MSSM Higgs Scenarios in Light of the Current LHC Constraints

Roberto Corral López, University of Granada, Spain

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Abstract

We discuss the possibility of physics beyond the Standard Model within Supersymmetry (SUSY) theory, more concretely within the so-called Minimal Supersymmetric Standard Model (MSSM). To accomplish this, we first present, at introductory level, the most basic aspects of the theory. Then we rather focus on the Higgs sector, we study with the help of the numerical code `FeynHiggs` some interesting features such as: the maximal mixing scenario, the decoupling limit and some of its possible decays. Finally we introduce two different tools `HiggsBounds` and `HiggsSignals` to perform a phenomenological analysis of the parameter space given the experimental measurements for what concerns the lightest MSSM Higgs boson decays into invisible states.

Contents

1 Introduction **3**

2 MSSM **3**

2.1 Imputs of the theory 4

2.2 Higgs sector of the MSSM 5

2.2.1 Maximal mixing scenario 7

2.2.2 Decoupling limit 8

3 Decays and branching ratios in the Higgs sector **10**

3.1 BR/BRSM ratio for the lightest Higgs boson decays 11

3.2 Heavy Higgs boson decays 11

3.3 Decays into neutralinos and charginos 13

4 HiggsBounds and HiggsSignal **16**

4.1 HiggsBounds 16

4.2 HiggsSignals 17

5 Compatibility of Higgs invisible decays with current LHC data **18**

6 Conclusion **21**

1 Introduction

We can state without any doubts that the Standard Model (SM) of particle physics is one of the most proven and successful theories in physics. However there are several reasons that lead us to think the SM is not the final theory of nature and that physics beyond the SM (BSM) is needed. We list a few of them: the strong CP problem, the Hierachy problem, the inclusion of gravity. Therefore one of the main issues of the particles physics community research nowadays is, within the measurements that we already have, explore the possibility of BSM physics existence.
There are many different theories trying to solve SM problems, one of the most popular and well-known is Supersymmetry (SUSY) which is the one we are going to focus on throughout this project, in particular the so-called Minimal Supersymmetric Standard Model (MSSM). We will focus on the Higgs sector of the theory. On the one hand we will study the MSSM Higgs bosons properties, in particular, their decays, couplings, behaviour depending on the parameters of the theory, and determinate how much parameter space is left within the measurements of the experiments. On the other hand we will also evaluate the possibility that the observed Higgs boson during Run I of the LHC is in fact the lightest Higgs of the MSSM.

2 MSSM

The Standard Model requires the Higgs mechanism to give masses to the massive particles of the theory without breaking down the gauge symmetry of the Lagrangian. Nevertheless among all the advantages that the Higgs mechanism brings, it also has some problems. We will not discuss this in detail (see intro in Ref. [1] and [2] for more) but the main problem that arises is that m_H^2 receives enormous quantum corrections from the virtual effects of every particle that couples, directly or indirectly, to the Higgs field.
To solve this problem we assume there is a symmetry between bosons and fermions called supersymmetry (SUSY) by which for every boson/fermion of the SM there exist a partner called "superpartner" which is actually a fermion/boson and thus composes the so-called Minimal Supersymmetric Standard Model (MSSM).

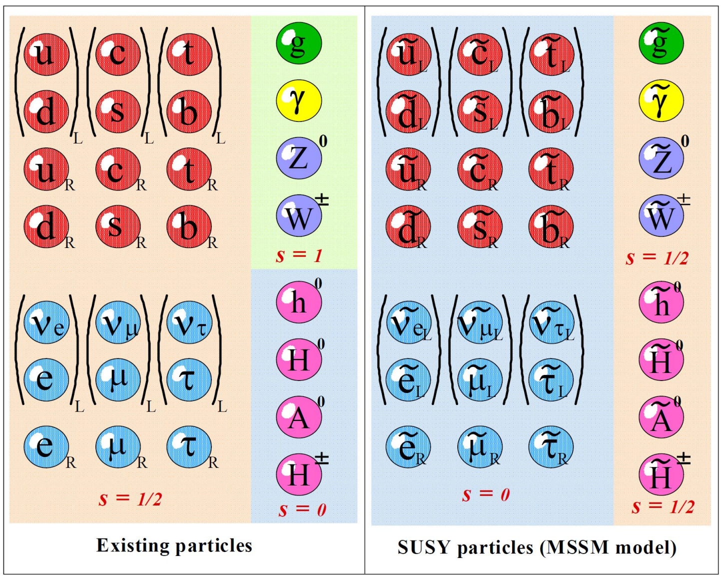

Figure 1. The MSSM particles taken from [3]

It may be a surprise that on the leftside of the picture (what it is supposed to be the SM) there are 5 different Higgs bosons (3 neutral and 2 charged), however this is due to the fact that the MSSM requires two Higgs doublets instead of just one to be consistent and successfuly give masses to all the fermions (more in Ref. [1] pg. 8).
We will talk more about the Higgs sector of the MSSM in the next section but first there is another issue we should address, the masses of the superpartners. If supersymmetry were unbroken, then there would have to be SUSY particles with exactly the same mass as the SM particles, nevertheless this would imply that we should have discovered some of these particles long time ago. Up to date none of these particles have been discovered, so then it's clear that supersymmetry must be broken. In other words this means that the underlying model should have a Lagrangian density that is invariant under supersymmetry, but a vacuum state that is not. In this way, supersymmetry is hidden at low energies in a manner analogous to the electroweak symmetry in the ordinary Standard Model. However a new question arises now, "How is supersymmetry broken?". The answer as usual is not unique (see section 7 of Ref. [1]) and actually there is no consensus on exactly how this should be done. One possibility which is useful from a practical point of view is just introduce extra terms that break supersymmetry explicitly in the effective MSSM Lagrangian. In order to prevent the reappearance of the quadratic divergences (the problem we mentioned in the first paragraph with the Higgs mechanism) the supersymmetry-breaking couplings should be soft (characteristic mass scale m_{soft} not much larger that 10^3 GeV) so we have:

$$
\begin{aligned}
\mathcal{L}_{soft}^{MSSM} &= -\frac{1}{2}\left(M_1 \widetilde{B}\widetilde{B} + M_2 \sum_{a=1}^{3} \widetilde{W}_a \widetilde{W}_a + M_3 \sum_{a=1}^{8} \widetilde{g}_a \widetilde{g}_a + c.c \right) \\
&\quad - \left(\widetilde{\overline{u}} \boldsymbol{a_u} \widetilde{Q} H_u - \widetilde{\overline{d}} \boldsymbol{a_d} \widetilde{Q} H_d - \widetilde{\overline{e}} \boldsymbol{a_e} \widetilde{L} H_d + c.c \right) \\
&\quad - \widetilde{Q}^\dagger \boldsymbol{m^2_Q} \widetilde{Q} - \widetilde{L}^\dagger \boldsymbol{m^2_L} \widetilde{L} - \widetilde{\overline{u}} \boldsymbol{m^2_{\overline{u}}} \widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}} \boldsymbol{m^2_{\overline{d}}} \widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}} \boldsymbol{m^2_{\overline{e}}} \widetilde{\overline{e}}^\dagger \\
&\quad - m^2_{H_u} H_u^* H_u - m^2_{H_d} H_d^* H_d - (b H_u H_d + c.c)
\end{aligned} \tag{1}
$$

where M_1, M_2 and M_3 are the bino, wino and gluino mass terms, the second line contains the trilinear couplings between the sfermions and the Higgs bosons with $\boldsymbol{a_i}$ $(i = u, d, e)$ complex 3×3 matrix in family space with dimensions of [mass]. The third line consists of sfermions mass terms and again each $\boldsymbol{m^2}$ is a 3×3 matrix in family space that can have complex entries, but they must be hermitian so that the Lagrangian is real[1]. Finally, in the last line of Eq. (1) we have supersymmetry-breaking contributions to the Higgs potential.

2.1 Imputs of the theory

In the general case the soft SUSY-breaking terms will introduce a huge number of unknown parameters (105), in addition to the 19 parameters of the SM, [8] which makes any phenomenological analysis in the MSSM very complicated. However a phenomenologically more viable MSSM can be defined making some assumptioms (see Ref. [4] pg 19) which leads to only 22 imput parameters:

- $\tan\beta$: the ratio of the VEVs of the two-Higgs doublet fields.

[1]To avoid clutter, we do not put tildes on the Q in $\boldsymbol{m^2_Q}$, etc.

- M_A, μ: the pseudoscalar Higgs boson mass and the higgsino mass parameter.
- M_1, M_2, M_3: the bino, wino and gluino mass parameters.
- $m_{\tilde{q}}$, $m_{\tilde{u}_R}$, $m_{\tilde{d}_R}$, $m_{\tilde{l}}$, $m_{\tilde{e}_R}$: the first/second generation sfermion mass parameters.
- A_u, A_d, A_e: the first/second generation trilinear couplings.
- $m_{\tilde{Q}}$, $m_{\tilde{t}_R}$, $m_{\tilde{b}_R}$, $m_{\tilde{L}}$, $m_{\tilde{\tau}_R}$: the third generation sfermion mass parameters.
- A_t, A_b, A_τ: the third generation trilinear couplings.

