Investigation of the Accuracy of Slowly Varying Envelope Approximation (SVEA) in THz-Pulse Generation and Comparison of Radiation Strengths of Continuous and Discontinuous Delay of the Excitation Matched to the Pulse Approaches in THz-Pulse Generation

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Abstract

In this report, we present two different studies. First study is investigation of the accuracy of slowly varying envelope approximation (SVEA) in THz-pulse generation. The second study is investigation of two different schemes to see which one gives the strongest radiation in THz-pulse generation. These two schemes are continuous delay of the excitation matched to the pulse, which is named as CDM in the report, and discontinuous delay of the excitation matched to the pulse, which is named as DDM. CDM represents the TPF using grating and DDM represents TPF using echelon.

Contents

1 Introduction

In this report, we present two different studies:

In the first one, we did some investigation on how accurate slowly varying envelope approximation (SVEA) is for Gaussian pulses with different number of cycles. In any wave, if the envelope of the wave is varying much slower than the carrier wave in time, then the second derivative term with respect to time can be neglected in complex wave equation, which results a simpler version of the wave equation to handle. That's why SVEA is often used. In the first study, we tried to see the relationship between the accuracy of the SVEA and the number of cycles of a Gaussian pulse.

In the second study, we did some investigation of single-cycle pulse generation using two different approaches: (1) continuous delay of the excitation matched to the pulse, which is named as CDM in the report, and (2) discontinuous delay of the excitation matched to the pulse, which is named as DDM. CDM represents the TPF using grating and DDM represents TPF using echelon. We compared these two approaches to see which one gives stronger radiation.

2 Theoretical Background

2.1 Formulation of Field Update Equation

$$\frac{\partial^2 V}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho(z,t)}{\varepsilon} \tag{1}$$

V: scalar potential field

ρ: charge density

$$\rho(z,t) = -q * \delta\big(z - z_s(t)\big) \tag{2}$$

q: charge of the electron

$z_s(t)$: position of the electron

Numerical Formulation:

For simplicity, let's make the following substitution for now:

$$f_z = -\frac{\rho(z,t)}{\varepsilon} \tag{3}$$

So, (1) is equivalent to the following:

$$\frac{\partial^2 V}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = f_z \tag{4}$$

Discretizing the equation, we obtain the following:

$$\frac{V_{i+1}^n - 2V_i^n + V_{i-1}^n}{dz^2} - \frac{1}{c^2}\frac{V_i^{n+1} - 2V_i^n + V_i^{n-1}}{dt^2} = {f_z}_i^n \tag{5}$$

$$V_i^n \triangleq V(i * dz, n * dt) \tag{6}$$

$${f_z}_i^n \triangleq f_z(i * dz, n * dt) \tag{7}$$

Rearranging the terms in (5), we get the following equation:

$$V_i^{n+1} = 2V_i^n - V_i^{n-1} + \left(\frac{c*dt}{dz}\right)^2 * (V_{i+1}^n - 2V_i^n + V_{i-1}^n) - (c*dt)^2 * {f_z}_i^n \tag{8}$$

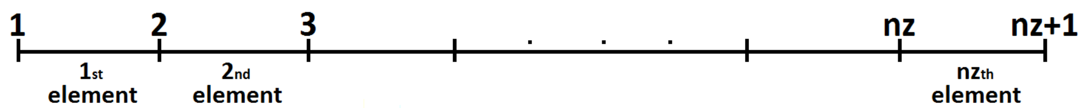


Figure 1: FDTD discretization

Now, we need to determine ${f_z}_i^n$ term in terms of our particle trajectory. f_z is a dirac-delta function at the position of the particle.

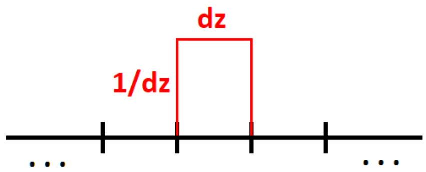


Figure 2: Dirac-Delta function representation in FDTD

The problem is that position of the particle might not be at the grid points (might be inside a cell) in our FDTD domain; however, only the grid points are defined in the computational domain. So, we will make an approximation described in the following figure. If particle is inside a cell, we will decompose the charge density into the neighbor grid points as reversely proportional to the distance between the particle and the grid point.

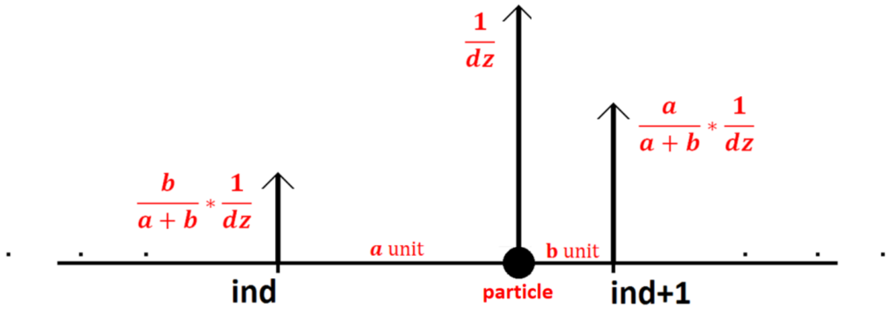


Figure 3: Charge density decomposition

Applying this approximation, we can calculate ${f_z}_i^n$ for a given particle trajectory.

2.2 Slowly Varying Envelope Approximation (SVEA)

Slowly varying envelope approximation (SVEA) is an approximation that applies when the envelope of a wave varies slowly in time and/or space compared to the wave itself. This approximation allows us to neglect the second order derivatives in the wave equation, which results simpler equation to handle. To apply SVEA, we need to go to complex phasor representation as follows:

$$\frac{\partial^2 V}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = f_z \tag{9}$$

Let's jump to the phasor domain taking the following equalities into account:

$$V = real\{V_0 e^{-jwt}\} \tag{10}$$

$$f_z = real\{f_0 e^{-jwt}\} \tag{11}$$

Plugging (10) and (11) into (9), our complex wave equation becomes as follows:

$$\frac{\partial^2}{\partial z^2}\left(V_0 e^{-jwt}\right) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(V_0 e^{-jwt}\right) = f_0 e^{-jwt} \tag{12}$$

It is equivalent to the following equation:

$$\frac{\partial^2 V_0}{\partial z^2} e^{-jwt} - \frac{e^{-jwt}}{c^2}\left[\frac{\partial^2 V_0}{\partial t^2} - j2w\frac{\partial V_0}{\partial t} - w^2 V_0\right] = f_0 e^{-jwt} \tag{13}$$

