



Signature rates of the Minimal Mirror Twin Higgs model

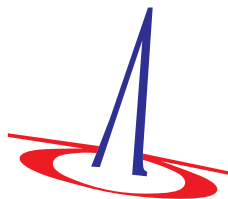
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Abstract

The Minimal Mirror Twin Higgs model offers some unique opportunities to solve problems in the Higgs sector of the Standard Model (SM) or in Dark Matter physics. It introduces a sector symmetric to the Standard Model, which contains a mirror version of all particles present in the SM. By allowing for mixing between the the SM Higgs bosons and the mirror Higgs boson, the coupling of the two sectors happens only via the Higgs bosons. In this report possible signatures, which might be accessible for the HL-LHC or future e^+e^- colliders, are presented for different ranges of model parameters. The Monte Carlo software WHIZARD is used to calculate the cross sections of the processes.



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1 Introduction

The idea of the Mirror World poses an intriguing solution to for some current challenges. It introduces an additional sector similar in structure to the Standard Model with mirror versions of all leptons, quarks and bosons. The mirror baryons are expected to fit all requirements of Dark Matter and therefore give an explanation to the cosmological energy problem.

It is consistent with collider measurements, which currently lack a signal of any kind of new physics with sufficient significance. Another feature of the Minimal Mirror Twin Higgs model is very little fine tuning present in the Higgs sector [1].

How such signals from the Minimal Mirror Twin Higgs model could look like in a detector will be part of the later sections. But first the model will be depicted further (section 3) and then tested using Monte Carlo software (section 2) against current limits deployed by the LHC and expected by the HL-LHC (section 4).

2 WHIZARD

WHIZARD (**W** **H**iggs **Z** **A**nd **R**espective **D**ecays) [2, 3] is a software package, designed to calculate multi-particle cross sections and simulate collision events. Matrix elements are generated by the build-in generator O'Mega. Various models are available by default like the SM and MSSM, but further models can be added by supplying the corresponding model file in various file formats. One possibility is the WHIZARD model file format, but in the following the Universal FeynRules Output (UFO) file format is used to provide the Minimal Mirror Twin Higgs model to WHIZARD. WHIZARDs interface for UFO files has just been developed and some part of the analysis are consistency checks.

The command language SINDARIN (**S**cripting **I**Ntegration, **D**ata **A**nalysis, **R**esults display and **I**nterfaces) is used to operate WHIZARD. Some basic knowledge about it will be useful to understand further remarks about the procedure [4].

Additional software can be linked together with WHIZARD to expand its capabilities. For shorter runtime for example, MPI or OpenMP parallelize the code execution.

More information can be found the WHIZARD MANUAL [4].

3 Minimal Mirror Twin Higgs Model

The Minimal Mirror Twin Higgs model consists of two parts, one is the Mirror World idea and the other is the Twin Higgs mechanism. Both will be described briefly in the following and the used parameter space will be defined.

3.1 Mirror World

This idea extends the Standard Model, which has a $SU(3) \times SU(2) \times U(1)$ symmetry, with another sector of the same interactions $SU(3)' \times SU(2)' \times U(1)'$. These two sectors each contain all the quarks, leptons and bosons known from the Standard Model and in the following the

mirror versions will be referred to with a “ ’ ” (e.g. W and W'). It is expected that the mirror particles are created and behave similar to the normal matter, but because there is no direct interaction between the two types, it has not been detected.

Those mirror particles could explain the cosmological energy problem in the early universe, because energy was transferred to the Mirror World. And because of the small cross section between normal matter and mirror particles, it is also a possible explanation for dark matter [1]. In order to find signals from the Mirror World, the Higgs sector has to be probed, because only the Higgs bosons couple to ordinary matter as well as to mirror matter. The Higgs sector is the connection to the Mirror World and the Twin Higgs mechanism plays an important role in understanding this connection.

3.2 Twin Higgs

The Twin Higgs model introduces two Standard Model like Higgs Doublets (H and H') and a common potential V_{eff} . In conjunction with the Mirror World, the heavier Higgs boson corresponds to the mirror Higgs.