Such a model, with this relatively few number of parameters has much more predictability and is much easier to investigate phenomenologically. Indeed when one looks at a given sector of the model, in general, only a small subset appears, for instance, the Higgs sector, in which we are interested, is mainly controlled by M_A and $\tan\beta$.

2.2 Higgs sector of the MSSM

As we said before, two Higgs doublets of complex scalar fields are needed in the MSSM:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$$

We now proceed to the description of electroweak symmetry breaking. The scalar potential[2] for the Higgs scalar fields in the MSSM is given by:

$$\begin{aligned} V = &\left(|\mu|^2 + m_{H_u}^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\ &+ \left[b\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + c.c\right] \\ &+ \frac{(g^2 + g'^2)}{8}\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 + \frac{g^2}{2}|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \end{aligned} \tag{2}$$

where μ is the higgsino mass parameter, g, $g'$are the coupling constant associated with the $SU(2)$ and $U(1)$ gauge groups respectively and b is a soft-SUSY breaking parameter related, as we will see later on, to M_A.
It's neccesary that the minimum of this potencial breaks electroweak symmetry down to electromagnetism $SU(2)_L \times U(1)_Y \to U(1)_{EM}$. Due to the freedom to make $SU(2)_L$ gauge transformation we can take, without loss of generality, $H_u^+ = 0$ at the minimum of the potential, which also leads to $H_d^- = 0$. In this way only H_u^0 and H_d^0 get non-zero vacuum expectation values (VEVs) and we can write:

$$v_u = \left\langle H_u^0 \right\rangle, \quad v_d = \left\langle H_d^0 \right\rangle$$

[2]The full scalar potential of the theory includes many terms involving the squarks and sleptons that we will ignore here since they do not get vacuum expectation values.

These VEVs are related to the mass of the Z^0 boson and the electroweak gauge couplings:

$$v_u^2 + v_d^2 = v^2 = 2\frac{m_Z^2}{(g^2 + g'^2)} \approx (174\,GeV)^2$$

And the ratio of them is written as:

$$\tan\beta \equiv \frac{v_u}{v_d} \tag{3}$$

The two Higgs doublet are composed by eight real scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Goldstone bosons G^0, $G^{\pm}$, which become the longitudinal modes of the Z^0 and $W^{\pm}$ massive vector bosons. The remaining five Higgs are: the neural scalars h^0 and H^0, the neutral pseudoscalar A^0 and the charged $H^{\pm}$. We can express them as follows:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} R_{\beta_0} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = R_{\beta_\pm} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

where R_α, R_{β_0} and $R_{\beta_\pm}$ are the rotation matrices, that diagonalize the mass matrices of the Higgs bosons . Then, provided that v_u, v_d minimize the tree-level potential one can find the masses for the other Higgs bosons[3]:

$$m_A^2 = \frac{2b}{sin(2\beta)} = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \tag{4}$$

$$m_{h,H}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{\left(m_A^2 - m_Z^2\right)^2 + 4m_A^2 m_Z^2 sin^2(2\beta)} \right) \tag{5}$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2 \tag{6}$$

And the mixing angle α is determined, at tree level, as a function of m_h,m_H, m_A, m_Z and β. (see [1] pg. 98)
The masses of A, H and $H^{\pm}$can be arbitrarly large, in principle, since they all grow with $\frac{b}{\sin(2\beta)}$. The maximal h boson mass is obtained when M_A and $\tan\beta$ take large values, however is not arbitrarly as the others, it is bounded above. From Eq. (3) it's possible to find at tree level:

[3]From now on until the end of the report we will suppress the 0 super index for the neutral Higgs boson to abbreviate the notation.

$$m_h < m_Z |\cos(2\beta)| \tag{7}$$

Since the mass of the Higgs discovered in 2012 is ≈ 125 GeV and $m_Z \approx 91$ GeV it's neccesary that the mass of the light Higgs h is subject to big quantum corrections in order to agree with the experimental measurements. The largest of such contributions typically comes from top and stop loops and we will talk about it in the next subsection.

2.2.1 Maximal mixing scenario

In the one-loop level, the mass matrix of the neutral scalar Higgs bosons takes the form:

$$M^2_{h,H} = \begin{pmatrix} -bcot\beta + M_Z^2 \sin^2\beta + \Delta M^2_{11} & b - M_Z^2 \sin\beta\cos\beta + \Delta M^2_{12} \\ b - M_Z^2 \sin\beta\cos\beta + \Delta M^2_{21} & -b\tan\beta + M_Z^2\cos^2\beta + \Delta M^2_{22} \end{pmatrix} \tag{8}$$

where ΔM^2_{ij} are the first loop corrections to the Higgs boson masses. If only contributions of the top Yukawa coupling are taken into account one can obtain a very simple analytical expression:

$$\Delta M^2_{22} \sim \Delta M^2_{21} \sim \Delta M^2_{12} \sim 0$$

$$\Delta M^2_{11} \sim \epsilon = \frac{3m_t^4}{2\pi^2 v^2 \sin^2\beta}\left[log\frac{M_S^2}{m_t^2} + \frac{X_t^2}{2M_S^2}\left(1 - \frac{X_t^2}{6M_S^2}\right)\right] \tag{9}$$

where $M_S = \frac{1}{2}\left(m_{\tilde{t_1}} + m_{\tilde{t_2}}\right)$ is the arithmetic average of the stop masses, m_t is the top quark mass and $X_t = A_t - \mu\tan\beta$ is the stop mixing parameter which has to do with the mass matrices of stops 1 and 2 (see [4] pg 23-27 for further info). One can see that the radiative corrections are enhanced when the logarithm in the first term of Eq. (9) is large. Furthermore the corrections are largest and maximize the lightest boson mass h in the so-called "maximal mixing scenario" where $X_t = A_t - \mu\cot\beta \sim \sqrt{6}M_S$. On the other hand the radiative corrections are much smaller for small values of X_t close to the no mixing scenario where $X_t = 0$. Both features can be easily observed in the following plot obtained with the numerical code `FeynHiggs` [7]:

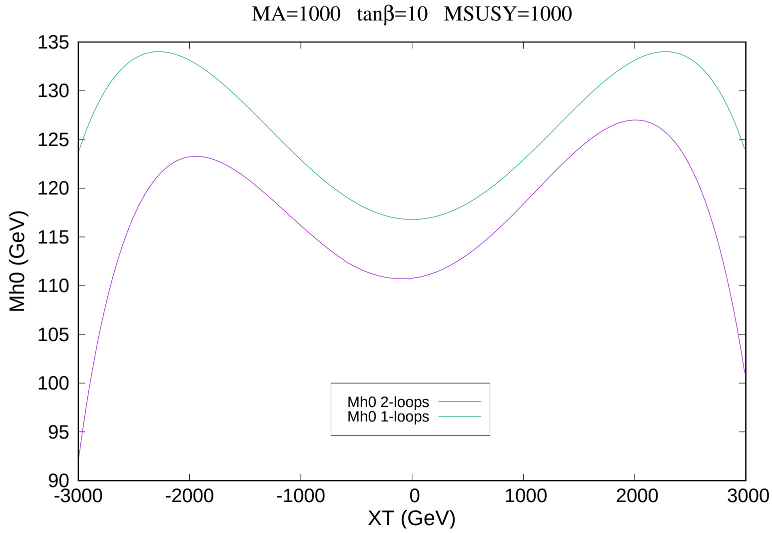

Figure 2. The lightest MSSM Higgs boson mass as a function of X_t calculated with `FeynHiggs2.13.0` for $\tan\beta = 10$, $M_s = M_A = 1$ TeV and $m_t = 172$ GeV and default values of the other parameters.

As one can see the maximum in the lightest Higgs boson mass at one-loop level is obtained for $X_t = \sqrt{6}M_S$ and for $X_t \simeq 2M_S$ at the two-loop level. The latter is due to the fact that `FeynHiggs` works in the on-shell scheme for what concerns the renormalization of the stop parameters.