Simplifying the exponentials, we get the following equation:

$$\frac{\partial^2 V_0}{\partial z^2} - \frac{1}{c^2}\left[\frac{\partial^2 V_0}{\partial t^2} - j2w\frac{\partial V_0}{\partial t} - w^2 V_0\right] = f_0 \tag{14}$$

SVEA assumption: $\left|\frac{\partial^2 V_0}{\partial t^2}\right| \ll 2w\left|\frac{\partial V_0}{\partial t}\right| \quad and/or \quad \left|\frac{\partial^2 V_0}{\partial t^2}\right| \ll w^2|V_0|$

Now let's apply SVEA only in time as follows:

$$\frac{\partial^2 V_0}{\partial z^2} - \frac{1}{c^2}\left[\cancel{\frac{\partial^2 V_0}{\partial t^2}} - j2w\frac{\partial V_0}{\partial t} - w^2 V_0\right] = f_0 \tag{15}$$

So, our approximated equation is

$$\frac{\partial^2 V_0}{\partial z^2} - \frac{1}{c^2}\left[-j2w\frac{\partial V_0}{\partial t} - w^2 V_0\right] = f_0 \tag{16}$$

Rearranging the terms, we get the following equation:

$$\frac{\partial V_0}{\partial t} = j\frac{w}{2}V_0 + j\frac{c^2}{2w}\left(\frac{\partial^2 V_0}{\partial z^2} - f_0\right) \tag{17}$$

Discretizing our equation, we get the following numerical equation:

$$\frac{{V_0}_i^{n+1} - {V_0}_i^n}{dt} = j\frac{w}{2}{V_0}_i^n + j\frac{c^2}{2w}\left(\frac{{V_0}_{i+1}^n - 2{V_0}_i^n + {V_0}_{i-1}^n}{dz^2} - {f_0}_i^n\right) \tag{18}$$

Rearranging the terms, we obtain the following update equation:

$${V_0}_i^{n+1} = {V_0}_i^n + j\frac{w\,dt}{2}{V_0}_i^n + j\frac{c^2 dt}{2w}\left(\frac{{V_0}_{i+1}^n - 2{V_0}_i^n + {V_0}_{i-1}^n}{dz^2} - {f_0}_i^n\right) \tag{19}$$

2.3 Absorbing Boundary Conditions

2.3.1 Without SVEA

2.3.1.1 Left Boundary ($\boldsymbol{z = 0}$)

$$\frac{\partial V}{\partial z} - \frac{1}{c}\frac{\partial V}{\partial t} = 0 \tag{20}$$

$$\frac{\partial V}{\partial t} = c\frac{\partial V}{\partial z} \tag{21}$$

Discretizing the equation, we get the following numerical equation:

$$\frac{V_1^{n+1} - V_1^n}{dt} = c\frac{V_2^n - V_1^n}{dz} \tag{22}$$

Rearranging the terms, we obtain boundary-update equation:

$$V_1^{n+1} = V_1^n + \left(\frac{cdt}{dz}\right)(V_2^n - V_1^n) \tag{23}$$

2.3.1.2 Right Boundary ($\boldsymbol{z = m * dz}$)

$$\frac{\partial V}{\partial z} + \frac{1}{c}\frac{\partial V}{\partial t} = 0 \tag{24}$$

$$\frac{\partial V}{\partial t} = -c\frac{\partial V}{\partial z} \tag{25}$$

Discretizing the equation, we get the following numerical equation:

$$\frac{V_m^{n+1} - V_m^n}{dt} = c\frac{V_m^n - V_{m-1}^n}{dz} \tag{26}$$

Rearranging the terms, we obtain boundary-update equation:

$$V_m^{n+1} = V_m^n + \left(\frac{cdt}{dz}\right)(V_m^n - V_{m-1}^n) \tag{27}$$

2.3.2 With SVEA

2.3.2.1 Left Boundary $(\boldsymbol{z = 0})$

$$\frac{\partial V}{\partial z} - \frac{1}{c}\frac{\partial V}{\partial t} = 0 \tag{28}$$

$$\frac{\partial V}{\partial t} = c\frac{\partial V}{\partial z} \tag{29}$$

$$V = real\{V_0 e^{-jwt}\} \tag{30}$$

Plugging (30) into (29),

$$\frac{\partial}{\partial t}\left(V_0 e^{-jwt}\right) = c\frac{\partial}{\partial z}\left(V_0 e^{-jwt}\right) \tag{31}$$

Taking derivatives and canceling the exponentials,

$$\frac{\partial V_0}{\partial t} - jwV_0 = c\frac{\partial V_0}{\partial z} \tag{32}$$

Discretizing the equation, we get the following numerical equation:

$$\frac{{V_0}_1^{n+1} - {V_0}_1^{n}}{dt} - jw{V_0}_1^{n} = c\frac{{V_0}_2^{n} - {V_0}_1^{n}}{dz} \tag{33}$$

Rearranging the terms, we obtain boundary-update equation:

$${V_0}_1^{n+1} = {V_0}_1^{n} + jwdt{V_0}_1^{n} + \left(\frac{cdt}{dz}\right)\left({V_0}_2^{n} - {V_0}_1^{n}\right) \tag{34}$$

2.3.2.2 Right Boundary ($z = m * dz$)

$$\frac{\partial V}{\partial z} + \frac{1}{c}\frac{\partial V}{\partial t} = 0 \tag{35}$$

$$\frac{\partial V}{\partial t} = -c\,\frac{\partial V}{\partial z} \tag{36}$$

$$V = real\{V_0 e^{-jwt}\} \tag{37}$$

Plugging (37) into (36),

$$\frac{\partial}{\partial t}\left(V_0 e^{-jwt}\right) = -c\,\frac{\partial}{\partial z}(V_0 e^{-jwt}) \tag{38}$$

Taking derivatives and canceling the exponentials,

$$\frac{\partial V_0}{\partial t} - jwV_0 = -c\,\frac{\partial V_0}{\partial z} \tag{39}$$

Discretizing the equation, we get the following numerical equation:

$$\frac{{V_0}_m^{n+1} - {V_0}_m^n}{dt} - jw{V_0}_m^n = -c\,\frac{{V_0}_m^n - {V_0}_{m-1}^n}{dz} \tag{40}$$

Rearranging the terms, we obtain boundary-update equation:

$${V_0}_m^{n+1} = {V_0}_m^n + jwdt{V_0}_m^n - \left(\frac{cdt}{dz}\right)\left({V_0}_m^n - {V_0}_{m-1}^n\right) \tag{41}$$

3 Simulations

3.1 1st Simulation

3.1.1 Description

In this simulation, we simulated the following case:

There is a particle in space, and it oscillates around the origin. Since particle oscillates, it radiates electric and magnetic field. We simulated this radiation solving the Maxwell's equation in two different ways: In the 1st way, we solved the equations in real domain and discretized the resulting equation by FDTD. In the second way, we solved the same equations by complex phasor representation and applied slowly varying envelope approximation (SVEA). So, we checked how accurate the SVEA is. We checked the accuracy of the SVEA for different number of cycles.