The Higgs potential for the two Higgs fields and the effect Lagrangian is then given by [5]:

$$V_{eff} = \lambda \left(|H|^2 + |H'|^2 - \frac{f_0^2}{2} \right)^2 + \kappa (|H|^4 + |H'|^4) - \sigma f_0^2 |H|^2 + \rho |H|^4 \quad (1)$$

$$\Leftrightarrow V_{eff} = -[\sigma + \lambda] f_0^2 |H|^2 - \lambda f_0^2 |H'|^2 + \frac{\lambda}{4} f_0^4 + [\kappa + \rho + \lambda] |H|^4 + [\lambda + \kappa] |H'|^4 + 2\lambda |H|^2 |H'|^2 \quad (2)$$

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \mathcal{L}_{mirror} - V_{eff} \quad (3)$$

\mathcal{L}_{SM} refers to the Lagrangian of the Standard Model excluding the Higgs sector and \mathcal{L}_{mirror} is the same for all the mirror particles. For the Mirror Model considered here, ρ is assumed to be zero, such that both Higgs sectors have the same coefficient in front of the quartic Higgs term. This is motivated by the fact that the mirror sector is supposed to act similar to the Standard Model, which also includes coupling constants being similar.

This means there are 4 free parameters left f_0, λ, σ and κ . For further studies, it is more convenient to change into a more physical basis, where for example the Standard Model Higgs mass can be set directly to fit measurements. The new parameters are:

$$m_h, \quad m_{h'}, \quad f \equiv \sqrt{\text{vev}^2 + \text{vev}'^2}, \quad \text{vev} \quad (4)$$

vev and vev' refer to the vacuum expectation values of the Higgs fields, $\langle H \rangle$ and $\langle H' \rangle$. m_h and $m_{h'}$ are the masses of the physical Higgs bosons. A transition between the two parameter sets

is given by,

$$f_0 = f \sqrt{1 + \frac{\kappa}{\lambda}(1 - \xi^2)} \quad \text{using} \quad \xi^2 = \text{vev}^2 / f^2 \quad (5)$$

$$\sigma = \frac{-\lambda \kappa (1 - 2\xi^2)}{\lambda + \kappa (1 - \xi^2)} \quad (6)$$

$$\lambda = \frac{\Delta}{4f^2 \xi \sqrt{1 - \xi^2}} \quad \text{using} \quad \Delta = \sqrt{4(m_{h'}^2 + m_h^2)^2 \xi^2 (1 - \xi^2) - 4m_{h'}^2 m_h^2} \quad (7)$$

$$\kappa = \frac{2(m_{h'}^2 + m_h^2)}{4f^2} - \frac{\Delta}{4f^2 \xi \sqrt{1 - \xi^2}} \quad (8)$$

Constraints on the possible range of parameters of this model are the following:

$$0 < \lambda \leq 4\pi, \quad |\sigma| \leq \lambda, \quad |\kappa| \leq \lambda, \quad \text{and} \quad f_0^2 > 0 \quad (9)$$

additionally the Standard Model Higgs mass $m_h = 125$ GeV and vacuum expectation value $\text{vev} = 246$ GeV are given. Exclusion due to experiments can be taken from literature [5] and the specific choice of parameters for the analysis is discussed in subsection 4.1.

One feature of this model is the resulting mixing of the Higgs and mirror Higgs boson. This mechanism is responsible for the coupling between the mirror world and ordinary matter. In particular the physical Standard Model Higgs field h for example is given by [1, 5]:

$$h = \cos(\gamma) h_0 - \sin(\gamma) h'_0 \quad h' = \sin(\gamma) h_0 + \cos(\gamma) h'_0 \quad (10)$$

$$\text{using} \quad H^{(\prime)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \text{vev}^{(\prime)} + h_0^{(\prime)} \end{pmatrix} \quad \text{and} \quad \sin(\gamma)^2 = \frac{\lambda^2 \text{vev}^2}{(\lambda + \kappa)^2 f^2} \quad (11)$$

γ acts as the mixing angle between the two Higgs bosons and is given by the model parameter. That makes the Higgs sector the link between the normal and the Mirror World, because any Higgs vertex of the Standard Model now also exists for the mirror Higgs. The mirror Higgs is therefore the only mirror particle that interacts with non-prime particles directly. For that reason, signatures from this model are best studied in processes, which are sensitive to this mixing. If the mixing is reduced to zero, the Standard Model results are recovered.