2.2.2 Decoupling limit

If we diagonalize the mass matrix given in Eq. (8) with the approximation made in Eq. (9), the mass of the Higgs boson are simply given by [9]:

$$M^2_{h,H} = \frac{1}{2}\left(M_A^2 + M_Z^2 + \epsilon\right)\left[1 \mp \sqrt{1 - 4\frac{M_Z^2 M_A^2\cos^2 2\beta + \epsilon\left(M_A^2\sin^2\beta + M_Z^2\cos^2\beta\right)}{\left(M_A^2 + M_Z^2 + \epsilon\right)^2}}\right] \tag{10}$$

In this approximation, the charged Higgs mass does not receive radiative corrections, however a very simple expression for the corrected charged Higgs boson mass can be found [10]:

$$M_{H^\pm} = \sqrt{M_A^2 + M_W^2 - \epsilon_+} \quad \text{with} \quad \epsilon_+ = \frac{3G_\mu M_W^2}{4\sqrt{2}\pi^2}\left[\frac{m_t^2}{\sin^2\beta} + \frac{m_b^2}{\cos^2\beta}\right] log\left(\frac{M_S^2}{m_t^2}\right) \tag{11}$$

Now we can analyze what happens with those masses, for a given $\tan\beta$ value, in the limit of $M_A \gg M_Z$, the so-called decoupling limit region:

$$M_h \overset{M_A \gg M_Z}{\longrightarrow} \sqrt{M_Z^2\cos^2 2\beta + \epsilon\sin^2\beta}\left[1 + \frac{\epsilon M_Z^2\cos^2\beta}{2M_A^2\left(M_Z^2 + \epsilon\sin^2\beta\right)} - \frac{M_Z^2\sin^2\beta + \epsilon\cos^2\beta}{2M_A^2}\right] \tag{12}$$

$$M_H \overset{M_A \gg M_Z}{\longrightarrow} M_A \left[1 + \frac{M_Z^2 \sin^2 2\beta + \epsilon \cos^2 \beta}{2M_A^2}\right] \tag{13}$$

$$M_{H^\pm} \overset{M_A \gg M_Z}{\longrightarrow} M_A \left[1 + \frac{M_W^2}{2M_A^2}\right] \tag{14}$$

Eq. (13) and (14) explain about the degeneration of the masses of the heavy and charged MSSM Higgs boson which are close in mass $M_H \simeq M_{H^\pm} \simeq M_A$ for high M_A. Another interesting feature that we can observe is how the mass of the lightest Higgs boson get satured as M_A grows. Using `FeynHiggs` it's possible to plot those masses as a function of M_A for given values of $\tan\beta$ and actually see these effects:

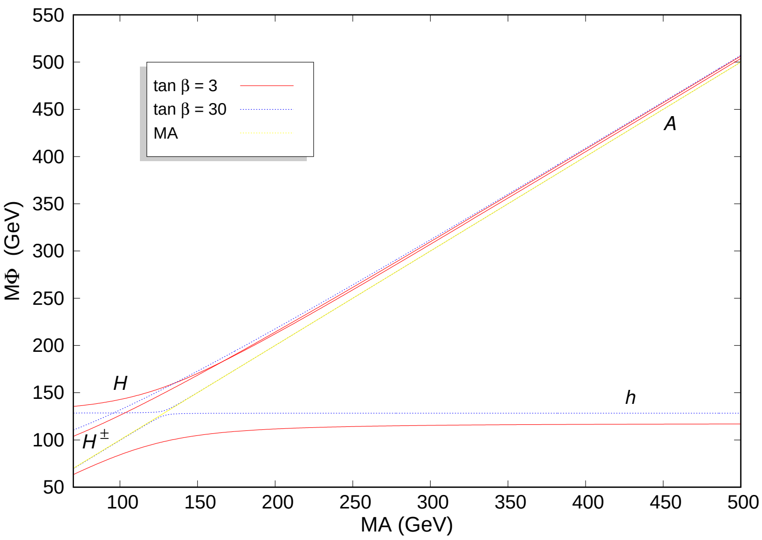

Figure 3. Masses of the MSSM Higgs bosons as a function of M_A calculated with `FeynHiggs2.13.0` for $\tan\beta = 3$ (red line), $\tan\beta = 30$ (blue dash line), $\mu = 500$ GeV, $M_S = 1.5$ TeV, $X_t = 3250$ GeV and the other parameters set as default, to show the so-called decoupling limit.

Here one can easily distinguish 3 different regions: the first one where the lightest Higgs is too light to be the SM-like[4] Higgs boson and the Heavy Higgs is the more SM-like Higgs boson. Then there is a second region in which none of them, one for being too light and the other too heavy, are SM-like. Finally there is a third and larger region where is the lightest Higgs the more SM-like. However these regions have also a dependence on $\tan\beta$ as we can observe in the Fig. 3 since for instance region 2 is considerably larger with $\tan\beta = 3$. Therefore to complete the view and have a full understanding of the behaviour of this MSSM Higgs boson masses, we have plotted a colour plot of those masses in the M_A and $\tan\beta$ plane:

[4]What we mean with being SM-like is basically having the same properties as the SM Higgs, i.e., same production and decays modes.

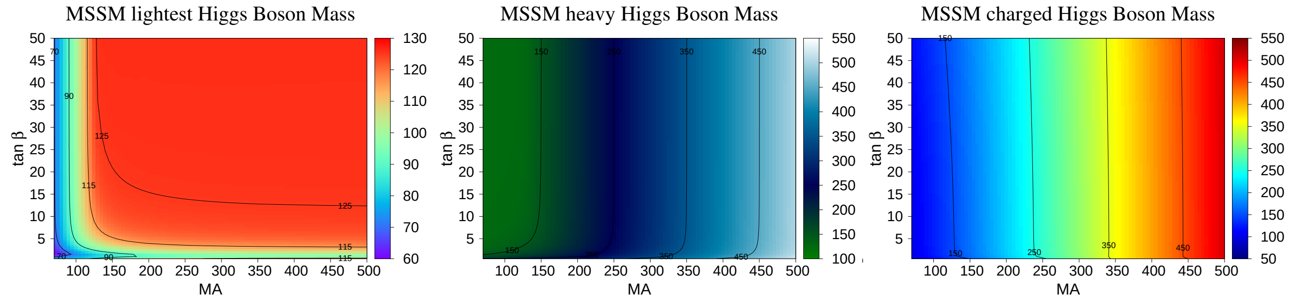

Figure 4. Colour plots of the masses of the MSSM Higgs bosons (left: the lightest, center: heavy, right charged) in the M_A[5] $\tan\beta$ plane. Calculated with `FeynHiggs2.13.0` for $\mu = 500$ GeV, $M_S = 1.5$ TeV, $X_t = 3250$ GeV and the other parameters set as default.

Fig. 4 show a little bit more in detail what we have already described above. The Heavy and charged Higgs boson mass barely depend on $\tan\beta$, only for low values one can see a dependence. Contrary to the lightest Higgs which 'needs' both large $\tan\beta$ and large M_A to reach high values.

3 Decays and branching ratios in the Higgs sector

Another interesting aspect we would like to take a look at are the couplings of the MSSM particles to the Higgs sector. At tree level the dependences on the angles β and α of the neutral MSSM Higgs bosons couplings to bosons and fermions are given by [1]:

$$\begin{aligned} hW^+W^-,\ hZZ,\ ZHA,\ W^\pm HH^\pm &\propto \sin(\beta-\alpha) \xrightarrow{DL} 1 \\ HW^+W^-,\ HZZ,\ ZhA,\ W^\pm hH^\mp &\propto \cos(\beta-\alpha) \xrightarrow{DL} 0 \end{aligned} \tag{15}$$

$$\begin{aligned} hb\bar{b},\ h\tau^+\tau^- &\propto -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha) \xrightarrow{DL} 1 \\ ht\bar{t} &\propto \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) - \cot\beta\cos(\beta-\alpha) \xrightarrow{DL} 1 \\ Hb\bar{b},\ H\tau^+\tau^- &\propto \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) - \tan\beta\sin(\beta-\alpha) \xrightarrow{DL} -\tan\beta \\ Ht\bar{t} &\propto \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha) \xrightarrow{DL} -\frac{1}{\tan\beta} \\ Ab\bar{b},\ A\tau^+\tau^- &\propto \tan\beta \\ At\bar{t} &\propto \cot\beta \end{aligned} \tag{16}$$

where $\xrightarrow{DL}$means in the decoupling limit.

[5] Unless we say the opposite, the units of M_A are always in GeV

3.1 BR/BRSM ratio for the lightest Higgs boson decays

The first question we can ask ourselves is whether the branching ratios of these decays are similar or differ from the ones in the SM. In order to answer that, we have used `FeynHiggs` to compute the branching ratios (BRs) in the MSSM and SM and plotted their ratio as a colour plot in the M_A, $\tan\beta$ plane. With BRSM we denote the branching ratio of a SM Higgs with the same mass as the corresponding MSSM Higgs boson. Thus one can easily visualize the regions where the couplings are more SM-like and where not:

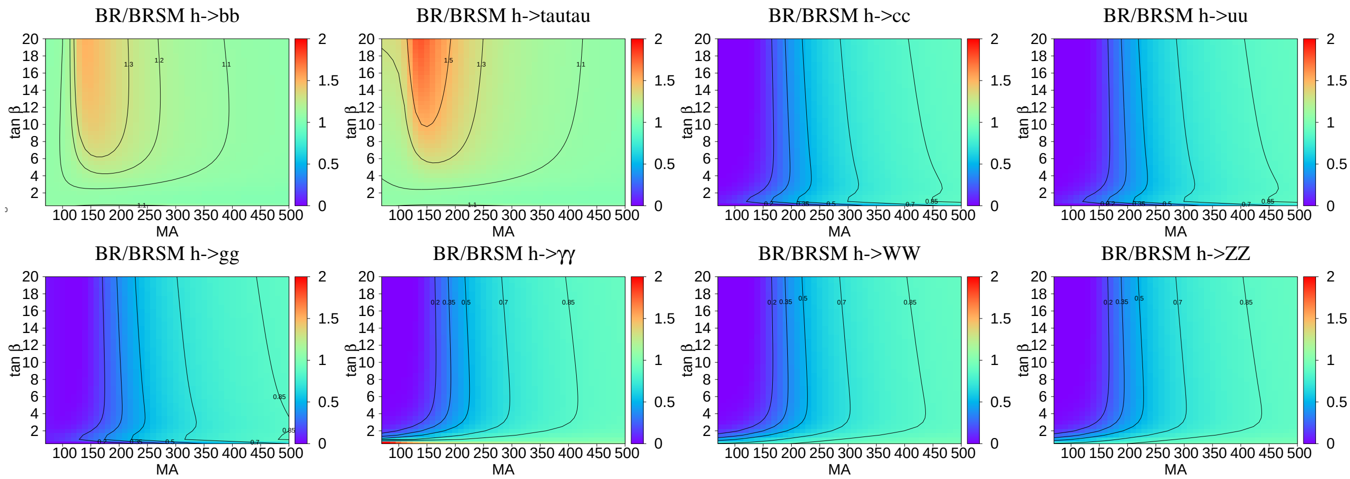

Figure 5. Colour plots of BR/BRSM ratio for differents decays of the lightest Higgs boson in the M_A $\tan\beta$ plane. Calculated with `FeynHiggs2.13.0` for $\mu = 500$ GeV, $M_S = 1.5$ TeV, $X_t = 3250$ GeV and the other parameters set as default.

As we can see, all figures show the decay modes of the lightest Higgs and none of them the Heavy Higgs since as we comment before the h boson is more SM-like in a larger region and so then it is more likely to explain the 125 GeV signal that was measured by LHC which is actually one of our objetives. Nonetheless the possibility that the H boson can explain the signal is still viable as it is demonstrated in Ref. [5]. We will discuss more on this later with the help of `HiggsSignals` and `HiggsBounds` though.

We can observe how some of the plots look similar, for example $bb \sim \tau\tau$ or $cc \sim uu$. This has to do with the fact that b and τ are downtype fermions (they both go down in the $SU(2)$ doublet), u and c are uptype fermions. Besides one can see how they all share a similar behaviour: the larger M_A gets, the closer the BR/BRSM ratio gets to 1, which corroborates what we said before in figure 3.

3.2 Heavy Higgs boson decays

On the other hand we may also be interested in how the heavy Higgs boson decay and the branching ratios of those. Thus we have plotted some of the decay modes in the M_A, $\tan\beta$ plane with the BR as colour and also the BR of all of these decays as a function of M_A for different fixed values of $\tan\beta$.

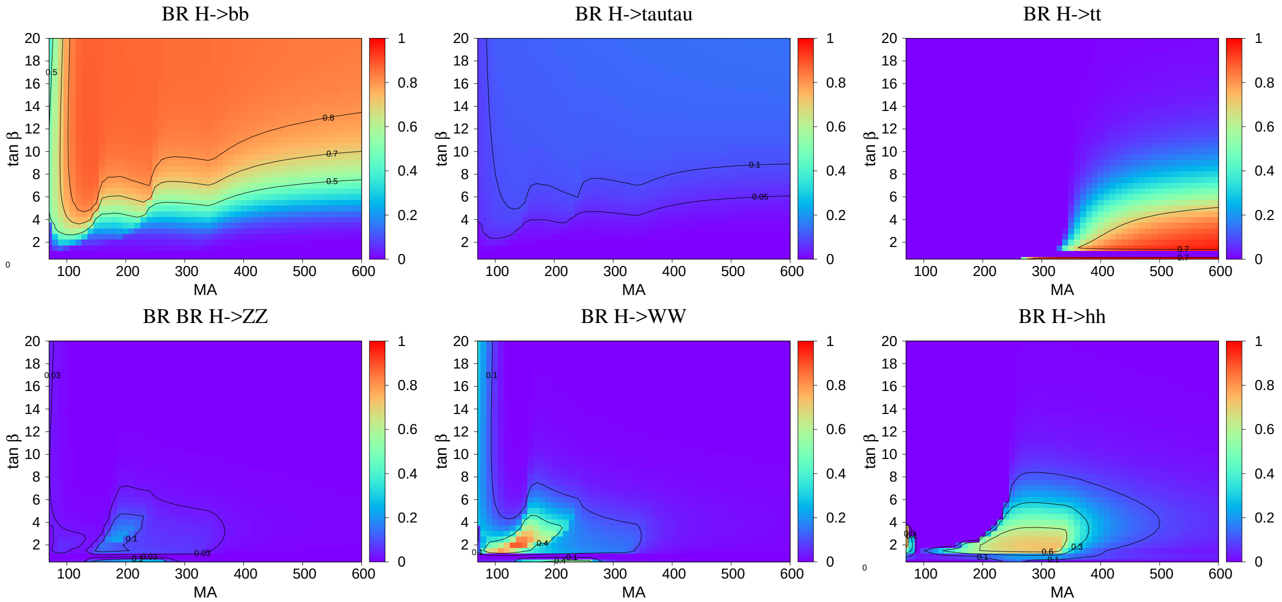

Figure 6. Colour plots of the branching ratio for differents decays of the heavy Higgs boson in the M_A,$\tan\beta$ plane. Calculated with `FeynHiggs2.13.0` for $\mu = 500$ GeV, $M_S = 1.5$ TeV, $X_t = 3250$ GeV and the other parameters set as default.

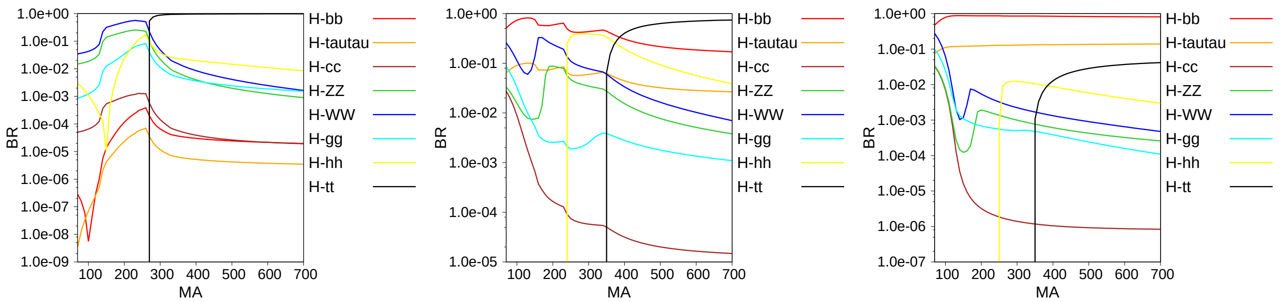

Figure 7. Branching ratio for differents decays of the heavy Higgs boson as a funtion of M_A calculated with `FeynHiggs2.13.0` for $\tan\beta = 0.5$ (left), $tan\,\beta = 5$ (center), $tan\,\beta = 15$ (right $\mu = 500$ GeV, $M_S = 1.5$ TeV, $X_t = 3250$ GeV and the other parameters set as default.

First taking a look into the $H \to bb$ or $H \to \tau\tau$ plot, we can observe how if $tan\,\beta$ increases the branching ratio does it too. This accords perfectly with Eq. (16) since if the coupling constant g_{Hdd} goes like $\propto \tan\beta$ in the DL, so the BR does. We can also see 3 different wiggles in the BR contour line shape for 3 specific masses $\sim 2 \cdot 80/2 \cdot 90$, $\sim 2 \cdot 125$ and $\sim 2 \cdot 172$ GeV,[6] which has to do with the fact that for those masses the channels of $H \to WW/ZZ$, $H \to hh$ and $H \to tt$ open kinematically as it can be observed in the other colour plots. Even though this is not completely true since, as one can see, the W channel for instance is oppened for values of the H mass under 150 GeV or, for low values of $\tan\beta$, the top channel already before 300 GeV since `FeynHiggs` incorporates off-shell effects, i.e. final state particles are considered to be unstable. Finally one can also appreciate those features in figure 7 and even more obviously, the behaviour of the couplings see Eq. (15) and (16), how for example the branching ratio of the top and charm quark decrease and the bottom quark and tau lepton ones are enhanced for large values of $\tan\beta$.