Aim: to obtain the relationship of the accuracy of SVEA and number of cycles.

3.1.2 Formulation

Particle Trajectory

$$z_s(t) = A_z * \sin(wt) * e^{-2\ln 2\frac{(t-t_0)^2}{\tau^2}} \tag{42}$$

z_s : position of the particle

$\frac{2\pi}{w}$: cycle duration

τ : pulse duration

$$number\ of\ cycles = \frac{\tau}{\frac{2\pi}{w}}$$

3.1.2.1 Without SVEA

Let's start with (8), which is the update equation without SVEA:

$$V_i^{n+1} \overset{(8)}{=} 2V_i^n - V_i^{n-1} + \left(\frac{c*dt}{dz}\right)^2 * (V_{i+1}^n - 2V_i^n + V_{i-1}^n) - (c*dt)^2 * {f_z}_i^n \tag{43}$$

We need to determine the f_z in terms of the particle trajectory.

$$f_z \overset{(3)}{=} -\frac{\rho(z,t)}{\varepsilon} \overset{(2)}{=} -\frac{q}{\varepsilon} * \delta\big(z - z_s(t)\big) \tag{44}$$

Applying dirac delta approximation described in theoretical background, we can calculate ${f_z}_i^n$ as follows:

$$ind = floor\left(\frac{z_s(ndt)}{dz}\right) \tag{45}$$

$${f_z}_i^n = \begin{cases} -\frac{q}{\varepsilon} * \frac{1}{dz} * \left[ind + 1 - \frac{z_s(ndt)}{dz}\right] & if \quad i = ind \\ -\frac{q}{\varepsilon} * \frac{1}{dz} * \left[\frac{z_s(ndt)}{dz} - ind\right] & if \quad i = ind + 1 \\ 0 & else \end{cases} \tag{46}$$

3.1.2.2 With SVEA

Let's start with (19), which is the update equation with SVEA:

$${V_0}_i^{n+1} \overset{(19)}{=} {V_0}_i^n + j\frac{wdt}{2}{V_0}_i^n + j\frac{c^2 dt}{2w}\left(\frac{{V_0}_{i+1}^n - 2{V_0}_i^n + {V_0}_{i-1}^n}{dz^2} - {f_0}_i^n\right) \tag{47}$$

We need to determine the f_0 in terms of the particle trajectory.

$$f_z \overset{(11)}{=} real\{f_0 e^{-jwt}\} \tag{48}$$

We know that f_z is determined by $z_s(t) = A_z * \sin(wt) * e^{-2\ln 2\frac{(t-t_0)^2}{\tau^2}}$

To determine f_0 , let's first find an imaginary particle trajectory such that:

$$z_s(t) = real\{z_{sc}(t)e^{-jwt}\} \tag{49}$$

So, the solution is clearly as follows:

$$z_{sc}(t) = A_z * e^{-2\ln 2\frac{(t-t_0)^2}{\tau^2}} * e^{\pi/2} \tag{50}$$

Note: $e^{\pi/2}$ appears in $z_{sc}(t)$ for phase correction because we have $\sin(wt)$ term in $z_s(t)$ instead of $\cos(wt)$.

Then we can determine f_0 as follows:

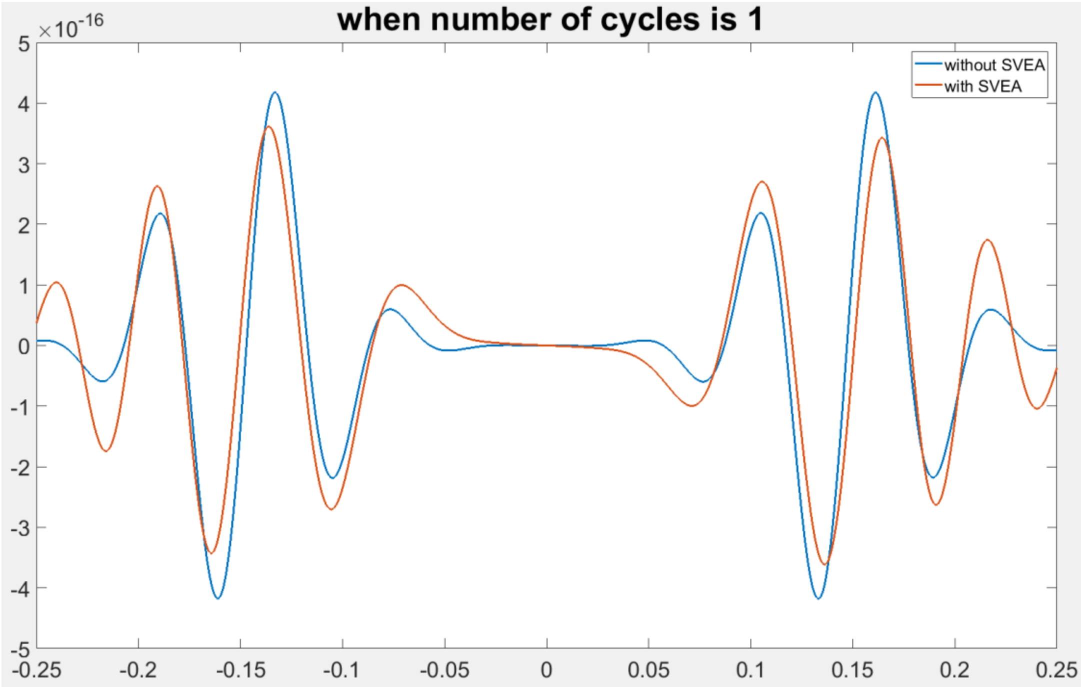
$$f_0(z,t) = -q * \delta\big(z - z_{sc}(t)\big) \tag{51}$$

