Compensation of Thermal Lens Effect in Yb:YAG Thin-Disk Amplifier by Simple Tunable-Focus High-Reflector

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Abstract

Thermal lens effect in high averaged power lasers is one of the actual problems nowadays. In this report it is shown that a very simple tunable-focus high-reflector, which consists of flat mirror laying on top of vacuum chamber, is able to compensate thermal lens effect preventing decadence of laser beam properties. Depending on pressure inside the vacuum chamber mirror acts like convex or concave mirror with tunable focus. Numerical calculations we present are based on theory of small deflections of thin plates. According to this theory, a sample mirror with the most appropriate parameters was chosen. Radius of curvature of the sample mirror was evaluated for different pressures using Michelson interferometer. Experimental results are in good agreement with numerical predictions.

Contents

1 Theory

1.1 Thermal lens effect

Yb:YAG crystals find application in various laser systems due to its large mechanical strength and high optical quality. Yb:YAG thin-disk amplifier, which is in the lab of Ultrafast Optics in the Centre of Free Electron Laser (CFEL), serves for multi-pass amplifying of the laser beam ($\lambda = 1030\ nm$) typically from 2-3 μJ to 100 mJ of energy. The main problem of this optical scheme is the fact that the crystal heats up leading to thermal lens effect which can be described with the following equation:

$$n(\Delta T) = n_0 + \frac{dn}{dT}\Delta T, \tag{1}$$

where dn/dT is physical characteristic of medium, which is called thermal optical coefficient. n_0 is refractive index at point in the absence of laser beam. For Yb:YAG crystal $dn/dT = 7.3 \cdot 10^{-6}/°K$. Depending on mechanism of cooling of the crystal there are two main cases. The first one is when $dn/dT > 0$, which is equal to convergence lens. The second one refers to $dn/dT < 0$, which is similar to divergence lens. It should be noted that at a first approximation one can neglect the absorption of generated radiation, because the power of this radiation is 100 less than the power of the laser beam. However, this effect should be taken into account in cases of non-linear crystals, where parametric frequency conversion is studied.

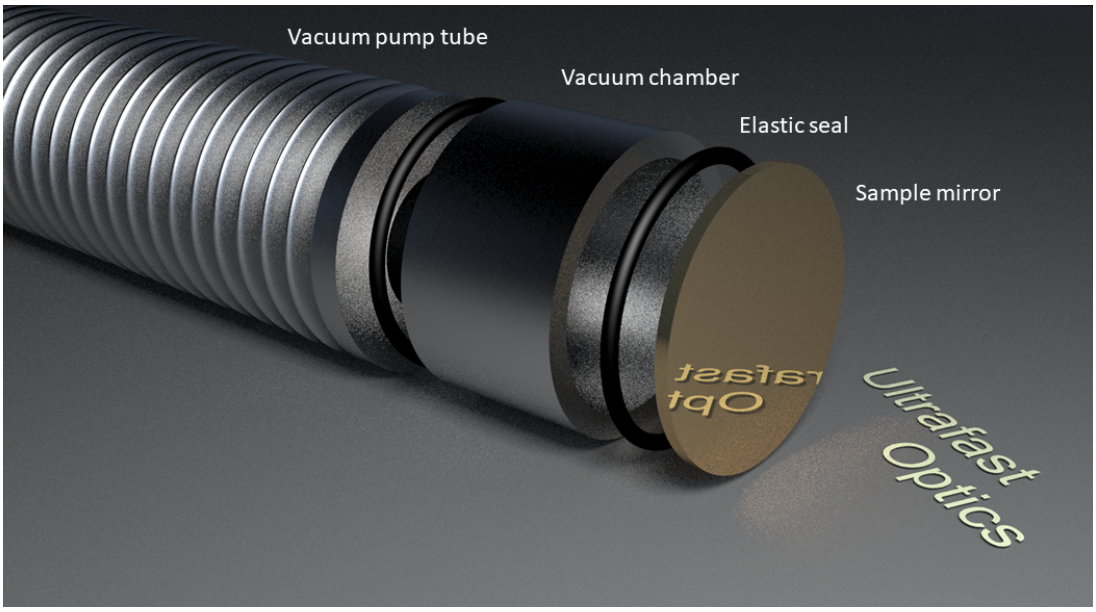

Figure 1: Simplified illustration of tunable-focus high-reflector.