[6]The masses in the graph are M_A masses and not H ones. However as we can see in figure 3, both are really similar from $M_A \simeq 150$ GeV

3.3 Decays into neutralinos and charginos

So far we have been studying the main decays of the neutral scalar Higgs bosons into SM particles. However it is also quite interesting to decays modes into new particles. Thus, if for instance we detect some of these in the experiments, it would be a conclusive proof of the existence of physics beyond the standard model.
The particles we are focusing on now are the neutral higgsinos ($\tilde{H}_u^0$ and $\tilde{H}_d^0$) and the neutral gauginos ($\tilde{B}$ and $\tilde{W}^0$) which, due to the effects of electroweak symmetry breaking, mix with each other to form four mass eigenstates called *neutralinos*. And also the charged higgsinos ($\tilde{H}_u^+$ and $\tilde{H}_d^-$) and winos ($\tilde{W}^+$ and $\tilde{W}^-$) mix to form two mass eigenstates with charge ± 1 called *charginos*. We will denote by $\tilde{\chi}_i \, (i = 1,2,3,4)$ and $\tilde{\chi}_i^\pm \, (i = 1,2)$ which by convention are labeled in ascending mass order. In the gauge-eigenstate basis $\psi^0 = \left(\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0\right)$ the neutralino mass part of the lagrangian is:

$$\mathcal{L}_{neut\,mass} = -\frac{1}{2}\left(\psi^0\right)^T \boldsymbol{M}_{\tilde{\boldsymbol{\chi}}} \left(\psi^0\right) + c.c \tag{17}$$

with

$$\boldsymbol{M}_{\tilde{\boldsymbol{\chi}}} = \begin{pmatrix} M_1 & 0 & -g'\frac{v_d}{\sqrt{2}} & g'\frac{v_u}{\sqrt{2}} \\ 0 & M_2 & g\frac{v_d}{\sqrt{2}} & -g\frac{v_u}{\sqrt{2}} \\ -g'\frac{v_d}{\sqrt{2}} & g\frac{v_d}{\sqrt{2}} & 0 & -\mu \\ g'\frac{v_u}{\sqrt{2}} & -g\frac{v_u}{\sqrt{2}} & -\mu & 0 \end{pmatrix} \tag{18}$$

where the entries M_1, M_2 comes directly from the MSSM soft Lagrangian (see eq. (1)) while the $-\mu$ entries are the supersymmetric higgsino mass terms and the terms proportional to g, g' are the result of Higgs-higgsino-gaugino couplings, where the Higgs was replaced by its vacuum expectation value. The latter determine the branching ratio of the decays of the lightest Higgs boson into the neutralinos so, as $g^2 \simeq 3.5 g'^2$, one can expect that the final states of the decays are more likely wino-like if kinematic effects are neglected. Finally by diagonalizing that matrix one can obtain the neutralinos as the rotated ψ^0 vector and their masses as the eigenvalues. If we call N to the rotation matrix, we have:

$$(\tilde{\chi}) = N\left(\psi^0\right)$$

where N is a 4×4 matrix. Thus each state $\tilde{\chi}_i$ is given by

$$\tilde{\chi}_i = N_{i1}\tilde{B} + N_{i2}\tilde{W}^0 + N_{i3}\tilde{H}_u^0 + N_{i4}\tilde{H}_d^0$$

Due to supersymmetry, only interactions of the Higgs between$\tilde{B}, \tilde{W}^0$ and $\tilde{H}_u^0, \tilde{H}_d^0$ are allowed and those are exactly proportional to g and g' as it was said before. This means that in the e.g. decay $H \to \tilde{\chi}_1\tilde{\chi}_1$ (th decay into the second neutralino is similar) needs to be some Higgsino-fraction in$\tilde{\chi}_1$, which is encoded in N_{13} and N_{14}. This fraction, that is to say, the mixing, is larger the closer μ gets to M_1 which is directly related to a larger branching ratio of the decay. On the other hand we have the chargino spectrum wich can be analyzed in a similar way. In the gauge-eigenstate basis $\psi^\pm = \left(\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-\right)$ the chargino mass terms in the lagrangian are:

$$\mathcal{L}_{charg\,mass} = -\frac{1}{2}\left(\psi^{\pm}\right)^{T}\boldsymbol{M}_{\tilde{\chi}^{\pm}}\left(\psi^{\pm}\right) + c.c \tag{19}$$

where, in a 2×2 block form,

$$\boldsymbol{M}_{\tilde{\chi}^{\pm}} = \begin{pmatrix} 0 & \boldsymbol{X^T} \\ \boldsymbol{X} & 0 \end{pmatrix} \tag{20}$$

with

$$\boldsymbol{X} = \begin{pmatrix} M_2 & g v_u \\ g v_d & \mu \end{pmatrix} \tag{21}$$

The mass eigenstates are related to the gauge eigenstates by two unitary 2×2 matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ according to

$$\begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = \boldsymbol{V}\begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} \quad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = \boldsymbol{U}\begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix} \tag{22}$$

Note that the mixing matrix for the positively charged left-handed fermions is different from that for the negatively charged left-handed fermions. They are chosen such that

$$\boldsymbol{U}^*\boldsymbol{X}\boldsymbol{V^{-1}} = \begin{pmatrix} m_{\tilde{\chi}_1^{\pm}} & 0 \\ 0 & m_{\tilde{\chi}_2^{\pm}} \end{pmatrix} \tag{23}$$

As one can see here, the parameters that play an important role in this sector of the MSSM are M_1, M_2 and μ, because they will mostly control the neutralinos and charginos masses which will open or not the decay channels. Therefore we have plotted the branching ratio of the $h \to \tilde{\chi}_i\tilde{\chi}_j$ $(i,\, j = 1,2)$ and $h \to \tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\pm}$ as contour lines in the M_1, M_2 plane with the corresponding masses of the neutralino or chargino as colour plot.

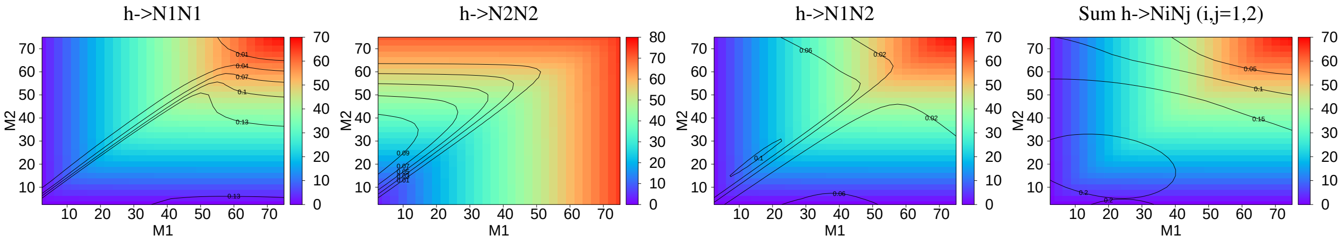

Figure 8. Decays of the lightest Higgs boson into Neutralinos with the branching ratio as contour lines (the fourth one is the sum of all) and the mass $M_{\tilde{\chi}1}$(for the first, third and fourth plot) or $M_{\tilde{\chi}2}$ (for the second plot) in GeV as colour plot in the M_1, M_2 plane in GeV as well. Calculated with `FeynHiggs2.13.0` for $\mu = 500$ GeV, $M_S = 1.5$ TeV, $X_t = 3250$ GeV, $M_A = 200$ GeV, $\tan\beta = 10$ and the other parameters set as default.

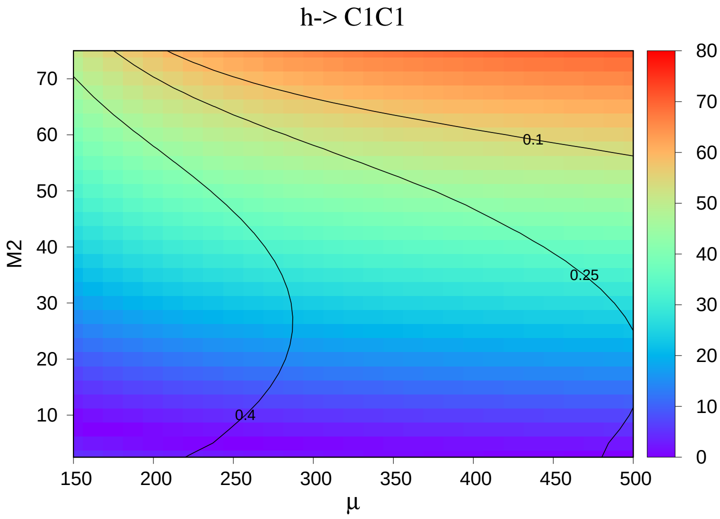

Figure 9. Decay of the lightest Higgs boson into the lightest chargino with the branching ratio as contour lines and the mass $M_{\tilde{\chi}_1^{\pm}}$ in GeV as colour plot in the μ, M_2 plane in GeV as well. Calculated with `FeynHiggs2.13.0` for $M_1 = 45$ GeV, $M_S = 1.5$ TeV, $X_t = 3250$ GeV, $M_A = 200$ GeV, $\tan\beta = 10$ and the other parameters set as default.

