



Ablation study in the capillary discharge of an electrothermal gun

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Abstract

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1 Introduction

This work has centered in improving the understanding of the ablation rate for an electrothermal gun, in the context of plasma wakefield acceleration. With the physics being the same as for an open cavity, we've used this model instead for two reasons:

- It has just one open boundary, so the numerical implementation is easier.
- There's extensive literature to compare it to.

1.1 Electrothermal gun

An electrothermal gun consists of a plasma source and a chemical propellant system. In this system, the propellant is usually ignited with plasma introduced by a jet from a capillary discharge or by an arc from an exploding wire inside the propellant charge. We are interested in the first one.

The discharge happens in a cylindrical tube closed at one end, where the anode is located, and open at the other, with the cathode (Fig. 1). A discharge is applied, either in a gas-filled capillary or in an ablation-controlled capillary, expelling plasma through the open end as a result.

On their own, electrothermal guns are mostly interesting because of their short ignition delay time, of about 2 ms, compared to conventional igniters, that take about 8 ms, but in our case they are just a handy model.

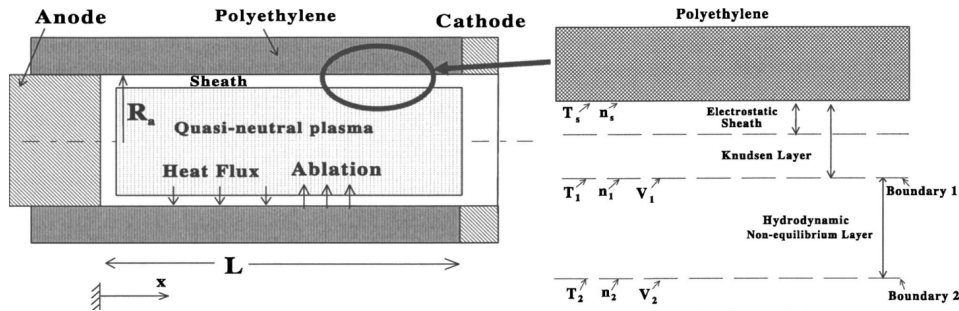


Fig. 1: Electrothermal gun and schematic of multilayer structure near the ablated surface.

1.2 Ablation

Ablation is the process of evaporating a material, with a subsequent ionization. There's many possible processes involved (Fig. 2) in it, but in our work we just care about two facts:

1. The material gets degraded

2. The newly ionized molecules contribute to the already existent plasma

In our case, ablation happens because of the high temperature achieved inside the capillary.

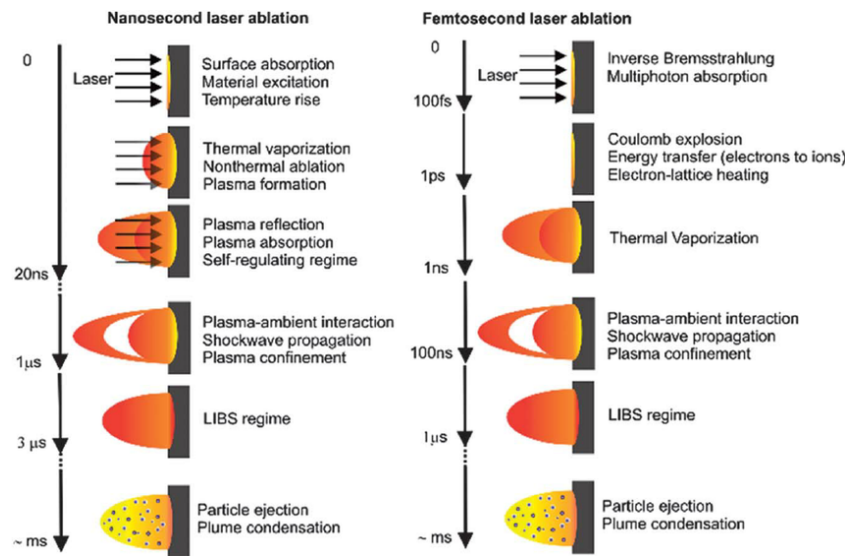


Fig. 2: Direct laser ablation. Different

2 Previous work

This have been the most important papers in our research.