Numerical formulation of $f_0(z,t)$:

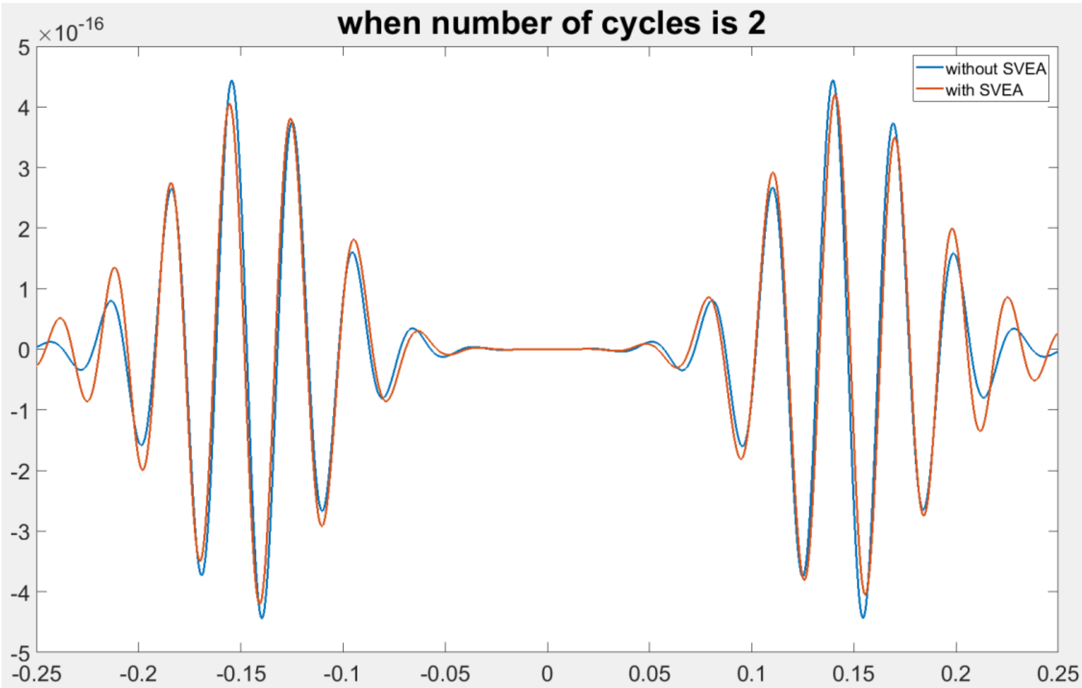
$$indc = floor\left(\frac{z_{sc}(ndt)}{dz}\right) \tag{52}$$

$$f_{0}{}_i^n = \begin{cases} -\frac{q}{\varepsilon} * \frac{1}{dz} * \left[indc + 1 - \frac{z_{sc}(ndt)}{dz}\right] & if \quad i = indc \\ -\frac{q}{\varepsilon} * \frac{1}{dz} * \left[\frac{z_{sc}(ndt)}{dz} - indc\right] & if \quad i = indc + 1 \\ 0 & else \end{cases} \tag{53}$$

