Generation of narrow spectrum IR radiation for production of THz radiation.

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September 5, 2018

Abstract

In the present work generation of a narrow-line spectral components for production of THz radiation was investigated. Two approaches for this purpose were considered. The approach based on use of spectral filter studied both theoretically and experimentally. The approach connected with use of cascaded four-wave mixing process was studied theoretically. Required setup parameters for the first approach were obtained. Dependence of generated harmonics power on propagation distance in photonic crystal fiber was obtained for the second approach.

Contents

1 Introduction

The presented work is connected with the AXSIS project. In the framework of this project there is being developed compact, THz driven, attosecond X-ray source. In the developed setup, that will have tabletop size, accelerated electrons produce X-ray radiation with keV energies of photons. Multicycle THz radiation is used as undulator and as accelerating component in the developed setup. Why is the THz radiation? Radiation with this frequency allows to use higher amplitudes and achieve higher gradients of electric field in accelerating wave in comparison with microwave radiation, that causes damage of metal details in a setup under high energies of a wave. So, it seems to be that we should use radiation with the highest possible frequency. However, the higher frequency, the smaller wavelength of radiation, and therefore it becomes difficult to put a bunch of accelerated electrons onto the peak of a wave - in this case one should use precise timing with few-femtosecond accuracy. Thus, THz radiation is a kind of compromise between emitted energy of electrons and precision of timing in proposed system.

THz radiation used to accelerate and wiggle electrons should be a multicycle, i.e. it should have a very narrow spectrum. There are considered two possible ways to produce such kind of THz radiation. The first one implies direct selection of narrow spectral components with use of spectral filter and following use of difference frequency generation phenomenon. Second one of them implies use of cascaded four-wave mixing process between two pump lasers with very narrow spectra and, as in previous case, following difference frequency generation. First approach was studied both theoretically and experimentally and the second one was studied theoretically.

2 First approach: spectral filter

2.1 Calculations

In order to produce narrow spectral lines for difference frequency generation of THz radiation one can use spectral filter. The aim is to tranform pulses with spectrum width $\Delta\lambda_{1/e} = 22\,nm$ into pulses with spectrum width $\Delta\lambda_{1/e} = 1.13\,pm$ and central wavelength 1030 nm. In our investigations there was chosen filter based on use of diffraction grating. The proposed filter consists of diffration grating, focusing lens and slit:

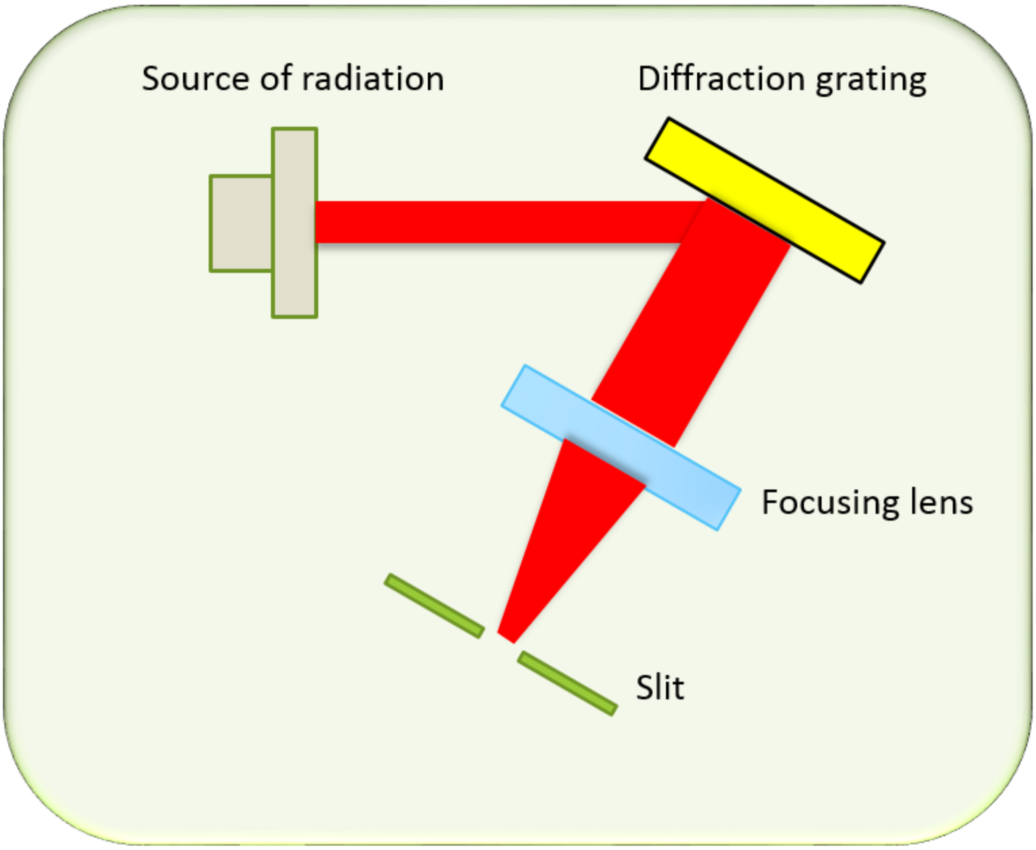


Figure 1: Simplified scheme of experimental setup.

Let's consider the idea of proposed method. Radiation from IR femtosecond laser hits onto diffraction grating. After diffraction harmonics pass through a focusing lens and are focused. Narrow slit is placed at the focal plane, at the focusing place of certain harmonic, that will be the central wavelength of a beam passing through the slit. This slit allows to pass only narrow spectral range of harmonics. Thus in output after slit we have a signal with narrow spectral line. In order to achieve desired spectral width in output ($\Delta\lambda = 1.13\ pm$), experimental setup should be configured precisely with a concrete parameters. In order to select suitable parameters there was written a Matlab program. This program calculates width of signal spectrum passing through the slit. Let's discuss theoretical aspects of calculations contained in this program.
The equation of reflective diffraction grating is:

$$sin\theta_m(\lambda) + sin\theta_i = \frac{m\lambda}{d}$$

where $\theta_m(\lambda)$ is an m-order reflactive angle for given λ, θ_i is an incident angle, m is an order of difraction, d is the distance from the center of one slit to the center of the adjacent slit. With use of this equation there were conducted calculations of spot sizes at the focal plane for different harmonics and its sizes. Calculations were conducted in accordance with wave and ray optics for beam with gaussian shape of intensity profile. Beam sizes before lens were obtained with use of ray optics laws in accordance with assumption that wave optics effects don't emerge on small distances (~0.1m). Beam sizes after lens were calculated in accordance with wave optics, because finite sizes of focused beams should be taken into account for accurate calculations of output spectrum. For all calcualtions I assumed that there are no astigmatism and abberations of a lens.

If initial beam falling onto the grating has a gaussian spectrum and gaussian intensity distribution, then after passing a lens each of harmonics forms a gaussian shape distribution of intensity at focal plane:

$$I(x,\lambda) = I_0(\lambda)e^{-(\frac{x-x_0(\lambda)}{\Delta x_{1/e}})^2}$$

By integration of this distribution over a slit width we obtain energy times meter passed through a fiber:

$$E_{passed}(\lambda) = \int\limits_{x_s}^{x_s+d} I(x,\lambda)\ dx$$

Where x_s - coordinate of left slit border at the focal plane, d - its width. After integration we have:

$$E_{passed}(\lambda) = I_0(\lambda)w_{spot}\ \frac{\sqrt{\pi}}{2\sqrt{2}}\left[erf\Big(\sqrt{2}\frac{x_s+d-x_{spot}}{w_{spot}}\Big) - erf\Big(\sqrt{2}\frac{x_s-x_{spot}}{w_{spot}}\Big)\right]$$

Where $I_0(\lambda)$ - spectral distribution of intensity, $w_{spot} = w_{spot}(\lambda)$ is $1/e^2$ beam radius of given harmonic on lens plane, $x_{spot} = x_{spot}(\lambda)$ is a center coordinate at the focal plane of a beam with wavelength λ. Distribution $E_{passed}(\lambda)$ is not a finite function. However, it is possible to estimate its size, because it has significantly different from zero values only in a finite region. By obtaining of width of this distribution we, in fact, obtain spectral line width of a signal passed through a slit.