2.1 Keidar, Beilis (2006) [1]

The main **objective** is testing the assumption of thermal equilibrium for ions, electrons and neutral particles ($T_i = T_e = T_a$).

2.1.1 Assumptions

- Axisymmetric steady-state problem.
- $L \gg R$ (uniform plasma parameters).
- Heavy particles are in equilibrium ($T_i = T_a = T_h$)
- They use **polyethylene**, $(C_2H_4)_n$.
- They take the velocity at the Knudsen layer as a **free parameter**.

- Saha equilibrium. No significant flow in radial direction, therefore $V_a \ll C_s$ and constant capillary pressure.

They use an empirical model for the vapour pressure

$$P_v = 10^5 \exp(A \times [1/B - 1/T_s]) \quad (1)$$

And assuming a constant pressure:

$$P(r) = n_a T_a + n_e T_e + n_i T_i = P_v \quad (2)$$

2.1.2 Results

There's not always thermal equilibrium. Defining the parameter $\alpha = V_a / \sqrt{2T_a/m_a}$ they find that for $\alpha \gtrsim 0.05$ there's equilibrium. For $\alpha = 0.001$ the electron temperature profile is almost constant.

Also 0.05 is about the maximum value for α , so the usual case will *not* be in equilibrium. We can only make that assumption for high ablation rates.

2.2 Keidar, Boyd, Beilis (2000) [2]

Objective: provide (upstream) boundary conditions in a time-dependent model of an electrical discharge ($t \sim 10\mu s$) in a teflon gas-filled capillary.

2.2.1 Assumptions

- They claim that the initial ignition plasma contributes to a few % on m and E . They take the **initial plasma density** as a **free parameter**.
- Plasma bulk is quasi-neutral and in equilibrium (LTE), so they only use one temperature.
- All plasma parameters uniform within plasma column.

2.2.2 Model

The model has two parts: the plasma bulk and the sheaths near the walls, both for the anode and the teflon (dielectric) cavity itself.

The current provided by the diffuse discharge on the anode surface is carried by electrons and controlled by the potential drop in the anode sheath:

$$U_a = -K_B T \log(I_{th}/I) \quad (3)$$

with I_{th} the random (thermal) electron current and I the circuit current. The potential drop for the dielectric sheath follows an similar equation, with I_i , the ion current, instead of I . They end up with an equation for the electron energy balance, where the unknowns are only T and n_e . The temperature can be obtained with the heat transfer equation.

2.3 Keidar Boyd (2006) [3]

Here they use a model with more layers (Fig. 1), with the central region occupying almost all the capillary. From interior to exterior:

- Quasineutral plasma
- Hydrodynamic non-equilibrium layer
- Knudsen layer
- Electrostatic sheath
- Polyethylene wall

This is the model we tried to reproduce.

3 Our model

Our model consists on three main parts:

1. Plasma bulk
2. Knudsen layer
3. Wall

3.1 Quasineutral plasma

Assuming all the parameters vary just in the axial direction x and time t , our mass, momentum and energy balance equations are:

$$A \left(\frac{\partial \rho}{\partial t} + \frac{\partial \rho V}{\partial x} \right) = 2\pi R_a \Gamma, \quad (4)$$

$$\rho \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \right) = -\frac{\partial P}{\partial x}, \quad (5)$$

$$\rho \left(\frac{\partial \varepsilon}{\partial t} + V \frac{\partial \varepsilon}{\partial x} \right) = -P \frac{\partial V}{\partial x} + Q, \quad (6)$$

with:

- ρ : mass density
- V : velocity
- R_a : capillary radius
- A : capillary cross section
- Γ : ablation rate
- P : pressure
- $\varepsilon = \frac{3}{2} \frac{T}{m} + \frac{V^2}{2}$
- T : temperature
- Q : heat

There's 3 contributions to the heat exchange ($Q = Q_j - Q_r - Q_f$):

- Joule heating: $Q_j = j^2 / \sigma$
- Radiation cooling: Q_r
- Cooling from energy losses: particle flux to the wall: Q_f (due to the electrostatic sheath)

The approach to solve the equations has been an iterative one (Fig. 3). Simply solving them one by one (see 4).

