Inelastic Dark Matter at Neutrino Experiments

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Abstract

In this report we investigate the phenomenology implied by a model for dark matter including a Majorana fermions with different masses and a massive gauge boson. First we compute relic abundances for given sets of mass parameters and coupling constants. Then we proceed to calculate the predicted sensitivities of direct detection using neutrino experiments. Finally, we relate our results to known bounds coming from collider experiments to identify new viable regions for future searches.

Contents

1 Introduction

Understanding the nature of dark matter (DM) is one of the biggest challenges of modern physics. Constraints on its non-gravitational interactions with the standard model (SM) have been obtained at collider experiments for masses of a few GeV. More recently, [1] proposed a new method to probe sub-GeV DM at neutrino experiments such as Super-Kamiokande (Super-K). This regime is classically difficult to probe due to the low energy at the average DM velocity $v \approx 10^{-3}$. However there is an unavoidable fraction of DM which is scattered by cosmic ray electrons, making it relativistic and hence able to be probed by Super-K.

This project builds up on this proposal by considering an extra particle in the dark sector with a small mass splitting. This implies that one can decay into the other and has non-trivial consequences. As described in more detail in Section 5 the addition of the second particle can lead to two extra Cherenkov radiation signals, which has no known background, making it promising for potential searches. Additionally, the cosmology of this model naturally yields correct relic abundances in relevant regions of the parameter space without any further assumptions.

The report is organised as follows. Section 2 introduces the basic cosmology tools used to calculate DM relic abundances. Section 3 describes the computation of cross-sections for the relevant interactions in this project. Sections 4 and 5 present our numerical results for viable parameters of the model against experimental constraints, and a conservative computation of the Super-K sensitivity to these regions. Finally we conclude and discuss further work in Section 6. We use natural units ($\hbar = c = k_B = 1$) throughout this report.

2 Cosmology

As the Universe expands and its temperature decreases, the number of massive particles of a given species in thermal equilibrium decreases according to the Boltzmann distribution. However when the rate of interactions Γ becomes comparable to the rate of expansion H the particles stop interacting and effectively *freeze-out* i.e. their total number stays constant from then on.

The time evolution of dark matter in the early Universe can be described by the Boltzmann equation [2]:

$$\frac{1}{a^3}\frac{d(n_X a^3)}{dt} = C[\{n_X\}] \tag{1}$$

where the term on the r.h.s is the collision term, which accounts for particle interactions, a is the scale factor and n_X is the dark matter number density. The form of the collision term depends on the specific interactions, but in the simple case of

$$X + \bar{X} \leftrightarrow l + \bar{l}$$

Eq.1 can be rewritten as [3]:

$$\frac{dN_X}{dx} = -\frac{\lambda}{x^2}[N_X^2 - (N_X^{eq})^2] \tag{2}$$

where we have defined the number of particles in a comoving volume $N_X \equiv n_X/s$, $x \equiv M_X/T$ and

$$\lambda \equiv \frac{2\pi^2}{45} g_{*S} \frac{M_X^3 \langle \sigma v \rangle}{H(M_X)} \tag{3}$$

where s is the entropy density, T is the temperature, M_X is the dark matter mass, g_{*S} is the number of entropic degrees of freedom and $H(M_X)$ is the Hubble constant evaluated at M_X.
We take N_X^{eq} to follow the Fermi-Dirac distribution in the non-relativistic limit (since freeze out occurs in the non-relativistic regime $x \sim \mathcal{O}(10)$) such that

$$N_X^{eq} = \frac{g}{s} \left(\frac{M_x^2}{2\pi x} \right)^{\frac{3}{2}} e^{-x} \tag{4}$$

where g is the number of internal degrees of freedom and the entropy is

$$s = \frac{2\pi^2}{45} g_{*s} \left(\frac{M_X}{x} \right)^3 . \tag{5}$$

In general Eq.2 has to be solved numerically and its shape is shown in Figure 1.