2.2 Experimental investigations

In order to achieve desired output spectrum and compare experimental results with results of calculations, there was conducted a set of experiments. The structure of experimental setup is shown below:

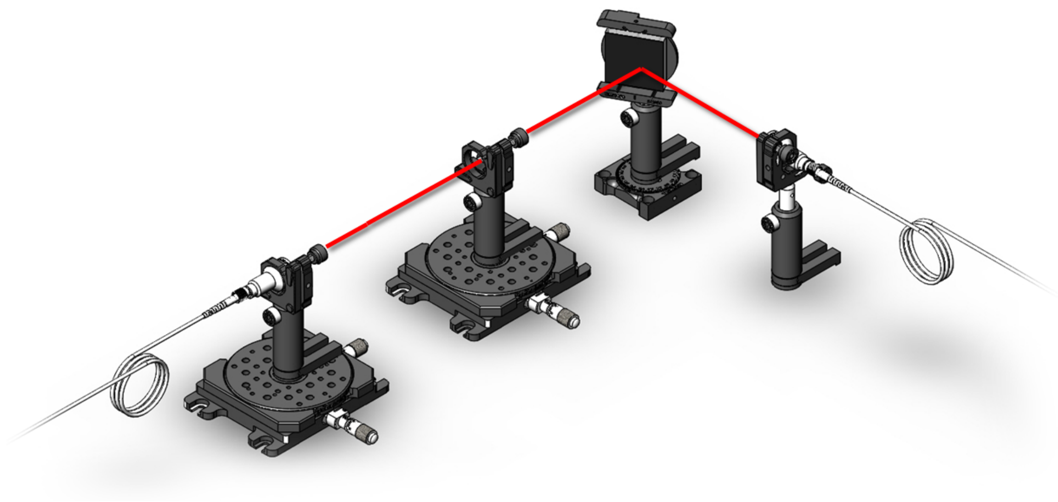


Figure 2: Scheme of experimental setup.

Femtosecond pulses from IR laser source (central wavelength 1033 nm, pulse duration 30 fs, repetition rate 70 MHz) hit onto the diffraction grating. After diffraction, waves with different wavelengths propagate under different angles and pass through a focusing lens. At the focal plane of this lens there is placed pinhole and after it there is mounted a fiber, connected to the spectrometer. Measurements were conducted with use of IR femtosecond laser source with spectrum showed in Figure 3. By approximation of this spectrum by gaussian curve, we obtain its parameters:

- Central wavelength $x_c = 1032.769 \pm 0.026nm$
- $1/e^2\ radius = 10.99 \pm 0.06nm$

In our experiments there was used a diffraction grating with parameters:

- $d^{-1} = 1400\ lines/mm$
- Size $50x50x10\ mm$

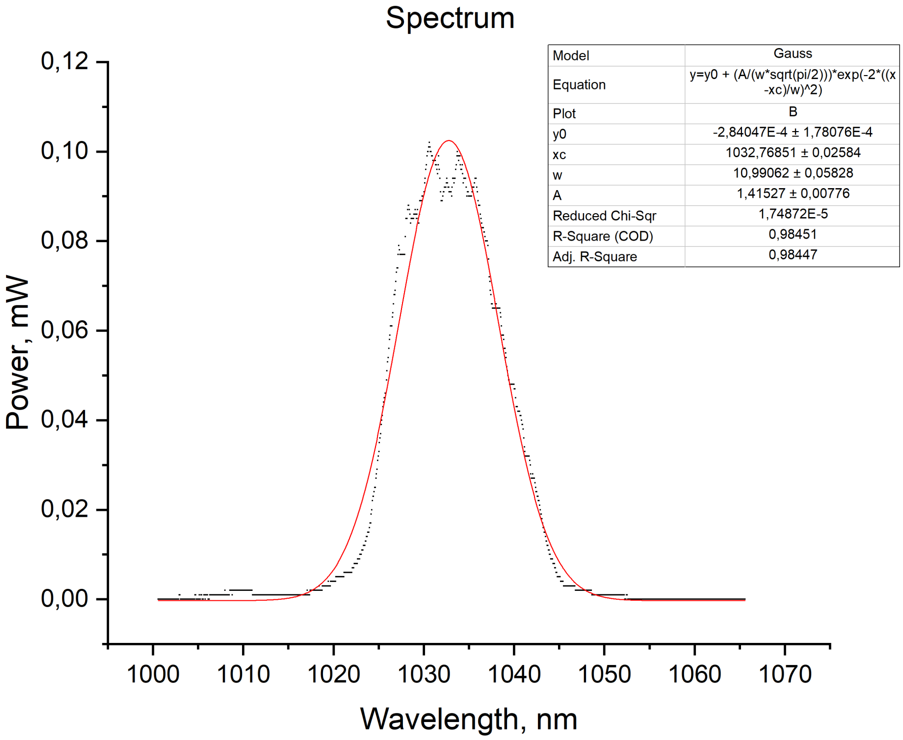


Figure 3: Spectrum of used laser.

