Energy Dependence of p_{T}^{0} in Pythia8

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Abstract: The problem of divergence of the interaction cross section at low transverse momentum (p_{T}) in hard processes is very well known because the cross section is inverse proportional to p_{T}^{2}. The complementary way to deal with it is to introduce a free parameter, called p_{T}^{0}, by simply adding it to the denominator to prevent the zero value to occur. p_{T}^{0} turns out to be energy-dependent of which the more precise relation will yield the more accurate of predictions in Pythia event generator. The current relation implemented in Pythia8 is a power relation described by 2 parameters $\left(p_{\mathrm{T}}^{0} = aE^{b}\right)$. Other possible relations which better fit with data, using 2 or 3 parameters, are investigated in this work. It is showed that we can actually improve the relation. Most of the functions which have been found to be better fit are composed of the power term. Modifications of the default function also yields better result.

Acknowledgments

I would like to thank my supervisors: Hannes Jung and Paolo Gunnelini for giving me a chance to learn many new things from the group. Their warm welcome, their encouragement and the way they took care of me and my friends in the group eliminated all my worries about work that I have to deal with. Paolo always make everything sound interesting that somehow impressed me. Although it was just two months, the whole experiences that I got from them is invaluable. Furthermore, I would like to thank Maximilian We who often waited to have lunch and have a break with me. And thanks to Arathi Ramesh who stayed working in the office with me until night many times. It was a fascinating summer school for me at DESY to have an opportunity to know many people across the globe — thank you to this program. And special thanks to Olaf Behnke who provided every kind of comfortabilities, conveniences and entertainments , and suggested many trips for us. I would absolutely suggest this program to others to let them know how cool this program is!

1 Introduction

In hadron-hadron collisions ($p\overline{p}$ collisions in Tevatron and pp collisions in LHC), the differential parton-level cross section which describes hard scattering or multiparton interaction(MPI)[1], can be written as a function of p_{T}^2 :

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_{\mathrm{T}}^2} = \frac{8\pi\alpha_{\mathrm{s}}^2(p_{\mathrm{T}}^2)}{9p_{\mathrm{T}}^4} \quad . \tag{1}$$

For constant α_{s} , after integrating (1), we will obtain:

$$\hat{\sigma} \propto \frac{1}{p_{\mathrm{T}}^2} \quad . \tag{2}$$

It can cause a problem when p_{T} is very small; the cross section will be infinitely large. Moreover, it actually does not fit very well with the observable data in the small-p_{T} region. One way to regularize this problem is to multiply equation (1) by the following factor

$$\frac{\alpha_{\mathrm{s}}^2\left(p_{\mathrm{T}}^2 + (p_{\mathrm{T}}^0)^2\right)}{\alpha_{\mathrm{s}}^2(p_{\mathrm{T}}^2)} \frac{p_{\mathrm{T}}^4}{\left(p_{\mathrm{T}}^2 + (p_{\mathrm{T}}^0)^2\right)^2} \quad . \tag{3}$$

After regularizing, equation (1) will be changed to:

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_{\mathrm{T}}^2} = \frac{8\pi\alpha_{\mathrm{s}}^2(p_{\mathrm{T}}^2 + (p_{\mathrm{T}}^0)^2)}{9\left(p_{\mathrm{T}}^2 + (p_{\mathrm{T}}^0)^2\right)^2} \quad . \tag{4}$$

This is responsible for a change in the partonic cross section

$$\hat{\sigma} \propto \frac{1}{(p_{\mathrm{T}}^2 + (p_{\mathrm{T}}^0)^2)^2} \quad . \tag{5}$$

The denominator apparently cannot be zero so that the divergence is completely eliminated. p_{T}^0 serves as a free parameter, which cannot be obtained from first principle, but must be tuned to data. According to CDF and CMS data for different center-of-mass energy, it is found that p_{T}^0, which is to be specified as an input for Pythia event generator, that yields a good agreement with those data is energy-dependent. The dependence is believed to be some power of the energy and this power relation is currently used in Pythia8 as a default function. More precisely, the default function can be further described by two additional parameters: $p_{\mathrm{T,ref}}^0$ and E_{pow}.

$$p_{\mathrm{T}}^0 = p_{\mathrm{T,ref}}^0 \left(\frac{E(\mathrm{TeV})}{7}\right)^{E_{\mathrm{pow}}} \quad , \tag{6}$$

where $p_{\mathrm{T,ref}}^0$ can be seen as a p_{T}^0 at a reference energy (7 TeV). These parameters can be found from fitting with the data. But there is no guarantee that this function is the correct one; it could be another one. The CDF data (0.3, 0.9 and 1.96 TeV) and CMS data (7 TeV) were formerly used as reference data for fitting. However, there are new data at 13 TeV center-of-mass energy from CMS that can be added to give a more precise value in the two parameters or to make a comparison between the default function and other functions. For the sake of simplicity in this work the following expression is used instead of (6):

$$p_{\mathrm{T}}^0 = aE^b \quad . \tag{7}$$

2 Methods

This work is mainly about exploring functions that can be well fitted with the data compared to the default function. But the data themself have to be explored first. Because p_{T}^0 is not an observable that can be directly obtained from the experiment, the way to obtain their values will be mentioned in the following section and then the strategy to explore the fitting functions will be discussed.