It appeared that direct cooling of the crystal is not easy for high averaged power lasers. That is why, there is an idea of creating simple optical system, which is meant to compensate thermal lens effect and improve the quality of laser beam. This system consists of flat mirror, which lays on the top of vacuum chamber (Fig. 1). The mirror is coated with thin film of dielectric material with high reflectivity. Using vacuum pump it is possible to control the pressure inside the chamber. Under some pressure the mirror deflects and acts like a bi-concave or bi-convex mirror.

1.2 Thin plate theory

To describe the behavior of the mirror under some pressure it is necessary to resort to the theory of circular thin plates. For the axially symmetrical bending the deflection depends upon the radial position r only. The differential equation of the deflected surface of the circular plate for this case [1]

$$\nabla_r^4 w = -D\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) = \frac{p}{D}, \tag{2}$$

where D is flexural rigidity

$$D = \frac{Eh^3}{12(1-\nu^2)}. \tag{3}$$

The solution is the sum of the complementary solution of the homogeneous differential equation, w_h, and particle solution w_p

$$w = w_h + w_p. \tag{4}$$

The complementary solution of the Eq. (2) is given by

$$w_h = C_1 lnr + C_2 r^2 lnr + C_3 r^2 + C_4, \tag{5}$$

where C_i can be evaluated from the boundary conditions. The particular solution can be obtained from integration the Eq.(2)

$$w_p = \int \frac{1}{r} \int r \int \frac{1}{r} \int \frac{rp(r)}{D} drdrdrdr. \tag{6}$$

If the plate is under a uniform loading $p = p_0 =$ const, the particular solution is

$$w_p = \frac{p_0 r^4}{64D} \tag{7}$$

2 Results

2.1 Calculations

According to the thin plate theory, which was mentioned before, in real case parameters of the deflective mirror can be calculated. We used a model of thin plate with simply supported edge (Fig. 2), which lays on the elastic seal. Boundary conditions for this

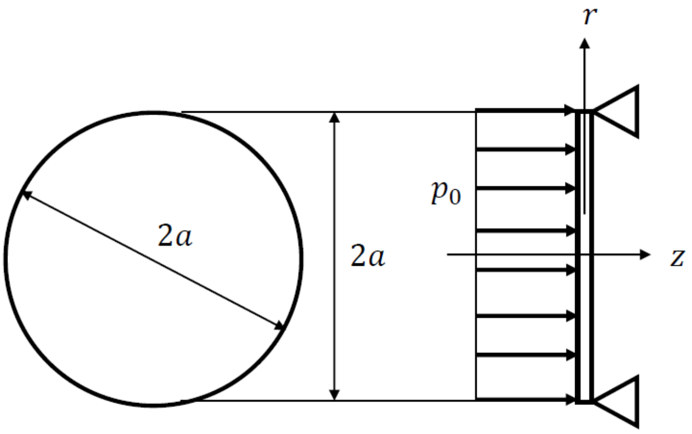

Figure 2: Plate with simply supported edge under a uniform load p_0.

case are

$$w|_{r=a} = 0, M_{r=a} = 0, \tag{8}$$

where M_r is radial bending moment

$$M_r = -D\left[-C_1\frac{1-\nu}{r^2} + 2C_2(1+\nu)lnr + C_2(3+\nu) + 2C_3(1+\nu) + \frac{p_0 r^2}{64D}\right] \tag{9}$$