Under incident angle $\theta_i = 82°$, focusing lens focal length $F = 0.2\ m$, initial beam radius $w_{in} = 5.23\ mm$ and with use a pinhole with diameter $D = 100\ \mu m$ as a slit there was obtained output spectrum shown on Figure 4.
The $1/e^2$ width of this spectrum in a gaussian approximation is $2w = 895.0 \pm 3.2\ pm$.Under this parameters by calculations there was obtained output spectrum width $2w_{calc} = 328\ pm$. Result obtained by calculations is smaller than experimental one, but we can see concurrence in order of values. Possible issue can be insufficient resolution of spectrometer. The set resolution was 50 pm, but this value hasn't been checked in experiment.

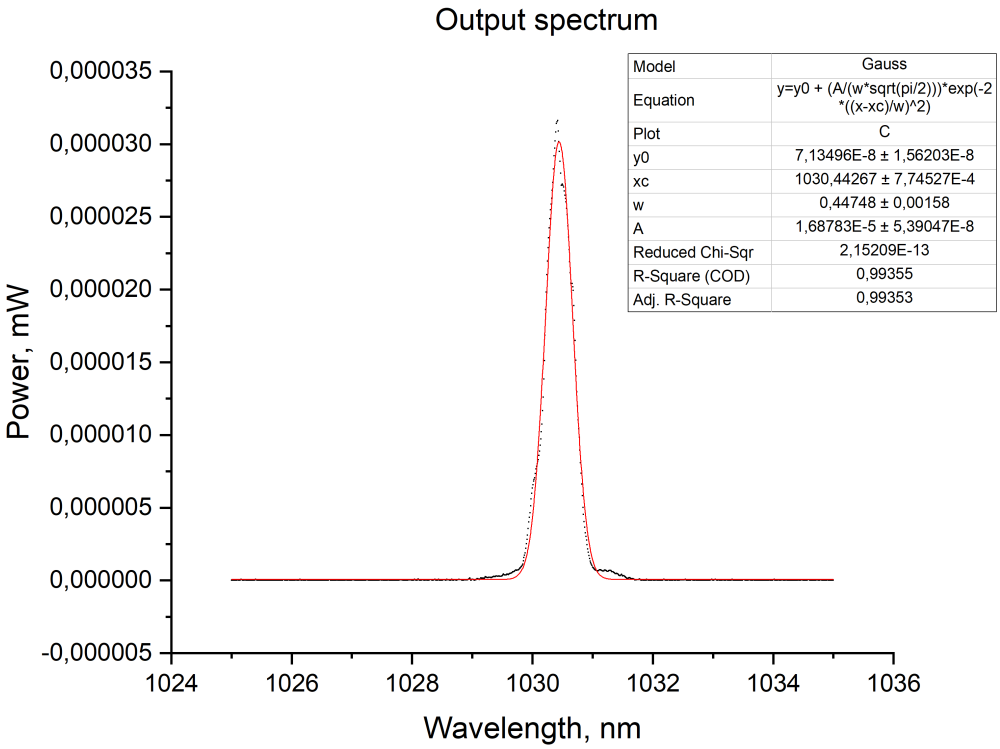


Figure 4: Output spectrum of the 1st experiment.

Under incident angle $\theta_i = 82°$, focusing lens focal length $F = 0.2\ m$, initial beam radius $w_{in} = 5.23\ mm$ and with use a hole with diameter $D = 1\ mm$ as a slit there was obtained output spectrum shown on Figure 5.
The $1/e^2$ width of this spectrum in a gaussian approximation is $2w = 1075.74 \pm 2.38\ pm$. By calculations there was obtained $2w_{calc} = 3198\ pm$. The result of calculations is bigger than the result of experiment. The reason of this divergence isn't clear. It seems like something after the hole limits the input beam diameter.