2.1 Investigating p_{T}^0 belonging to each energy

Because of the energy-dependent behavior, each energy has its own p_{T}^0 value. To find such values, the Pythia8 event generator is used to generate at most 30 predictions, each of which is assigned different value of p_{T}^0. Then a Rivet analysis[2] is employed to analyse the results by creating histogram. Afterwards, all the predictions are compared to the data using the Professor framework[3]. The aim is to find the value of p_{T}^0 that makes the prediction look most similar to the data; practically, one looks at the value of p_{T}^0 that yields the lowest value of χ^2. To do so, the interpolation method in Professor is used to make a continuous function of χ^2 versus p_{T}^0, and then the tuning method is used to find the value of p_{T}^0 that minimizes χ^2. By redoing the same process for other energies, one will obtain all values of p_{T}^0 associated with them. Moreover, we also try to find p_{T}^0 for other PDF(parton distribution functions) sets other than just leading order(LO) namely next to leading order(NLO) and next to next to leading order (NNLO) PDF sets. Figure 1 is the result after generating 30 predictions and using Rivet analysis to compare to the data of some particular energy and observable.

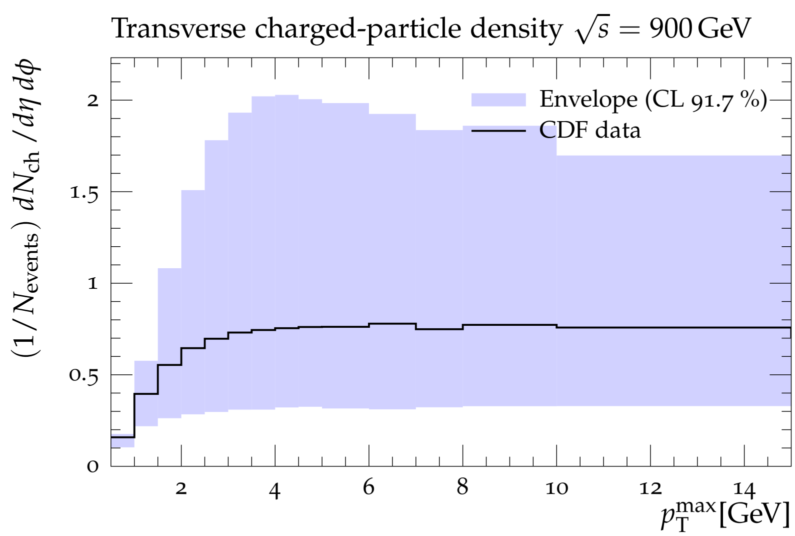

Figure 1: *This graph shows the data that are almost coverd by the 30 predictions which are generated by Pythia. All predictions have p_T^0 associated with them. It means that the value of p_T^0 which best fits the data exists in this range. And it can be determined by using Professor framework.*

2.2 Investigating fitting functions

After obtaining the values of p_{T}^0 for each energy and each PDF sets, the next step is to find fitting functions. We explore any functions within 2-3 parameters that are better fitting with the data than the default one. Again the χ^2 is the value for comparing the default function to others.

Other than arbitrary functions, starting to modify the default one seems to be easier to work with. Three kinds of modification of (7) are introduced by thinking of the default as being able to multiply by some factor or to be written in perturbative form as in the following:

$$p_{\mathrm{T}}^0 = aE^b f(E;c) \quad , \tag{8}$$

$$p_{\mathrm{T}}^0 = aE^b + g(E;c) \quad , \tag{9}$$

$$p_{\mathrm{T}}^0 = aE^b \left(1 + h(E;c)\right) \quad , \tag{10}$$

which will be called f-modification, g-modification and h-modification respectively where $f(E;c)$, $g(E;c)$ and $h(E;c)$ are functions (with parameter c) to be explored.

3 Results

This section will show all the results of our work. First, we will show, for each energy and PDF set, the values of p_{T}^0 that will be fitted. Then all of the fitting functions will be revealed.

3.1 Result from investigating p_{T}^0

Utilizing Professor program, we will obtain all values of p_{T}^0. As previously mentioned, p_{T}^0's are determined by looking at the χ^2 from comparing the generated events to the experimental data. Table 1 illustrates the values of them and their lowest possible values of χ^2 from a fit belonging to each energy and PDF sets. The energy-dependent of p_{T}^0 can be obviously seen in figure 2 which will be used as data points for fitting.

energy (TeV)	**LO**		**NLO**		**NNLO**	
	χ^2	p_{T}^0	χ^2	p_{T}^0	χ^2	p_{T}^0
0.3	0.595	$1.535^{+0.024}_{-0.023}$	0.645	$1.437^{+0.022}_{-0.020}$	0.623	$1.425^{+0.022}_{-0.021}$
0.9	0.462	$1.740^{+0.018}_{-0.055}$	0.636	$1.559^{+0.020}_{-0.018}$	0.708	$1.524^{+0.018}_{-0.018}$
1.96	0.506	$1.956^{+0.021}_{-0.021}$	0.172	$1.663^{+0.017}_{-0.016}$	0.525	$1.632^{+0.030}_{-0.027}$
7	0.684	$2.346^{+0.026}_{-0.026}$	0.469	$1.889^{+0.035}_{-0.032}$	1.280	$1.870^{+0.052}_{-0.049}$
13	0.311	$2.571^{+0.020}_{-0.019}$	1.180	$1.958^{+0.028}_{-0.027}$	1.830	$1.939^{+0.029}_{-0.028}$

Table 1: *All of the values of p_T^0 and their χ^2 that indicates how good they fit the experimental data in each energy and PDF sets*

