Differential distribution of Z/γ^* and $W^{\pm}$ decay products

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Abstract: In this project, the Drell-Yan production and subsequent decay of vector bosons into leptons was investigated. Specifically, the decays were treated in the lab-frame of the colliding protons. The matrix elements for the processes were decomposed in terms of the hadronic and leptonic tensor for reusability. The lepton transverse momentum spectrum obtained this way was calculated at leading order and compared to data simulated by a Monte Carlo code.

Contents

1 Introduction

1.1 Motivation

The Standard Model (SM) is a gauge theory based on the SU(3)×SU(2)×U(1) symmetry group describing a wide range of phenomena in nature, as we know it, to very high precision. One of the interactions the SM predicts is the electroweak (EW) interaction mediated by particles known as the Z boson, $W^\pm$ bosons and the photon (γ). The first three are massive and the masses have been experimentally determined to considerable accuracy [1, 2]. This is important, for example, in EW precision tests that allow to test the SM for hints of Beyond-the-SM (BSM) physics [3]. The most current values for Z and W masses are given by [4]:

$$m_Z = 91.1876 \pm 0.0021\,\mathrm{GeV} \tag{1.1}$$

$$m_W = 80.3850 \pm 0.0150\,\mathrm{GeV}. \tag{1.2}$$

Evidently, the uncertainty on Z mass is an order of magnitude lower than that of W. Hence, currently, the latter is limiting the constraints for BSM fits. One of the reasons why m_W is known less precisely than m_Z lies in the way they decay. The leptonic decay $Z \to l\bar{l}$ produces a pair of a lepton and an anti-lepton, however, the leptonic decay $W^{\pm} \to l\nu_l$ produces a pair of a lepton and an anti-neutrino or vice versa. The latter decay channel was used in the W mass measurement [2]. Since the neutrino escapes the detector, it requires evaluation of missing momenta of the decay to reconstruct the lepton center-of-mass (COM) frame. This is a challenging task itself as it requires precise tracking of leftover QCD radiation (jets) that were produced in hadronic collisions which created W and Z in the first place. However, this same decay could potentially be treated in the laboratory (lab) frame of the colliding hadrons that produce the boson in the first place. In this way, the measured observables are also the variables of the lab frame.

Additionally, it is important to mention that when higher order corrections to this decay are considered, one must include processes where the colliding partons, for example, radiate a gluon. What is more, the additional radiation provides recoil, so the tree-level case $\vec{q}_T = 0$ for the vector boson no longer holds true. Specifically, one may write

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \delta(q_T^2) + O(\alpha_S), \tag{1.3}$$

where σ is the cross section of the process and α_S is the strong coupling constant. In the high-q_T region ($q_T \gtrapprox m_W$) the perturbative expansion of the differential cross section is convergent. However, in the $q_T \ll m_W$ region the convergence is spoiled by the presence of high-order logarithmic terms which can schematically be given as $\alpha_s^n \ln^m\left(\frac{m_W}{q_T}\right)$, where $2n \geq m$. Nonetheless, it is possible to sum these logarithms to all orders which allows to obtain predictions for all regions of q_T [5]. This technique is known as q_T-resummation. In practice, the experiments use Monte Carlo event generators to reconstruct q_T of the W in lab frame. Hence, the perturbative accuracy of m_W determination becomes limited by Monte Carlo method accuracy. Since q_T is a natural variable of the lab frame, finding a high-precision relation between lepton transverse momentum $p_{T\ell}$ and q_T will help to improve the accuracy.

This project will analyse the theoretical background of the Z and W decays into two leptons, by separately considering the hadronic production of an intermediate boson and its subsequent decay into leptons. This is done by decomposing the cross section in terms of leptonic and hadronic tensors.

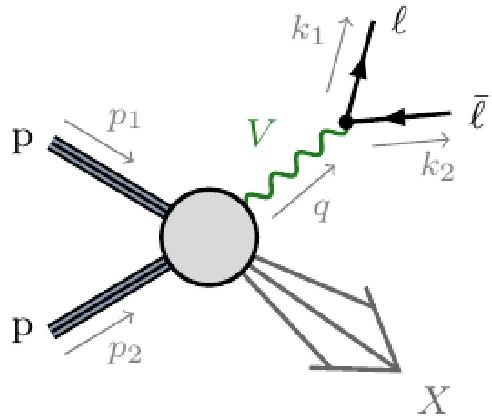

Figure 1. A schematic example of the Drell-Yan process [6].

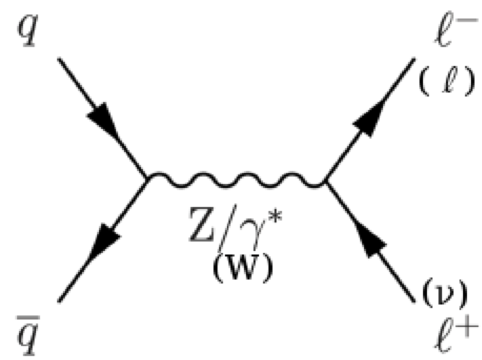

Figure 2. Feynman diagram for Drell-Yan processes.

1.2 Leptonic and Hadronic tensors

The matrix element for any process can be deduced from the Feynman rules for that process. Consider the case of a Drell-Yan process $p_1(P_a)+p_2(P_b) \to V(q)+X \to \ell_1(k_1)+\ell_2(k_2)+X$ (Fig. 1), where $p_{1,2}$ are incoming hadrons with momenta $P_{a,b}$, $\ell_{1,2}$ are outgoing lepton states with momenta $k_{1,2}$ and V is the intermediate boson with momentum q. The X is an unconstrained state outgoing of hadronic radiation. The matrix element in such a case will be built from two independent currents:

$$\mathcal{M} \sim J_{q\mu} J_\ell^\mu, \tag{1.4}$$

where q stands for "quark" and ℓ stands for "lepton". The $\sim$ sign is used to denote all the factors that come in front of the two currents and depend on the process. The cross section is proportional to the square of the matrix element which goes as

$$\mathcal{M}^\dagger \mathcal{M} = |\mathcal{M}|^2 \sim J_{q\mu} J_{q\nu}^\dagger J_\ell^\mu J_\ell^{\nu\dagger}. \tag{1.5}$$

Note that in the case of W and Z these currents can be broken down further into a vector and axial component $J^\mu = J_V^\mu - J_A^\mu$. Hence, the most general tensor should also include helicity indices. Since Eq. (1.4) may be separated into the solely hadronic (i.e. q) part and into the leptonic (i.e. ℓ) part, one can write:

$$|\mathcal{M}|^2 = \sum_{hh'} H_{hh'\mu\nu} L_{hh'}^{\mu\nu}, \tag{1.6}$$

where all the factors previously denoted as $\sim$ were now incorporated into the hadronic tensor $H_{hh'\mu\nu} \sim J_{q\mu} J_{q\nu}^\dagger$ and leptonic tensor $L_{hh'}^{\mu\nu} \sim J_\ell^\mu J_\ell^{\nu\dagger}$. It is conventional to demand that all the couplings and gauge boson propagators be incorporated into only one of the tensors and in this report all prefactors will be moved to the leptonic tensor. The full cross section can then be decomposed as [7]

$$\sigma = \frac{1}{2E_{\mathrm{CM}}^2} \sum_{hh'} H_{hh'\mu\nu} L_{hh'}^{\mu\nu}. \tag{1.7}$$

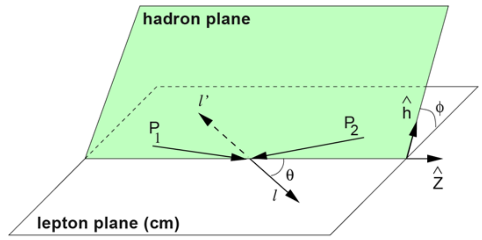

Figure 3. The Collins-Soper frame [9]. This frame is expressed in terms of two variables $\cos\theta$ and ϕ as depicted. Note that the proton momenta are not generally equal in any direction. Taken from [10].

1.3 Photon Exchange in the COM frame of leptons

For the Drell-Yan process (Fig. 2), only the vector current for the lepton part is present and the elements of leptonic current are given by [7]:

$$J^{\mu}_{\ell f} = (k_1, k_2, q) = \sum_{\text{spins}} -Q_f \frac{4\pi\alpha}{q^2} \left(\bar{u}(k_1)\gamma^{\mu} v(k_2)\right), \tag{1.8}$$

where u and v are the appropriate spinors for the particles in the process, $\alpha_{\text{em}} \equiv \alpha = \frac{e^2}{4\pi}$ is the fine structure constant and Q_f is the charge of a quark with flavour f. As in (1.4), the leptonic tensor is formed by squaring the leptonic current from Eq. (1.8). Specifically, this yields:

$$\begin{aligned} L^{\mu\nu}_{ff'}(k_1, k_2, q) &= J^{\mu\dagger}_{\ell f}(k_1, k_2) J^{\nu}_{\ell f'}(k_1, k_2) \\ &= \sum_{s,r=1}^{2} \left(\frac{4\pi\alpha}{q^2}\right)^2 Q_f Q_{f'} \left[\bar{v}^s(k_2)\gamma^{\mu} u^r(k_1)\right] \left[\bar{u}^r(k_1)\gamma^{\nu} v^s(k_2)\right], \end{aligned} \tag{1.9}$$

where s, r are the spin states of the spinors. Taking advantage of the sum and using the completeness relations for the spinors $\sum_{s=1}^{2} u_s \bar{u}_s = \not{p} + m$ and $\sum_{s=1}^{2} \bar{v}_s v_s = \not{p} - m$ [8], one can reduce the equation to

$$\begin{aligned} L^{\mu\nu}_{ff'}(k_1, k_2, q) &= \left(\frac{4\pi\alpha}{q^2}\right)^2 Q_f Q_{f'} \mathrm{Tr}[\not{k}_1 \gamma^{\mu} \not{k}_2 \gamma^{\nu}] \\ &= 4 \left(\frac{4\pi\alpha}{q^2}\right)^2 Q_f Q_{f'} \Big(k_1^{\mu} k_2^{\nu} + k_1^{\nu} k_2^{\mu} - g^{\mu\nu}(k_1 \cdot k_2)\Big). \end{aligned} \tag{1.10}$$