As long as by calculations spot radius of wave with $\lambda = 1030\ nm$ is $w_{spot} = 1.96\ \mu m$ $(1/e^2)$ and pinhole radius is 50 μm, than a lot of waves with different wavelengths pass through the pinhole. Radii of spots w_{spot} and linear dispersion $\frac{dx}{d\lambda}$ are proportional to focal length of a lens F. Therefore, since we have a pinhole with radius 50 μm it does make sense to use lens with bigger focal length in order to increase spot sizes at focal plane and decrease amount of waves passed through a pinhole. The new lens was used in the next experiment.
In this experiment there was used achromatic lens with focal length $F = 1\ m$. Initial beam radius was $w_{in} = 0.9\ mm$, incident angle was $\theta_i = 84°$. Under these conditions by calculations spot size of wave with $\lambda = 1030\ nm$ is $w_{spot} = 42.5\ \mu m$ $(1/e^2)$, that

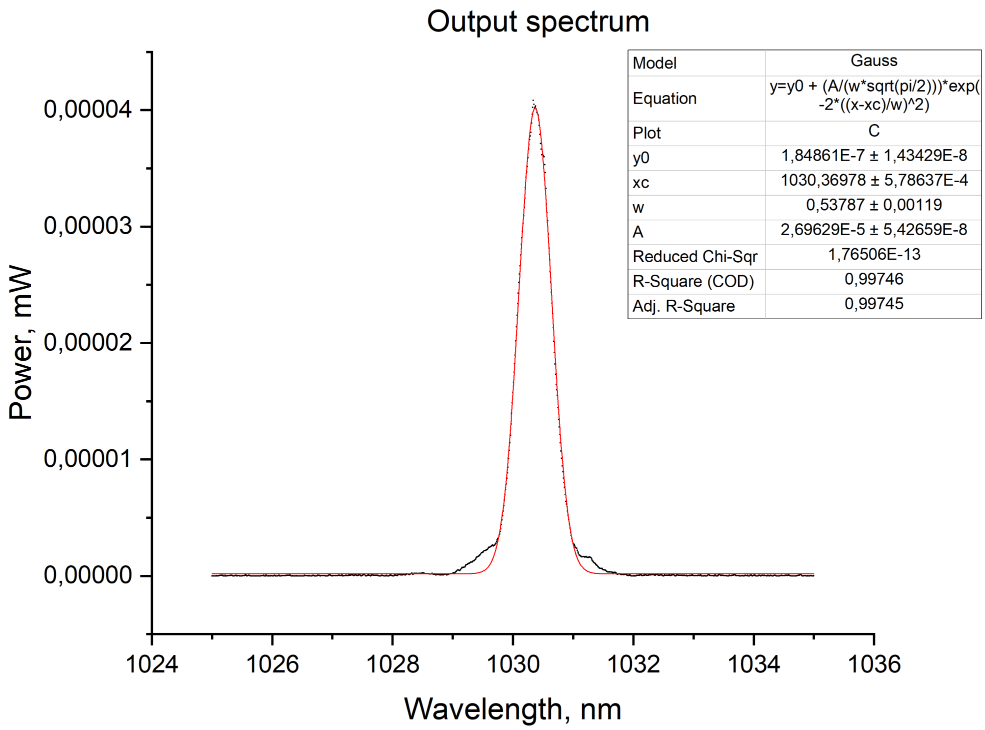


Figure 5: Output spectrum of the 2nd experiment.

is almost the same as radius of a pinhole. The obtained output spectrum is shown on Figure 6.
The $1/e^2$ width of this spectrum in a gaussian approximation is $2w = 750.9 \pm 3.4\ pm$. Under this parameters by calculations there was obtained output spectrum width $2w_{calc} = 692\ pm$. The divergence, again, can be a consequence of insufficient spectrometer resolution.

So, let's summarize the obtained results:

Experiment #	F, m	r_{hole}, μm	w_{in}, mm	$\Delta\lambda^{exp}_{1/e^2}$, pm	$\Delta\lambda^{calc}_{1/e^2}$, pm
1	0.2	50	5.23	895.0 ± 3.2	328
2	0.2	500	5.23	1075.74 ± 2.38	3198
3	1	50	0.9	750.9 ± 3.4	692

The obtained results allow to make a several conclusions. Firstly, increase of output hole diameter causes increase in output spectrum width. Secondly, one can see, that program gives results coinciding with experimental results in order. The divergences in results for 1st and 3rd experiments can be consequences of insufficient resolution of