In the following a certain choice of parameters is used to simulate data. The Standard Model Higgs mass and the vacuum expectation value are known and therefore fixed. That leaves only the mirror Higgs mass and f available for variation.

In order to produce a Higgs resonance at a consistent energy and at the same time vary the mixing between the two Higgs bosons, the mirror Higgs vacuum expectation value vev' can be used. vev' is not a part of physical parameter set defined above. But with a fixed vev , varying $f = \sqrt{\text{vev}^2 + \text{vev}'^2}$ fulfills the same purpose. A high value of f results in a high value of vev' , since vev is fixed, and therefore creates a smaller mixing. This is illustrated in Figure 1.

From theoretical constraints follows that vev' must be bigger or equal to vev . The influence of the Mirror World gets stronger for smaller vev' and the smallest value results for $f^2 = 2 \text{vev}^2 \Leftrightarrow f/\text{vev} = \sqrt{2}$. This suggests giving the value of f in terms of vev or simply giving f/vev .

The decay width of the mirror Higgs can also be calculated and depends on the Higgs mixing and therefore on the model parameter f [5]. The different decay width depending on the decay channel are shown in Figure 1. Summing all contribution gives the total decay width $\Gamma_{\text{total}}^{h'}$, which is used in all further cross sections calculations.

Figure 1 also shows some constrains on the parameter space. The mass of the mirror Higgs and its vacuum expectation value can not be chosen independently, otherwise the mixing angle might leave the physical reasonable range, e.g. with a mass of $m_{h'} = 800$ GeV, f can not exceed ~ 6.4 vev.

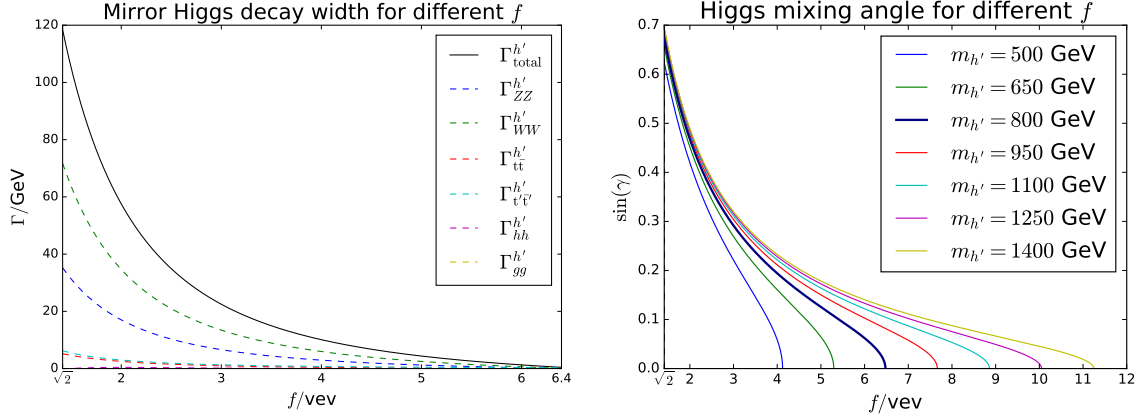


Figure 1: Calculated decay width and Higgs mixing. For the decay into mirror and SM fermions the heaviest fermions are chosen, because those have the biggest influence.

4 Analysis

Next the specific model parametrisation used for the calculation is discussed and the results are presented. Subsection 4.1 starts by giving some general information about the implementation and the parameters, before the results are shown in the later subsections.

4.1 WHIZARD configuration

The Minimal Mirror Twin Higgs model is compared to the Standard Model as its limit. Both models are loaded into WHIZARD via the UFO interface to ensure comparability. Whereas the Minimal Mirror Twin Higgs model is created with FeynRules specifically for this analysis, the Standard Model UFO was made available by the FeynRules model database [6].

In contrast to the build-in Standard Model for WHIZARD, both UFO models do not set the electron or light quark mass to zero. Also they contain all Higgs vertices, even those coupling the Higgs to light particles like electrons.

