser inner subdetector of IceCube, that helps to detect low energy neutrinos and neutrino oscillations.

When neutrinos interact with ice they produce charged moving secondary particles, that cause Cerenkov emission, which can be detected by photo-multipliers (PMT).

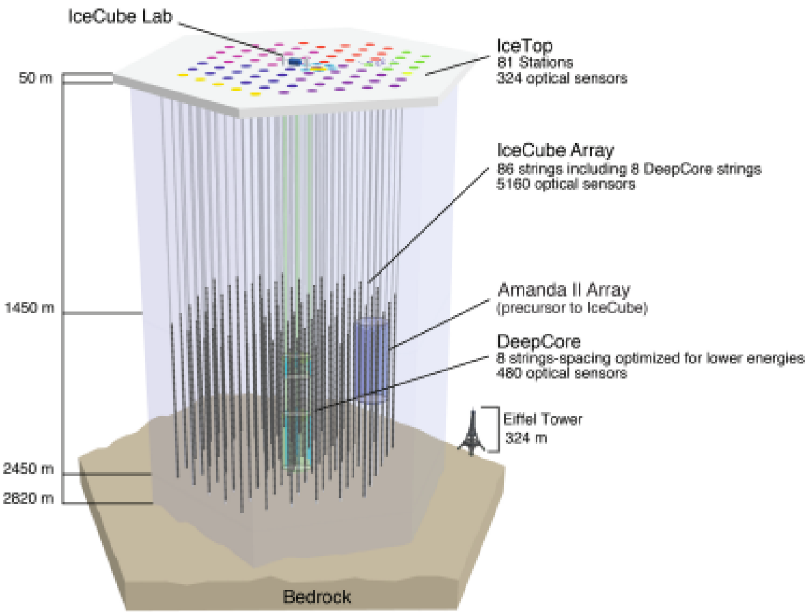

Figure 2: Scheme of IceCube detector with IceTop, main IceCube array and DeepCore

3 Results

3.1 Octant Angle

A precise measurement of ν_μ oscillation probability to ν_e around $5GeV$, would give a hint which octant do neutrinos have (HO or LO).

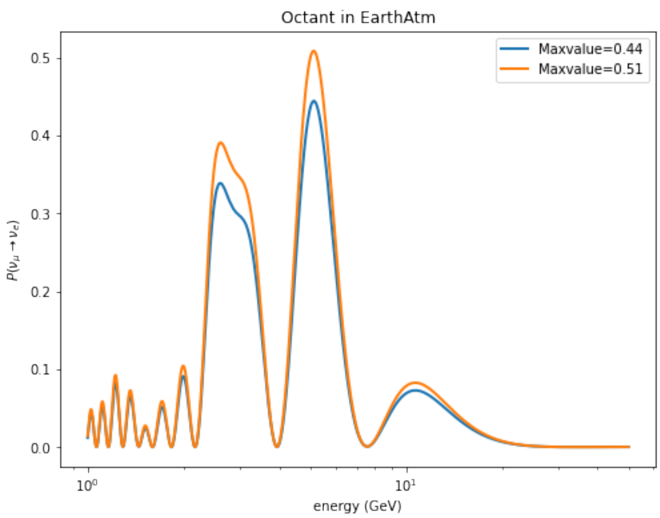

Figure 3: The plot gives the oscalltion probability from ν_μ to ν_e. The orange curve represents the high octant $\theta = 47$, while the blue represents the low octant $\theta = 43$.

3.2 Mass Hierarchy

In normal mass hierarchy both octant angles can be resolved also the oscillation probability is measurable. On the other hand, inverted mass hierarchy has probability that almost vanish at $5GeV$ and above also the octant angles is not resolved as shown in the figure below.