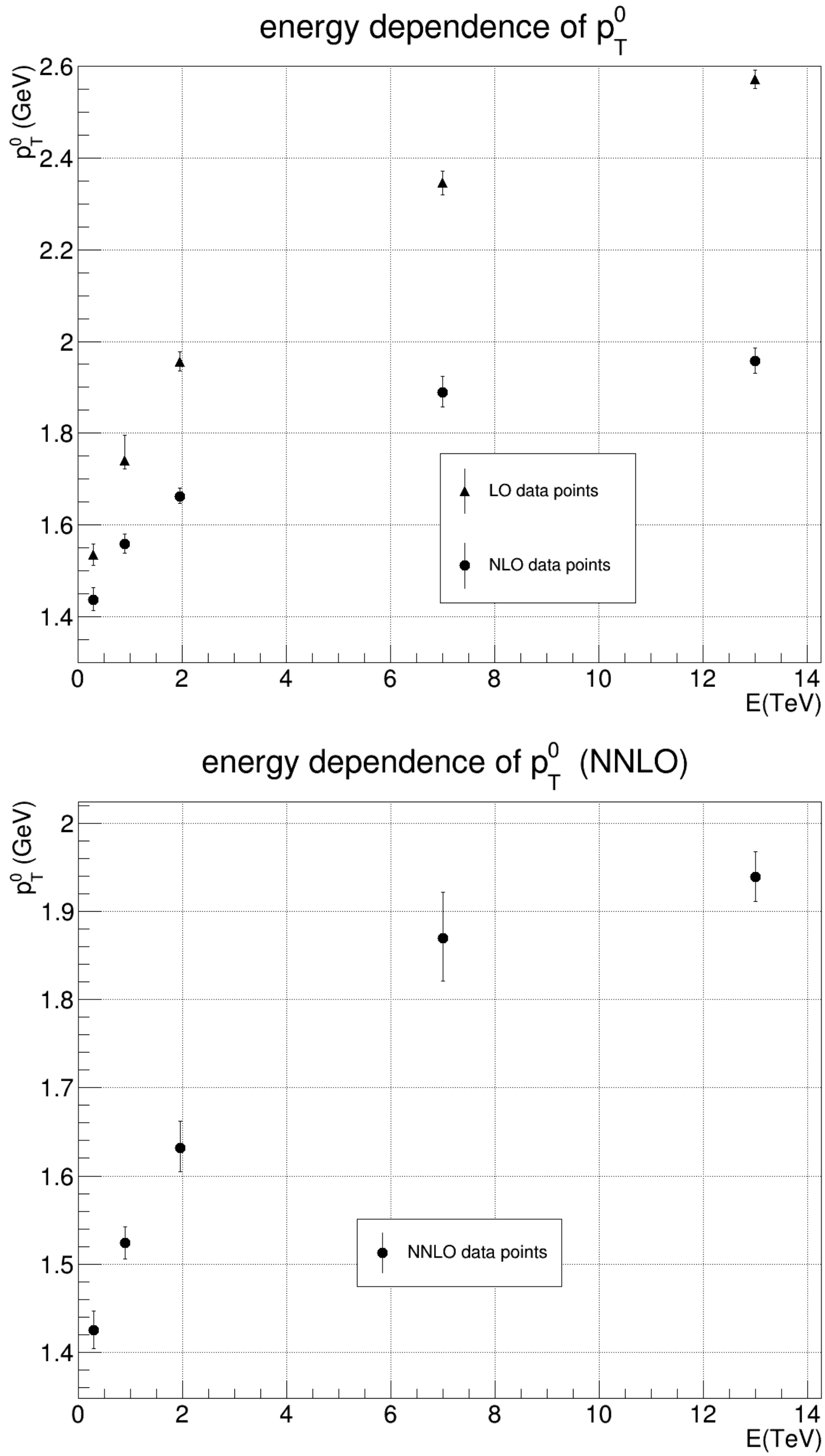

Figure 2: *Graph of p_T^0 versus center-of-mass energy in each PDF sets*

3.2 Result from fitting

To begin with, the behavior of the default function implemented in Pythia8 in fitting the data is illustrated in figure 3. The χ^2's imply that this function does not fit the data very well.