The model parameters are derived in Equation 4: $m_h, m_{h'}, f, v$. The vacuum expectation value and mass of the SM Higgs are set according to the current experimental results, $m_h = 125$ GeV and $v = 246$ GeV [5]. In order to compare different realisations of the Minimal Mirror Twin

Higgs model a mirror Higgs mass of $m_{h'} = 800 \text{ GeV}$ is chosen and different values of f are compared. This approach is more useful than comparing multiple mirror Higgs masses, because in this way Higgs resonances keep the same energy even while varying parameters.

The possible parameter space is limited by the constraints described in subsection 3.2. As presented by Aqeel Ahmed [5], the LHC has additionally excluded some part of the possible parameters and the HL-LHC is going to exclude even more. This is done by the direct searches $h' \rightarrow ZZ$ of the LHC and by the projected reach for the HL-LHC.

For a heavy Higgs with a mass of $m_{h'} = 800 \text{ GeV}$, $f = 2$ is excluded by the SM Higgs signal-strength measurements at LHC run-1. $f = 3$ and $f = 4$ will be probed by the HL-LHC via $h' \rightarrow ZZ$. Any higher values of f will create even smaller deviations from the Standard Model and therefore will be even harder to probe. Those values of f will be compared in the following, to give an impression on what precision needs to be reached.

In order to avoid divergences and unwanted areas of the phase space, WHIZARD can apply cuts. Cuts are restrictions which are imposed onto the process and keep out disruptive influences. All following analyses deploy this cut to avoid divergences caused by particles moving collinear to the beam:

```
cuts = all abs(Eta) < 5 [b:"b~"]
```

This syntax sets the maximum of the absolute value of the pseudorapidity to 5 and discards all phase space points with larger values.

The uncertainties are taken from WHIZARD, but the error bars are very small.

4.2 $\mu^+, \mu^- \rightarrow b, \bar{b}$

Firstly, the resonances of the Higgs and the mirror Higgs will be looked at.

The process $\mu^+, \mu^- \rightarrow b, \bar{b}$ is used, because the electron-Higgs coupling for an e^+, e^- beam is too weak to produce a noticeable signal. Because the Higgs coupling depends on the mass of the particle, the Higgs-muon coupling is much stronger. Detecting the cross section of the mirror Higgs poses a challenge even with muons.

First taking a look at the results for the Standard Model in Figure 2, represented by the black line. The peak at the energy of 125 GeV can be seen clearly and is relatively narrow, because the decay width of Higgs boson in the Standard Model is extraordinary small ($\sim \text{keV}$).

Secondly comparing this to results for the Minimal Mirror Twin Higgs model in Figure 2 and 3, one notices differences and similarities. The peak at 125 GeV is similar to the Standard Model peak, since both models contain the Standard Model Higgs boson. But the height of the peak differs due to the mixing of the Higgs bosons. The amount of mixing is controlled via the vacuum expectation value of the mirror Higgs boson and therefore f . Increasing the value of f recovers the Standard Model solution as the mixing vanishes.

Furthermore the additional peak around 800 GeV is due to the heavy mirror Higgs boson. This peak is much wider than the SM Higgs resonance and also the width shrinks for higher values of f . This confirms the expectations from Figure 1.

In contrast to before the height of the peak increases for larger f and smaller mixing. In Equation 1 the different dependence on the mixing angle γ can be seen directly. The exact

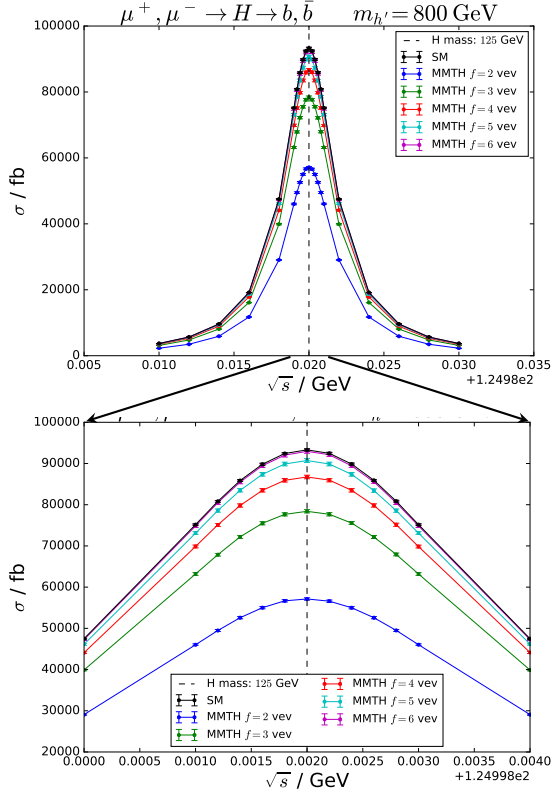


Figure 2: Resonance at the Higgs mass shown with two different levels of detail.