Note that in Eq. (1.10), the lepton tensor is still manifestly covariant as no assumptions have been made about the frame of reference. Moving into the lepton COM frame [9] (see Fig. 3 for definitions of variables in Collins-Soper (CS) frame), the 4-vectors are parametrised as [6]

$$k^{\mu}_{1,2} = \frac{Q}{2}(1, \pm\sin\theta\cos\phi, \pm\sin\theta\sin\phi, \pm\cos\theta), \qquad Q = \sqrt{q^2}. \tag{1.11}$$

Note that Eq. (1.10) is differential in k_1^μ and k_2^μ, but their kinematics may be fully described by the six independent variables $\{\cos\theta, \phi, q^\mu\}$. Generally, this can be written in terms of some measurement function of the leptonic phase space $M(\Phi_L)$ as

$$L^{\mu\nu}(q, M) = \int \mathrm{d}\Phi_L L^{\mu\nu}(k_1, k_2, q^2) M(\Phi_L) \delta^4(q - k_1 - k_2). \tag{1.12}$$

For $M = \mathbb{1}$, one integrates over all of the decay phase. On the other hand, for $M = \delta(\cos\theta - \cos[\theta(\Phi_L)])\delta(\phi - \phi(\Phi_L))$ one retains variables $\{\cos\theta, \phi\}$. Writing this explicitly yields:

$$\begin{aligned}
\int \mathrm{d}\Phi_L L^{\mu\nu}(k_1, k_2, q^2) &= \int \frac{\mathrm{d}^4 k_1^\mu \mathrm{d}^4 k_2^\nu}{(2\pi)^2} \delta^4(q - k_1 - k_2)\Theta(k_1^0)\delta((k_1)^2)\Theta(k_2^0)\delta((k_2)^2 M L^{\mu\nu}(k_1, k_2, q^2) \\
&= \frac{1}{32\pi^2} \int \mathrm{d}\cos\theta \mathrm{d}\phi L^{\mu\nu}(k_1(\cos\theta, \phi), k_2(\cos\theta, \phi), q^2) = \\
&= \int \mathrm{d}\cos\theta \mathrm{d}\phi L^{\mu\nu}(\cos\theta, \phi, q^2),
\end{aligned} \tag{1.13}$$

where the prefactors were absorbed into

$$L_{ff'}^{\mu\nu}(\cos\theta, \phi, q^2) = \frac{2\alpha^2}{q^4} Q_f Q_{f'} \Big(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu}(k_1 \cdot k_2) \Big). \tag{1.14}$$

In the lepton COM frame the intermediate boson is stationary. Hence, with $\vec{q} = 0$, the hadronic tensor in such a reference frame can be expressed as [6]:

$$H^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & H_{11} & H_{12} & H_{13} \\ 0 & H_{21} & H_{22} & H_{23} \\ 0 & H_{31} & H_{32} & H_{33} \end{pmatrix} \tag{1.15}$$

Dotting the lepton tensor Eq. (1.14) with Eq. (1.15) and by using Eq. (1.11), one acquires the full scattering cross section for such process. For the diagonal elements:

$$\begin{aligned}
L^{11} H_{11} &= \frac{2\alpha^2}{q^4} Q_f Q_{f'} \left(-\frac{Q^2}{4} \sin^2\theta \cos^2\phi - \frac{Q^2}{4} \sin^2\theta \cos^2\phi - (-1)\frac{Q^2}{2} \right) H_{11} \\
&= \frac{\alpha^2}{q^2} Q_f Q_{f'} (1 - \sin^2\theta \cos^2\phi) H_{11}
\end{aligned} \tag{1.16}$$

And, hence, similarly:

$$L^{22} H_{22} = \frac{\alpha^2}{q^2} Q_f Q_{f'} (1 - \sin^2\theta \sin^2\phi) H_{22} \tag{1.17}$$

$$L^{33} H_{33} = \frac{\alpha^2}{q^2} Q_f Q_{f'} (1 - \cos^2\theta) H_{33} \tag{1.18}$$

Repeating the same process, starting from Eq. 1.10 for the off-diagonal elements yields

$$\begin{aligned} L^{12}H_{12} &= \frac{2\alpha^2}{q^4}Q_fQ_{f'}\left(-\frac{Q^2}{4}\sin^2\theta\cos\phi\sin\phi - \frac{Q^2}{4}\sin^2\theta\sin\phi\cos\phi\right)H_{12} \\ &= -\frac{\alpha^2}{2q^2}Q_fQ_{f'}\sin^2\theta\sin 2\phi H_{12}, \end{aligned} \tag{1.19}$$

And so:

$$L^{21}H_{21} = -\frac{\alpha^2}{2q^2}Q_fQ_{f'}\sin^2\theta\sin 2\phi H_{21} \tag{1.20}$$

$$L^{23}H_{23} = -\frac{\alpha^2}{2q^2}Q_fQ_{f'}\sin 2\theta\sin\phi H_{23} \tag{1.21}$$

$$L^{32}H_{32} = -\frac{\alpha^2}{2q^2}Q_fQ_{f'}\sin 2\theta\sin\phi H_{32} \tag{1.22}$$

$$L^{13}H_{13} = -\frac{\alpha^2}{2q^2}Q_fQ_{f'}\sin 2\theta\cos\phi H_{13} \tag{1.23}$$

$$L^{31}H_{31} = -\frac{\alpha^2}{2q^2}Q_fQ_{f'}\sin 2\theta\cos\phi H_{31} \tag{1.24}$$

All the expressions (Eqs.(1.16)-(1.24)) can be summed up to get the full differential cross section of the γ decay. Eqs.(1.16)-(1.18) can be modified in order to be expressed in terms of spherical harmonics (see Appendix A). Then summing up the diagonal elements expressed in terms of spherical harmonics and using appropriate factors from Eqs.(1.16)-(1.18) gives:

$$\begin{aligned} &L^{11}H_{11} + L^{22}H_{22} + L^{33}H_{33} = \\ &= \frac{\alpha^2}{2q^2}Q_fQ_{f'}\Big[(1+\cos^2\theta)(H_{11}+H_{22}+H_{33}) \\ &+ (1-3\cos^2\theta)H_{33} + (H_{22}-H_{11})\sin^2\theta\cos 2\phi\Big]. \end{aligned} \tag{1.25}$$

By integrating the leptonic tensor of Eq. (1.14) over the free variables $\cos\theta$ and ϕ, one arrives at the “unpolarised” inclusive leptonic tensor in the decay phase space [7]:

$$L^{\mu\nu}_{ff'}(q^2) = \frac{8\pi\alpha^2}{3q^2}Q_fQ_{f'}\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right). \tag{1.26}$$

This construct can be used to further simplify Eqs.(1.16)-(1.24). Dotting $L^{\mu\nu}_{ff'}(q^2)$ result with Eq. (1.15) yields the unpolarised cross section of this process

$$\frac{d\sigma^{\text{unpol}}}{d^4q} = \frac{8\pi\alpha^2}{3q^2}Q_fQ_{f'}(H_{11}+H_{22}+H_{33}). \tag{1.27}$$

Defining a set of angular coefficients $A_{0\ldots7}$ (motivated by [6]):

$$\begin{aligned} &A_0 = 2H_{33} && A_1 = -(H_{13}+H_{31}) && A_2 = 2(H_{22}-H_{11}) \\ &A_3 = 0 && A_4 = 0 && A_5 = -(H_{12}+H_{21}) \\ &A_6 = -(H_{23}+H_{32}) && A_7 = 0, && \end{aligned} \tag{1.28}$$

one gets the final form for Eq. (1.25):

$$\begin{aligned}
&L^{11}H_{11}+L^{22}H_{22}+L^{33}H_{33}= \\
&=\frac{3}{16\pi}\frac{d\sigma^{\text{unpol}}}{d^4q}\Big[(1+\cos^2\theta)+\frac{1}{2}A_0(1-3\cos^2\theta)+\frac{1}{2}A_2\sin^2\theta\cos 2\phi\Big].
\end{aligned} \tag{1.29}$$

If the same procedure is repeated for all the off-diagonal elements (Eqs.(1.19)-(1.24)) as well, and then summed with Eq. (1.29) (see Appendix B), one arrives at the polarised differential cross section given below. The expression agrees with the result of [6], after taking the limit where vector and axial coupling vanish:

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\mathrm{d}\cos\theta\mathrm{d}\phi}=\frac{3}{16\pi}\frac{\mathrm{d}\sigma^{\text{unpol}}}{\mathrm{d}^4q}\Big[&(1+\cos^2\theta)+\frac{1}{2}A_0(1-3\cos^2\theta) \\
&+A_1\sin 2\theta\cos\phi+\frac{1}{2}A_2\sin^2\theta\cos 2\phi \\
&+A_5\sin^2\theta\sin 2\phi+A_6\sin 2\theta\sin\phi\Big].
\end{aligned} \tag{1.30}$$

2 General leptonic and hadronic tensors for vector bosons

2.1 The leptonic tensor for Z/γ^* exchange

The leptonic tensor in the case of Z/γ^* exchange is now given by interference of vector-vector (VV), vector-axial (VA) and axial-axial (AA) currents. Furthermore, the Z is not massless, which means the line shape of the Z must be accounted for. The reduced (dimensionless) propagator for a vector boson is given by [11]

$$P_X(q^2)=\frac{1}{1-\frac{m_X^2}{q^2}+i\frac{\Gamma_X m_X}{q^2}}, \tag{2.1}$$

where $X=Z,W,\gamma$. For the γ case ($m=0$) this simply yields $P_\gamma=1$.