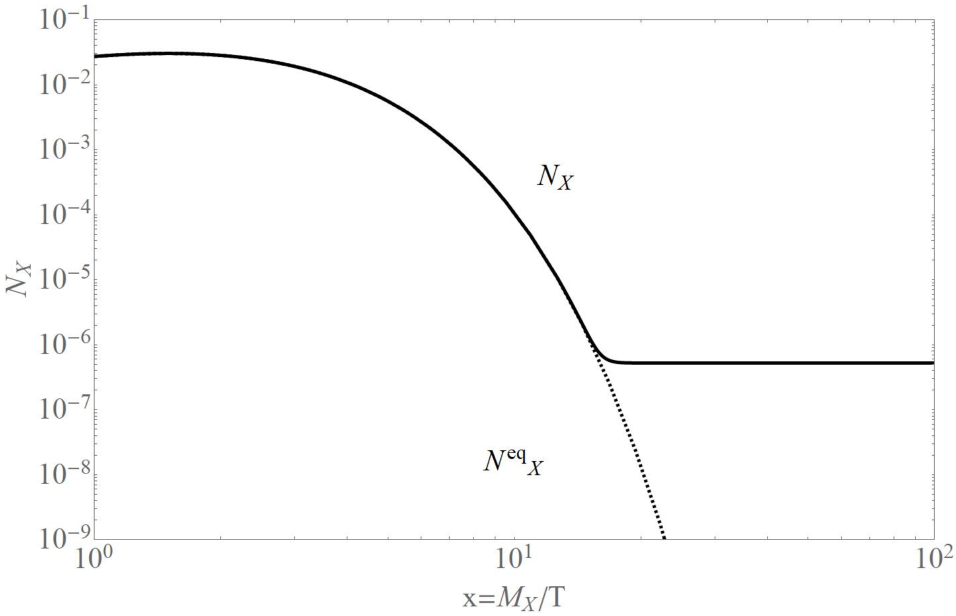


Figure 1: Full line: Numerical solution to the Boltzmann equation. Dashed line: Boltzmann distribution in the non-relativistic regime.

Once we know the relic number of dark matter particles N_X^{∞}, it is straightforward to calculate the relic abundance $\Omega_X h^2$:

$$\Omega_X h^2 = \frac{M_X N_X^{\infty}}{\rho_{crit,0}} \frac{2\pi^2}{45} g_{*s} T_0^3 \tag{6}$$

where $\rho_{crit,0}$ is the current day critical density and T_0 is temperature of the cosmic microwave background.

3 Co-annihilation cross section

In this project we considered a model for Majorana fermionic dark matter χ_1 and χ_2, of mass m_1 and m_2 respectively, whose interactions are mediated by a vector field V of mass m_v which will be referred to as *dark photon* from now onwards. We consider a Lagrangian density including

$$\mathcal{L}_{kin} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F_D^{\mu\nu}F_{D\mu\nu} - \frac{\epsilon}{2}F^{\mu\nu}F_{D\mu\nu} - \frac{m_v^2}{2}V^\mu V_\mu$$

$$\mathcal{L}_{Dint} = -i\bar{\chi_1}\gamma^5\gamma^\mu V_\mu g_D \chi_2 - i\bar{\chi_2}\gamma^5\gamma^\mu V_\mu g_D \chi_1$$

$$\mathcal{L}_{EMint} = -iqJ^\mu A_\mu$$

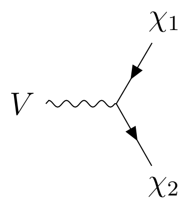
where $F^{\mu\nu}$ is the usual Field strength, $F_D^{\mu\nu} = \partial_\mu V^\nu - \partial_\nu V^\mu$ is the "dark" field strength, J^μ is the standard model current, g_D is the dark coupling constant and ϵ is the kinetic mixing parameter.
It is possible to diagonalize the kinetic terms by redefining the electromagnetic vector as $\hat{A}^\mu \to A^\mu + \epsilon V^\mu$. This means that the kinetic term can now simply be written as

$$\mathcal{L}_{kin} \supset -\frac{1}{4}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} - \frac{1}{4}F_D^{\mu\nu}F_{D\mu\nu} - \frac{m_v^2}{2}V^\mu V_\mu$$