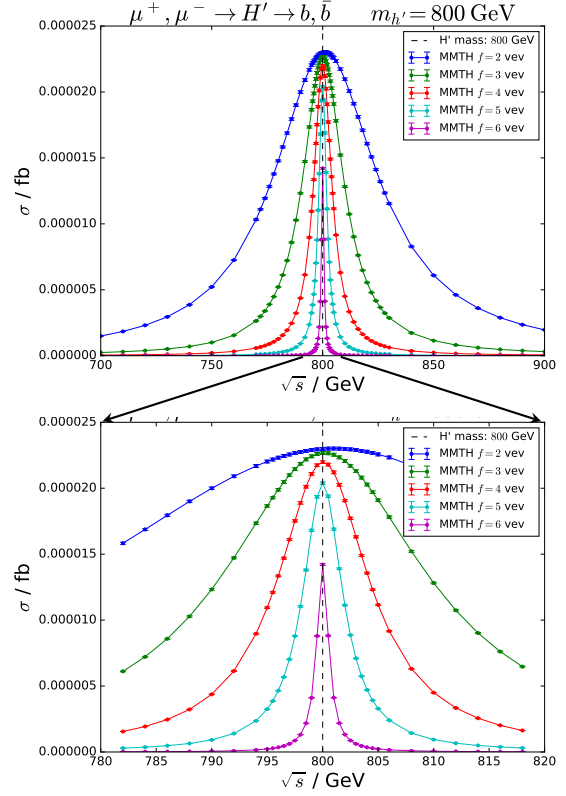


Figure 3: Resonance at the mirror Higgs mass shown with two different levels of detail. The slight tilt in the data points, which are centered symmetrically around 800 GeV, is due to the cut. The SM cross section is exactly zero, because there is no H' propagator.

location of this peak depends on $m_{h'}$ and is therefore model dependent, but in this case is fixed to $m_{h'} = 800$ GeV.

In Figure 3 and 2 the propagator was restricted to the Higgs bosons in order to make the peak visible. Although the resonance of the SM Higgs produces a reasonable cross section and could be seen even without the restriction, the mirror Higgs is significantly weaker and vanishes with enough background. This is expected as the width of the mirror Higgs is also orders of magnitude larger than the width of the SM Higgs as predicted by theory. Figure 4 shows the unrestricted process, where the peak can no longer be seen.

4.3 Standard Model vs. Mirror World

Next the connection between the normal and the mirror sector is further explored by comparing different process channels. In order to do that the process $\mu^+, \mu^- \rightarrow b, \bar{b}$ is chosen and all possible versions from the Mirror World are compared. Either the beam particles can be mirror

particles or the final particles can. Because the Higgs couples to both sectors even a transition is possible. Only a mixing within the final (or initial) state is not possible (e.g. $\mu^+, \mu^- \rightarrow b', \bar{b}$), as there is no vertex of the sort: $H^{(\prime)} \rightarrow b, \bar{b}'$. In this section, additionally to $m_{h'} = 800$ GeV also $f = 4$ vev is chosen, because that represents a parameter set, which is not yet excluded by measurements, but has the possibility to be reached by future technology.

One feature of this comparison seen in Figure 4 is, that the cross section for the SM to mirror (*turquoise*) and vice versa process (*green*) is the same. This is expected, as the process should be symmetric. In contrast the cross section of the SM only and mirror only process do not equal. This is due to the difference between the SM sector and the mirror sector, which couple differently to the intermediate particle and the mirror sector is preferred because the exchanged particle is the mirror Higgs.

For this analysis, the propagator was limited to the mirror Higgs, because the underground is too strong to get a clear signal otherwise. The plots without this limitation do not show the mirror Higgs resonance significantly within its uncertainties.