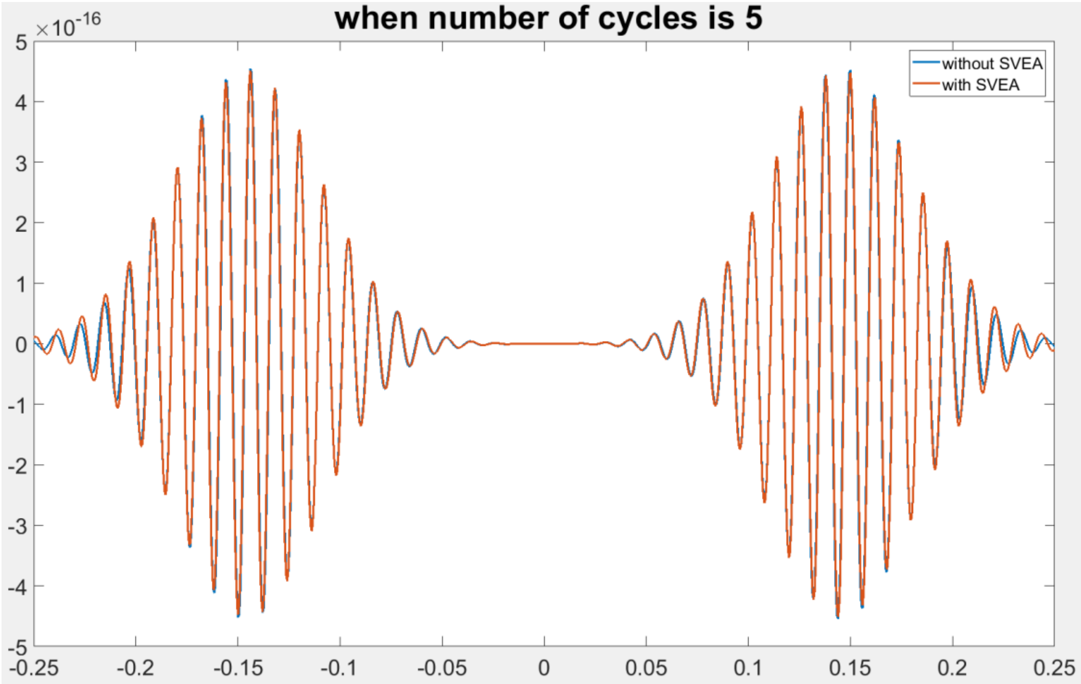
3.1.3 Results



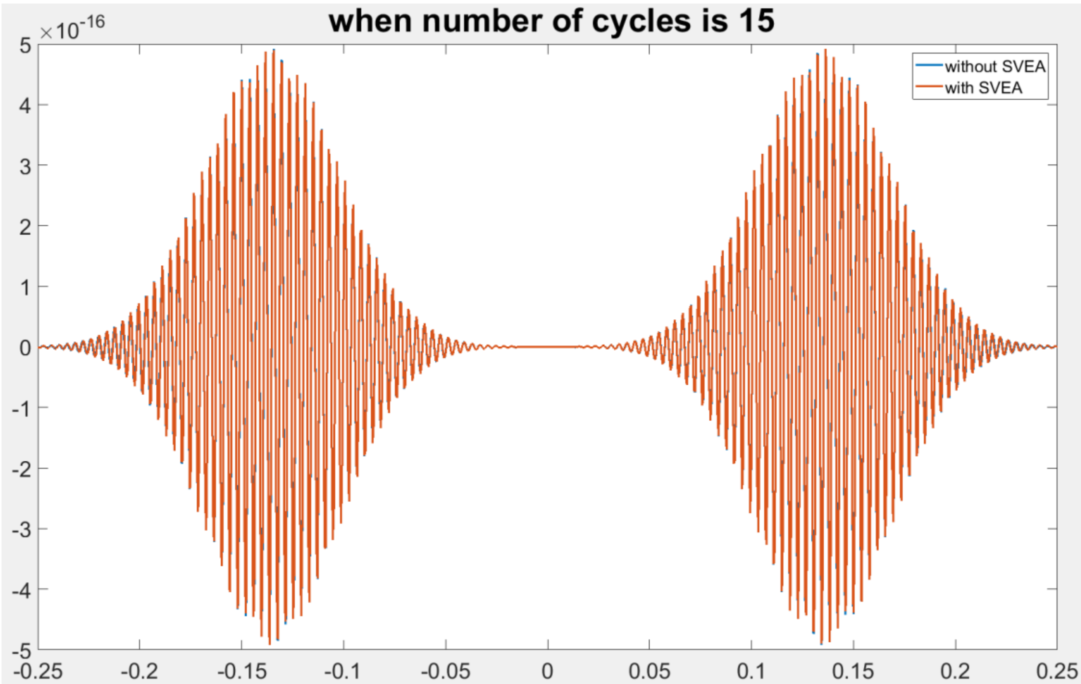
Graph 1: Radiation strength vs Space coordinates after the radiated for $\boldsymbol{T_0}$ seconds



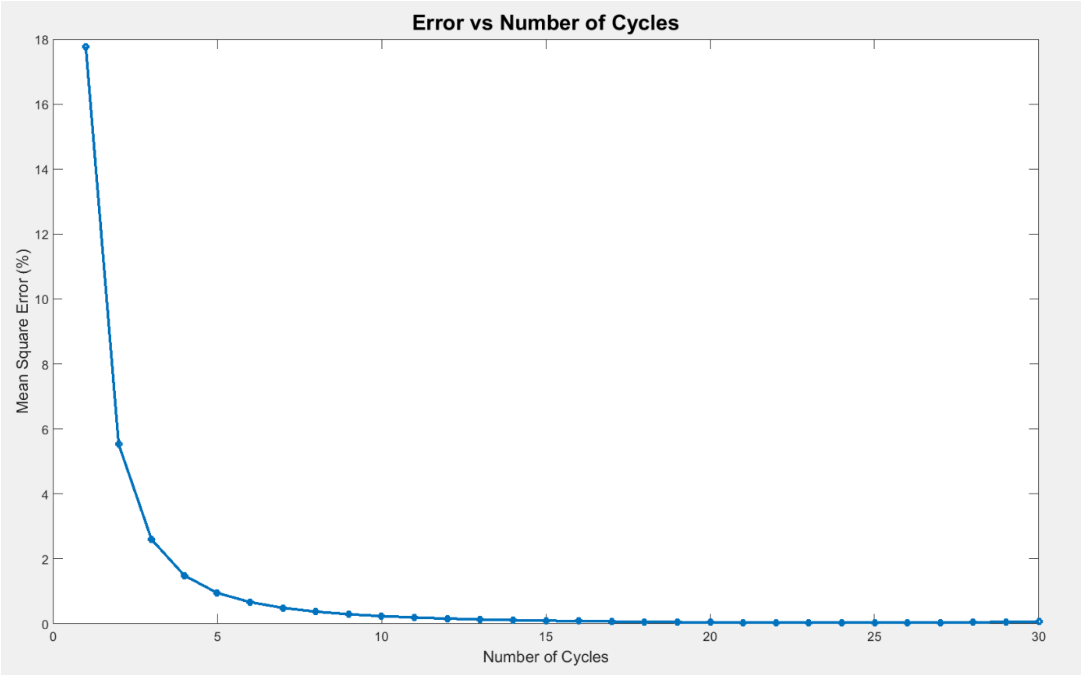
Graph 2: Radiation strength vs Space coordinates after the radiated for $\boldsymbol{T_0}$ seconds



Graph 3: Radiation strength vs Space coordinates after the radiated for $\boldsymbol{T_0}$ seconds



Graph 4: Radiation strength vs Space coordinates after the particle radiated for $\boldsymbol{T_0}$ seconds



Graph 5: Error of the SVEA method vs number of cycles of the Gaussian pulse

3.1.4 Conclusion of the 1st Simulation

Results (see Graph 5) clearly show that SVEA in time becomes more and more accurate as the number of cycles of the Gaussian pulse increases. So, this conclusion should be taken into account in single-cycle pulse generation.

3.2 2nd Simulation

3.2.1 Description

In this simulation, we did some investigation of single-cycle pulse generation using two different approaches: (1^{st}) continuous delay of the excitation matched to the pulse, which is named as CDM in the report, and (2^{nd}) discontinuous delay of the excitation matched to the pulse, which is named as DDM. CDM represents the TPF using grating and DDM represents TPF using echelon.

This time we do not have a moving charged particle, instead we have energizing units in space and they energize the field.

Aim: to obtain which approach, CDM or DDM, generates stronger radiation in single-cycle pulse generation

3.2.2 Formulation of CDM

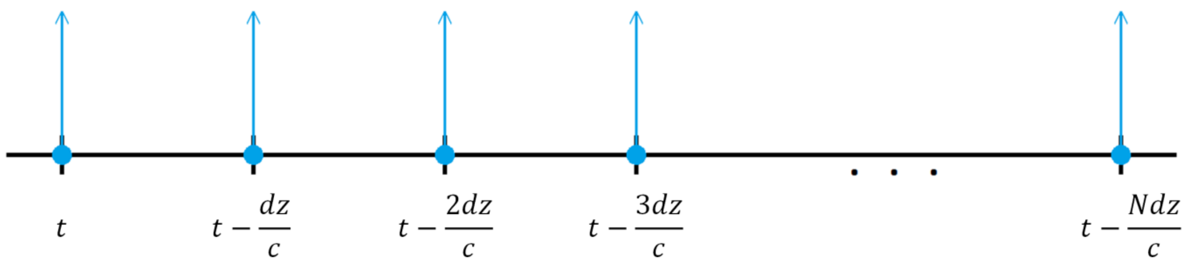


Figure 4: Energizin Scheme for CDM

$$\rho(t,z) = -q * \cos(wt - k_z z) * e^{-2\ln 2\frac{\left(t-\frac{z}{c}\right)^2}{\tau^2}} \tag{54}$$