In the neutralino decays one can observe how the decay into the wino is prefered over the bino. Since for instance in the first figure for the decay into the lightest neutralino, when M_1 is larger than M_2 the branching ratio is larger as well and is this region the lightest neutralino mass is mainly defined by M_2 so it is more wino-like. Indeed the ratio between the branching ratios in the more wino-like zone ~ 0.13 and the more bino-like one ~ 0.4 is close to the 3.5 factor between the couplings we mentioned before . In the second plot, the branching ratio is larger when M_2 is larger than M_1, then M_2 mainly defined the $\tilde{\chi}_2$ mass which means that is more wino-like again. Furthermore we can see in all of the plots how for large values of $M_{\tilde{\chi}_1}$ or $M_{\tilde{\chi}_2}$ the branching ratios decrease because the decay kinematically closes for $m_h \leq M_{\tilde{\chi}_i} + M_{\tilde{\chi}_j}$
On the other hand we can take a look into the chargino decays: the first interesting feature is that there is one single decay, which is due to the fact that, as we can see in Eq. (21), the second neutralino mass is controlled by μ then for those values of μ the lightest Higgs boson cannot decay into the second chargino. We could have also change the range of μ into a smaller one in order to have the other possible decays, nevertheless those plots and also the one we have done are purely pedagogical since LEP excluded charginos up to ~ 110 GeV and in this region

the maximum mass is ~ 80 GeV. Thus, if we want study phenomenologically viable processes we should check them against existing experimental bounds. For the Higgs sector we will do this in the following sections.

4 HiggsBounds and HiggsSignal

So far everything we have done has been mainly theoretical. We have used the program `FeynHiggs` to study some of the MSSM phenomenology and we have got used to the Higgs sector and its most important parameters. Nevertheless what we are really interested in is whether all of these particles and decays can actually exist or not. As we said before none of the MSSM features has been discoverd yet so we would like to know in which areas of the MSSM parameter space we should focus. Therefore as we are interested mostly in the Higgs sector we will use `HiggsBounds` and `HiggsSignals`, two powerful tools that we will explain in the followings subsections [7].

4.1 HiggsBounds

What `HiggsBounds` essentially does is to check whether the existence of the MSSM heavy Higgs boson is allowed or excluded for some specific input parameters. To do so it checks with the experimental measuremets that have already been done at LHC, LEP and Tevatron.

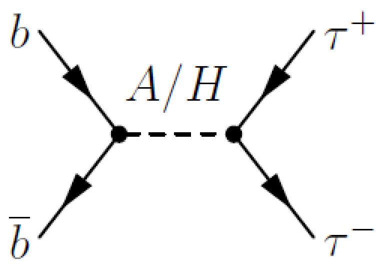

Figure 10. Feynman diagram of the $b\bar{b} \xrightarrow{H^0/A^0} \tau^+\tau^-$ process.

For example, the amplitude in the $b\bar{b} \xrightarrow{H^0/A^0} \tau^+\tau^-$ process goes like $|M|^2 \propto \tan^4\beta$ and so the cross section does, whichs means that for large values of $\tan\beta$, that process should be observed. However checking with the measurements, the heavy Higgs has never been observed, so if $\tan\beta$ is large enough we can discard such input values. Doing that in the M_A, $\tan\beta$ plane one can obtain the result presented in Fig. 11:

[7]Both of this explanations are far from being ful descriptions of `HiggsBounds` and `HiggsSignals`. To get a broad overview take a look into the manual and useful references given in Ref. [6].

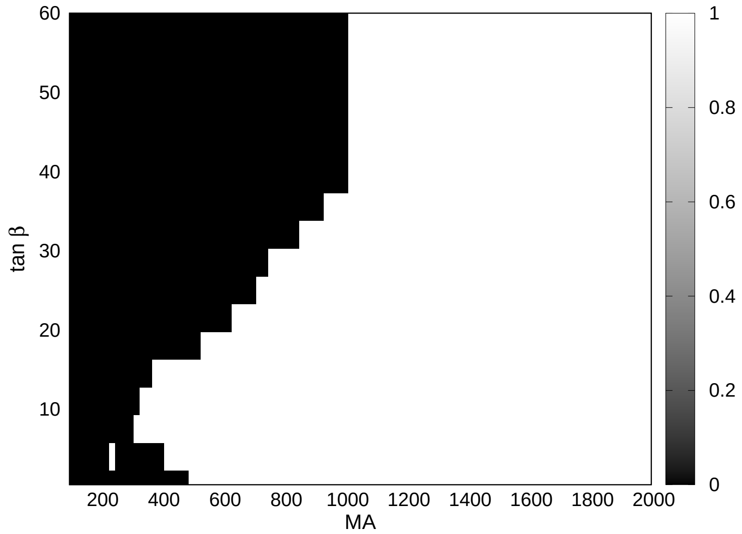

Figure 11. `HiggsBounds` test in the M_A, $\tan\beta$ plane where black is excluded and white allowed. Calculated with `HiggsBounds4.3.1`, `FeynHiggs2.13.0` for $\mu = 200$ GeV, $M_S = 1150$ GeV, $X_t = 2500$ GeV, $M_1 = 1000$ GeV, $M_2 = 1000$ GeV , $M_3 = 2500$ GeV and the other parameters set as default.

As we can see, the larger $\tan\beta$ gets the more excluded the region becomes. However we may say that this plot is not up to date with the latest results from ATLAS or CMS, but the tendency of the excluded region is to go further in M_A and lower in $\tan\beta$.

4.2 HiggsSignals

On the other hand we are also interested in how much is the lightest Higss boson of the MSSM compatible with the 125 GeV signal observed during Run I of the LHC. To classify this, `HiggsSignals` give us two output parameters:

- $\chi^2(m_h)$ which tell us whether the lightest MSSM Higgs boson mass fit to the 125 GeV signal measured. For our results, values of $7 \sim 8$ are considered as compatible, and the larger it gets the less compatible it becomes.
- $\chi^2(\mu)$ which tell us if the lightest Higgs boson couplings are similar to the measured ones. For our results, values of $60 \sim 75$ are considered as compatible, and the larger it gets the less compatible it becomes.

Here is an example of both values plotted as colour in the M_A, $\tan\beta$ plane.

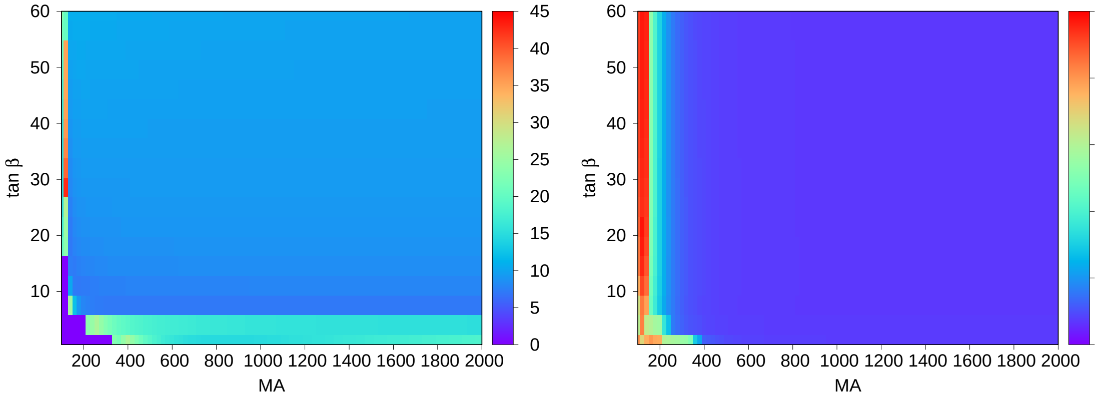

Figure 12. `HiggsSignals` test in the M_A, $\tan\beta$ plane with $\chi^2(m_h)$ (left) and $\chi^2(\mu)$ (right) as colour plots. Calculated with `HiggsSignals1.4.0`, `FeynHiggs2.13.0` for $\mu = 200$ GeV, $M_S = 1150$ GeV, $X_t = 2500$ GeV, $M_1 = 1000$ GeV, $M_2 = 1000$ GeV , $M_3 = 2500$ GeV and the other parameters set as default.

In both we can observe almost the same behaviour, for small values of M_A, no matter how large tanβ gets, all the region are excluded. On the other hand we can see a slightly difference since for $\chi^2(m_h)$ when tanβ is under ~ 5 every value of M_A is excluded which as one can see does not happen with $\chi^2(\mu)$ since m_h deviates too much from 125 GeV. Finally we take a look again into figure 3 so we can observe how in the decoupling limit and tanβ not too small, the lightest MSSM Higgs boson really agrees with the signal found at the LHC (both the mass and the couplings).

5 Compatibility of Higgs invisible decays with current LHC data

At this point we have all the tools to make a rigorous analysis of every decay process in the Higgs sector of the MSSM. We are rather interested in the lightest Higgs boson decays into neutralinos because the lightest neutralino is an ideal candidate of being one of the dark matter particles, which if discovered could shed some light on this 'dark' issue. We first have set $M_A = 1000$ GeV, $M_S = 1100$ GeV, $X_t = 2500$ GeV, $M_3 = 2500$ GeV and all the other parameters as default in order to have a lightest Higgs boson not so different from the ~ 125 GeV measured one in the region of the M_1, M_2 plane we are interested in.