The terms involving the logarithms in Eq. 5 yield an infinite displacement and bending moment, and the shear force for all values of C_1 and C_2, except zero; therefore, $C_1 = C_2 = 0$. Substituting for w_p from Eq. 7 into Eq. 5, we obtain the equation for the deflected surface:

$$w = c_3 r^2 + c_4 + \frac{p_0 r^4}{64D}, \tag{10}$$

and the expression for M_r may be obtained from Eq. (4.22) in case of $C_1 = C_2 = 0$:

$$M_r = -D\left[2C_3(1+\nu) + \frac{p_0 r^2}{64D}(3+\nu)\right] \tag{11}$$

Solving Eq. 2 with the boundary conditions Eq. 8 yields

$$C_3 = -\frac{p_0 a^2(3+\nu)}{32D(1+\nu)}, C_4 = \frac{p_0 a^4(5+\nu)}{64D(1+\nu)} \tag{12}$$

Substituting the above into Eq. 10 gives the plate deflection in the form

$$w = \frac{p_0(a^2 - r^2)}{64D} \left(\frac{5+\nu}{1+\nu} a^2 - r^2 \right) \tag{13}$$

The maximum deflection, which occurs at $r = 0$, is

$$w_{max} = \frac{p_0 a^4}{64D} \left(\frac{5+\nu}{1+\nu} \right) \tag{14}$$

The solution (Eq. 13) of the problem with parameters from table 1 is presented in Fig. 4. The parabolic fitting was calculated within 1 Joule laser aperture (16-mm diameter). The optical quality of such mirror depends on spherical aberrations, which are shown in Fig. 5. These aberrations were calculated and for different pressures in range from 0 to 2.5 atm. One can see that the less pressure inside the chamber, the more parabolic shape the mirror has. In the same way spherical aberrations are calculated with varying thickness of the mirror. In this case the opposite situation occurs. With increasing thickness, aberrations become less. Moreover, such ranges of pressures and thicknesses were chosen according to our experimental and manufacturing facilities.

The final cause of this work is the possibility to tune focus of the mirror with varying pressure inside the vacuum chamber. Expression which describes curvature of the mirror can be calculated using illustration in Fig. 3:

$$R = \frac{h^2 + a^2}{2h} \approx \frac{a^2}{2h} \tag{15}$$

On the basis of the theory of thin plates deflections, the dependence between focus length and applied pressure was calculated (Fig. 4b). It should be noted that at point of 2 atm the focus length is in range from 1 m to almost 5 m for different thicknesses. At the same time, the bigger thickness, the less spherical aberrations the mirror has. Therefore, the most appropriate parameters of our system exist.

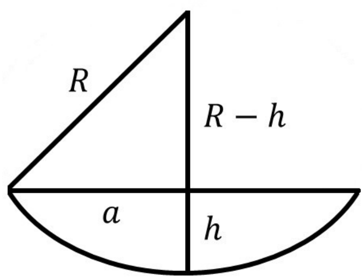

Figure 3: Schematic illustration of deflected mirror with radius a, depth (deflection) h and radius of curvature R.