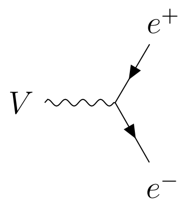
and therefore the EM interaction term now reads

$$\mathcal{L}_{EMint} = -iqJ^\mu A_\mu - iq\epsilon J^\mu V_\mu$$

This effectively introduces a new vertex that allows for the dark photon to interact with the standard model. Therefore we add the following vertices to our standard model quantum field theory:



$$= ig_D\gamma^\mu \tag{7}$$

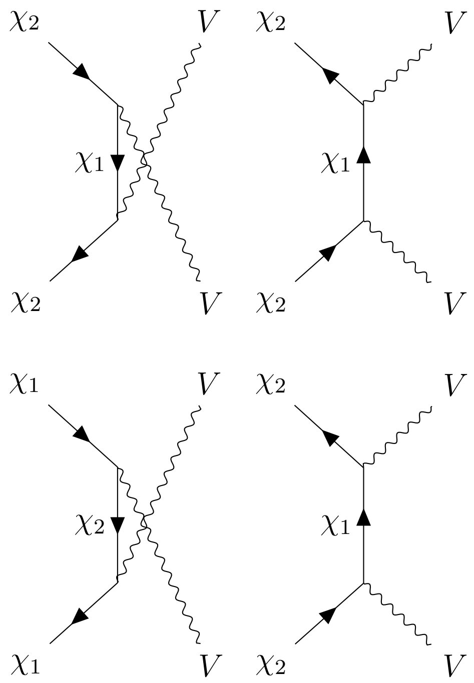


$$= i\epsilon e\gamma^\mu \tag{8}$$

These then allow us to draw 3 different tree level feynman diagrams which describe dark matter coannihilation into dark photons or electron positron pairs.
Now, the differential cross section for a scattering process involving two incoming and two outgoing particles is given by [4]

$$\frac{d\sigma}{d\Omega} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{|\mathbf{p_1}|}{(2\pi)^2 4E_{com}} |\mathcal{M}|^2 \tag{9}$$

where $\mathbf{p_1}$ is the momentum of either of the outgoing particles, E_A and E_B are the energies of the incoming particles and $\mathcal{M}$ is the matrix element associated with the process.
To calculate the matrix element $\mathcal{M}$ to first order we consider the tree level diagrams associated with these processes



taking care of averaging over all possible spins and polarizations of the particles involved. For the $\chi_1\chi_1 \to VV$ and $\chi_2\chi_2 \to VV$ processes, assuming $m_1, m_2 > m_v$, in the non-relativistic limit corresponds to

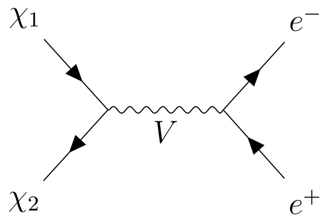
$$\langle \sigma v_{(\chi_1\chi_1 \to VV)} \rangle = \frac{8\pi\alpha_D^2}{9} \frac{[1 - m_v^2/m_1^2]^{\frac{1}{2}}}{m_1^2} \frac{(m_1^2 - m_v^2)(3m_2^2 - 2m_1m_2 + 3m_1^2 + 4mv^2)}{(m_2^2 - m_1^2 + m_v^2)^2} \tag{10}$$

$$\langle \sigma v_{(\chi_2\chi_2 \to VV)} \rangle = \frac{8\pi\alpha_D^2}{9} \frac{[1 - m_v^2/m_2^2]^{\frac{1}{2}}}{m_2^2} \frac{(m_2^2 - m_v^2)(3m_1^2 - 2m_2m_1 + 3m_2^2 + 4mv^2)}{(m_1^2 - m_2^2 + m_v^2)^2} \tag{11}$$

where $\alpha_D \equiv \frac{g_D^2}{4\pi}$. If, instead, the mass of the dark photon is larger than the chion masses, the decay is usually forbidden. However, due to the exponential tail in the energy distribution of these particles, a certain fraction of them will have enough energy to access this coannihilation process, and the relation of this "forbidden channel" cross section to the one just derived is given by

$$\langle \sigma v_{for} \rangle = \frac{n_v^{eq}}{n_\chi^{eq}} \langle \sigma v_{norm} \rangle \tag{12}$$

The results were checked by taking the $m_1 = m_2$ limit and comparing with the literature [5]. For the coannihilation into positron electron pair ($\chi_1\chi_2 \rightarrow e^+e^-$) the relevant Feynman diagram is



and applying the non-relativistic limit it leads to the following cross section.

$$\langle \sigma v_{(\chi_1\chi_2 \rightarrow e^+e^-)} \rangle = 4\pi\alpha\alpha_D\epsilon^2 \frac{\sqrt{(m_2+m_1)^2 - 4m_e^2}((m_1+m_2)^2 + 2me^2)}{(m_1+m_2)((m_1+m_2))^2 - m_v^2)^2} \tag{13}$$

which also reduces to results in the literature [5] for the $m_1 = m_2$ limit.