$\rho(t,z)$: charge density

$k_z = \frac{w}{c}$: wave number

$\frac{2\pi}{w}$: cycle duration

τ : pulse duration

$$number\ of\ cycles = \frac{\tau}{\frac{2\pi}{w}}$$

3.2.2.1 Without SVEA

Let's start with (8), which is the update equation without SVEA:

$$V_i^{n+1} \overset{(8)}{\cong} 2V_i^n - V_i^{n-1} + \left(\frac{c*dt}{dz}\right)^2 * (V_{i+1}^n - 2V_i^n + V_{i-1}^n) - (c*dt)^2 * {f_z}_i^n \tag{55}$$

We need to determine f_z in terms of the energizing scheme.

$$f_z \overset{(3)}{\cong} -\frac{\rho(z,t)}{\varepsilon} \overset{(54)}{\cong} -\frac{q}{\varepsilon} * \cos(wt - k_z z) * e^{-2\ln 2\frac{\left(t-\frac{z}{c}\right)^2}{\tau^2}} \tag{56}$$

Discretizing (56),

$${f_z}_i^n = -\frac{q}{\varepsilon} * \cos(wndt - k_z idz) * e^{-2\ln 2\frac{\left(t-\frac{idz}{c}\right)^2}{\tau^2}} \tag{57}$$

3.2.2.2 With SVEA

Let's start with (19), which is the update equation with SVEA:

$${V_0}_i^{n+1} \overset{(19)}{\cong} {V_0}_i^n + j\frac{wdt}{2}{V_0}_i^n + j\frac{c^2dt}{2w}\left(\frac{{V_0}_{i+1}^n - 2{V_0}_i^n + {V_0}_{i-1}^n}{dz^2} - {f_0}_i^n\right) \tag{58}$$

We need to determine f_0 in terms of our energizing scheme. To do that, let's consider the following equations:

$$f_z \overset{(56)}{\cong} -\frac{q}{\varepsilon} * \cos(wt - k_z z) * e^{-2\ln 2\frac{\left(t-\frac{z}{c}\right)^2}{\tau^2}} \tag{59}$$

$$f_z \overset{(11)}{\cong} real\{f_0 e^{-jwt}\} \tag{60}$$

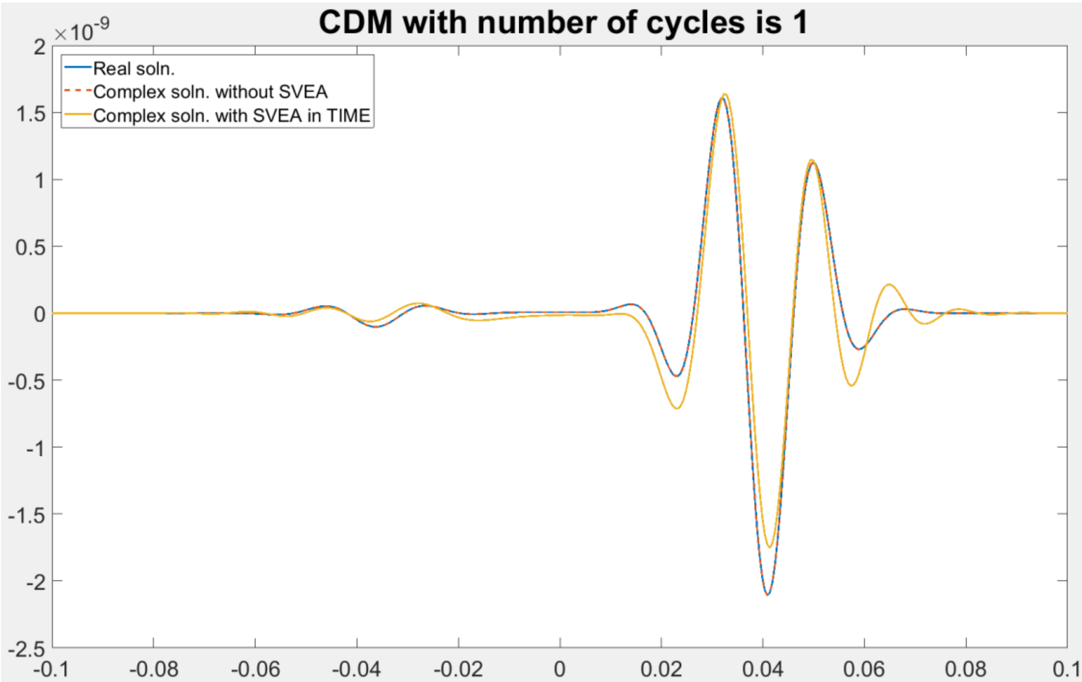
One can trivially determine f_0 as follows:

$$f_0 = -\frac{q}{\varepsilon} * e^{-2\ln 2\frac{\left(t-\frac{z}{c}\right)^2}{\tau^2}} * e^{jk_z z} \tag{61}$$

Discretizing (61), we get the numerical equation as follows:

$$f_{0_i}^{\,n} = -\frac{q}{\varepsilon} * e^{-2\ln 2\frac{\left(t-\frac{idz}{c}\right)^2}{\tau^2}} * e^{jk_z idz} \tag{62}$$

3.2.3 Results



Graph 6: Radiation strength vs Space coordinates for CDM ($\boldsymbol{T_0}$ seconds after the radiation)

3.2.4 Formulation of DDM

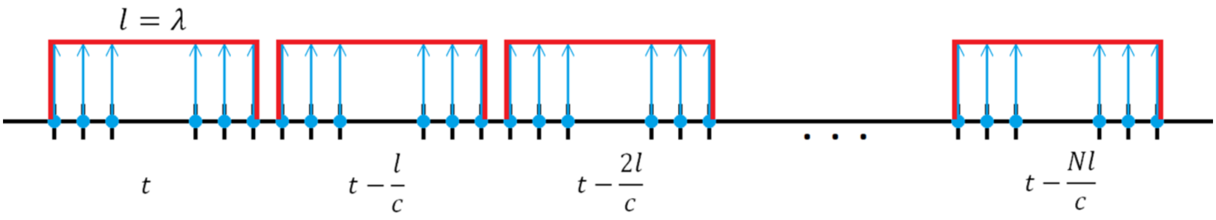


Figure 5: Energizing Scheme for DDM

$$\rho(t,z) = -q * cos\left(wt - k_z * floor\left(\frac{z}{L}\right) * L\right) * e^{-2\,ln\,2\frac{\left(t - \frac{floor\left(\frac{z}{L}\right)*L}{c}\right)^2}{\tau^2}} \tag{63}$$