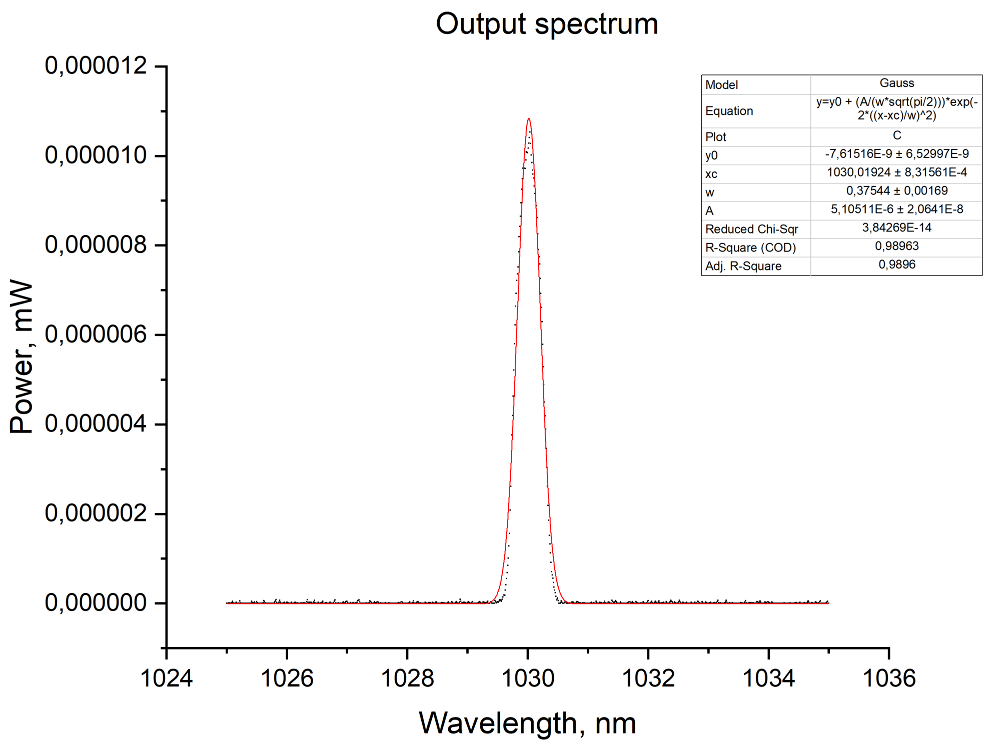


Figure 6: Output spectrum of the 3rd experiment.

used spectrometer. Divergence in the 2nd experiment can be caused by fiber input: it is possible, that input hole of a fiber is smaller, than hole used to implement filtering. Anyway, experimental results shows, that for selection of desired spectral range it is necessary to apply another configuration of experimental setup.

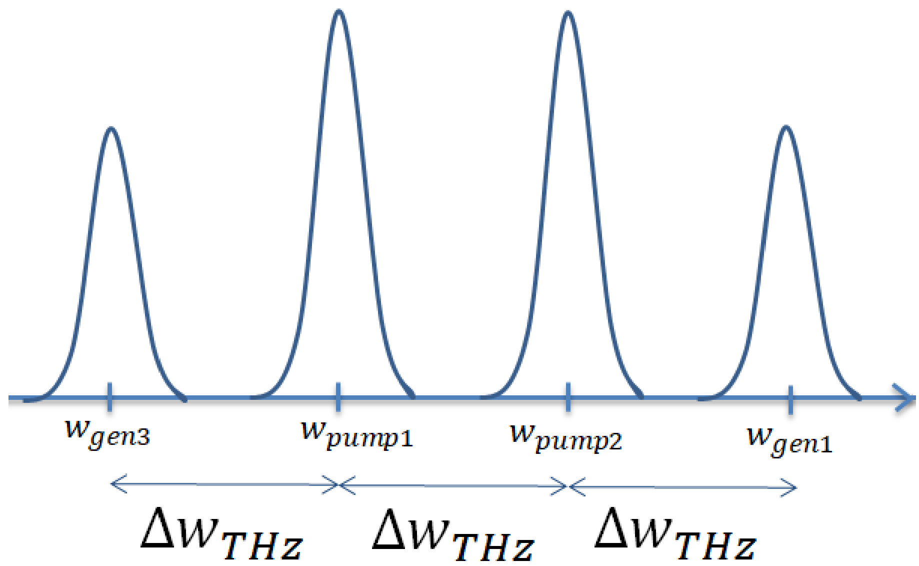
It can be shown, that output spectrum width is inversely proportional to number of grating lines, covered by input beam. And calculations presented in created program are in agreement with this fact. It was calculated in program, that under parameters

- $d^{-1} = 1900\ lines/mm$
- Length of grating = 75 cm
- Size of a beam on grating is equal to length of grating
- Focal length F = 10 cm
- Radius of a slit is 2 times smaller than spot size of a wave with wavelength $\lambda = 1030\ nm$