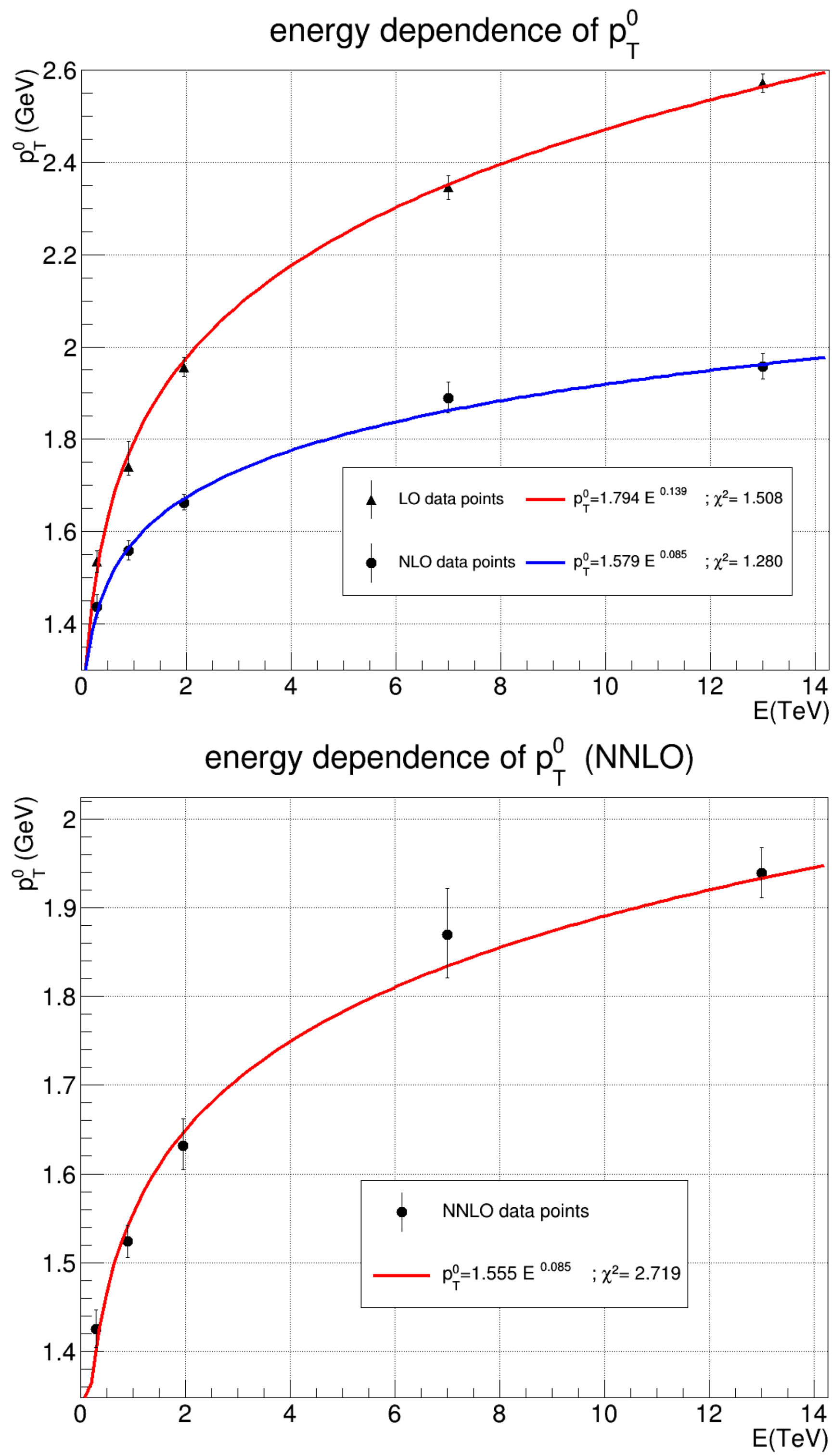

Figure 3: *Fitting curves from the default function*

3.2.1 2-parameter function

There is only one function with 2 parameters that gives better fitting result with respect to the default function which is $p_{\mathrm{T}}^0 = E^a + b$. The results are demonstrated in figure 4.