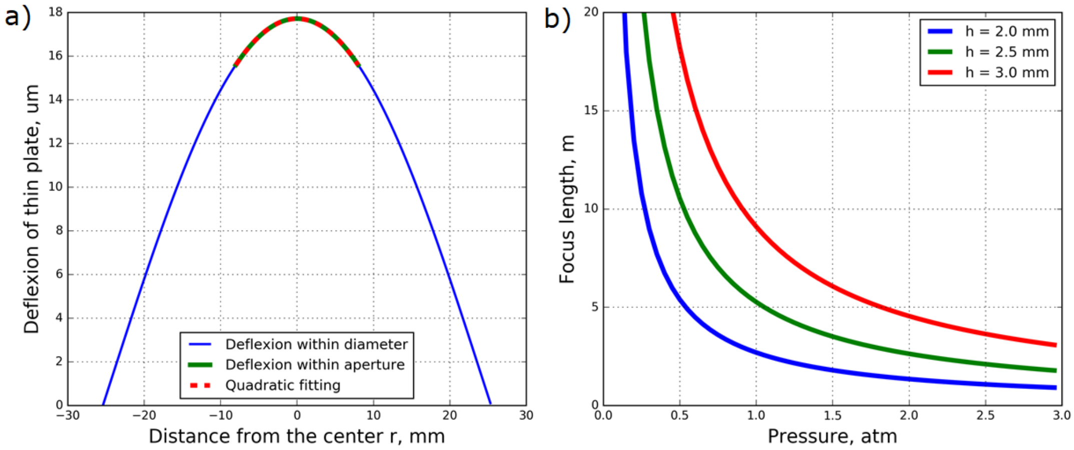

Figure 4: (a) Calculated deflection of the thin mirror under uniform load P_0. (b) Calculated dependence between focus length and applied pressure for different thicknesses. Calculation parameters for (a) and (b) of the thin mirror are presented in the table 1.

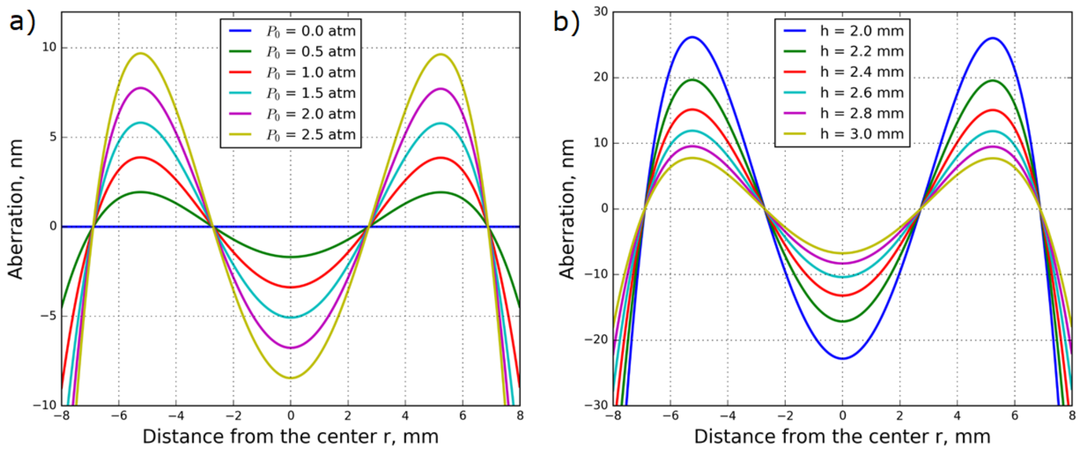

Figure 5: Calculated spherical aberrations for different pressures (a) and different thicknesses (b) within 1 Joule laser aperture (16-mm diameter) for the mirror with parameters presented in table 1.

2.2 Experimental set-up

For experimental work the sample (Fig. 7) was chosen with the following parameters: diameter is 51 mm, thickness is 3 mm. The mirror has gold 150-nm film to increase the reflectivity of the surface. The system of gold-coated mirror laying on the top of the vacuum chamber was installed into position of one of two mirrors which form arms of the interferometer. The experimental set-up is shown in Fig. 6. The beam propagates through the system of two lenses forming a telescope. The beam broadens up to 3.5 mm in diameter, and then it is split by a beamsplitter into two arms. Each of those light beams is reflected back toward the beamsplitter which then combines their amplitudes

Table 1: Parameters of the mirror

Diameter $2a$	51 mm
Young's modulus E	$73 \cdot 10^9$ Pa
Thickness h	3 mm
Pressure P_0	$2 \cdot 10^5$ Pa
Poisson's rate ν	0.17