The SM-mirror-mixing channels do not show any background, because only the Higgs bosons can act as mediator particles. Both SM-mirror-mixing channels give again the same result, exactly as before.

Reproducing those results in an experiment is difficult, as it requires a mirror muon beam on the one hand and detecting mirror quarks on the other hand. The SM-only channel is the only one avoiding these problems and therefore has been explored in more detail in subsection 4.2.

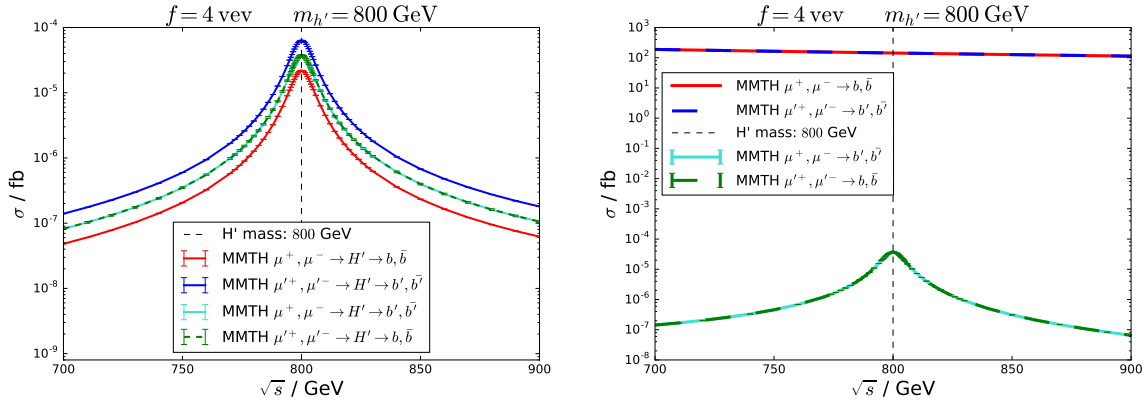


Figure 4: The cross section plotted for different combination of SM and mirror particles: *left* only mirror Higgs as the mediator particle and *right* unrestricted exchange particles. Both plots are drawn in log scale.

4.4 WW Fusion

An important Higgs production process is the WW fusion: $e^+, e^- \rightarrow \nu_e, \bar{\nu}_e, H$. The channel with two W bosons is only one among many possible channels, but it has the strongest influence in the energy range used in Figure 5. Another possible channel would be for example via Higgs radiation. This process is also influenced by the mixing as seen in Figure 5.

Due to Higgs mixing the cross section is overall smaller than in the Standard Model. The same phenomena could be seen at the Higgs resonance in Figure 2. Producing a Higgs is suppressed by large mixing and therefore the cross section goes down. As before the gap between the Minimal Mirror Twin Higgs model and the Standard Model can be closed by larger values of f .

This energy cannot be probed right now, but it would be possible for a future collider like the ILC [7]. Neutrinos are of course not visible in the detector, but they can be measured by missing momentum. In order to see anything in the detector, the decay products of the $H^{(\prime)}$ must be visible (e.g. not mirror particles).

Figure 5 compares cross sections of the Standard Model to different parametrisation of the Minimal Mirror Twin Higgs model. For small energies the influence of the Higgs channel is rather small and therefore the different parameters have very little effect. Going to larger energies the Higgs channel becomes more relevant and therefore different values for f clearly reach different cross sections. All data sets rise logarithmically for large energies as expected for vector boson fusion processes [8].

The general trend with a mirror Higgs in the final state is quite different. First of all there is no Standard Model data since it does not contain the mirror Higgs. Secondly the logarithmic rise is much more prominent, because the background is much smaller.