4 Calculation of relic abundances

Using the cross-sections calculated in Section 3 along with Eqs.2,6, we can compute relic abundances for regions of the 5-D parameter space. Since there is more than one process taking place, we need to be careful when defining $\langle \sigma v \rangle$. Following the approach in [6], we define an effective cross section $\langle \sigma v \rangle_{eff}$:

$$\langle \sigma v \rangle_{eff} = \sum_{i,j}^{N} \langle \sigma v \rangle_{ij} \frac{g_i g_j}{g_{eff}^2} (1+\Delta_i)^{\frac{3}{2}} (1+\Delta_j)^{\frac{3}{2}} e^{-x(\Delta_i + \Delta_j)} \tag{14}$$

where we have introduce the following:

$$\Delta_i = \frac{m_i - m_1}{m_1} \tag{15}$$

and

$$g_{eff} = \sum_{i}^{N} g_i (1+\Delta_i)^{\frac{3}{2}} e^{-x\Delta_i} \tag{16}$$

The indices $\langle \sigma v \rangle_{ij}$ correspond to the incoming particles χ_i, χ_j and we assume $\langle \sigma v \rangle_{ij} = \langle \sigma v \rangle_{ji}$.
Running the numerical simulation with $\langle \sigma v \rangle_{eff}$ for fixed α_D, $\frac{m_2}{m_1}$, $\frac{m_v}{m_1}$ and varying ϵ, m_v we obtain lines of correct relic abundance in the $m_v - \epsilon$ plane. These are shown in Figure 2 alongside constraints [7] in the shaded regions.

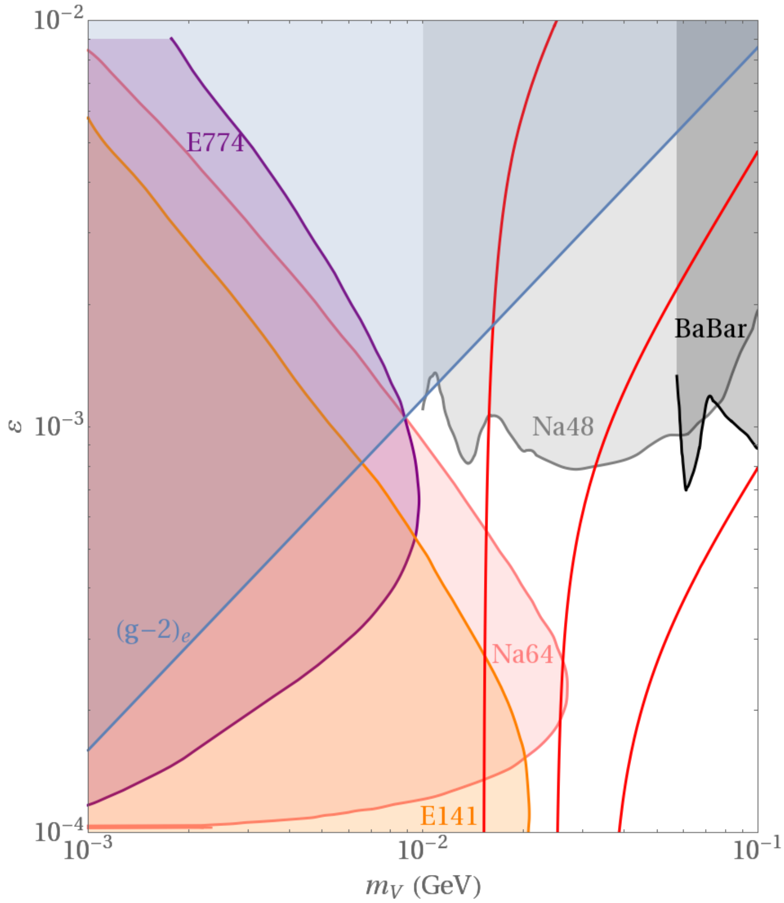
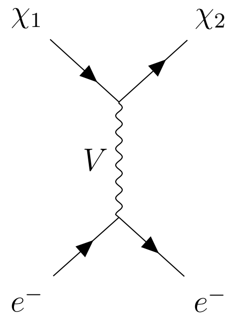
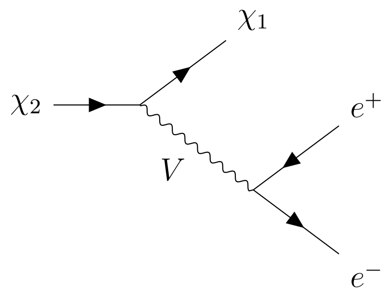