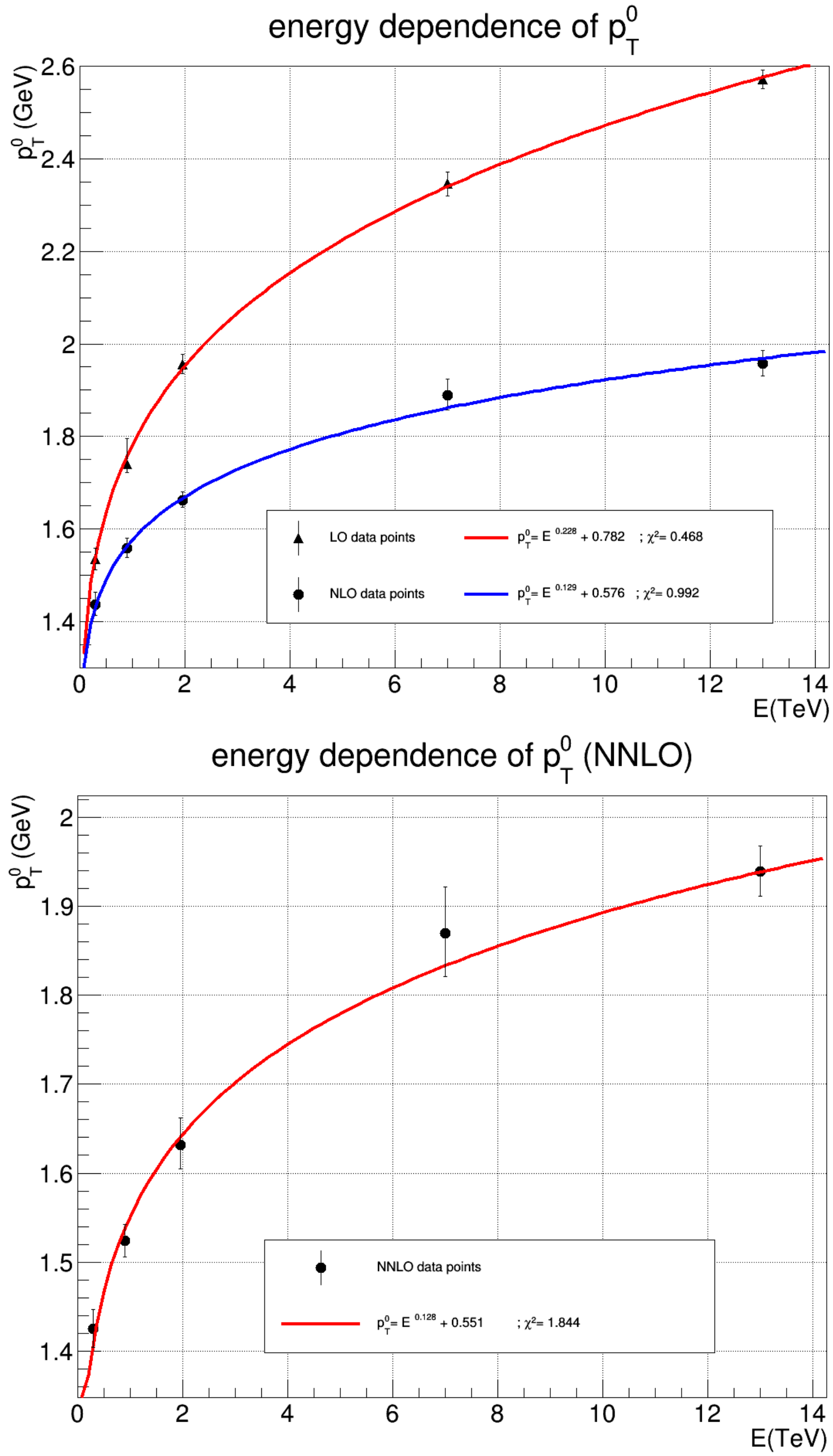

Figure 4: *Fitting curves from two-parameter function:* $p_T^0 = E^a + b$

3.2.2 f-modification

There are 3 functions of $f(E;c)$ in equation(8) namely $f(E;c) = \arctan(E+c)$, $f(E;c) = \arctan(E) + c$ and $f(E;c) = \ln(E) + c$ that result in better fitting which is illustrated in figure 5.

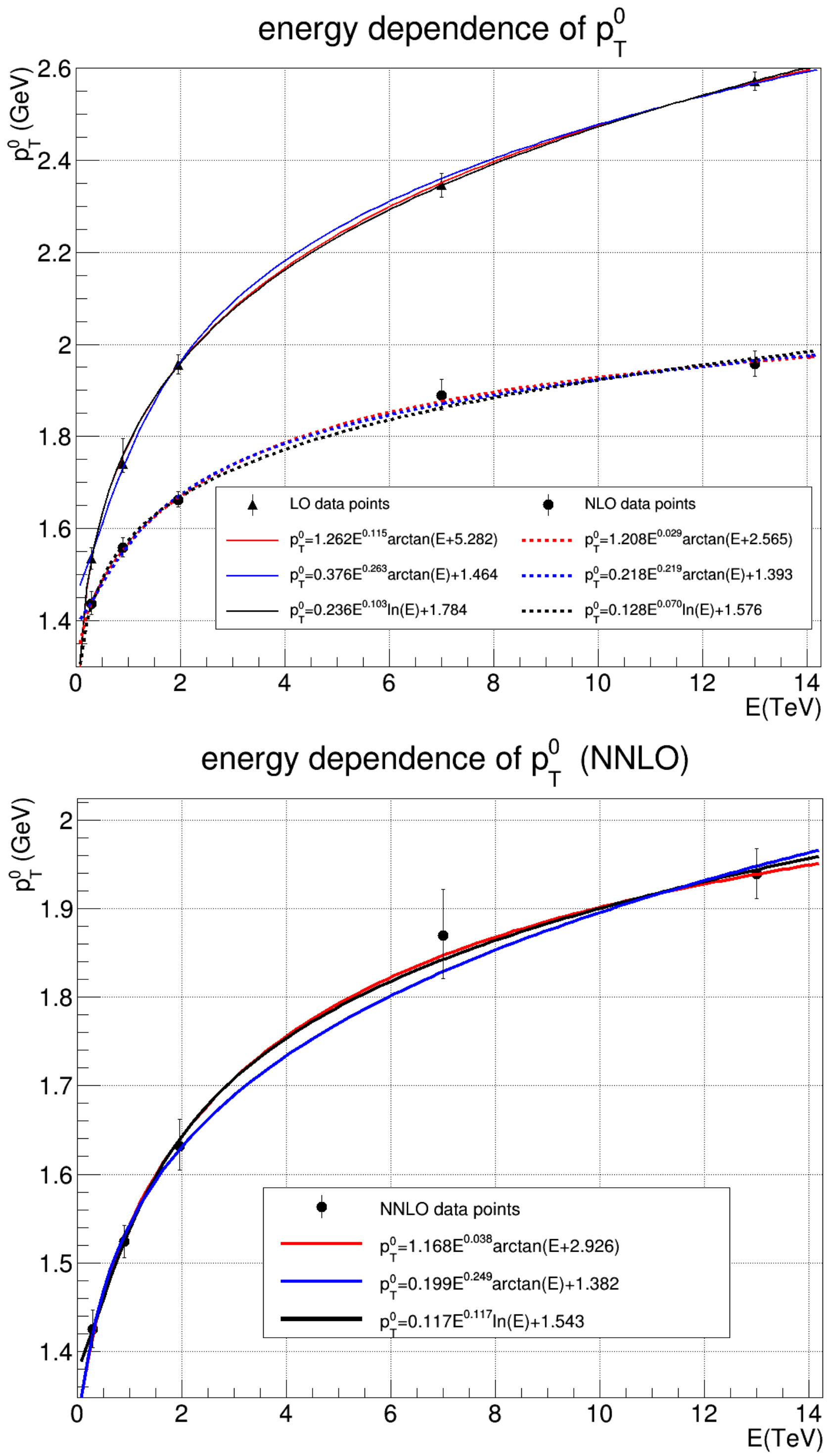

Figure 5: *Fitting curves from f-modification*

3.2.3 g-modification

Three functions of $g(E;c)$ in equation(9) are found to be better than the default function. Those functions are $g(E;c) = c$, $g(E;c) = 1 + cE$ and $g(E;c) = 1 + c$ as shown in figure 6.