Starting from leptonic currents for the joint Z/γ^* case (given explicitly in [7]), and including the decay width of the Z, one arrives at such expressions:

$$\begin{aligned}
L^{\mu\nu}_{VVff'}=\sum_{\text{spins}}\left(\frac{4\pi\alpha}{q^2}\right)^2\Bigg(&Q_fQ_{f'}[\bar{v}(k_1)\gamma^\mu u(k_2)][\bar{u}(k_2)\gamma^\nu v(k_1)] \\
&-\mathrm{Re}[P_Z]Q_{f'}v_f[\bar{v}(k_1)\gamma^\mu(v_l-a_l\gamma^5)u(k_2)][\bar{u}(k_2)\gamma^\nu v(k_1)] \\
&-\mathrm{Re}[P_Z]Q_f v_{f'}[\bar{v}(k_1)\gamma^\mu u(k_2)][\bar{u}(k_2)\gamma^\nu(v_l-a_l\gamma^5)v(k_1)] \\
&+|P_Z|^2v_fv_{f'}[\bar{v}(k_1)\gamma^\mu(v_l-a_l\gamma^5)u(k_2)][\bar{u}(k_2)\gamma^\nu(v_l-a_l\gamma^5)v(k_1)]\Bigg),
\end{aligned} \tag{2.2}$$

$$L^{\mu\nu}_{AAff'}=\sum_{\text{spins}}|P_Z|^2\left(\frac{4\pi\alpha}{q^2}\right)^2a_fa_{f'}[\bar{v}(k_1)\gamma^\mu(v_l-a_l\gamma^5)u(k_2)][\bar{u}(k_2)\gamma^\nu(v_l-a_l\gamma^5)v(k_1)], \tag{2.3}$$

$$L^{\mu\nu}_{VAf'f} = L^{\mu\nu}_{AVff'} = \sum_{\text{spins}} \left(\frac{4\pi\alpha}{q^2}\right)^2 \Bigg(\mathrm{Re}[P_Z] a_f Q_{f'} [\bar{v}(p_1)\gamma^\mu (v_l - \gamma^5 a_l) u(p_2)][\bar{u}(p_2)\gamma^\nu v(p_1)]$$
$$- |P_Z|^2 a_f v_{f'} [\bar{v}(k_1)\gamma^\mu (v_l - a_l\gamma^5) u(k_2)][\bar{u}(k_2)\gamma^\nu (v_l - a_l\gamma^5) v(k_1)] \Bigg). \tag{2.4}$$

Using Eq. (2.1), the $\mathrm{Re}[P_Z]$ and $|P_Z|^2$ are given by:

$$|P_{W,Z}|^2 = \frac{1}{\left(1 - \frac{m^2_{W,Z}}{q^2}\right)^2 + \left(\frac{m_{W,Z}\Gamma_{W,Z}}{q^2}\right)^2}, \tag{2.5}$$

$$Re[P_{W,Z}] = \frac{1 - \frac{m^2_{W,Z}}{q^2}}{\left(1 - \frac{m^2_{W,Z}}{q^2}\right)^2 + \left(\frac{m_{W,Z}\Gamma_{W,Z}}{q^2}\right)^2}. \tag{2.6}$$

Using properties of traces [8] and the commutation relations of γ^5 (see Appendix C) one reduces Eqs.(2.2)-(2.4) to

$$L^{\mu\nu}_{VVff'}(q^2, k_1, k_2) = 4\left(\frac{4\pi\alpha}{q^2}\right)^2 \Bigg[\Big(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu}(k_1 \cdot k_2)\Big) \times \Big(Q_f Q_{f'}$$
$$- v_l \mathrm{Re}[P_Z](v_f Q_{f'} + v_{f'} Q_f) + (v_l^2 + a_l^2)|P_Z|^2 v_f v_{f'}\Big) \tag{2.7}$$
$$- i\epsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} a_l \mathrm{Re}[P_Z](v_f Q_{f'} + v_{f'} Q_f - 2v_l \frac{v_f v_{f'}}{1 - \frac{m_Z^2}{q^2}}) \Bigg],$$

$$L^{\mu\nu}_{AAff'}(q^2, k_1, k_2) = 4\left(\frac{4\pi\alpha}{q^2}\right)^2 a_f a_{f'} |P_Z|^2 \Bigg[(v_l^2 + a_l^2)\Big(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu}(k_1 \cdot k_2)\Big)$$
$$+ 2i v_l a_l \epsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} \Bigg], \tag{2.8}$$

$$L^{\mu\nu}_{AVff'}(q^2, k_1, k_2) = L^{\mu\nu}_{VAf'f}(q^2, k_1, k_2) = 4\left(\frac{4\pi\alpha}{q^2}\right)^2 (-a_f)\mathrm{Re}[P_Z] \Bigg[\Big(- Q_{f'} v_l + \frac{v_{f'}(v_l^2 + a_l^2)}{1 - \frac{m_Z^2}{q^2}} \Big)$$
$$\times \Big(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu}(k_1 \cdot k_2)\Big) - i k_{1\rho} k_{2\sigma} \epsilon^{\mu\nu\rho\sigma} \Big(Q_{f'} a_l - 2v_{f'} \frac{v_l a_l}{1 - \frac{m_Z^2}{q^2}}\Big) \Bigg]. \tag{2.9}$$

In a similar procedure as described for the photon case these can be used to calculate the differential cross sections in the case of Z/γ^* [6]. Here, the coefficients A_3, A_4, A_7 in Eq. (1.28) no longer vanish, but instead receive contributions from AV, VA and AA terms.

By introducing the following notation:

$$
\begin{aligned}
K^{\mu\nu} &= k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu}(k_1 \cdot k_2), \\
\epsilon(\mu, \nu, k_1, k_2) &= k_{1\rho} k_{2\sigma} \epsilon^{\mu\nu\rho\sigma}, \\
L^+_{VVff'} &= Q_f Q_{f'} - v_l \mathrm{Re}[P_Z](v_f Q_{f'} + v_{f'} Q_f) + (v_l^2 + a_l^2)|P_Z|^2 v_f v_{f'}, \\
L^-_{VVff'} &= a_l \mathrm{Re}[P_Z](v_f Q_{f'} + v_{f'} Q_f) - 2 v_l a_l |P_Z|^2 v_f v_{f'}, \\
L^+_{AAff'} &= (v_l^2 + a_l^2)|P_Z|^2 a_f a_{f'}, \\
L^-_{AAff'} &= -2 v_l a_l |P_Z|^2 a_f a_{f'}, \\
L^+_{AVff'} &= L^+_{VAf'f} = v_l \mathrm{Re}[P_Z] a_f Q_{f'} - (v_l^2 + a_l^2)|P_Z|^2 a_f v_{f'}, \\
L^-_{AVff'} &= L^-_{VAf'f} = -a_l \mathrm{Re}[P_Z] a_f Q_{f'} + 2 v_l a_l |P_Z|^2 a_f v_{f'},
\end{aligned}
\tag{2.10}
$$

the Eqs.(2.7)-(2.9) simplify into a straightforward form:

$$
L^{\mu\nu}_{hh'ff'}(q^2, k_1, k_2) = 4\left(\frac{4\pi\alpha}{q^2}\right)^2 \left(K^{\mu\nu} L^+_{hh'ff'} - i\epsilon(\mu, \nu, k_1, k_2) L^-_{hh'ff'}\right). \tag{2.11}
$$

Here $\pm$ denotes the parity of the coefficient under $\mu \leftrightarrow \nu$ interchange. Note that in the limit of $a = 0$ and $v = 0$, the pure γ case is restored, as in Eq. (1.10). Furthermore, no assumptions have been made about the form of Eq. (2.11), or its arguments k_1, k_2, q, hence this form is manifestly covariant.

2.2 The leptonic tensor for $W^\pm$ exchange

The $W^\pm$ boson case differs from the two previous processes because it induces a flavour change for the quarks. Hence, moving all EW couplings and propagators from the hadronic current into the leptonic part, one gets a two-flavour-indexed leptonic current

$$
J^\mu_{\ell ud}(q^2, k_1, k_2) = \frac{4\pi\alpha V_{ud}}{8q^2 \sin^2\theta_\mathrm{w}} \bar{u}(k_1)\gamma^\mu(1-\gamma^5)d(k_2)P_W(q^2). \tag{2.12}
$$

Here, $\frac{e}{\sin\theta_\mathrm{w}} = g_\mathrm{w}$ and $e^2 = 4\pi\alpha$ were used and V_{ud} is an element of CKM matrix for an $u \to d$ type transition. Evidently, this is composed of two terms forming the $V - A$ structure of weak interaction with a common factor pulled out explicitly:

$$
J^\mu_{\ell ud}(q^2, k_1, k_2) = \frac{\pi\alpha V_{ud}}{2q^2 \sin^2\theta_\mathrm{w}}(J^\mu_V - J^\mu_A). \tag{2.13}
$$

In order to form the leptonic tensor, this equation needs to be squared like in the Z/γ^* cases. This is obtained, as given in Appendix C, in the special case where couplings are taken to be $a = 1$, $v = 1$ and $Q_f = 0$ (note that the result is not exactly the limit, due to a different prefactor):

$$
\begin{aligned}
L^{\mu\nu}_{udu'd'}(q^2,k_1,k_2) &= \left(\frac{\pi\alpha}{2q^2\sin^2\theta_{\rm w}}\right)^2 (J_V^{\mu\dagger}J_V^{\mu} + J_A^{\mu\dagger}J_A^{\mu} - J_V^{\mu\dagger}J_A^{\mu} - J_A^{\mu\dagger}J_V^{\mu}), \\
&= 2\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\rm w}}\right)^2 \frac{V_{ud}^* V_{u'd'}}{\left(1-\frac{m_W^2}{q^2}\right)^2 + \frac{m_{W^2}\Gamma_W^2}{q^4}} (K^{\mu\nu} - i\epsilon(\mu,\nu,k_1,k_2)), \quad (2.14) \\
&= 2\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\rm w}}\right)^2 V_{ud}^* V_{u'd'}|P_W|^2 (K^{\mu\nu} - i\epsilon(\mu,\nu,k_1,k_2)),
\end{aligned}
\tag{2.15}
$$

where $|P_W|^2$is given by Eq. (2.5). Just like in the $Z/\gamma^/$ boson case this can be, although trivially, simplified to a similar form like Eq. (2.11). Introducing the following notation

$$L^+_{VVudu'd'} = L^+_{AAudu'd'} = \left(\frac{1}{8\sin^2\theta_{\rm w}}\right)^2 V_{ud}^* V_{u'd'}|P_W|^2 \tag{2.16}$$

$$L^-_{VAudu'd'} = L^-_{AVudu'd'} = \left(\frac{1}{8\sin^2\theta_{\rm w}}\right)^2 V_{ud}^* V_{u'd'}|P_W|^2 \tag{2.17}$$

$$L^+_{VAudu'd'} = L^+_{AVudu'd'} = L^-_{VVudu'd'} = L^-_{AAudu'd'} = 0 \tag{2.18}$$

yields the exact same form as Eq. (2.11) up to interchange of indices $f \leftrightarrow ud$:

$$L^{\mu\nu}_{hh'udu'd'}(q^2,k_1,k_2) = 4\left(\frac{4\pi\alpha}{q^2}\right)^2 \left(K^{\mu\nu}L^+_{hh'udu'd'} - i\epsilon(\mu,\nu,k_1,k_2)L^-_{hh'udu'd'}\right) \tag{2.19}$$