$\rho(t,z)$: charge density

$k_z = \frac{w}{c}$: wave number

$\frac{2\pi}{w}$: cycle duration

τ : pulse duration

$number\ of\ cycles = \frac{\tau}{\frac{2\pi}{w}}$

$L = l$: energizing block length

$\lambda = c * \frac{2\pi}{w}$: wavelength

3.2.4.1 Without SVEA

Let's start with (8), which is the update equation without SVEA:

$$V_i^{n+1} \overset{(8)}{\widehat{=}} 2V_i^n - V_i^{n-1} + \left(\frac{c*dt}{dz}\right)^2 * (V_{i+1}^n - 2V_i^n + V_{i-1}^n) - (c * dt)^2 * {f_z}_i^n \tag{64}$$

We only need to change f_z term as follows:

$$f_z \overset{(3)}{\widehat{=}} -\frac{\rho(z,t)}{\varepsilon} \overset{(63)}{\widehat{=}} -\frac{q}{\varepsilon} * cos\left(wt - k_z * floor\left(\frac{z}{L}\right) * L\right) * e^{-2\ln 2 \frac{\left(t - \frac{floor\left(\frac{z}{L}\right)*L}{c}\right)^2}{\tau^2}} \tag{65}$$

Discretizing (65),

$$f_{z_i}^{\ n} = -\frac{q}{\varepsilon} * cos\left(wndt - k_z * floor\left(\frac{idz}{L}\right) * L\right) * e^{-2\ln 2 \frac{\left(t - \frac{floor\left(\frac{idz}{L}\right)*L}{c}\right)^2}{\tau^2}} \tag{66}$$

3.2.4.2 With SVEA

Let's start with (19), which is the update equation with SVEA:

$$V_{0_i}^{\ n+1} \overset{(19)}{\widehat{=}} V_{0_i}^{\ n} + j\frac{wdt}{2}V_{0_i}^{\ n} + j\frac{c^2dt}{2w}\left(\frac{V_{0_{i+1}}^{\ n} - 2V_{0_i}^{\ n} + V_{0_{i-1}}^{\ n}}{dz^2} - f_{0_i}^{\ n}\right) \tag{67}$$

We need to determine f_0 in terms of our energizing scheme. To do that, let's consider the following equations:

$$f_z \overset{(65)}{\widehat{=}} -\frac{q}{\varepsilon} * cos\left(wt - k_z * floor\left(\frac{z}{L}\right) * L\right) * e^{-2\ln 2 \frac{\left(t - \frac{floor\left(\frac{z}{L}\right)*L}{c}\right)^2}{\tau^2}} \tag{68}$$

$$f_z \overset{(11)}{\widehat{=}} real\{f_0 e^{-jwt}\} \tag{69}$$

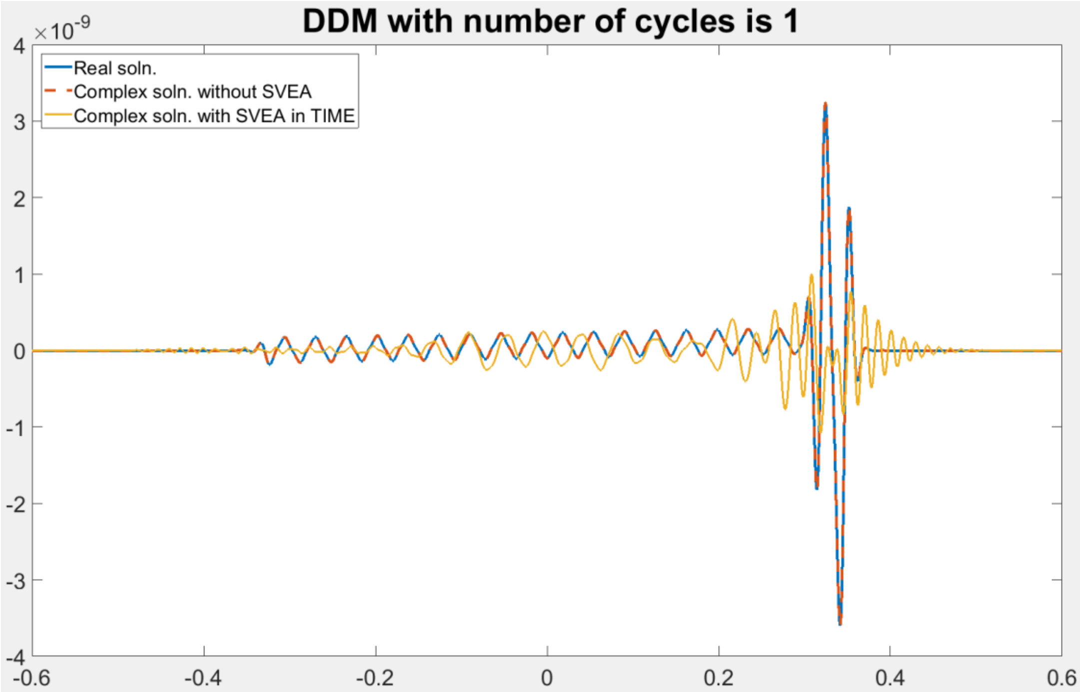
One can trivially determine f_0 as follows:

$$f_0 = -\frac{q}{\varepsilon} * e^{-2\ln 2 \frac{\left(t - \frac{floor\left(\frac{z}{L}\right)*L}{c}\right)^2}{\tau^2}} * e^{jk_z * floor\left(\frac{z}{L}\right)*L} \tag{70}$$