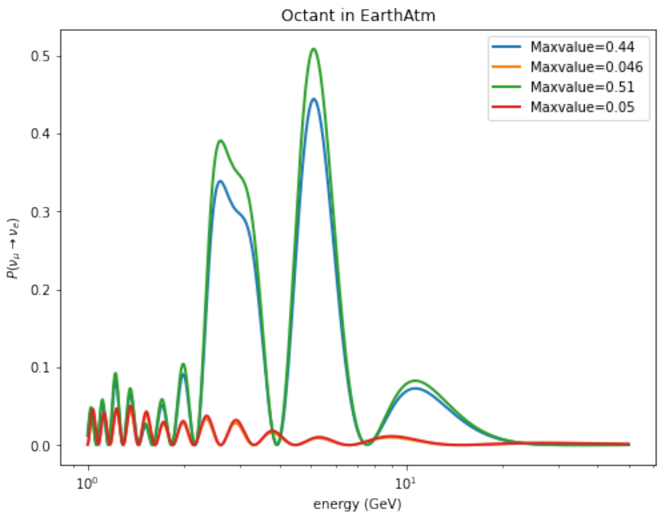

Figure 4: The plot gives the oscillation probability from ν_μ to ν_e. The green and blue curves are for normal H, while red and orange curved represents inverted H.

3.3 CP Phase

From equation (5), the δ_{CP} has a small effect on the oscillation probability. It gives the highest probability when it vanishes. Since, increasing the energy, $\Delta_{ij} = \Delta m^2_{ij} L/4E_\nu$ would decrease, so setting δ_{CP} to zero, makes $cos(\Delta_{ij} = \Delta m^2_{ij} L/4E_\nu + \delta_{CP})$ goes to 1.

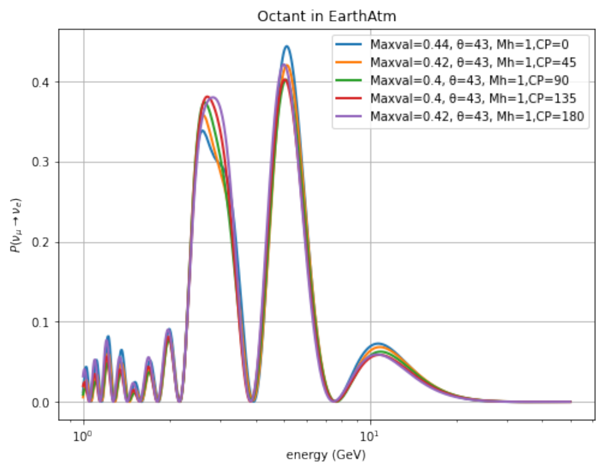

Figure 5: The plot gives the oscillation probability from ν_μ to ν_e. Here we have varied the phase to see its effect on the probability for normal hierarchy.

3.4 Anti-Neutrinos

Antineutrinos almost share the same probability as the inverted mass hierarchy of the neutrinos but with a phase around −180.

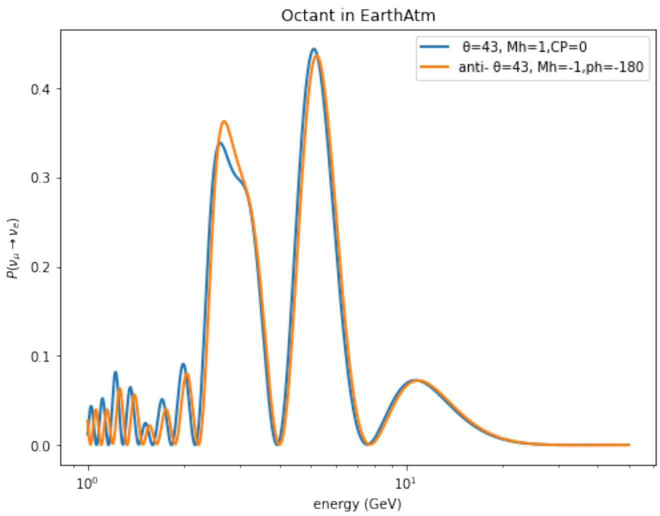

Figure 6: The blue curve for neutrinos in normal hierarchy and orange curve for antineutrinos in inverted hierarchy, with −180 they have the same probability curve.

4 Conclusions

In this work, we have examined how mass hierarchy, octant angle and cp phase affect the oscillations probability. Also, neutrinos and antineutrinos have around the same probability in opposite hierarchy mode with −180 phase. The actual values for these parameters would be decided by precise measurements in the future.

Acknowledgements

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