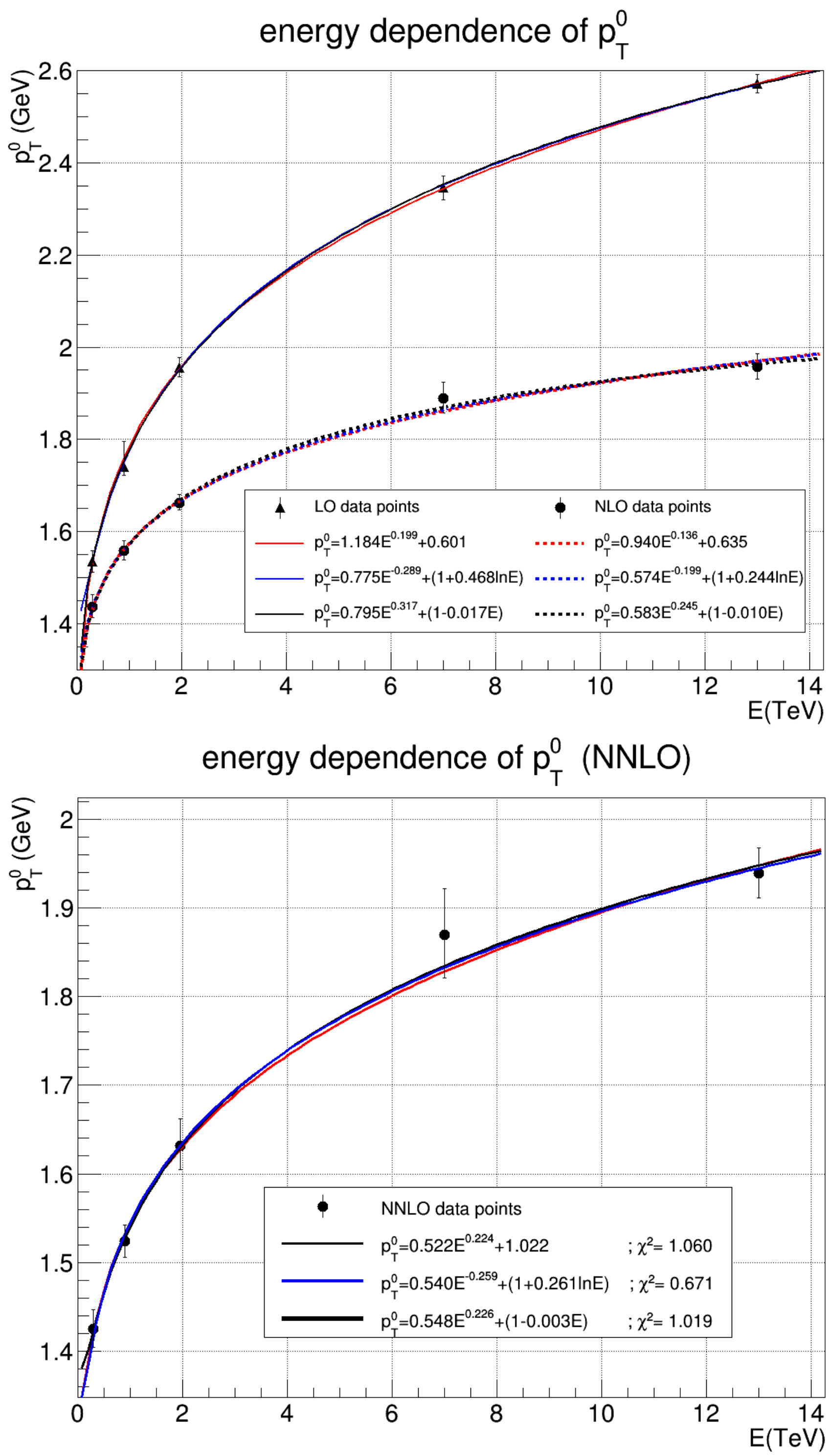

Figure 6: *Fitting curves from g-modification*

3.2.4 h-modification

Two functions of $h(E;c)$ namely $h(E;c) = 1 + cE$ and $h(E;c) = 1 + E^c$ are also found to be good as shown in figure 7.