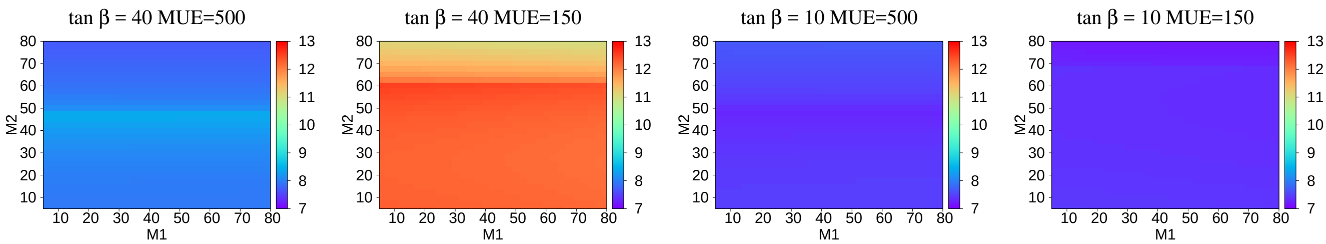

Figure 13. Colour plot of $\chi^2(m_h)$ in the M_1, M_2 plane for differents values of $\tan\beta$ and μ. Calculated with `HiggsSignals1.4.0`, `FeynHiggs2.13.0` for $M_A = 1000$ GeV, $M_S = 1000$ GeV, $X_t = 2500$ GeV, $M_3 = 2500$ GeV and the other parameters set as default.

We can see a little dependence of $\chi^2(m_h)$ with M_2, the larger the latter, the smaller the former. The parameters of the second plot are most excluded since low μ and large $\tan\beta$ have the largest effect on the light Higgs boson mass in the considered scenarios.

Thus we can now take a look into the branching ratios of the lightest Higgs boson decays into neutralinos for each different parameter combination and show the corresponding $\chi^2(\mu)$ in the following plots:

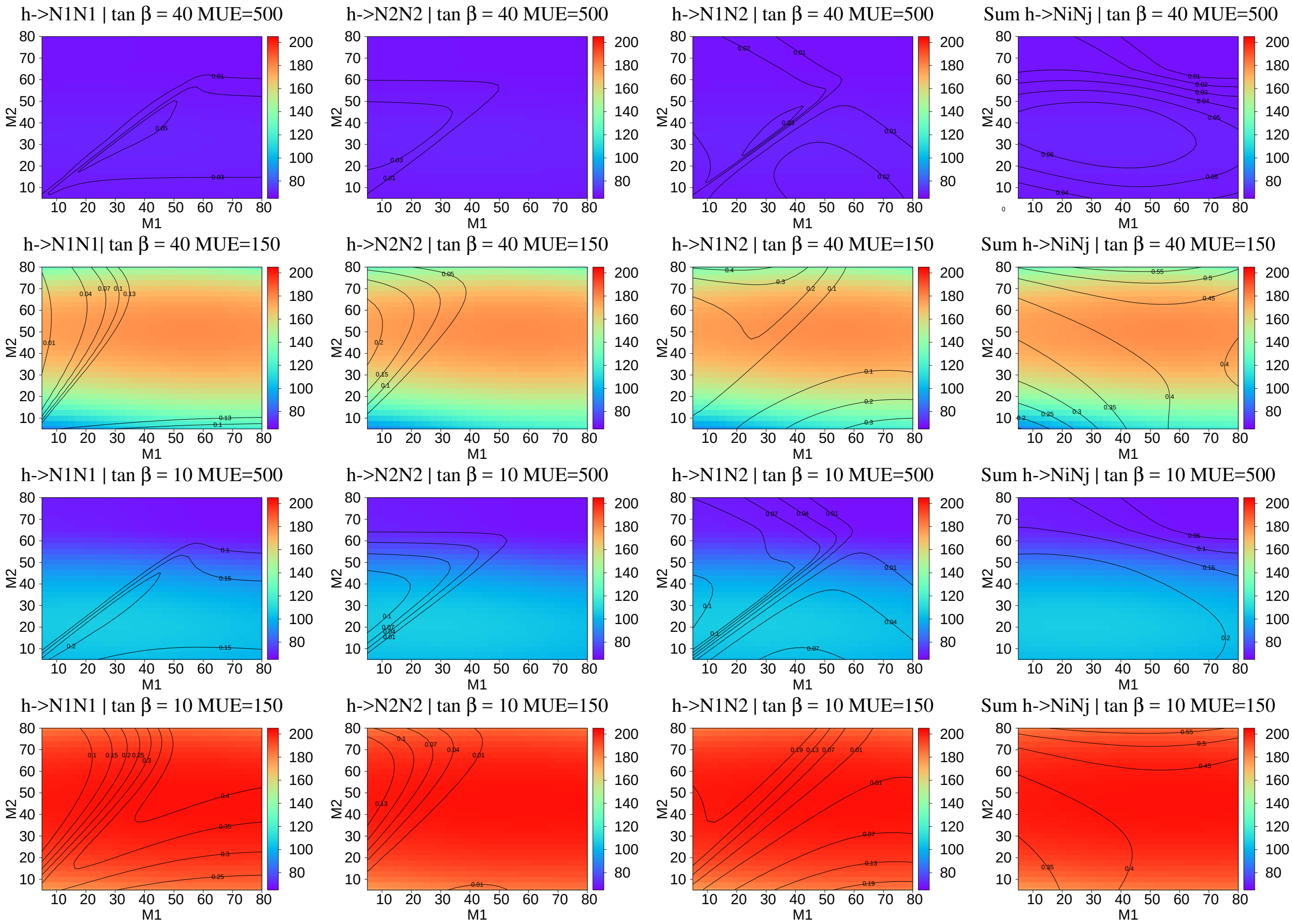

Figure 14. Decays of the lightest Higgs boson into Neutralinos with the branching ratio as contour lines (the fourth one in each row is the sum of all) and the $\chi^2(\mu)$ as colour plot in the M_1, M_2 plane for differents values of $\tan\beta$ and μ. Calculated with `HiggsSignals1.4.0`, `FeynHiggs2.13.0` for $M_A = 1000$ GeV, $M_S = 1000$ GeV, $X_t = 2500$ GeV, $M_3 = 2500$ GeV and the other parameters set as default.

As one can see here both tanβ and μ have an influence on the branching ratios of the decays. In the first row of plots both parameters are high enough so the BR does not exceed 5%. Thus we don't have a penalty of `HiggsSignals` and this region is still compatible. By lowering μ to 150 GeV we observe how all BRs increase, as we explained before in 3.3, and so almost every region in the M_1 ,M_2 plane is excluded (except from the lower left corner). If instead of μ we lower $\tan\beta$ down to 10 we observe also an increase of the BR however if M_2 is high enough we do not get a big penalty from `HiggsSignals` as one can see. Finally if we lower both parameters the BRs rise too much making the region completely excluded.
Given the dependence between the branching ratio and μ we want to work it out in a bit more

detail. Thus, with the same parameters but fixing $M_2 = 150$ GeV we have plotted the BRs in the M_1 , μ plane with $\chi^2(\mu)$ as coloured regions. Before we show $\chi^2(m_h)$ in that region:

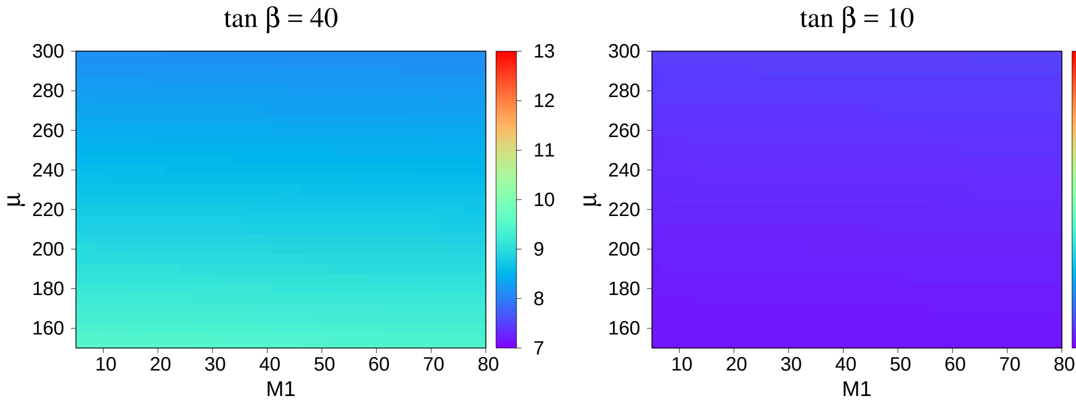

Figure 15. Colour plot of $\chi^2(m_h)$ in the M_1, μ plane for differents values of $\tan\beta$. Calculated with `HiggsSignals1.4.0`, `FeynHiggs2.13.0` for $M_A = 1000$ GeV, $M_S = 1000$ GeV, $X_t = 2500$ GeV, $M_3 = 2500$ GeV, $M_2 = 150$ GeV and the other parameters set as default.

Here unlike before, for low values of $\tan\beta$ the Higgs boson mass is more compatible and we also observe this feature if μ increases.