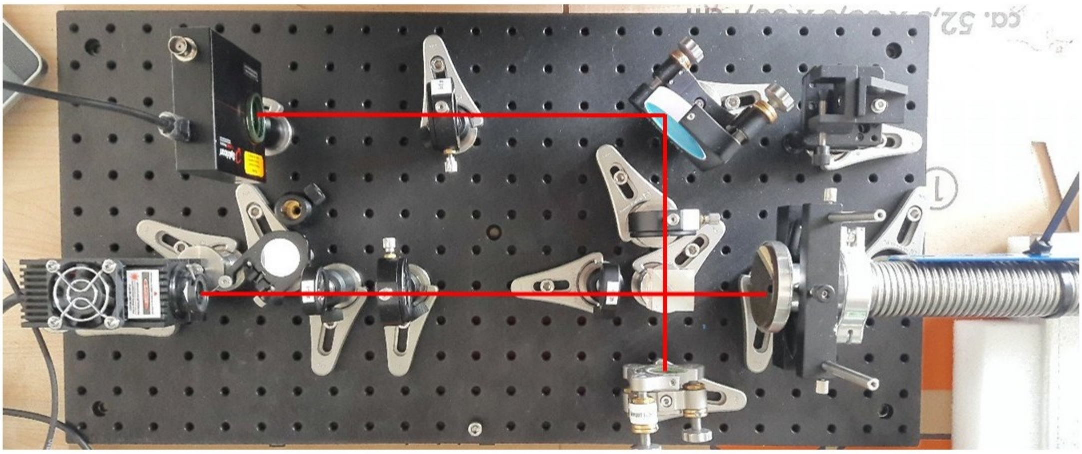

Figure 6: Experimental set-up: the Michelson Interferometer. A source of light is Nd:YAG laser ($\lambda = 1064$ nm)

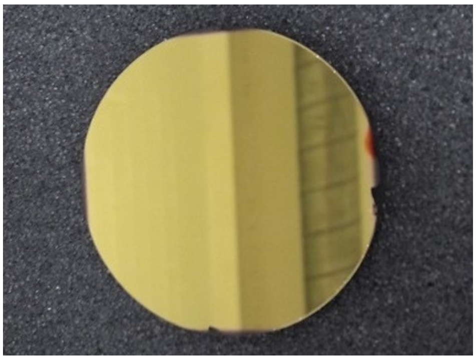

Figure 7: The sample: gold-coated mirror (diameter is 51 mm, thickness is 3 mm, thickness of gold film is 150 nm)

using the superposition principle. Finally, the beam comes to Spiricon CCD camera.

2.3 Measurements

It is possible to see how the interference fringes curve by varying pressure inside the vacuum chamber. In Fig. 8 an interference pattern in case of gold-coated mirror under 1 atm is shown. Fringes are slightly curved. It corresponds to the deflection of the mirror.

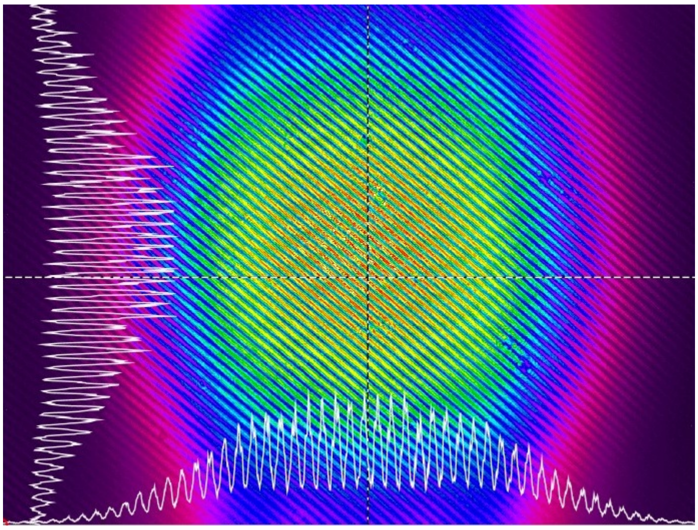

Figure 8: An interference pattern in case of gold-coated mirror under uniform load of 1 atm.