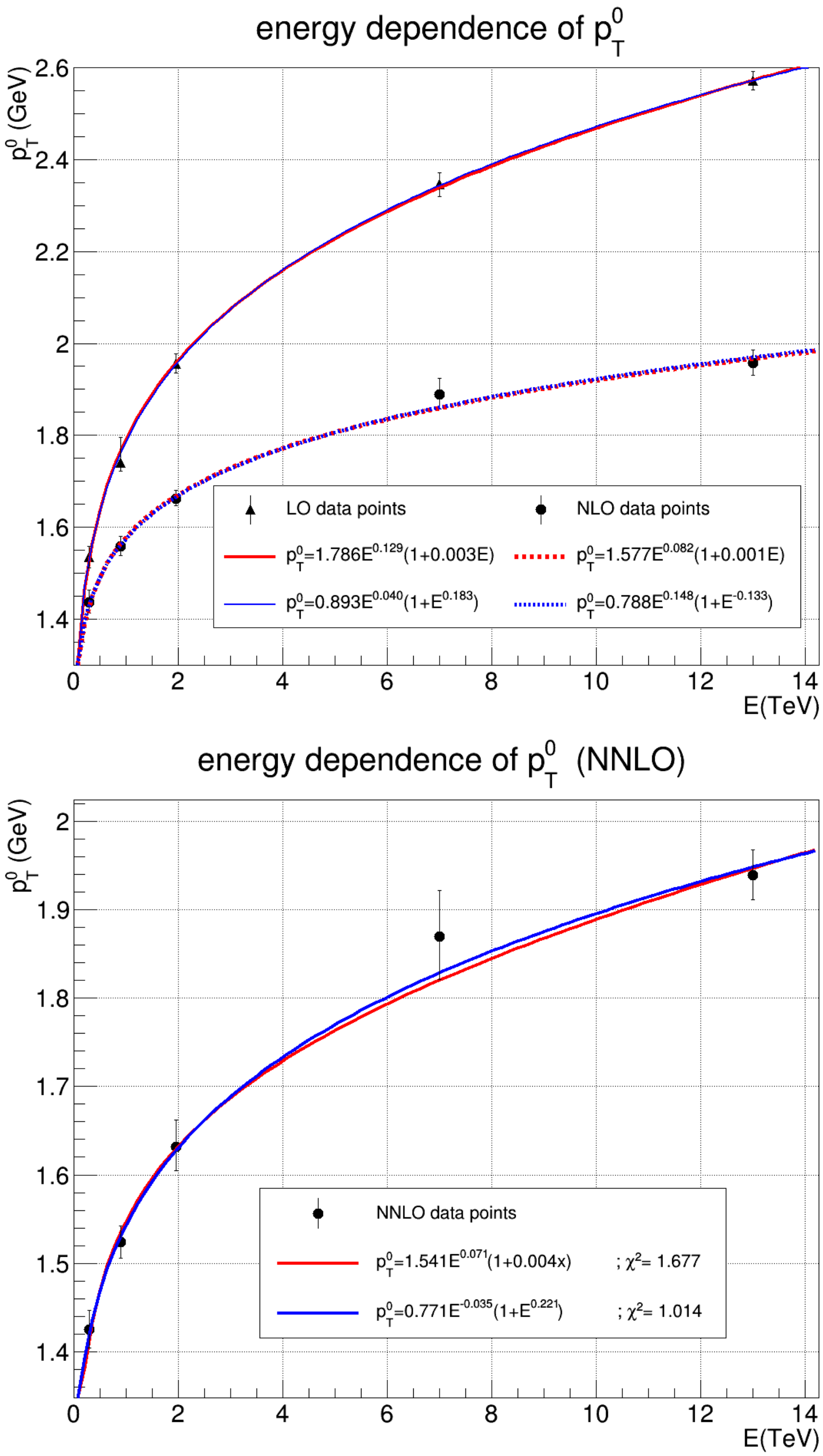

Figure 7: *Fitting curves from h-modification*

3.2.5 Other functions

There are many functions exclusive of those modifications previously mentioned that also give good χ^2, some of them plotted in figure 8.