We're assuming no viscosity or Lorentz force and $\frac{\partial T}{\partial x} = 0$, and the ideal gas law.

The tricky bit is imposing boundary conditions in the cathode. We've tried using ghost cells to make the pressure go to the atmospheric one in the far field limit, but haven't succeeded.

3.1.1 Saha equilibrium

From the plasma bulk we know the massive density, which can be converted to the massive species number density, due to electrons not contributing substantially. The Saha ionization equation gives the ionization state of a gas *in thermal equilibrium* as a function of temperature:

$$\frac{n_e n_i^+}{n_n} = B \frac{g_i}{g_n} T^{1.5} \exp(-I_i/T) \equiv A_i \quad (7)$$

where n_e , n_i and n_n are the electron, ion and neutral number densities.

Supplemented with conservation of nuclei and the quasineutrality condition:

$$\begin{aligned} 2(n_C + n_C^+) &= n_H + n_H^+, \\ n_e &= n_c^+ + n_H^+ \end{aligned} \quad (8)$$

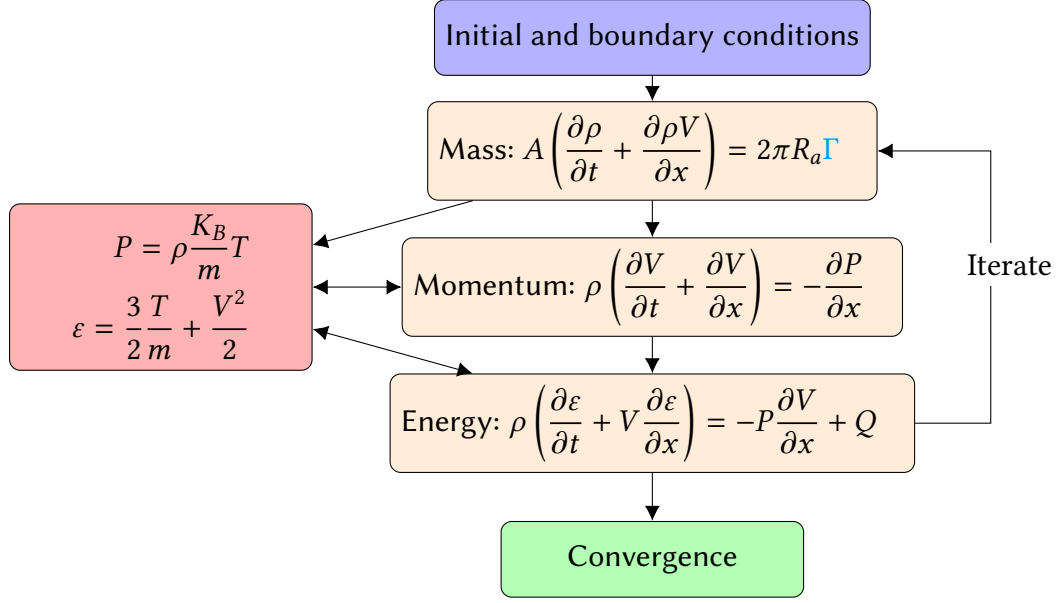


Fig. 3: Solution approach for plasma bulk.

both shown for polyethylene here, our ions being carbon and hydrogen. Following the method in [2] we find an equation for the electron density:

$$2n_e^2(n_e + A_H) + n_e^2(n_e + A_C) - (n_h - 2n_e) \times [2A_H(n_e + A_C) + A_C(n_e + A_H)] = 0 \quad (9)$$

with n_h the total heavy particle density $\propto \rho$.

3.2 Wall

The temperature at the wall follows:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial r^2} \quad (10)$$

with $a = \frac{\lambda}{c_p \rho}$ and boundary conditions:

$$\begin{aligned} -\lambda \frac{\partial T}{\partial r}(r = R_a) &= q(t) - \Delta H \cdot \Gamma \\ \lambda \frac{\partial T}{\partial r}(r = \infty) &= 0 \\ T(t = 0) &= T_0 \end{aligned} \quad (11)$$

$q(t)$ is the heat flux density and should include both convective and radiative parts, we're approximating it according to [4] as:

$$q(t) = \lambda_h \left(\frac{\partial T_h}{\partial r} + \frac{T_h}{r} \right) \quad (12)$$

with λ_h the ions thermal conductivity; and ΔH the ablation heat, the energy necessary for the wall material to ablate.