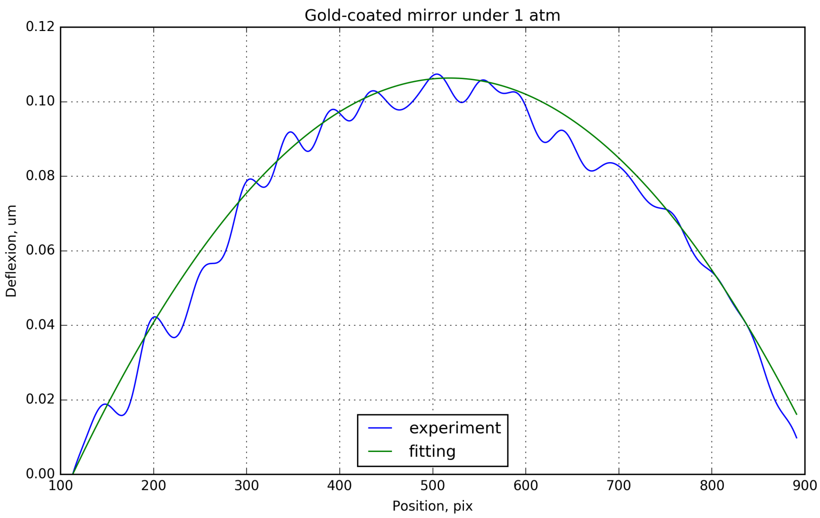

Figure 9: Corresponding deflection of the gold-coated mirror under uniform load of 1 atm (blue curve represents experimental data, the green one is fitting). Aperture is 3.5 mm

The curvature of the deflected mirror was calculated by using mathematical methods based on double Fourier transform of the intensity distribution of the pattern (Fig. 9). The depth (deflection) of the mirror equals 0.11 μm. Theoretical prediction is 0.10 μm. Measurements were carried out only for 1.0, 0.5 and ≈ 0.2 atm due to difficulty of varying pressure inside the vacuum chamber to a high accuracy. Nevertheless, numerical calculations are in good agreement with the experiment. This suggests that it is possible to use this simple system in future experiments with Yb:YAG thin-disk amplifier to compensate thermal lens effect by directly varying pressure inside the vacuum chamber.

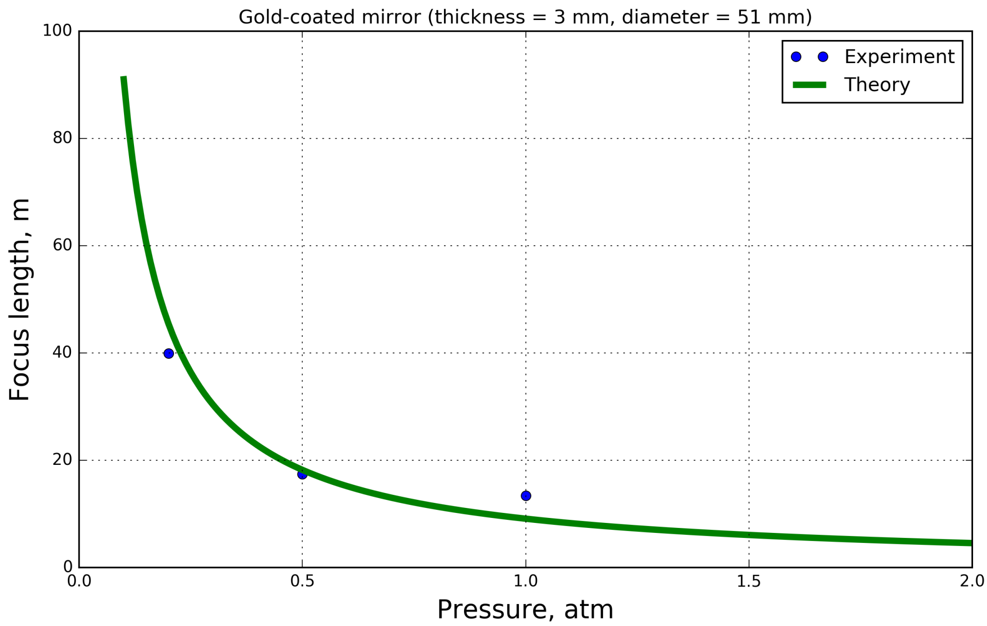

Figure 10: Dependence between focus and applied pressure. Green line represents numerical calculations, blue dots are experimental data