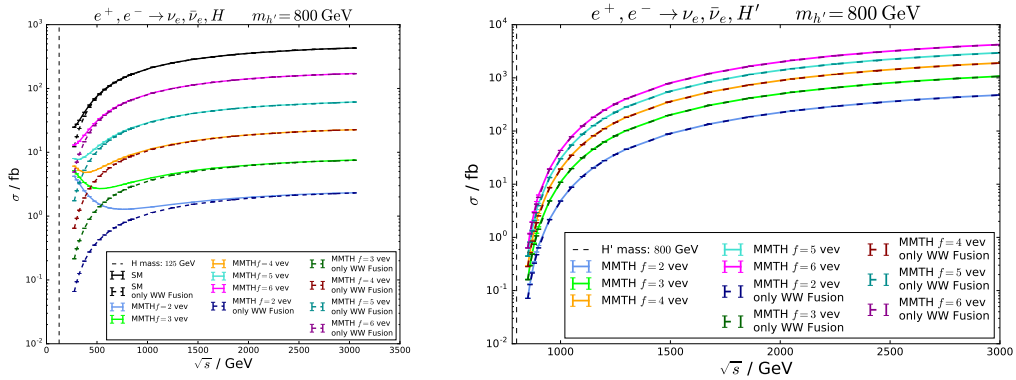


Figure 5: Cross section for WW fusion into Higgs and mirror Higgs in log-scale.

Looking explicitly at a Z boson in the final state in Figure 6, a different distribution develops. There is a dominant peak which falls off towards higher energies. For lower energies this final state cannot be achieved, because it falls below the combined mass of Z and the $H^{(\prime)}$. With the mirror Higgs in the final state, this peak is completely visible and much broader than the peak in the SM Higgs data. This is explained by the different decay widths of the two Higgs bosons and is also visible in Figure 7.

The electron to Higgs coupling is so weak, because of the low mass of electrons, that it can be neglected. Much more important for this process is the contribution from Higgs radiating off the Z boson, which is influenced by the ZZH vertex. As seen in Figure 5, this vertex increases logarithmically but in this process it is limited by the decay width of the resonance peak. This causes the different appearance for the SM and mirror Higgs in Figure 6.

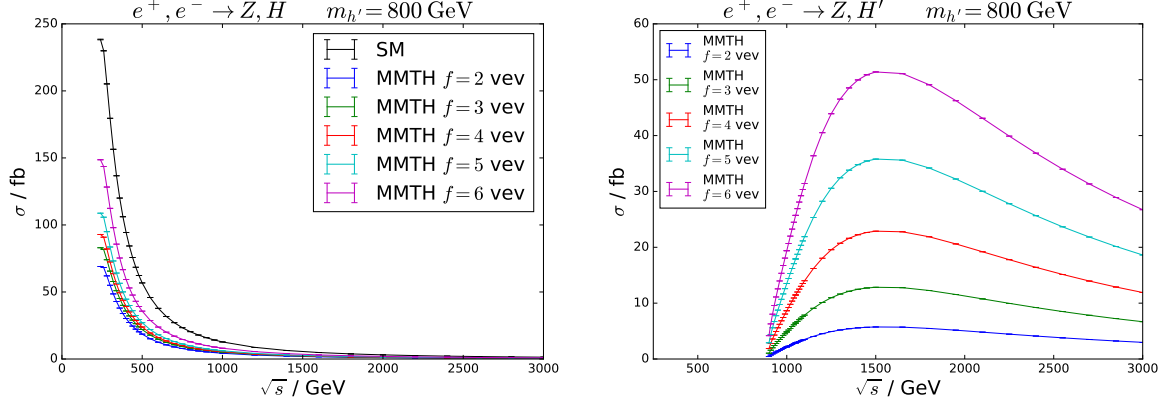


Figure 6: Cross section for $e^+, e^- \rightarrow Z, H^{(\prime)}$

4.5 $e^+, e^- \rightarrow b, \bar{b}, Z$

Lastly the same resonances as seen in subsection 4.2 can be found with an e^+, e^- beam and an additional Z . The process is $e^+, e^- \rightarrow b, \bar{b}, Z$ and in order to eliminate some background, there is an additional cut applied:

$$\begin{aligned} \text{cuts} = & \quad \text{all } \text{abs}(\text{Eta}) < 5 \text{ [b:"b~"}] \\ & \text{and all } M > 790 \text{ GeV [b,"b~"}] \end{aligned}$$

and with all $M > 120 \text{ GeV}$ for the Standard Model Higgs resonance respectively. This combines some previously studied processes and phenomena.

In the desired channel the bottom and anti-bottom quarks originate from a mirror Higgs boson, therefore their invariant mass must equal 800 GeV. As a reference, the same channel is also probed at the energy of the SM Higgs mass.