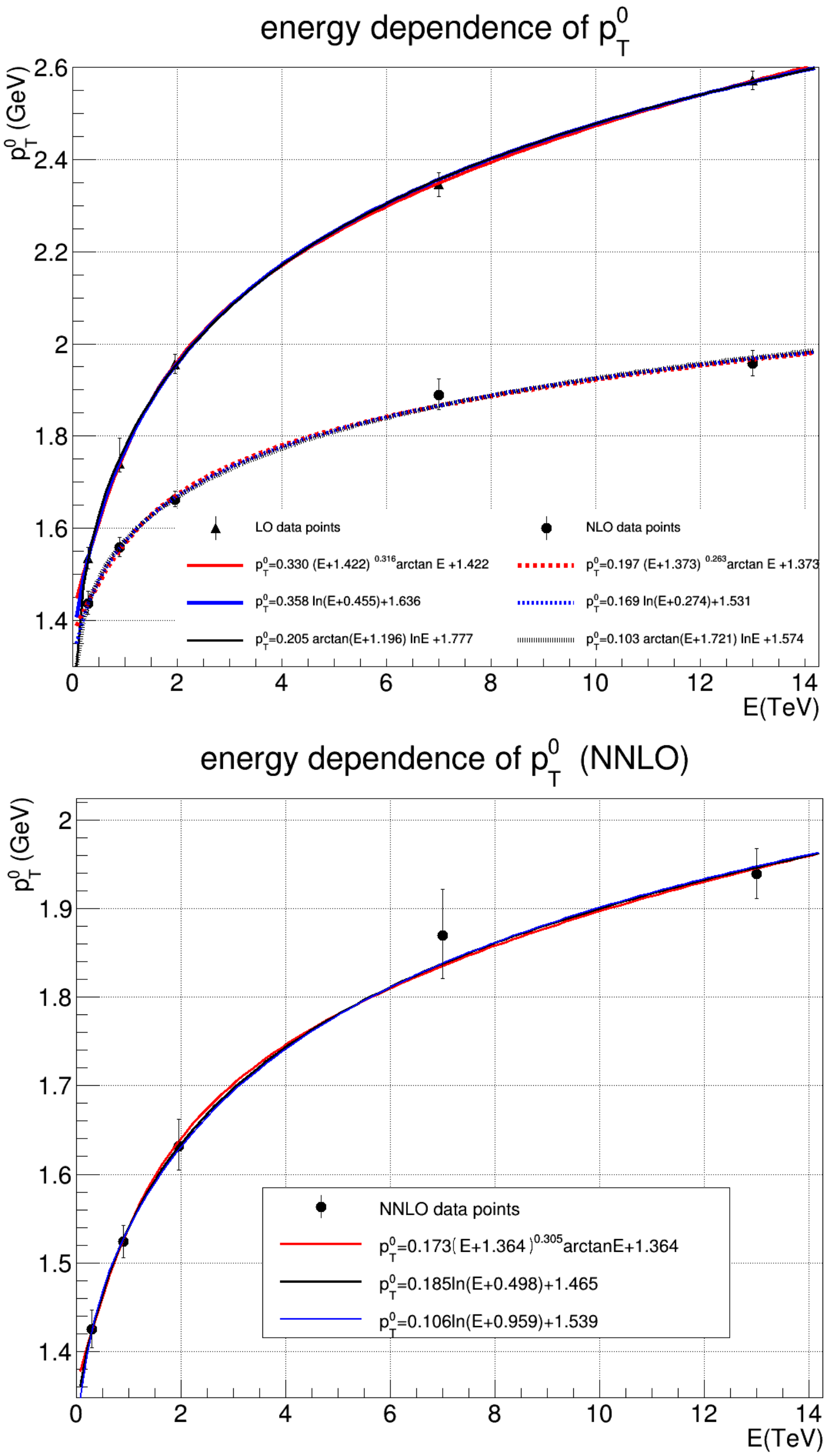

Figure 8: *Fitting curves from other functions*

fitting function	LO				NLO				NNLO			
	a	b	c	χ^2	a	b	c	χ^2	a	b	c	χ^2
$p_{\mathrm{T}}^0 = aE^b$	1.794	0.139	-	1.508	1.579	0.085	-	1.280	1.555	0.085	-	2.719
$p_{\mathrm{T}}^0 = E^a + b$	0.228	0.782	-	0.468	0.129	0.576	-	0.992	0.128	0.551	-	1.844
$p_{\mathrm{T}}^0 = aE^b \arctan(E + c)$	1.262	0.115	5.282	0.184	1.208	0.029	2.565	0.354	1.168	0.038	2.926	0.499
$p_{\mathrm{T}}^0 = aE^b \arctan(E) + c$	0.376	0.263	1.464	0.686	0.218	0.219	1.393	0.880	0.199	0.249	1.382	0.412
$p_{\mathrm{T}}^0 = aE^b \ln(E) + c$	0.236	0.103	1.784	0.142	0.128	0.070	1.576	0.968	0.117	0.117	1.543	1.010
$p_{\mathrm{T}}^0 = aE^b + c$	1.184	0.199	0.601	0.160	0.940	0.136	0.635	0.986	0.522	0.224	1.022	1.060
$p_{\mathrm{T}}^0 = aE^b + (1 + c\ln(E)$	0.775	-0.289	0.468	0.178	0.574	-0.199	0.244	0.824	0.540	-0.259	0.261	0.671
$p_{\mathrm{T}}^0 = aE^b + (1 + cE)$	0.795	0.317	-0.017	0.222	0.583	0.245	-0.010	0.558	0.548	0.226	-0.003	1.018
$p_{\mathrm{T}}^0 = aE^b(1 + cE)$	1.786	0.129	0.003	0.368	1.577	0.082	0.001	1.200	1.541	0.071	0.004	1.677
$p_{\mathrm{T}}^0 = aE^b(1 + E^c)$	0.893	0.040	0.183	0.168	0.788	0.148	-0.133	0.992	0.771	-0.035	0.221	1.014
$p_{\mathrm{T}}^0 = a\ln(E + b) + c$	0.358	0.455	1.636	0.223	0.169	0.274	1.531	0.681	0.185	0.498	1.465	0.535
$p_{\mathrm{T}}^0 = a\ln(E^b + c)$	1.797	0.355	1.696	0.118	1.543	0.231	1.776	0.934	1.219	0.341	2.546	0.950
$p_{\mathrm{T}}^0 = a(E + c)^b \arctan(E) + c$	0.330	0.316	1.422	0.024	0.197	0.263	1.373	0.936	0.173	0.305	1.364	0.607
$p_{\mathrm{T}}^0 = a\arctan(E + b)\ln(E) + c$	0.205	1.196	1.777	0.286	0.103	1.721	1.574	0.752	0.106	0.959	1.539	0.569
$p_{\mathrm{T}}^0 = a\arctan(E)\ln(E + b) + c$	0.271	4.095	1.419	0.062	0.138	5.176	1.371	0.980	0.137	3.984	1.366	0.571
$p_{\mathrm{T}}^0 = E^a \arctan(E^b) + c$	0.148	0.242	0.997	0.115	0.096	0.107	0.790	0.963	0.158	0.001	0.763	1.422

Table 2: *All of functions investigated in this work that better in fitting data compared to the default function (the topmost one)along with the values of all parameters and* χ^2 *associated with them*

4 Conclusions

According to table 2, the default function is obviously not the best function to use for describing the energy dependence of p_{T}^0. There are many functions with 3 parameters better but only one function with 2 parameters can be found in this work. Moreover, it shows that one way to obtain better functions is to modify the default function by introducing the new factor or perturbative term (f,g,h-modification). Even though these functions do not build from first principle, most of them remain composed of a power term which indirectly supports the assumption about the relation of p_{T}^0 and energy.

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