3 Conclusion

In summary, it was demonstrated how the problem of crystal heating in multi-passing Yb:YAG thin-disk amplifier can be solved. A simple system was proposed for this purpose. It consists of the flat mirror (HR) lying on the top of the vacuum chamber (Fig. 1). Varying pressure inside the chamber one can tune focus length of the mirror. Therefore, this focus length can be adjusted to the thermal lens curvature. Mirror such as this could produce a 4-meter focal length (under $P_0 = 2$ atm) with no more than 20 nm of peak-to-valley spherical aberration (1/50 of a wave). In the nearest future this simple system is meant to compensate thermal lens effect and improve the quality of the 1 Joule laser beam (16-mm diameter).

4 Acknowledgments

I would like to express gratitude to my supervisor Luis Zapata and his wife Kelly for introducing me to the field of ultrafast optics and high energy lasers, for providing me all the necessary and also for helping me in everything during the whole stay at DESY.

5 Supplementary material

5.1 Variable thickness

In our project we used a mirror with constant thickness (3 mm). However, there is another way to solve the problem of thermal lens effect. If one knows the curvature of thermal lens in crystal and it is a constant, it is easy to solve Eq. 2 for flexural rigidity $D(r)$ and then obtain an expression for thickness distribution $h(r)$. Implementation of this method is described in detail in [2].

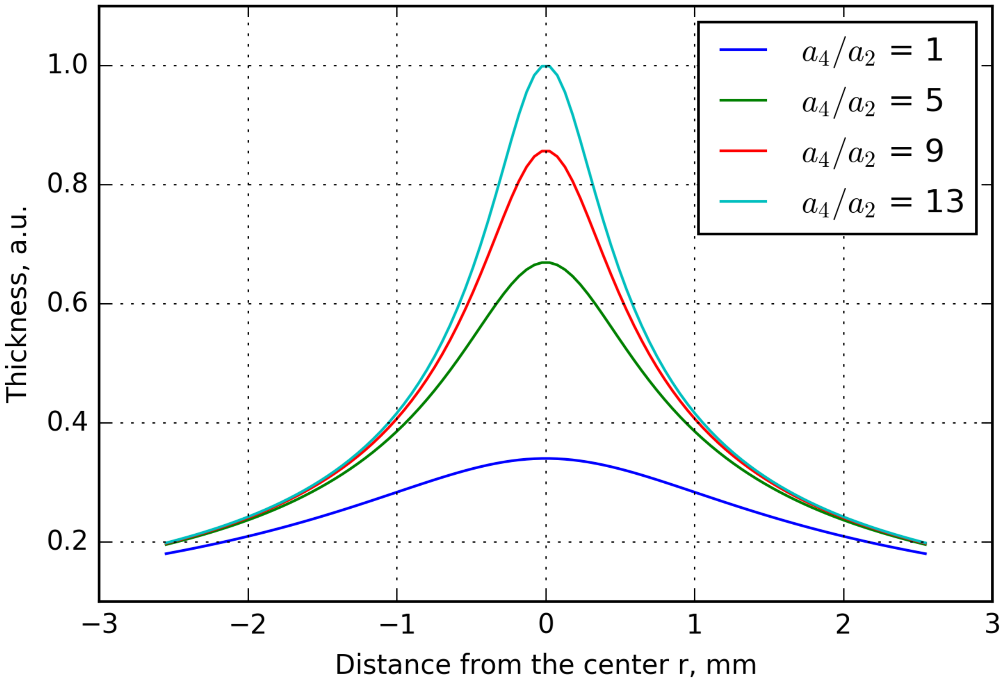

Figure 11: Thickness distribution of the mirror in case deflection w is a 4-order polynomial.