2.3 The hadronic tensor for proton collisions

By covariance, current conservation $q^\mu H_{\mu\nu} = 0$ and the CP properties of QCD, the hadronic tensor $H_{\mu\nu}$ for a Drell-Yan process can be decomposed into nine form factors as [6]:

$$
\begin{aligned}
H_{\mu\nu} = {} & H_1\tilde{g}_{\mu\nu} + H_2\tilde{P}_{a,\mu}\tilde{P}_{a,\nu} + H_3\tilde{P}_{b,\mu}\tilde{P}_{b,\nu} + H_4\left(\tilde{P}_{a,\mu}\tilde{P}_{b,\nu} + \tilde{P}_{b,\mu}\tilde{P}_{a,\nu}\right) \\
& + H_5\left(\tilde{P}_{a,\mu}\tilde{P}_{b,\nu} - \tilde{P}_{b,\mu}\tilde{P}_{a,\nu}\right) + iH_6\epsilon(\mu,\nu,P_a,q) + iH_7\epsilon(\mu,\nu,P_b,q) \\
& + H_8\left(\tilde{P}_{a,\mu}\epsilon(\nu,P_a,P_b,q) + \mu\leftrightarrow\nu\right) + H_9\left(\tilde{P}_{b,\mu}\epsilon(\nu,P_a,P_b,q) + \mu\leftrightarrow\nu\right),
\end{aligned}
\tag{2.20}
$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$ and $\tilde{P}_{a/b,\mu} = \tilde{g}_{\mu\nu}P^\nu_{a/b}$. The tensor elements such as H_{11} are not frame independent, however, the hadronic form factors H_{1-9} are. To get the structure of tensor elements, expressed as Eq. (1.15), in terms of these frame-independent form factors one needs to know the momentum of the protons in the lepton COM frame. However, that is not trivial to determine. In the lab frame, the colliding proton momenta can be written as

$$P_a^\mu = (E_{\rm CM}/2, 0, 0, E_{\rm CM}/2)^T \qquad P_b^\mu = (E_{\rm CM}/2, 0, 0, -E_{\rm CM}/2)^T \tag{2.21}$$

where $E_{\rm CM}$ is their center of mass energy. The exchanged boson momentum is then

$$q_\mu = (q_0, q_1, q_2, q_3) \tag{2.22}$$

Boosting into the CS frame (see Appendix D), one then has such relations between proton momenta $P^{\rm CS}_{a,b}$ in the CS frame:

$$P^\mu_a = e^{-Y} P^\mu \qquad\qquad P^{0,1,2}_b = e^{Y} P^{0,1,2} \qquad\qquad P^3_b = -e^{Y} P^3, \tag{2.23}$$

where $Y = \frac{1}{2}\ln\frac{q_0+q_3}{q_0-q_3}$ is the rapidity of the proton and P^μ is defined as

$$P^\mu \equiv E_{\rm CM}/2 \begin{pmatrix} \frac{\sqrt{Q^2+(q_1)^2}}{Q} \\ -\frac{q_1}{Q} \\ 0 \\ 1 \end{pmatrix}, \tag{2.24}$$

Note that here a rotation was made, such that $\vec{q}_T = q_1 \vec{e}_x$. At any point, full generality can be restored by changing $P_1 \to P_T$.

Returning to the decomposition of $H_{\mu\nu}$ given in Eq. (2.20), one can note that in the CS frame we always have $\tilde{g}_{0\nu} = \tilde{g}_{\mu 0} = 0$ and $\tilde{g}_{ij} = -\delta_{ij}$. Hence, noting that $P_{a,2}{=}P_{b,2} = 0$, some simplifications may be applied:

$$\tilde{P}_i = -P_i, \quad \text{for i=1,3,} \tag{2.25}$$

$$\tilde{P}_i = 0, \quad \text{for i=0,2,} \tag{2.26}$$

$$\epsilon(\mu,\nu,P,q) = 0, \quad \text{if } \mu=\nu \text{ or } \mu,\nu \neq 2, \tag{2.27}$$

$$\epsilon(\mu,P_a,P_b,q) = 0, \quad \text{if } \mu,\nu \neq 2. \tag{2.28}$$

Using the definitions of Eq. (2.23), this finally yields such form for all the elements of $H_{\mu\nu}$ in the CS frame:

$$H_{11} = -H_1 + (P_1)^2 \left(H_2 e^{-2Y} + H_3 e^{2Y} + 2H_4\right), \tag{2.29}$$

$$H_{22} = -H_1, \tag{2.30}$$

$$H_{33} = -H_1 + (P_3)^2 \left(H_2 e^{-2Y} + H_3 e^{2Y} - 2H_4\right), \tag{2.31}$$

$$H_{13} = P_1 P_3 (H_2 e^{-2Y} - H_3 e^{2Y} - 2iH_5), \tag{2.32}$$

$$H_{12} = 2Q(P_1)^2 P_3 (H_8 e^{-2Y} + H_9 e^{2Y}) + iQP_3(H_7 e^{2Y} - H_6 e^{-2Y}), \tag{2.33}$$

$$H_{32} = 2QP_1(P_3)^2 (H_8 e^{-2Y} - H_9 e^{2Y}) + iQP_1(H_7 e^{2Y} + H_6 e^{-2Y}), \tag{2.34}$$

where other off-diagonal elements are related to the ones given by $H^*_{ij} = H_{ji}$. Notice that at tree level, i.e. $\vec{q}_T = 0$, all terms with P_1 vanish, making the hadronic tensor much simpler. However, some terms, such as the second one in H_{33} will not vanish. If Lam-Tung relation, given as [6]

$$H_1 = \frac{1}{2} H^\mu{}_\mu, \tag{2.35}$$

is to be restored as $q_T \to 0$, there must be an intrinsic reason in the specific combination $H_2 e^{-2Y} + H_3 e^{2Y} - 2H_4$ forcing it to vanish.

3 Differential cross section for $W^{\pm}$ decays at tree level

3.1 Light cone coordinates

When working in the lab frame, the variables that are naturally simple to measure are the rapidities of particles and their transverse momenta. Hence, it is useful to parametrise coordinates as the so-called light-cone (LC) coordinates [12]:

$$k^{\mu} = \frac{k^{-}}{2} n^{\mu} + \frac{k^{+}}{2} \bar{n}^{\mu} + k_{\perp}^{\mu}, \tag{3.1}$$

where vectors n^{μ} and $\bar{n}^{\mu}$ are defined as any 4-vector pair to satisfy

$$n^2 = \bar{n}^2 = 0, \tag{3.2}$$

$$n\bar{n} = 2. \tag{3.3}$$

For proton-proton collisions $\vec{n} = -\vec{\bar{n}} = \vec{e}_z$ is chosen, which implies

$$k^{-} = k^0 + k^3, \tag{3.4}$$

$$k^{+} = k_0 - k_3, \tag{3.5}$$

$$k_{\perp}^{\mu} = (0, k_1, k_2, 0)^T. \tag{3.6}$$

For experimental applications it is useful to have the cross section expressed in terms of transverse momentum of one of the leptons $p_{T\ell}$, rapidities of the colliding protons Y, and the total energy of the intermediate boson Q. These variables are easily accessible in LC coordinates.

Calculating the cross section for a vector boson decay in the lab frame again requires the computation of $L^{\mu\nu} H_{\mu\nu}$. For experimental applications it is very interesting to have the cross section expressed in terms of rapidities of the colliding protons Y, transverse momentum of one of the leptons $p_{T\ell}$ and the total energy of the intermediate boson Q and most of these variables are directly accessible in LC coordinates. To calculate various contractions of tensors in $L^{\mu\nu} H_{\mu\nu}$ one may find it simpler to decompose the structures in terms of LC variables. Any tensor, can be rewritten as

$$\begin{aligned} T^{\mu\nu} = &\frac{1}{4} \left(T^{++} \bar{n}^{\mu} \bar{n}^{\nu} + T^{--} n^{\mu} n^{\nu} + T^{+-} \bar{n}^{\mu} n^{\nu} + T^{-+} n^{\mu} \bar{n}^{\nu} \right) \\ &+ \frac{1}{2} \left(T^{+\perp,\nu} \bar{n}^{\mu} + T^{\perp+,\mu} \bar{n}^{\nu} + T^{-\perp,\nu} n^{\mu} + T^{\perp-,\mu} n^{\nu} \right) \\ &+ T^{\perp\perp,\mu\nu}. \end{aligned} \tag{3.7}$$

3.2 Tree level decay of W

One of the ways to obtain the cross section differential in certain variables is to integrate over the phase space in a given order and introduce a measurement function, as done previously in Eq. (1.12). Consider the differential cross section expressed in such a way that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q \mathrm{d}p_l^+ \mathrm{d}p_l^-} = \int \frac{\mathrm{d}^4 k_1 \mathrm{d}^4 k_2}{(2\pi)^2} \delta(k_1^2) \delta(k_2^2) \delta^4(q - k_1 - k_2) H_{\mu\nu} L^{\mu\nu} M(p_\ell^+, p_\ell^-), \tag{3.8}$$

where $p_\ell^\pm$ stands for light cone momenta of one of the leptons. This means that the other lepton has to be fully integrated over. In the case of W decay this is preferred, as experimentally the neutrino is not detected. Choosing to integrate over all of the second lepton 4-momenta yields:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\mathrm{d}p_\ell^+\mathrm{d}p_\ell^-} = \int \frac{\mathrm{d}^4k_1}{(2\pi)^2}\delta(k_1^2)\delta(k_2^2)M(p_\ell^+,p_\ell^-))H_{\mu\nu}L^{\mu\nu}, \tag{3.9}$$

and a constraint $k_2^\mu = q^\mu - k_1^\mu$. The on-shell δ function for k_1 can be integrated trivially by going to LC variables $\mathrm{d}^4k = \frac{1}{2}\mathrm{d}k^+\mathrm{d}k^-\mathrm{d}^2\vec{k}_T$:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\mathrm{d}p_\ell^+\mathrm{d}p_\ell^-} &= \frac{1}{4}\int \frac{\mathrm{d}k_1^+\mathrm{d}k_1^-\mathrm{d}^2\vec{k}_{1T}}{(2\pi)^2}\delta(k_1^+k_1^- - |k_{1T}|^2)\delta(k_2^+k_2^- - |k_{2T}|^2)M(p_\ell^+,p_\ell^-))H_{\mu\nu}L^{\mu\nu}\\ &= \frac{1}{8}\int \frac{\mathrm{d}k_1^+\mathrm{d}k_1^-\mathrm{d}|k_{1T}|^2\mathrm{d}\phi}{(2\pi)^2}\delta(k_1^+k_1^- - |k_{1T}|^2)\delta(k_2^+k_2^- - |k_{2T}|^2)M(p_\ell^+,p_\ell^-))H_{\mu\nu}L^{\mu\nu}\\ &= \int \frac{\mathrm{d}k_1^+\mathrm{d}k_1^-\mathrm{d}\phi}{32\pi^2}\delta(k_2^+k_2^- - |k_{2T}|^2)M(p_\ell^+,p_\ell^-))H_{\mu\nu}L^{\mu\nu},\end{aligned} \tag{3.10}$$

giving another constraint $k_1^+k_1^- = |k_{1T}|^2$. Note that, at this point, the remaining δ is just shorthand for writing $\delta(k_2^+k_2^- - |k_{2T}|^2) = \delta((q^+ - k_1^+)(q^- - k_1^-) - k_1^+k_1^- - |q_T|^2 + 2|q_T||k_{1T}|\cos\phi)$.