As seen in Figure 7, both plots show a similar course. The control plot of the Standard Model around the $H+Z$ resonance shows a distinct peak slowly falling off due to the momentum distribution. Because there are now three particles in the final state, the momentum can actually be distributed and the peak becomes wider in comparison to Figure 3 and 2. The data points on the left side of the plot, which are exactly zero, result from the cut prohibiting any decays in this energy range.

The data of the Minimal Mirror Twin Higgs model has the same overall trend as the Standard Model around the SM Higgs mass. Additionally there is a peak at $H'+Z$ mass energy. In the same way as before in Figure 3 and 2, the mixing decreases the cross section around 125 GeV and increases it around 800 GeV. Also similar is the ordering of the lines. The data set with the smallest value of f has the strongest Standard Model discrepancies and higher values get closer to it.

The influence of the mirror Higgs on the background cross section around 800 GeV is so small, that the specific mirror Higgs channel was singled out in Figure 7. For better comparison, the same has been done for the SM Higgs channel.

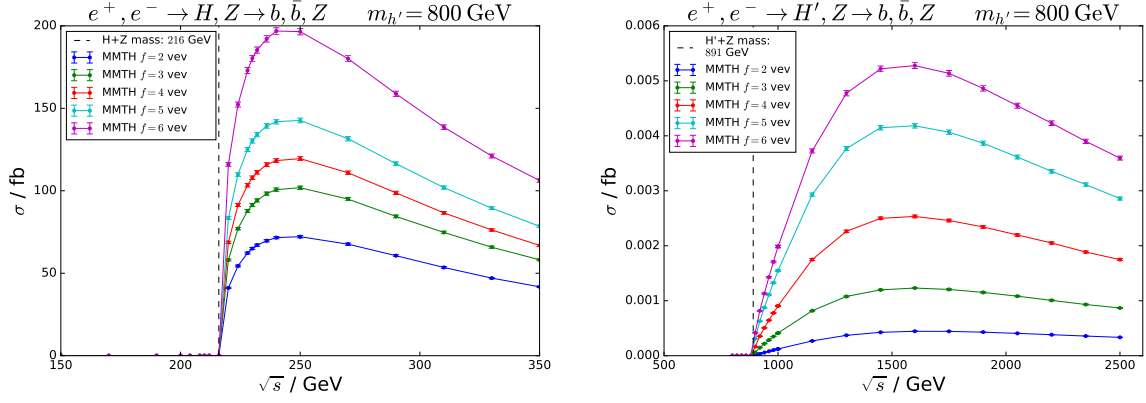


Figure 7: The cross section for $e^+, e^- \rightarrow b, \bar{b}, Z$ restricted to only use H or H' as exchange particle ($\rightarrow H', Z$).

5 Conclusion

The Minimal Mirror Twin Higgs model introduces signatures for future lepton-antilepton-colliders which of course depend on the specific choice of parameter. In this case a Mirror World with nearly identical properties as the SM is assumed and a range of different mirror Higgs parametrisations are compared.

In general, the results of the Standard Model can be reproduced by choosing low mixing between the SM Higgs and the mirror Higgs, which manifests in a high vacuum expectation value for the mirror Higgs. Theoretical constraints restrict the parameter space, such that the following values have been chosen to represent different regions of the available parameters: $f/\text{vev} = 2, 3, 4, 5, 6$. Whereat $f/\text{vev} = 2$ has already been excluded by measurements at the LHC and $f/\text{vev} = 3, 4$ are inside the projected reach of the HL-LHC.

The mixing of the Higgs bosons results in a change of the WW Fusion and $b\bar{b}$ production cross section in e^-e^+ experiments.

Looking at the process $\mu^+\mu^-$ to $b\bar{b}$ could enable direct detection of mirror matter via a Higgs process. Regarding this process, it is also very interesting to consider processes that have mirror particles in their final state and can therefore not be detected directly.

Testing the capabilities of the UFO interface did not yield any open issues and gave unambiguous results compared to the build-in SM model. Constructing the Minimal Mirror Twin Higgs model in Mathematica [9] with FeynRules [10] resulted in a working UFO file. This file is then tested in different scenarios and produced consistent results in all test cases (mainly e^-e^+ processes).

Acknowledgements

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