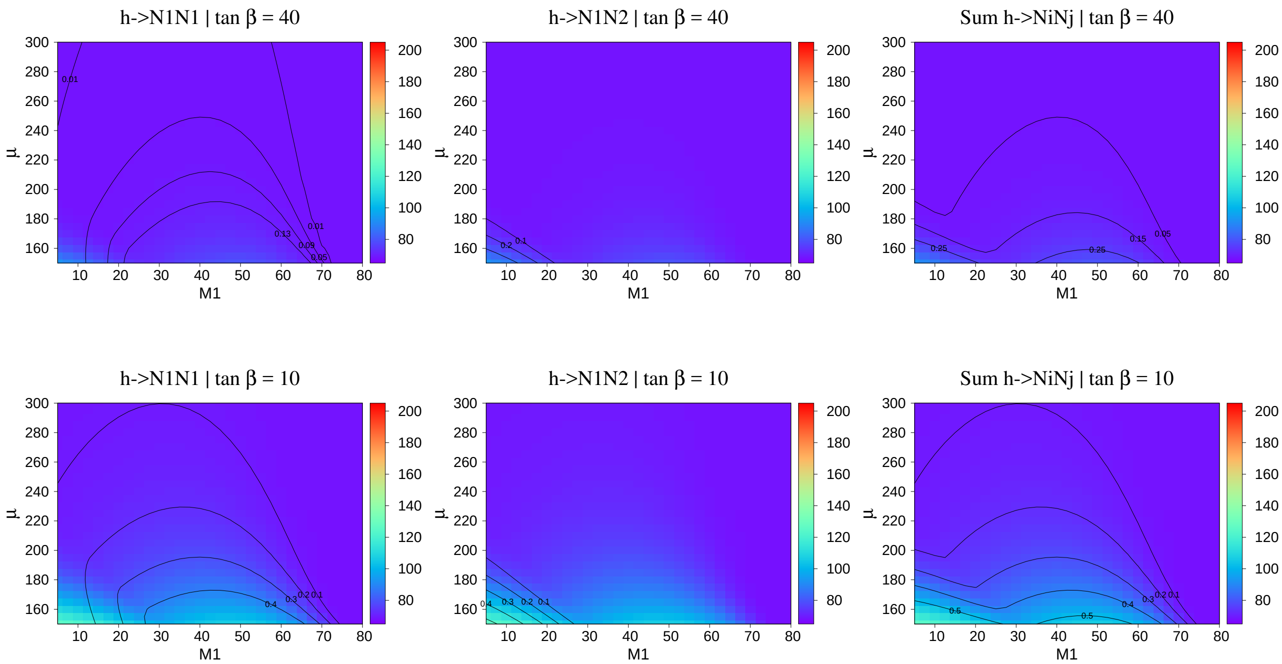

Figure 16. Decays of the lightest Higgs boson into Neutralinos with the branching ratio as contour lines (the third one in each row is the sum of all) and $\chi^2(\mu)$ as colour plot in the M_1, μ plane for different values of $\tan\beta$. Calculated with `HiggsSignals1.4.0`, `FeynHiggs2.13.0` for $M_A = 1000$ GeV, $M_S = 1000$ GeV, $X_t = 2500$ GeV, $M_3 = 2500$ GeV, $M_2 = 150$ GeV and the other parameters set as default.

First we realize that we do not have $\tilde{\chi}_2\tilde{\chi}_2$ decay kinematically open since the M_2 mass is fixed to 150 GeV. Nevertheless for low values of M_1 we still observe the decay into $\tilde{\chi}_1\tilde{\chi}_2$ as one can see in the left bottom corner of the second plot (in both rows). Similar to the previous variation of M_2 from 5 to 80 GeV we again find a penalty from `HiggsSignals` for lower values of μ, however it is not as pronounced.

6 Conclusion

In our project we have studied many differents aspects of the MSSM, more precisely in its Higgs sector. We have seen how the lightest Higgs boson of the MSSM is in most parts of the parameter space SM-like and thus can explain the Higgs-like state at $\sim$ 125 GeV measured by the LHC. There is a small corner left for the heavy Higgs boson to explain the measured state. Then we have focused on the decays of the lightest Higgs boson into neutralinos, observed considerable different features of those decays and analysed their dependence on the parameters of the theory. As a result, we have obtained a noteworthy dependence of the μ parameter on Higgs decays into Neutralinos that could be interesting to explore in future projects.

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References

[1] S. P. Martin, *A Supersymmetry primer*, Adv. Ser. Direct. High Energy Phys. **21** (2010) 1 [Adv. Ser. Direct. High Energy Phys. **18** (1998) 1] [hep-ph/9709356].

[2] BSM Felix Brümmer Lectures at the DESY Summer Student Programme 2017. `https://summerstudents.desy.de/hamburg/e246457/e246487/BSM_DESY_1.pdf`

[3] `http://francis.naukas.com/2015/06/25/el-modelo-supersimetrico-minimo-i-la-teoria/`

[4] A. Djouadi, *The Anatomy of electro-weak symmetry breaking II. The Higgs bosons in the minimal supersymmetric model*, Phys. Rept. **459** (2008) 1 [hep-ph/0503173].

[5] P. Bechtle, H. E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, *The Light and Heavy Higgs Interpretation of the MSSM*, Eur. Phys. J. C **77** (2017) no.2, 67 [arXiv:1608.00638 [hep-ph]].

[6] `HiggsBounds` and `HiggsSignals` manuals `http://higgsbounds.hepforge.org`

[7] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein and K. E. Williams, *HiggsBounds: Confronting Arbitrary Higgs Sectors with Exclusion Bounds from LEP and the Tevatron*, Comput. Phys. Commun. **181** (2010) 138 [arXiv:0811.4169 [hep-ph]].

[8] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein and K. E. Williams, *HiggsBounds 2.0.0: Confronting Neutral and Charged Higgs Sector Predictions with Exclusion Bounds from LEP and the Tevatron*, Comput. Phys. Commun. **182** (2011) 2605 [arXiv:1102.1898 [hep-ph]].

[9] P. Bechtle, O. Brein, S. Heinemeyer, O. Stal, T. Stefaniak, G. Weiglein and K. Williams, *Recent Developments in HiggsBounds and a Preview of HiggsSignals*, PoS CHARGED **2012** (2012) 024 [arXiv:1301.2345 [hep-ph]].

[10] P. Bechtle, O. Brein, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and K. E. Williams, HiggsBounds-4*: Improved Tests of Extended Higgs Sectors against Exclusion Bounds from LEP, the Tevatron and the LHC*, Eur. Phys. J. C **74** (2014) no.3, 2693 [arXiv:1311.0055 [hep-ph]].

[11] P. Bechtle, S. Heinemeyer, O. Stal, T. Stefaniak and G. Weiglein, *Applying Exclusion Likelihoods from LHC Searches to Extended Higgs Sectors*, Eur. Phys. J. C **75** (2015) no.9, 421 [arXiv:1507.06706 [hep-ph]].

[12] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, *HiggsSignals: Confronting arbitrary Higgs sectors with measurements at the Tevatron and the LHC*, Eur. Phys. J. C **74** (2014) no.2, 2711 [arXiv:1305.1933 [hep-ph]].

[13] O. Stål and T. Stefaniak, *Constraining extended Higgs sectors with HiggsSignals*, PoS EPS **-HEP2013** (2013) 314 [arXiv:1310.4039 [hep-ph]].

[14] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, *Probing the Standard Model with Higgs signal rates from the Tevatron, the LHC and a future ILC*, JHEP **1411** (2014) 039 [arXiv:1403.1582 [hep-ph]].

[15] H. Bahl and W. Hollik, *Precise prediction for the light MSSM Higgs boson mass combining effective field theory and fixed-order calculations*, Eur. Phys. J. C **76** (2016) no.9, 499 [arXiv:1608.01880 [hep-ph]].

[16] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *High-Precision Predictions for the Light CP -Even Higgs Boson Mass of the Minimal Supersymmetric Standard Model*, Phys. Rev. Lett. **112** (2014) no.14, 141801 [arXiv:1312.4937 [hep-ph]].

[17] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *The Higgs Boson Masses and Mixings of the Complex MSSM in the Feynman-Diagrammatic Approach*, JHEP **0702** (2007) 047 [hep-ph/0611326].

[18] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, *Towards high precision predictions for the MSSM Higgs sector*, Eur. Phys. J. C **28** (2003) 133 [hep-ph/0212020].

[19] S. Heinemeyer, W. Hollik and G. Weiglein, *The Masses of the neutral CP - even Higgs bosons in the MSSM: Accurate analysis at the two loop level*, Eur. Phys. J. C **9** (1999) 343 [hep-ph/9812472].

[20] S. Heinemeyer, W. Hollik and G. Weiglein, *FeynHiggs: A Program for the calculation of the masses of the neutral CP even Higgs bosons in the MSSM*, Comput. Phys. Commun. **124** (2000) 76 [hep-ph/9812320].

[21] S. Dimopoulos and D. Sutter, Nucl. Phys. B452 (1995) 496; see also the discussion given by H.E. Haber, hep-ph/9709450.

[22] Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85 (1991) 1; ibid. Phys. Lett. B262 (1991) 54; J.R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B257 (1991) 83; ibid. Phys. Lett. B262 (1991) 477; H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815.

[23] E. Boos, A. Djouadi, M. Muhlleitner and A. Vologdin, Phys. Rev. D66 (2002) 055004