3.3 Knudsen layer

To connect the plasma bulk with the wall we need a distributional approach, we can't use average quantities due to the layer not being in equilibrium.

We approximate the distribution function within the discontinuity region by the sum of distributions before and after it [5].

$$f(x, \mathbf{v}) = \alpha(x)f_1(\mathbf{v}) + [1 - \alpha(x)]f_2(\mathbf{v}) \quad (13)$$

At the outer edge of the Knudsen layer a Maxwellian distribution is assumed:

$$f_2(\mathbf{v}) = n_1 \left(\frac{m}{2\pi K_B T_1} \right)^{3/2} \exp \left(-m \frac{(v_x - V_1)^2 + v_y^2 + v_z^2}{2K_B T_1} \right) \quad (14)$$

While at the phase interface:

$$f_1(\mathbf{v}) = \begin{cases} n_0 \left(\frac{m}{2\pi K_B T_0} \right)^{3/2} \exp \left(-\frac{mv^2}{2K_B T_0} \right), & v_x > 0 \\ \beta f_2(\mathbf{v}), & v_x < 0 \end{cases} \quad (15)$$

With $M = V_1 / \sqrt{\gamma RT} = m\sqrt{2/\gamma}$

Solving the conservation laws:

$$\begin{aligned} \int d\mathbf{v} v_x f(x, \mathbf{v}) &= C_1, \\ \int d\mathbf{v} v_x^2 f(x, \mathbf{v}) &= C_2, \\ \int d\mathbf{v} v_x v^2 f(x, \mathbf{v}) &= C_3. \end{aligned} \quad (16)$$

We have the next system of equations [6]:

$$\begin{aligned} \frac{n_0}{2\sqrt{\pi d_0}} &= n_1 V_1 + \beta \frac{n_1}{2\sqrt{\pi d_1}} \left[\exp(-\alpha^2) - \alpha \sqrt{\pi} \operatorname{erfc}(\alpha) \right] \\ \frac{n_0}{4d_0} &= \frac{n_1}{2d_1} \left\{ (1 + 2\alpha^2) - \beta \left[(0.5 + \alpha^2) \operatorname{erfc}(\alpha) - \alpha \exp(-\alpha^2)/\pi^{0.5} \right] \right\} \\ \frac{n_0}{(\pi d_0)^{1.5}} &= \frac{n_1}{\pi d_1^{1.5}} \left\{ \alpha(2.5 + \alpha^2) - \beta/2 \left[(2.5 + \alpha^2) \alpha \operatorname{erfc}(\alpha) - (2 + \alpha^2) \exp(-\alpha^2)/\pi^{0.5} \right] \right\} \end{aligned} \quad (17)$$

Which can be simplified [7] to end up with expressions for T_1 and ρ_1 , that we note are just functions of V_1 :

$$\begin{aligned} \frac{T_1}{T_s} &= \left[\sqrt{1 + \pi \left(\frac{\gamma - 1}{\gamma + 1} \frac{m}{2} \right)^2} - \sqrt{\pi} \frac{\gamma - 1}{\gamma + 1} \right]^2 \equiv \frac{T_1}{T_s}(V_1) \\ \frac{\rho_1}{\rho_s} &= \sqrt{\frac{T_s}{T_1}} \left[\left(m^2 + \frac{1}{2} \right) e^{m^2} \operatorname{erfc}(m) - \frac{m}{\sqrt{\pi}} \right] \equiv \frac{\rho_1}{\rho_s}(V_1) \end{aligned} \quad (18)$$

Also, combining:

$$n_1 V_1 = n_2 V_2 \quad (19)$$

$$n_1 K_B T + m n_1 V_1^2 = n_2 K_B T_2 + m n_2 V_2^2 \quad (20)$$

We find:

$$V_1^2 = \frac{K_B}{m} \frac{n_2 T_2 - n_1 T_1}{n_1 - (n_1^2/n_2)} \quad (21)$$

With a real V_1 if $n_1 T_1 > n_2 T_2$ and $n_1 > n_2$.