Discretizing (70),

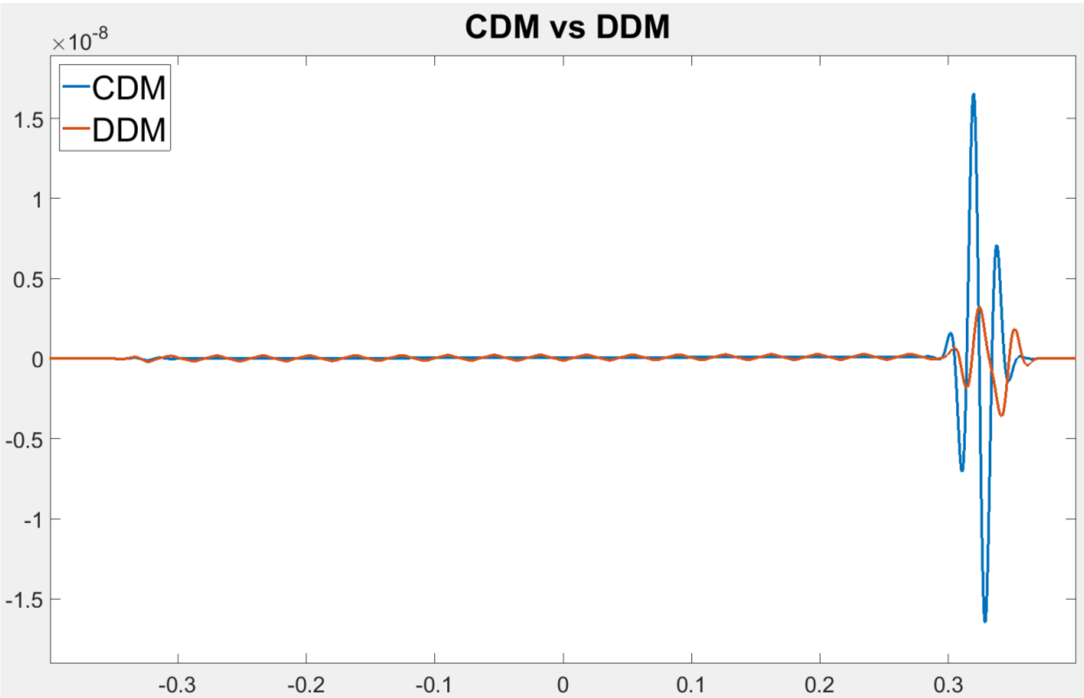
$$f_{0_i}^{\ n} = -\frac{q}{\varepsilon} * e^{-2\ln 2 \frac{\left(ndt - \frac{floor\left(\frac{idz}{L}\right)*L}{c}\right)^2}{\tau^2}} * e^{jk_z * floor\left(\frac{idt}{L}\right)*L} \tag{71}$$

3.2.5 Results



Graph 7: Radiation strength vs Space coordinates for DDM ($\boldsymbol{T_0}$ seconds after the radiation)

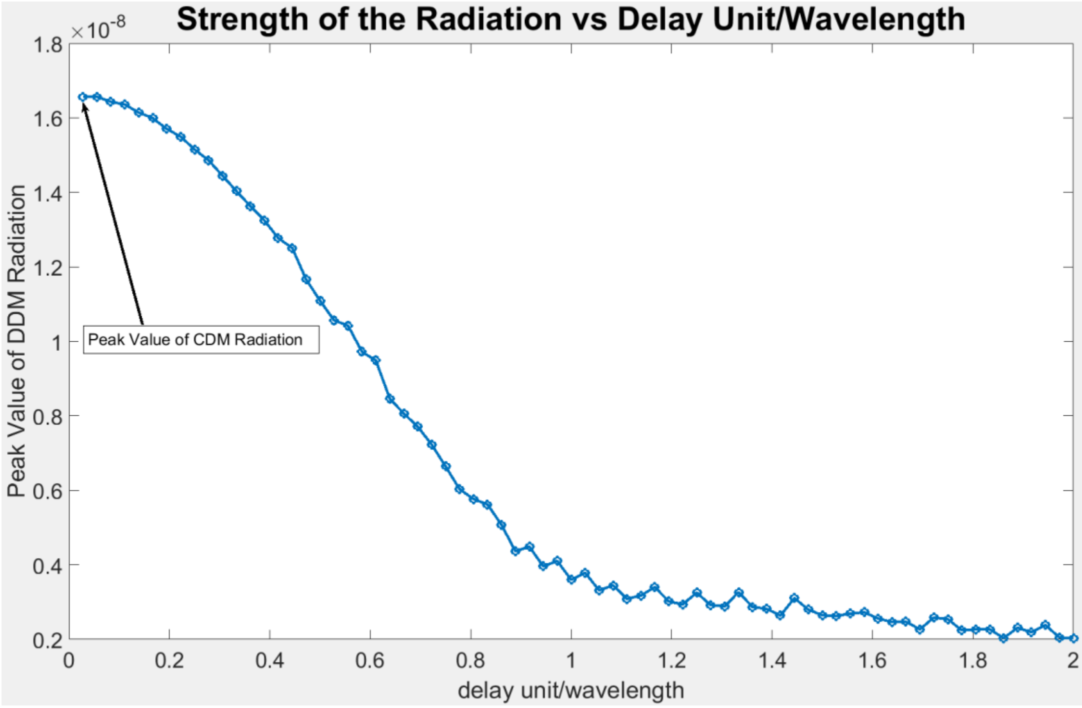
3.2.6 Comparison of CDM and DDM



Graph 8: Radiation strength vs Space coordinates for CDM and DDM

($\boldsymbol{T_0}$ seconds after the radiation)

We did some investigation to reveal what size of the energizing block unit, L, gives the strongest radiation in DDM. In Graph 7, L was set to be a wavelength. Now we varied L/λ ratio keeping λ constant to see what ratio gives the strongest radiation.



Graph 9: Peak value of the radiation for DDM vs delay unit/wavelength ($\mathbf{L/\lambda}$)

3.2.7 Conclusion of the 2nd Simulation

There are several conclusions here.

Graph 6 shows that SVEA can be used in CDM approach.

Graph 7 shows that SVEA is a very bad approximation for DDM approach.

Graph 8 shows that CDM is a much better approach than DDM to generate the strongest radiation.

Graph 9 strongly verifies the conclusion made for **Graph 8** because **Graph 9** shows that the smaller the size of the energizing blocks chooses the stronger radiation is obtained. One needs to notice that when the size of the energizing blocks in DDM gets smaller and smaller, then DDM converges to CDM. So, **Graph 9** verifies that CDM is the best version of DDM to generate the strongest field.

4 References

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[2] Taflove, A., & Hagness, S. C. (2010). *Computational electrodynamics: The finite-difference time-domain method*. Boston, MA: Artech House.