Figure 2: Red lines: Correct relic abundances for various parameters; from left to right: $\alpha_D = 1$, $\frac{m_2}{m_1} = 1.7$, $\frac{m_v}{m_1} = 1.72$; $\alpha_D = 1$, $\frac{m_2}{m_1} = 1.45$, $\frac{m_v}{m_1} = 1.68$; $\alpha_D = 1$, $\frac{m_2}{m_1} = 1.3$, $\frac{m_v}{m_1} = 1.65$. Shaded lines: constraints from collider experiments and the anomalous magnetic moment $(g-2)_e$.

5 Detection

For detection purposes we consider the following process: a highly relativistic χ_1 particle enters the detector tank and scatters off an electron, turning into a χ_2. Whereas this might seem like an extra requirement, it is actually a direct consequence of the model described in section 3. If, dark matter were to be able to be directly detected, a fraction of the particles constituting the galactic halo would also have to interact with cosmic rays and be accelerated to relativistic speeds, allowing for a transfer of momentum in the interaction with an electron high enough to give off Cherenkov radiation [1]. The χ_2 particle then decays into χ_1 and a dark photon V, which in turn decays into an electron positron pair.





Notice how both of the Feynman diagrams involved in this process are the same as the one used for the $\chi_1\chi_2 \to e^+e^-$ coannihilation process previously discussed. In this case, the inherently relativistic nature of the process makes the calculation of the matrix element rather non trivial. However, we can factorize it as follows in order to simplify the calculations [8] [9]. The matrix element associated with the process is

$$\mathcal{M}_{tot} = \epsilon g_D e \bar{u}_{(p_{\chi 1})}\gamma^5\gamma^\mu u_{(p_{\chi 2})}\frac{\eta_{\mu\nu} - q_\mu q_\nu}{(q^2 - m_v^2)}\bar{u}_{(p_-)}\gamma^\nu v_{(p_+)} \tag{17}$$

but we recognize that the propagator can be written as a sum over polarization vectors [10] so that

$$\frac{\eta_{\mu\nu} - q_\mu q_\nu}{(q^2 - m_v^2)} = \frac{1}{3}\sum_a \epsilon^{*a}_{\mu(q)}\epsilon^a_{\nu(q)} \tag{18}$$

which in turn allows us to write the matrix element as

$$\mathcal{M}_{tot} = \frac{1}{(q^2 - m_v^2)}\frac{1}{3}\sum_a \mathcal{M}_{(\chi_2\to\chi_1 V^a)}\mathcal{M}_{(V^a\to e^+e^-)} \tag{19}$$

We still have to factorize the rest of the differential decay rate. The general formula [4] is

$$d\Gamma = \frac{1}{2E_{\chi_2}}\frac{d^3p_{\chi_1}}{(2\pi)^3}\frac{1}{2E_{\chi_1}}\frac{d^3p_+}{(2\pi)^3}\frac{1}{2E+}\frac{d^3p_-}{(2\pi)^3}\frac{1}{2E_-}|\mathcal{M}_{tot}|^2(2\pi)^4\delta^{(4)}(p_{\chi_2} - p_{\chi_1} - p_- - p_-) \tag{20}$$