Now it's possible to find a V_1 that's consistent with (18) and (21). And $\Gamma = m n_1 V_1$, so we found a relation between plasma bulk and wall properties and the ablation rate.

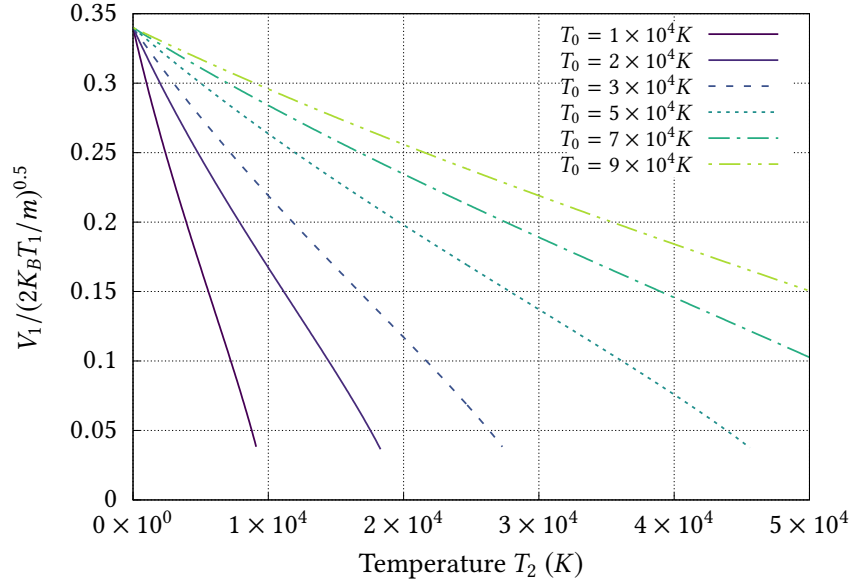


Fig. 4: $V_1(T_2)$ with surface temperature as a parameter and $n_2 = 10^{23}$, $n_1 = 10^{24}$

In Fig. 4 we solve this relations for a set of bulk and wall temperatures and a fixed densities.

4 Numerical methods used

All the model has been implemented in a fortran program. To solve the system of equations in 3.1 each has been written like:

$$\frac{\partial u}{\partial t} + k \frac{\partial v}{\partial x} = S(t, x). \quad (22)$$

So u and v don't have to be equal, and k can be equal to V . Each variable is discretized in a lattice representing position and time, where a finite difference upwind scheme [8] can be applied. After having ρ for each grid point, (9) is solved to get n_e .

As boundary conditions we set all gradients to zero at $x = 0$ and take $V(t = 0, x) = 0$, $\rho(t = 0, x) = ctt$, $T(t = 0) = T_{lab}$. We're still not sure how to implement the exit boundary condition: pressure must tend to the atmospheric one in the far field limit, but the way it does that affects the solution, so further research is needed.

5 Further work

Solving the whole model in a self-consistent manner is still not possible, so we suggest using a third party code for solving the plasma bulk, like ENZO (O'Shea et al. 2005), FLASH (Fryxell et al. 2000), RAGE (Gittings et al. 2008) or CASTRO (Almgren et al. 2010), even though they are focused on astrophysics.

The wall material should be changed to sapphire to be useful for the FLASHForward experiment.

Species conservation. In the future we would like to be able to add different gas species. In that case we may need an extra equation.

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}) = \sum_j S_j R_{ji} \quad (23)$$

with n_i the density of species i , S_j the reaction rate for reaction j and R_{ji} the stoichiometric constant of the production of particle i by reaction j . Technically, we already needed it, so it would be worth checking if it makes a difference.

Lorentz force. The current in the axial direction generates a magnetic field in the ϕ direction. This results in F_{Lor} (in the negative r -direction) of magnitude

$$F_{Lor} = J \times \frac{\mu J r}{2}. \quad (24)$$

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