The measurement functions, however, are now trivial in terms of $k_1^\pm$ and integrating over them simply swaps with the variables $p_l^\pm$. Hence the final result is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\mathrm{d}p_\ell^+\mathrm{d}p_\ell^-} = \int \frac{\mathrm{d}\phi}{32\pi^2}\delta(k_2^+k_2^- - |k_{2T}|^2))H_{\mu\nu}L^{\mu\nu}, \tag{3.11}$$

or equivalently

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^+\mathrm{d}q^-\mathrm{d}p_\ell^+\mathrm{d}p_\ell^-} = \int \frac{\mathrm{d}^2\vec{q}_T\mathrm{d}\phi}{64\pi^2}\delta(k_2^+k_2^- - |k_{2T}|^2))H_{\mu\nu}L^{\mu\nu}. \tag{3.12}$$

For the hadronic tensor this holds [7] at tree level:

$$\int \mathrm{d}^2\vec{q}_T H^{\mu\nu}_{hh'qq'}(q^2, Y, \vec{q}_T^{\,2}) = h^{\mu\nu}_{hh'qq'}f_q(x_a)f_{q'}(x_b) + h^{\mu\nu}_{hh'q'q}f_{q'}(x_a)f_q(x_b) + O(\alpha_s), \tag{3.13}$$

where Y is the proton rapidity, $x_{a,b}$ are the Bjorken x, f_q - parton distribution functions (PDFs) and $h^{\mu\nu}_{hh'qq'}$ is the hard function, given as:

$$\begin{aligned}h^{\mu\nu}_{VVff'qq'} &= h^{\mu\nu}_{AAff'qq'} = -\frac{1}{2N_c}\left(g^{\mu\nu} - \frac{1}{2}(n^\mu\bar{n}^\nu + n^\mu\bar{n}^\nu)\right)\delta_{fq}\delta_{f'q'} + O(\alpha_s),\\ h^{\mu\nu}_{VAff'qq'} &= h^{\mu\nu}_{AVff'qq'} = \frac{1}{4N_c}\left(i\epsilon^{\mu\nu\lambda\kappa}n_\lambda\bar{n}_\kappa\right)\delta_{fq}\delta_{f'q'} + O(\alpha_s),\end{aligned} \tag{3.14}$$

where N_c is the number of quark colors and only leading order terms are considered. Essentially, since at this order integration over q_T becomes trivial, and in the no-recoil case $\vec{q}_T = 0$, one can write Eq. (3.12) by using Eqs.(3.13) and (3.14) as:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}q^+\mathrm{d}q^-\mathrm{d}p_\ell^+\mathrm{d}p_\ell^-} &= \int \frac{\mathrm{d}^2\vec{q}_T\mathrm{d}\phi}{64\pi^2}\delta((q^+-p_\ell^+)(q^--p_\ell^-)-p_\ell^+p_\ell^- - |q_T|^2+2|q_T||p_{T\ell}|\cos\phi)H_{\mu\nu}L^{\mu\nu} \\ &\stackrel{\vec{q}_T=0}{=} \sum_{hh'}\int\frac{\mathrm{d}\phi}{32\pi^2}\delta((q^+-p_\ell^+)(q^--p_\ell^-)-p_\ell^+p_\ell^-) \\ &\times\left(h_{hh'ud,\mu\nu}f_u(x_a)f_{\bar{d}}(x_b)+h_{hh'du,\mu\nu}f_{\bar{d}}(x_a)f_u(x_b)\right)L^{\mu\nu}_{hh'}.\end{aligned} \tag{3.15}$$

Light cone coordinates are convenient here because for massless particles one may express $p_\ell^\pm = |p_{T\ell}|e^{\mp\eta}$, where η is the rapidity. Hence, trading $\pm$ variables $\mathrm{d}q^+\mathrm{d}q^-\mathrm{d}p_\ell^+\mathrm{d}p_\ell^-$ for $Q, Y, p_{T\ell}, \eta$ gives

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}p_{T\ell}^2\mathrm{d}\eta} &= \sum_{hh'}\int\frac{\mathrm{d}\phi}{32\pi}\delta((Qe^{-Y}-p_{T\ell}e^{-\eta})(Qe^{Y}-p_{T\ell}e^{\eta})-p_{T\ell}^2)H_{\mu\nu}L^{\mu\nu} \\ &\times\left(h_{hh'ud,\mu\nu}f_u(x_a)f_{\bar{d}}(x_b)+h_{hh'du,\mu\nu}f_{\bar{d}}(x_a)f_u(x_b)\right)L^{\mu\nu}_{hh'}\end{aligned} \tag{3.16}$$

This allows us to freely integrate over the ϕ angle, leaving

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}p_{T\ell}^2\mathrm{d}\eta} = \frac{1}{16\pi}\sum_{hh'}\delta(Q^2-2Qp_{T\ell}\cosh(Y-\eta)) \tag{3.17}$$

$$\times\left(h_{hh'ud,\mu\nu}f_u(x_a)f_{\bar{d}}(x_b)+h_{hh'du,\mu\nu}f_{\bar{d}}(x_a)f_u(x_b)\right)L^{\mu\nu}_{hh'}. \tag{3.18}$$

where the Dirac δ was expanded by multiplying out the brackets. Another simplification is to integrate over the δ function with respect to η, which yields

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}p_{T\ell}^2} = \frac{1}{16\pi}\sum_{hh'}\frac{\left(h_{hh'ud,\mu\nu}f_u(x_a)f_{\bar{d}}(x_b)+h_{hh'du,\mu\nu}f_{\bar{d}}(x_a)f_u(x_b)\right)L^{\mu\nu}_{hh'}}{2Qp_{\ell T}\sinh(Y-\eta)}, \tag{3.19}$$

and a constraint for η, being now defined as

$$\eta = Y - \cosh^{-1}\frac{Q}{2p_{T\ell}}. \tag{3.20}$$

Evidently this also requires that $p_{T\ell} < \frac{Q}{2}$, i.e. the transverse momentum of leptons may only take certain values limited by this relation. Note that this only holds true at leading-order, i.e., when $\vec{q}_T = 0$. Introducing Θ as the Heaviside function and using the leptonic tensor for the W case from Eq. (2.14) yields

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}p_{lT}^2} = \frac{1}{16\pi}\sum_{hh'}\Bigg(&\frac{(h_{hh'ud,\mu\nu}f_u(x_a)f_{\bar{d}}(x_b)+u\leftrightarrow\bar{d})L^+_{hh'ud}K^{\mu\nu}}{2Qp_{T\ell}\sinh(Y-\eta)} \\ &-i\frac{(h_{hh'ud,\mu\nu}f_u(x_a)f_{\bar{d}}(x_b)+u\leftrightarrow\bar{d})L^-_{hh'ud}\epsilon(\mu,\nu k_1,k_2)}{2Qp_{T\ell}\sinh(Y-\eta)}\Bigg)\Theta(\frac{Q}{2}-p_{T\ell}).\end{aligned} \tag{3.21}$$

Since, for the W case, all helicity combinations produce the same prefactor, and due to $|V_{ud}|^2 = |V_{du}|^2$, all the PDFs have the same common factor here, up to a sign. Hence, summing up all the products (see Appendix E) of hard functions and leptonic tensor as given by (3.14) and (2.14) respectively, yields

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}p_{T\ell}^2} = \frac{1}{16\pi}\frac{f_u(x_a)f_{\bar{d}}(x_b) + u \leftrightarrow \bar{d}}{2Qp_{T\ell}\sinh(Y-\eta)}\Theta(\frac{Q}{2} - p_{T\ell}) \\ \times 2\frac{|V_{ud}|^2|P_W|^2}{N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_\mathrm{w}}\right)^2\left(p_{T\ell}Qe^{Y-\eta} - p_{T\ell}^2\right).\end{aligned} \tag{3.22}$$

Putting in the constraint $p_{T\ell} < \frac{Q}{2}$ in the form of $\eta = Y - \cosh^{-1}\frac{Q}{2p_{T\ell}}$, one arrives at the differential following cross section:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}p_{T\ell}^2} &= \frac{1}{16\pi}\frac{f_u(x_a)f_{\bar{d}}(x_b) + u \leftrightarrow \bar{d}}{Qp_{T\ell}\sinh(\cosh^{-1}\frac{Q}{2p_{T\ell}})}\Theta(\frac{Q}{2} - p_{T\ell}) \\ &\quad\times 2\frac{|V_{ud}|^2|P_W|^2}{N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_\mathrm{w}}\right)^2\left(p_{T\ell}Qe^{\cosh^{-1}\frac{Q}{2p_{T\ell}}} - p_{T\ell}^2\right) \\ &= \frac{1}{16\pi E_\mathrm{CM}^2}\frac{f_u(x_a)f_{\bar{d}}(x_b) + u \leftrightarrow \bar{d}}{Q\sqrt{Q^2 - 4p_{T\ell}^2}}\Theta(\frac{Q}{2} - p_{T\ell}) \\ &\quad\times 2\frac{|V_{ud}|^2|P_W|^2}{N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_\mathrm{w}}\right)^2\left(p_{T\ell}Qe^{\cosh^{-1}\frac{Q}{2p_{T\ell}}} - p_{T\ell}^2\right)\end{aligned} \tag{3.23}$$