Assuming the deflection of thermal lens can be described with a 4-order polynomial

$$w = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4, \tag{16}$$

but usually odd coefficients a_1 and a_3 equals zero, because cross section of the beam is almost circular, therefore, Eq. 16 is represented, as follows

$$w = a_0 + a_2 x^2 + a_4 x^4. \tag{17}$$

Then, substituting above in Eq. 2 and solve it for $D(r)$, one obtains

$$D(r) = const \left[\frac{a_2(1+\nu)}{2a_4(3+\nu)} + r^2 \right]^{\frac{-4}{3+\nu}}, \tag{18}$$

where *const* can be evaluated from the boundary condition for flexural rigidity. Furthermore, using Eq. 3, we obtain expression for thickness distribution

$$h(r) = \left[\left[\frac{12(1-\nu^2)}{E}\right]\left(const\left[\frac{a_2(1+\nu)}{2a_4(3+\nu)}+r^2\right]^{\frac{-4}{3+\nu}}\right)\right]^{\frac{1}{3}} \tag{19}$$

Assuming $a = 25.5$ mm, $\nu = 0.17$, $P_0 = 0$, $E = 70 \cdot 10^9$ Pa, and $const = 1$, one can see that the bigger a_4, the sharper distribution we have (Fig. 11).

This method seems to be relatively simple, however, in practice it is not flexible and it is quite expensive to fabricate a mirror with the defined thickness distribution.

5.2 Elastic foundation

There is one more way to simulate deflection of the mirror. We consider circular plates loaded symmetrically with respect to their center and resting on a elastic foundation, so called Wynkler-type. Taking into account the foundation reaction $q(r) = k \cdot w(r)$, governing differential equation of bending of circular plates resting on a elastic foundation:

$$\nabla_r^4 w = -D\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right) = \frac{p-kw(r)}{D} \tag{20}$$

Denoting

$$l = \left(\frac{D}{k}\right)^{\frac{1}{4}}, \tag{21}$$

and then introducing the dimensionless coordinate $\zeta = r/l$, one can write Eq. 20 in the form

$$\left(\frac{d^2}{d\zeta^2}+\frac{1}{\zeta}\frac{d}{d\zeta}\right)^2 w + w = \frac{pl^4}{D} \tag{22}$$

or in the form

$$\nabla_r^4 w + w = \frac{pl^4}{D} \tag{23}$$

For $p = 0$, the homogeneous equation (Eq. 23) is reduced to a system of the following two second-order differential equations:

$$\frac{d^2w}{d\zeta^2}+\frac{1}{\zeta}\frac{dw}{d\zeta} \pm iw = 0. \tag{24}$$

The solution of this system of differential equations is given by

$$w = C_1J_0(\zeta\sqrt{i}) + C_2J_0(\zeta\sqrt{-i}) + C_3H_0^1(\zeta\sqrt{i}) + C_4H_0^1(\zeta\sqrt{-i}), \tag{25}$$

where $J_0(\zeta\sqrt{\pm i})$ is the Bessel function of the first kind of the zero order and $H_0^1(\zeta\sqrt{\pm i})$ is the Hankel function of the first kind of zero order. The particular solution also can be found [1].

Mathematical approach described above is quite applicable to implementation. However, this theory looks a little more complicated in comparison with the case of mirror lying on elastic seal. Solution does not look like a simple polynomial with even indexes in power in its terms. Thus, spherical aberrations might be much bigger than in our approach. This issue requires a more detailed study.

References

[1] Thin Plates and Shells: Theory, Analysis, and Applications. *Eduard Ventsel and Theodor Krauthammer*

[2] Deformable mirrors for intra-cavity use in high-power thin-disk lasers *Stefan Piehler, Tom Dietrich, Philipp Wittmuss, Oliver Sawodny, Marwan Abdou Ahmed, and Thomas Graf*