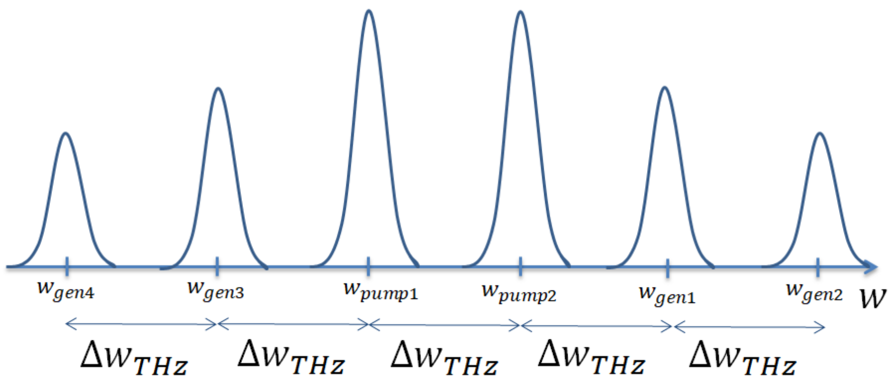
it is possible to obtain spectrum with width $\Delta\lambda_{1/e^2} = 1.2\ pm$. Experiments with these parameters can be a next step in investigations of proposed method.

3 Second approach: cascaded four wave mixing process

Four wave mixing (FWM) is a 3^{rd} order nonlinear process of new harmonics generation in medium. Under this process interaction between 4 photons occurs. There are two possible results of this interaction: three photons annihilate and create a new photon or two photons annihilate and produce two another photons. We want to apply 2nd case of this process for THz generation.
Let's consider proposed method of THz generation. Using two pump lasers with narrow spectra with central wavelengths at 1029.4 nm and 1030 nm we can start a FWM in photonic crystal fiber (PCF). The spectral distance between this harmonics is equal to THz frequency. FWM process of generation new spectral harmonics will look like:



Each couple of the spectral lines will produce two new harmonics and so on. As a result we will have the following situation, that is called cascaded FWM:



The factor, that limits amount of generated spectral components and determine spacing between them in spectrum, is so-called phase-matching conditions. From mathematical point of view it looks like:

$$\begin{cases} \Delta k_M + \Delta k_W + \Delta k_{NL} = 0 \\ w_1 + w_2 = w_3 + w_4 \end{cases}$$

where $\Delta k_M, \Delta k_W$ and Δk_{NL} represents wavenumber mismatch between initial and generated photons occuring as a result of material dispersion, waveguide dispersion and nonlinear effects, respectively. w_1, w_2 - frequencies of initial photons, w_3, w_4- frequencies of generated photons

From physical point of view these conditions are, in fact, momentum conservation law and energy conservation law for the system of four photons, respectively. The momentum conservation law selects waves generated in each point of initial wave propagation trajectory that interfere constructively. The energy conservation law determines frequencies of generated waves. Equations shown above are perfect phase matching conditions, which allow two pump harmonics generate only two certain harmonics. In practice we have a less strict phase matching conditions due to the fact, that constructive interference can occur for a range of harmonics. Taking into account, that intensity of generated harmonic can be expressed as [1]:

$$I_{gen} = I_{gen}^{max} sinc(\frac{\Delta k_{total} L}{2})$$

One can write phase-matching conditions as

$$\begin{cases} \Delta k_M + \Delta k_W + \Delta k_{NL} \leq \frac{2\pi}{L} \\ w_1 + w_2 = w_3 + w_4 \end{cases}$$

where L - length of radiation propagation medium. Therefore, as a result we obtain spectrum consisting of lines with finite width. In order to produce THz radiation efficiently, one should generate as much as possible harmonics to achieve higher peak power

of a signal. Since phase matching conditions depend on length of a medium, thus it is nessesary to determine dependence of generated harmonics power on this length. Let's discuss calculations allowing obtain this dependence.
Wave propagation in dispersive nonlinear medium is governed by nonlinear Scroedinger equation [2]:

$$-i\frac{\partial A}{\partial z} = \beta(i\frac{\partial}{\partial \tau})A + \gamma|A|^2 A$$

where A is the slowly-varying wave amplitude, τ is the retarded time $t - \beta_1 z$, where β_1 is the group slowness, $\beta = \sum_{n=2}^{\infty} \beta_n (i\frac{\partial}{\partial \tau})^n/n!$ is the Taylor expansion of the dispersion function and γ is the Kerr nonlinearity coefficient. Near the zero dispersion frequency one can assume $\beta = 0$. In this case the solution is:

$$A(\tau, z) = A(\tau, 0)exp[i\gamma|A(\tau, 0)|^2 z]$$

Consider the two-frequency boundary condition

$$A(\tau, 0) = \sqrt{P_{ref}}\left[\rho_+ exp(i\phi) + \rho_- exp(-i\phi)\right]$$

where $\rho_+ = \sqrt{\frac{P_1}{P_{ref}}}$, $\rho_- = \sqrt{\frac{P_2}{P_{ref}}}$, P_1 - initial peak power of 1st pump harmonic, P_2 - initial peak power of 2nd pump harmonic, $\phi = w\tau + \phi_1$, that corresponds to two pumps with frequency difference 2w and initial phase difference $2\phi_1$,

one can write the solution as:

$$A(\tau, z) = \sum_{n=1}^{\infty} A_n(2\rho_+\rho_-\gamma P_{ref} z) exp(in\phi)$$

where

$$A_n(z) = \sqrt{P_{ref}}\left[i^{\frac{n-1}{2}}\rho_+ J_{\frac{n-1}{2}}(2\rho_+\rho_-\gamma P_{ref} z) + i^{\frac{n+1}{2}}\rho_- J_{\frac{n+1}{2}}(2\rho_+\rho_-\gamma P_{ref} z)\right]$$

and

$$P_n(z) = |A_n(z)|^2$$

By plotting this function with the parameters $P_{ref} = 1\ W,\ P_1 = 75\ W, P_2 = 75\ W, \gamma = 0.0108\ \frac{1}{Wm}$ we obtain dependence of power of one pump harmonic and nine generated harmonics on propagation distance (Figure 7):

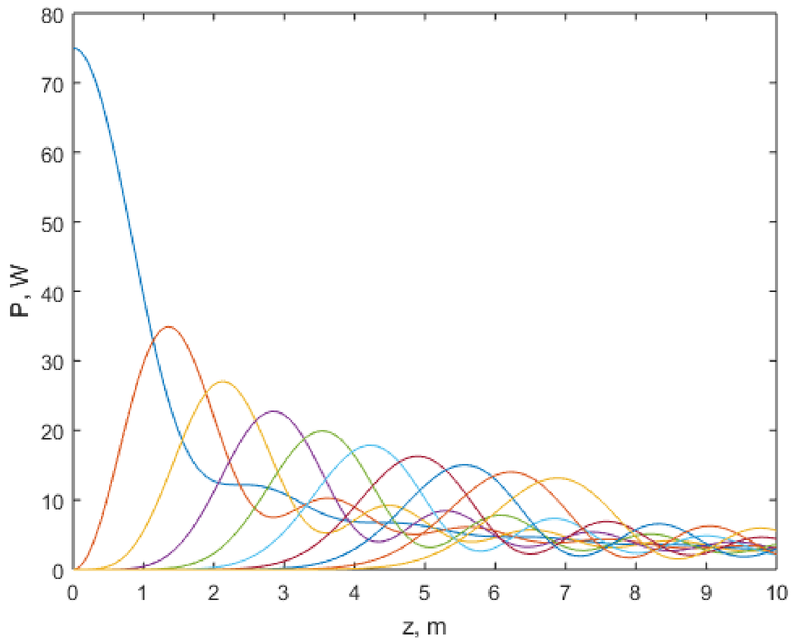


Figure 7: Dependence of power of one pump harmonic and nine generated harmonics on propagation distance in PCF

With use of this dependence one can determine suitable fiber length in order to generate certain amount of harmonics. Experimental observation of proposed cascaded FWM process is an object of future investigations.

4 Acknowledgments

I would like to thank organizing team of DESY Summer Student Program 2018. I thank Olaf Behnke for informing and constant participation in life of summer students. I thank Ute Micheelsen for her coordination of summer students during application period. I thank all summer students for their energy and creative ideas. I thank scientific group FS-CFEL2 for their cozy company and comfortable working atmosphere. And of course, I would like to thank my supervisor Nicholas Matlis for the provided opportunity to work on fascinating project, for his scientific advicement and his outstanding ability to explain difficult things clear. Due to him I've learned a lot of new.

References

[1] Nonlinear optics, p. 79 - *Robert W. Boyd*

[2] Four-wave-mixing cascades near the zero-dispersion frequency - *C. J. McKinstrie and M. G. Raymer*

[3] Nonlinear fiber optics - *Govind P. Agrawal*

[4] Numerous explanations by *Nicholas H. Matlis*