An intuitive way to check this result is to plot $\mathrm{d}\sigma/\mathrm{d}p_{Tl}$. Hence, integrating the previous result over two of the three variables, produces the following equation form:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}p_{T\ell}} = \int \mathrm{d}Q\mathrm{d}Y \cdot 4p_{T\ell}\frac{1}{16\pi E_\mathrm{CM}^2}\frac{f_u(x_a)f_{\bar{d}}(x_b) + u \leftrightarrow \bar{d}}{\sqrt{Q^2 - 4p_{lT}^2}}\Theta(\frac{Q}{2} - p_{T\ell}) \\ \times 2\frac{|V_{ud}|^2|P_W|^2}{N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_\mathrm{w}}\right)^2\left(p_{T\ell}Qe^{\cosh^{-1}\frac{Q}{2p_{T\ell}}} - p_{T\ell}^2\right)\end{aligned} \tag{3.24}$$

Note that $x_{a,b}$ can be related to Q and Y as $x_{a,b} = \frac{Q}{E_\mathrm{CM}}e^{\pm Y}$. Furthermore, to get the cross section for annihilating quarks of all flavours, this needs to be summed up for all u and d indices, where $u = \{u, c\}$ and $d = \{d, s, b\}$.

The sketch of this graph agrees with the expected behaviour (see Fig. 4 and Fig. 5). Furthermore, one can see that in the narrow width approximation (NWA), i.e. $\Gamma_W \to 0$ (see Appendix F), there is a sharp cut-off at $m_W/2$, which comes from the constraint on $p_{T\ell}$. Nonetheless, for a finite width, some spill over happens into the $p_{T\ell} > m_W/2$ region. The PDFs used for this calculation were provided by LHAPDF [13] and interfaced by SCETlib [14].

However, when comparing the graph directly with the points from Monte Carlo-based fixed order code MCFM 8.2 [15], some disagreement can be clearly seen (see Fig. 6). One

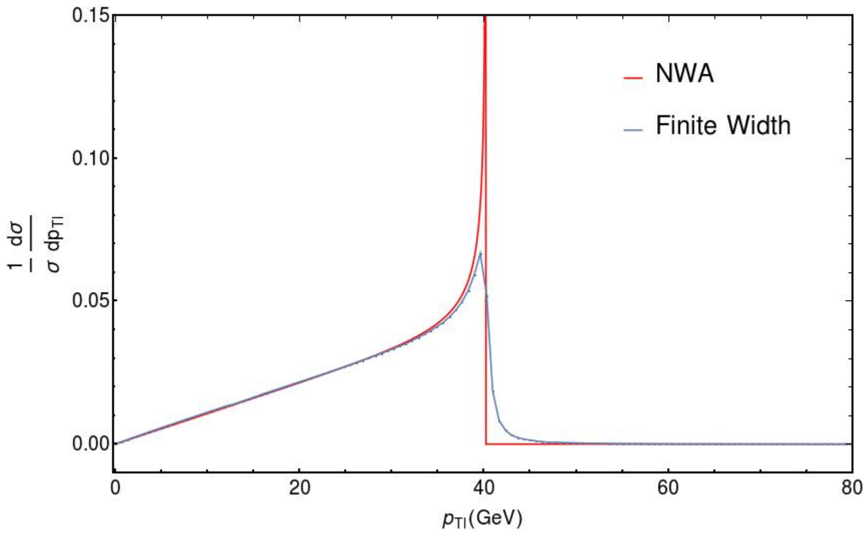

Figure 4. Sketch of $\frac{\mathrm{d}\sigma}{\mathrm{d}p_{T\ell}}$ as given by Eq. (3.24). A sharp cut-off is seen at $m_W/2$ in NWA case, but some spill over happens in the finite width case (see Fig. 5).

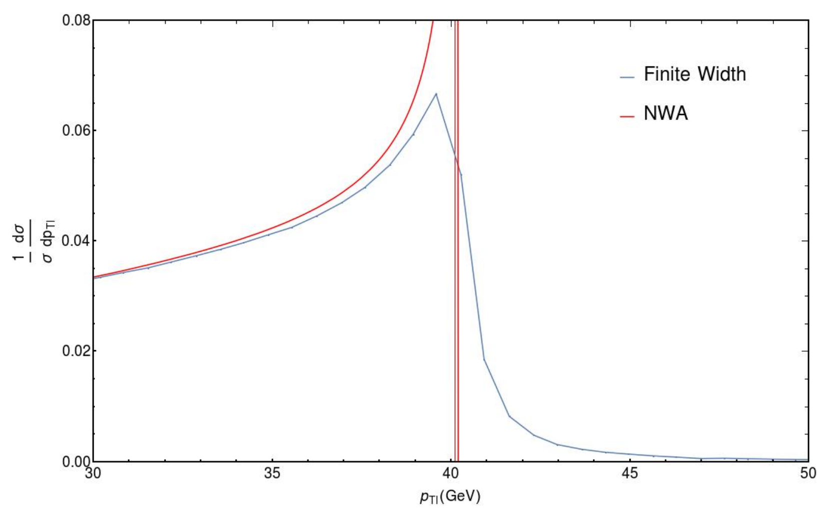

Figure 5. Sketch of differential sp[ectrum $\frac{\mathrm{d}\sigma}{\mathrm{d}p_{T\ell}}$ as given by Eq. (3.24), zoomed in on finite width graph. Spill over due to finite width can clearly be seen.

of the causes, may be binning artifacts. Monte Carlo points correspond to binned points from 2 GeV wide regions, whereas the graphs produced here were integrated in the whole region. Nonetheless, this does not explain the mismatch in low-$p_{T\ell}$ region, where any binning artifacts should be irrelevant. Furthermore, integrating Eq. (3.24) over the same 2 GeV wide bins and plotting bin by bin does not reproduce the Monte Carlo result (see Fig. 7). Finally, inefficient integration over the point close to the fall-off ($p_{T\ell} = m_W/2$) may also produce a mismatch, however, again this does not provide a good explanation for the disagreement in the lower $p_{T\ell}$ region. Further analysis of the approach described in Section 3.2 will be needed, however, due to time constraints of the project will not be presented here.

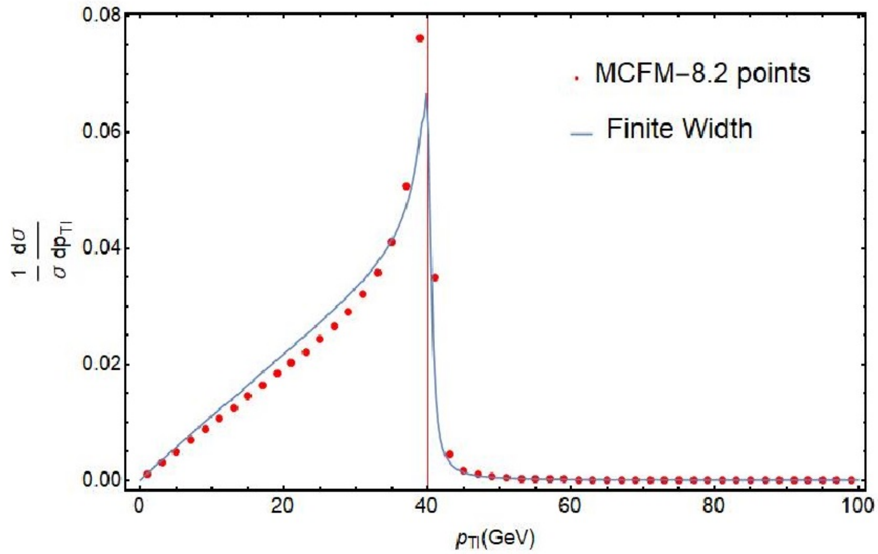

Figure 6. Monte Carlo data compared with the differential spectrum $\frac{\mathrm{d}\sigma}{\mathrm{d}p_{T\ell}}$ given by Eq. (3.24). A mismatch can be seen.

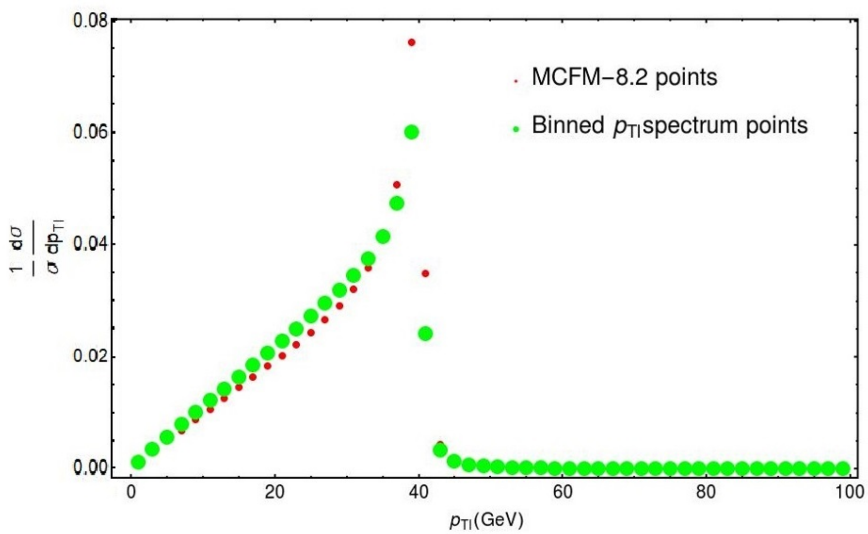

Figure 7. Monte Carlo data compared with the points from 2 GeV wide integrated regions of $\frac{\mathrm{d}\sigma}{\mathrm{d}p_{T\ell}}$ given by Eq. (3.24). The selected bins were centered at MCFM data points. A mismatch can be seen.

4 Summary and Outlook

The W boson mass is a very important parameter in the Standard Model that is currently challenging to directly measure in collider experiments. One of the limiting factors is the treatment of the W decay in the leptonic center-of-mass frame, where the momenta of both leptons must be known for efficient reconstruction of the frame. In this project, the foundations for a treatment of the decay in the lab frame were analysed.