In order to factorize it in a useful way, we multiply it by unity written as follows [9]

$$1 = \int \frac{d^3q}{(2\pi)^3 2E_v} ds_v (2\pi)^4\delta^{(4)}(q - p_{\chi_2} - p_{\chi_1}) \tag{21}$$

where we have introduced the invariant mass of the dark photon $s_v = q^2$, with q being the 4-momentum carried by the virtual dark photon. Since the δ function in this equation requires $q = p_{\chi_2} + p_{\chi_1}$ we write the decay rate in the lab frame by integrating over the momenta of all the outgoing particles to give

$$
\begin{aligned}
\Gamma = \frac{1}{6E_{\chi_2}} \int \frac{ds_v}{2\pi(s_v - m_v^2)^2} \\
&\times \int \frac{d^3 p_{\chi_1}}{(2\pi)^3 2E_{\chi_1}} \int \frac{d^3 q}{(2\pi)^3 2E_v} (2\pi)^4 \delta^{(4)}(p_{\chi_2} - p_{\chi_1} - q) \sum_a |\mathcal{M}_{(\chi_2 \to \chi_1 V^a)}|^2 \\
&\times \int \frac{d^3 p_+}{(2\pi)^3 2E_+} \int \frac{d^3 p_-}{(2\pi)^3 2E_-} (2\pi)^4 \delta^{(4)}(q - p_- - p_+) \sum_a |\mathcal{M}_{(V^a \to e^+ e^-)}|^2
\end{aligned}
$$

Both the second and the third line of this equation are Lorentz invariant. We therefore choose to compute the second one in the χ_2 rest frame ($p_{\chi_2} = (m_2, \mathbf{0})$)and the third one in the V rest frame ($q = (\sqrt{s_v}, \mathbf{0})$). Collapsing the integrals on the δ functions, taking care of introducing the relevant factors in order to change their arguments, [11] ensures conservation of Energy and momentum at each vertex of the interaction, leading to the following

$$
\begin{aligned}
\Gamma = \frac{g_D^2 \epsilon^2 e^2}{192\pi^3} \int \frac{dcos\theta}{2} \int_{4m_e^2}^{(m_2 - m_1)^2} ds_v \frac{((m_1 + m_2)^2 - s_v)((m_2 - m_1)^2 + 2s_v)}{E_{\chi_2}(s_v - m_v^2)^2} \left(1 + \frac{2me^2}{s_v}\right) \\
\times \left[\left(1 - \frac{2(m_1^2 + s_v)}{m_2^2} + \frac{(m_1^2 - s_v)^2}{m_2^4}\right) \left(1 - \frac{4m_e^2}{s_v}\right) \right]^{1/2}
\end{aligned}
$$

To evaluate the integral, we operate a variable substitution in order to render the angular dependence of s_v explicit

$$
s_v = q^2 = (p_{\chi_2} - p_{\chi_1})^2 = m_2^2 + m_1^2 - 2(E_{\chi_2} E_{\chi_1} - |\mathbf{p}|_{\chi_2} ||\mathbf{p}_{\chi_1}| cos(\theta)) \tag{22}
$$

After integrating over the angle θ, we effectively get the differential decay rate $d\Gamma/dK_{\chi_1}$ where $K_{\chi_1} = E_{\chi_1} - m_1$ is the kinetic energy of the outgoing χ_1 particle which, due to conservation of energy, has a one to one relationship with the energy of the electron positron pair. The probability of detection of an interaction is therefore given by

$$
P = \int_0^{K_{max}} \frac{1}{\Gamma} \frac{d\Gamma}{dK_{\chi_1}} \tag{23}
$$

where $K_{max} = E_{\chi_2} - m_1 - E_{threshold}$, with $E_{threshold} \approx 10MeV$ being the minimum energy necessary for the electron pair to give off Cherenkov radiaton and be detected at the Super-K experiment. This accounts for the fact that it is less likely for low energy interactions to be detected and more likely for more energetic ones.

The expected number of events that should have been detected is then given by

$$
N = \int_t^{t+T} dt \int dK_1 \int dK_e \quad \Phi_{(K_1)} \frac{d\sigma}{dK_1 dK_e} P_{(K_1, K_e)} \tag{24}
$$

where T is the run time of the experiment. Since $\sigma \propto \epsilon^2$ and $\Phi \propto \epsilon^2$ we have $N \propto \epsilon^4$. This effectively enables us to draw a sensitivity curve in $m_v - \epsilon$ parameter space for a given choice of mass ratios. As it can be seen in Figure 3, this overlaps with the correct relic abundance lines that we have computed for some mass ratios.