The Drell-Yan process of two quarks annihilating into an intermediate boson and its subsequent decay into two leptons was treated by encoding the quark annihilation process in the hadronic tensor, and the decay process in the leptonic tensor. The most general leptonic tensors were calculated for the Z/γ and $W^{\pm}$ decay cases. We show that they may be written in a similar form by decomposing them in terms of different parities. In addition, the elements of the hadronic tensor in the center-of-mass frame of leptons were expressed in terms of the coefficients of a general Lorentz decomposition of the hadronic tensor.

The produced leptonic tensors may be used to calculate the cross section at any order. However, due to time constraints only the leading order was considered. The calculation of the leading order differential cross section of the transverse spectrum of the lepton momenta $\frac{\mathrm{d}\sigma}{\mathrm{d}p_{T\ell}}$ produced the correct qualitative behaviour of the function, explicitly showing a fall-off at $m_W/2$, which was very sharp in the case of narrow width approximation. Nonetheless, some spill over to the $p_{T\ell} > m_W/2$ in the finite width case could be seen, as expected. The data points from a Monte Carlo simulation at leading-order showed some deviations from the expected distributions here, however. Further research is needed to clarify why the results produced here show disagreement with the Monte Carlo data generated by MCFM 8.2 code.

Appendices

A Expressing unpolarised cross section in spherical harmonics

Consider Eqs.(1.16)-(1.18). Rewriting $\cos 2\phi = 1 - 2\sin^2\phi = 2\cos^2\phi - 1$ and dropping the common factor in front of each term, one can write:

$$\begin{aligned}
&(1-\sin^2\theta\cos^2\phi)H_{11}\\
&=(1-\sin^2\theta\cdot\frac{1}{2}(\cos 2\phi+1))H_{11}\\
&=(1-\frac{1}{2}\sin^2\theta\cos 2\phi-\frac{1}{2}\sin^2\theta)H_{11}\\
&=(1-\frac{1}{2}\sin^2\theta\cos 2\phi-\frac{1}{2}(1-\cos^2\theta))H_{11}\\
&=(1-\frac{1}{2}\sin^2\theta\cos 2\phi-\frac{1}{2}(1-\cos^2\theta))H_{11}\\
&=\frac{1}{2}(1+\cos^2\theta-\sin^2\theta\cos 2\phi)H_{11}
\end{aligned} \tag{A.1}$$

and

$$(1-\sin^2\theta\sin^2\phi)H_{22}=\frac{1}{2}(1+\cos^2\theta+\sin^2\theta\cos 2\phi)H_{22} \tag{A.2}$$

$$(1-\cos^2\theta)H_{33}=\frac{1}{2}(2+\cos^2\theta-3\cos^2\theta)H_{33}, \tag{A.3}$$

which expresses the the coefficients in Eqs. (1.16)-(1.18) in terms of spherical harmonics.

B Expressing $H_{\mu\nu}L^{\mu\nu}$ in terms of angular coefficients

Consider Eq. (1.25). One can take out the unpolarised cross section in Eq. (1.27) as the common factor to yield:

$$\begin{aligned}
&L^{11}H_{11}+L^{22}H_{22}+L^{33}H_{33}=\\
&=\frac{8\pi\alpha^2}{3q^2}Q_fQ_{f'}(H_{11}+H_{22}+H_{33})\frac{3}{16\pi}\Big((1+\cos^2\theta)\\
&+(1-3\cos^2\theta)\frac{H_{33}}{H_{11}+H_{22}+H_{33}}\\
&+\sin^2\theta\cos 2\phi\frac{H_{22}-H_{11}}{H_{11}+H_{22}+H_{33}}\Big).
\end{aligned} \tag{B.1}$$

Which reproduces exactly Eq. (1.29). Consider Eqs.(1.19)-(1.24). Summing them up yields:

$$\begin{aligned}
&L^{12}H_{12}+L^{21}H_{21}+L^{13}H_{13}+L^{31}H_{31}+L^{23}H_{23}+L^{32}H_{32}=\\
&-\frac{\alpha^2}{2q^2}Q_fQ_{f'}\Big(\sin^2\theta\sin 2\phi H_{21}+\sin^2\theta\sin 2\phi H_{12}\\
&\qquad+\sin 2\theta\sin\phi H_{23}+\sin 2\theta\sin\phi H_{32}\\
&\qquad+\sin 2\theta\cos\phi H_{13}+\sin 2\theta\cos\phi H_{31}\Big)
\end{aligned} \tag{B.2}$$

Again taking out the unpolarised cross section as the common prefactor:

$$= \frac{8\pi\alpha^2}{3q^2} Q_f Q_{f'} (H_{11} + H_{22} + H_{33}) \frac{3}{16\pi} \Big(\sin^2\theta \sin 2\phi \frac{-(H_{12} + H_{21})}{H_{11} + H_{22} + H_{33}} + \sin 2\theta \sin\phi \frac{-(H_{23} + H_{32})}{H_{11} + H_{22} + H_{33}} + \sin 2\theta \cos\phi \frac{-(H_{13} + H_{31})}{H_{11} + H_{22} + H_{33}} \Big) \tag{B.3}$$

Given the set of angular coefficients in Eq. (1.28), one can immediatelly sum Eqs. (B.1) and (B.3) to get Eq. (1.30).

C Trace Evaluations for Z/γ leptonic tensor

As in Eq. (1.10), one can reduce the rather complicated structures of Eqs.(2.2)-(2.4) to the evaluation of traces. In the structures, four types of traces will appear:

$$\sum_{\text{spins}} [\bar{v}(k_1)\gamma^\mu u(k_2)][\bar{u}(k_2)\gamma^\nu v(k_1)] = \mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu] \tag{C.1}$$

$$\sum_{\text{spins}} [\bar{v}(k_1)\gamma^\mu (v_l - a_l\gamma^5) u(k_2)][\bar{u}(k_2)\gamma^\nu v(k_1)] = v_l \mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu] - a_l \mathrm{Tr}[\not{k}_1\gamma^\mu\gamma^5 \not{k}_2\gamma^\nu] \tag{C.2}$$

$$\sum_{\text{spins}} [\bar{v}(k_1)\gamma^\mu u(k_2)][\bar{u}(k_2)\gamma^\nu (v_l - a_l\gamma^5) v(k_1)] = v_l \mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu] - a_l \mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu\gamma^5] \tag{C.3}$$

$$\sum_{\text{spins}} [\bar{v}(k_1)\gamma^\mu (v_l - a_l\gamma^5) u(k_2)][\bar{u}(k_2)\gamma^\nu (v_l - a_l\gamma^5) v(k_1)] = v_l^2 \mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu] + a_l^2 \mathrm{Tr}[\not{k}_1\gamma^\mu\gamma^5 \not{k}_2\gamma^\nu\gamma^5] - v_l a_l \left(\mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu\gamma^5] + \mathrm{Tr}[\not{k}_1\gamma^\mu\gamma^5 \not{k}_2\gamma^\nu] \right) \tag{C.4}$$

By using the commutation properties of γ^5 and $\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4i\epsilon^{\mu\nu\rho\sigma}$ [8] all the traces can be evaluated to yield:

$$\mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu] = 4(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu}(k_1 \cdot k_2)) \tag{C.5}$$

$$\mathrm{Tr}[\not{k}_1\gamma^\mu\gamma^5 \not{k}_2\gamma^\nu] = 4ik_{1\rho}k_{2\sigma}\epsilon^{\rho\mu\sigma\nu} = -4ik_{1\rho}k_{2\sigma}\epsilon^{\mu\nu\rho\sigma} \tag{C.6}$$

$$\mathrm{Tr}[\not{k}_1\gamma^\mu \not{k}_2\gamma^\nu\gamma^5] = -4ik_{1\rho}k_{2\sigma}\epsilon^{\mu\nu\rho\sigma} \tag{C.7}$$

$$\mathrm{Tr}[\not{k}_1\gamma^\mu\gamma^5 \not{k}_2\gamma^\nu\gamma^5] = 4(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu}(k_1 \cdot k_2)). \tag{C.8}$$

Using the relations in Eq. (C.5)-(C.8) with Eqs.(2.2)-(2.4) will yield the Eqs.(2.7)-(2.9).

D Boosting into the Collins-Soper frame

Consider two protons with momenta given in Eq. (2.21) and a boson with momentum given in Eq. (2.22). It is always possible to do such a rotation so that one of the spatial components of q_μ would vanish. In this case, $q_2 = 0$ is chosen. To express Eq. (2.21) in the leptonic COM frame (CS frame [9]), two Lorentz boosts are required: a longitudinal and a transverse one. In this particular frame, the intermediate boson must be at rest, so that

$$q_\mu{}' = (q_0{}', 0, 0, 0). \tag{D.1}$$

Applying a boost in the z direction with velocity $\beta_L = \frac{q^3}{q^0}$ and again in x direction with velocity $\beta_T = \frac{q^1}{\sqrt{(q^0)^2-(q^3)^2}}$, one arrives at

$$q^{\mu\prime} = (\sqrt{(q^0)^2 - (q^1)^2 - (q^3)^2}, 0, 0, 0) = (Q, 0, 0, 0). \tag{D.2}$$

Hence, applying the same boosts to Eq. (2.21) will yield an expression for the proton momentum in the same frame. Applying the first boost in the z direction for P_a:

$$\begin{aligned} P_a^{\text{lab}} = \begin{pmatrix} E_{\text{CM}}/2 \\ 0 \\ 0 \\ E_{\text{CM}}/2 \end{pmatrix} &\rightarrow \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E_{\text{CM}}/2 \\ 0 \\ 0 \\ E_{\text{CM}}/2 \end{pmatrix} \\ &= E_{\text{CM}}/2 \begin{pmatrix} \sqrt{\frac{1-\beta}{1+\beta}} \\ 0 \\ 0 \\ \sqrt{\frac{1-\beta}{1+\beta}} \end{pmatrix} = e^{-Y} E_{\text{CM}}/2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \end{aligned} \tag{D.3}$$

where $Y = \frac{1}{2} \ln \frac{1+\beta}{1-\beta} = \frac{1}{2} \ln \frac{q^3+q^0}{q^3-q^0}$ is the rapidity of the protons. Applying the second boost in the x direction yields

$$\begin{aligned} e^{-Y} E_{\text{CM}}/2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} &\rightarrow e^{-Y} E_{\text{CM}}/2 \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \\ &= e^{-Y} E_{\text{CM}}/2 \begin{pmatrix} \frac{\sqrt{Q^2+(q^1)^2}}{Q} \\ -\frac{q_1}{Q} \\ 0 \\ 1 \end{pmatrix} \equiv P_a^{\text{CS}}. \end{aligned} \tag{D.4}$$