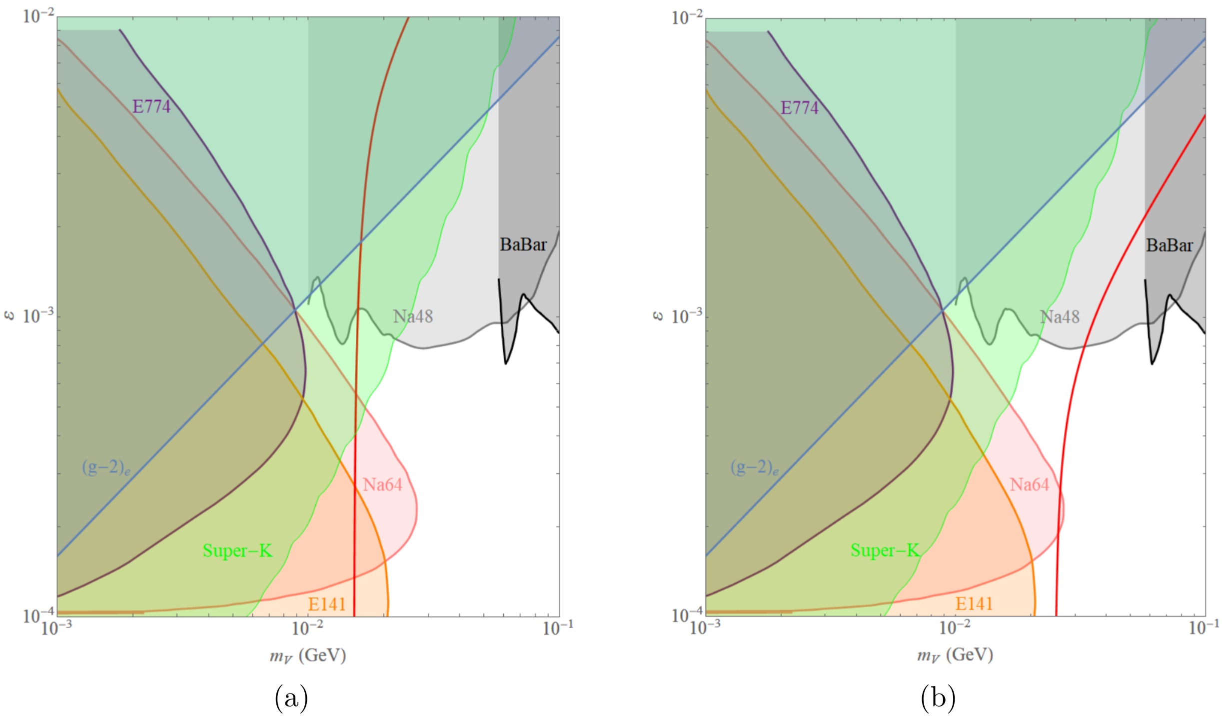


Figure 3: Red lines: Correct relic abundance for (a) $\alpha_D = 1$, $\frac{m_2}{m_1} = 1.7$, $\frac{m_v}{m_1} = 1.72$, (b) $\alpha_D = 1$, $\frac{m_2}{m_1} = 1.45$, $\frac{m_v}{m_1} = 1.68$. Shaded lines: constraints from collider experiments and the anomalous magnetic moment $(g-2)_e$. Green Shaded line: Super-Kamiokande sensitivity.

6 Conclusion

In conclusion we have shown how a model for dark matter with an effective Lagrangian such as the one described in section 3 succeeds in reproducing the observed relic abundances for regions of parameter space overlapping with the sensitivity of direct detection searches using neutrino experiments. This opens up the possibility to probe dark matter masses at a scale previously inaccessible to direct detection in a virtually background free environment. Similar calculations could also be performed considering a larger range of mass ratios opening up the possibility to test the aforementioned models, and one could also think to introduce a wider range of new particles in the dark sector and use the same techniques to infer detector sensibilities.

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