Similarly, for the other proton momentum P_b one arrives at

$$P_b^{\text{lab}} = \begin{pmatrix} E_{\text{CM}}/2 \\ 0 \\ 0 \\ E_{\text{CM}}/2 \end{pmatrix} \rightarrow e^{Y} E_{\text{CM}}/2 \begin{pmatrix} \frac{\sqrt{Q^2+(q^1)^2}}{Q} \\ -\frac{q_1}{Q} \\ 0 \\ -1 \end{pmatrix} \equiv P_b^{\text{CS}} \tag{D.5}$$

The two momenta $P^{\rm CS}_{a,b}$ can still be related. Defining

$$P^\mu \equiv E_{\rm CM}/2 \begin{pmatrix} \frac{\sqrt{Q^2+(q^1)^2}}{Q} \\ -\frac{q_1}{Q} \\ 0 \\ 1 \end{pmatrix}, \tag{D.6}$$

one then has such relations between $P^{\rm CS}_{a,b}$:

$$P_a^\mu = e^{-Y} P^\mu \qquad\qquad P_b^{0,1,2} = e^Y P^{0,1,2} \qquad\qquad P_b^3 = -e^Y P^3. \tag{D.7}$$

Using these to construct the hadronic form factors in the CS frame explicitly from Eq. (2.20):

$$\begin{aligned}
H_{11} &= -H_1 + H_2(P_{a,1})^2 + H_3(P_{b,1})^2 + 2H_4 P_{a,1}P_{b,1}, \\
H_{22} &= -H_1, \\
H_{33} &= -H_1 + H_2(P_{a,3})^2 + H_3(P_{b,3})^2 + 2H_4 P_{a,3}P_{b,3}, \\
H_{13} &= H_2(P_{a,1}P_{a,3}) + H_3(P_{b,1}P_{b,3}) + H_4(P_{a,1}P_{b,3} + P_{b,1}P_{a,3}), \\
&\quad + iH_5(P_{a,1}P_{b,3} - P_{b,1}P_{a,3}) \\
H_{12} &= -iH_6 P_{a,3}Q - iH_7 P_{b,3}Q + H_8\Big(- P_{a,1}Q(P_{a,1}P_{b,3} - P_{a,3}P_{b,1})\Big), \\
&\quad + H_9\Big(- P_{b,1}Q(P_{a,1}P_{b,3} - P_{a,3}P_{b,1})\Big) \\
H_{32} &= iH_6 P_{a,1}Q + iH_7 P_{b,1}Q + H_8\Big(- P_{a,3}Q(P_{a,1}P_{b,3} - P_{a,3}P_{b,1})\Big), \\
&\quad + H_9\Big(- P_{b,3}Q(P_{a,1}P_{b,3} - P_{a,3}P_{b,1})\Big),
\end{aligned} \tag{D.8}$$

where applying the simplifications given in Eqs.(D.7) allows to write the hadronic form factors as Eqs.(2.29)-(2.34).

E W Lepton tensor contracted with the leading order hard functions

Looking at the products of hard functions given by Eqs.(3.14) and the decomposed leptonic tensor of W decay given by Eq. (2.19), one gets:

$$\begin{aligned}
h^{\mu\nu}_{AAud} L^+_{AAud} K^{\mu\nu} = h^{\mu\nu}_{VVud} L^+_{VVud} K^{\mu\nu} &= -\frac{1}{2N_c}\left(g^{\mu\nu} - \frac{1}{2}(n^\mu \bar{n}^\nu + n^\mu \bar{n}^\nu)\right) L_{VVud} K^{\mu\nu} \\
&= -\frac{|V_{ud}|^2 |P_W|^2}{2N_c}\left(\frac{\pi\alpha}{q^2 \sin^2\theta_{\rm w}}\right)^2 \Bigg(2(k_1\cdot k_2) - 4(k_1\cdot k_2) \\
&\quad - \frac{1}{2}(2k_1^- k_2^+ + 2k_1^+ k_2^- - 4(k_1\cdot k_2))\Bigg) \\
&= \frac{|V_{ud}|^2 |P_W|^2}{2N_c}\left(\frac{\pi\alpha}{q^2 \sin^2\theta_{\rm w}}\right)^2 \left(k_1^- k_2^+ + k_1^+ k_2^- \right) \\
&= \frac{|V_{ud}|^2 |P_W|^2}{N_c}\left(\frac{\pi\alpha}{q^2 \sin^2\theta_{\rm w}}\right)^2 \left(\frac{p_{T\ell}Qe^{Y-\eta} + p_{T\ell}Qe^{-Y+\eta} - 2p_{Tl}^2}{2}\right),
\end{aligned} \tag{E.1}$$

and

$$
\begin{aligned}
h^{\mu\nu}_{AVud} L^{-}_{AVud} i\epsilon(\mu,\nu,k_1,k_2) &= h^{\mu\nu}_{VAud} L^{-}_{VAud} i\epsilon(\mu,\nu,k_1,k_2) \\
&= i\frac{|V_{ud}|^2|P_W|^2}{4N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\mathrm{w}}}\right)^2 \epsilon_{\mu\nu\lambda\kappa} n^\lambda \bar{n}^\kappa \epsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} \\
&= i\frac{|V_{ud}|^2|P_W|^2}{4N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\mathrm{w}}}\right)^2 2(\delta^\rho_\lambda\delta^\sigma_\kappa - \delta^\sigma_\lambda\delta^\rho_\kappa) n^\lambda \bar{n}^\kappa k_{1\rho} k_{2\sigma} \\
&= i\frac{|V_{ud}|^2|P_W|^2}{2N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\mathrm{w}}}\right)^2 (n^\rho\bar{n}^\sigma - n^\sigma\bar{n}^\rho) k_{1\rho} k_{2\sigma} \\
&= i\frac{|V_{ud}|^2|P_W|^2}{2N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\mathrm{w}}}\right)^2 (k_1^+ k_2^- - k_1^- k_2^+) \\
&= i\frac{|V_{ud}|^2|P_W|^2}{N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\mathrm{w}}}\right)^2 \left(\frac{p_{T\ell} Q e^{Y-\eta} - p_{T\ell} Q e^{-Y+\eta}}{2}\right)
\end{aligned}
\tag{E.2}
$$

Note that here $k_2^\pm = q^\pm - k^\pm = q^\pm - p_\ell^\pm$ and $k_{1T} = k_{2T} = p_{T\ell}$ were used in both expressions. Summing up the terms from Eqs.(E.2) with (E.1):

$$
\begin{aligned}
&(h^{\mu\nu}_{AAud} L^{+}_{AAud} + h^{\mu\nu}_{VVud} L^{+}_{VVud}) K^{\mu\nu} - i(h^{\mu\nu}_{AVud} L^{-}_{AVud} + h^{\mu\nu}_{VAud} L^{-}_{VAud})\epsilon(\mu,\nu,k_1,k_2) \\
&= 2\frac{|V_{ud}|^2|P_W|^2}{N_c}\left(\frac{\pi\alpha}{q^2\sin^2\theta_{\mathrm{w}}}\right)^2 \left(p_{T\ell} Q e^{-Y+\eta} - p_{T\ell}^2\right)
\end{aligned}
\tag{E.3}
$$

Using these relations, Eq. (3.21) can be expressed as Eq. (3.22).

F Narrow width approximation

Consider the limit representation of the Dirac Delta function, as a Lorentzian going to zero

$$\delta(x) = \frac{2}{\pi}\lim_{\epsilon\to 0}\frac{\epsilon}{x^2+\epsilon^2}. \tag{F.1}$$

In the Narrow Width Approximation (NWA), one has to consider vanishing width, i.e. $\Gamma \to 0$. The propagator given in Eq. (2.5) can be written as:

$$|P_W|^2 = \frac{1}{\left(1-\frac{m_W^2}{q^2}\right)^2 + \left(\frac{m_W\Gamma_W}{Q^2}\right)^2} = \frac{Q^4}{(Q^2-m_W^2)^2 + (m_W\Gamma_W)^2}. \tag{F.2}$$

Using Eq. (F.1), in the limit of NWA this then becomes:

$$\lim_{\Gamma\to 0}|P_W|^2 = \frac{\pi}{2}\frac{Q^4}{(m_W\Gamma_W)}\delta(Q^2-m_W^2) = \frac{\pi}{2}\frac{Q^4}{(m_W^2\Gamma_W)}\delta(Q-m_W). \tag{F.3}$$

Consider a cross section differential only in Q:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q} = A(Q)|P_W|^2, \tag{F.4}$$

where $A(Q)$ is a function dependent only on the variable Q. Hence, in the NWA limit:

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{NWA}}{\mathrm{d}Q} &= A(m_W) \times \frac{\pi}{2} \frac{Q^4}{(m_W^2 \Gamma_W)} \delta(Q - m_W) \\
&= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q} \frac{1}{|P_W|^2} \right) \Bigg|_{Q=m_W} \times \frac{\pi}{2} \frac{Q^4}{(m_W^2 \Gamma_W)} \delta(Q - m_W) \\
&= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q} \frac{(Q^2 - m_W^2)^2 + (m_W \Gamma_W)^2}{Q^4} \right) \Bigg|_{Q=m_W} \times \frac{\pi}{2} \frac{Q^4}{(m_W^2 \Gamma_W)} \delta(Q - m_W) \\
&= \frac{\mathrm{d}\sigma}{\mathrm{d}Q} \Bigg|_{Q=m_W} m_W^2 \Gamma_W^2 \times \frac{\pi}{2} \frac{1}{(m_W^2 \Gamma_W)} \delta(Q - m_W) \\
&= \delta(Q - m_W) \frac{\pi}{2} \Gamma_W \frac{\mathrm{d}\sigma}{\mathrm{d}Q} \Big|_{Q=m_W}
\end{aligned} \tag{F.5}$$

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