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Abstract:

To make predictions of the power spectrum to a higher precision, more accurate descriptions of the processes that define the power spectrum must be brought forth. Predictions of the power spectrum and unequal time correlators have been tested using several different approaches. In this research project, we use the eikonal approximation which resums the impact of soft long-wavelength modes and generalize it to those of unequal times. As well, we perform 1-loop corrections on the eikonal expression and test its validity in limits of small and large k-values. We compare the results of our unequal time correlated functions with those of equal time correlated measurements.

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1. INTRODUCTION

In present models of how large-scale structure develops, cosmologists use a hydrodynamical approach to dark matter. With this approach, one can use standard perturbation theory as a solution to the hydrodynamical equations and find the equations of motion and thus, the evolution of the system. Starting with density contrast:

$$\delta \equiv \frac{\rho}{\bar{\rho}} - 1$$

$$\delta(x) = \int d^3k\ \delta(k)\ e^{ik\cdot x}$$

we can define the power spectrum P(k) as the Fourier transform of $\xi(r)$ where:

$$\xi(r) = <\delta(x)\ \delta(x+r)>$$

Then the Fourier transform would be:

$$\int \frac{d^3x}{(2\pi)^3}\frac{d^3r}{(2\pi)^3}\xi(r)e^{[-i(k+k')\cdot x - ik'\cdot r]}$$

$$= \ \delta_D(k+k')\int \frac{d^3r}{(2\pi)^3}\xi(r)e^{\mathrm{ik}\cdot r}$$

$$=<\delta(k)\ \delta(k')>$$

$$\equiv \delta_D(k+k')P(k)$$

Given the equations of motion:

$$\frac{\partial\tilde{\delta}(k,\tau)}{\partial\tau} + \tilde{\theta}(k,\tau) = -\int d^3k_1 d^3k_2 \delta_D(k-k_{12})\alpha(k_1,k_2)\tilde{\theta}(k_1,\tau)\tilde{\delta}(k_2,\tau);$$

$$\frac{\partial\tilde{\theta}(k,\tau)}{\partial\tau} + H(\tau)\tilde{\theta}(k,\tau) + \frac{3}{2}\Omega_m H^2(\tau)\tilde{\delta}(k,\tau)$$
$$= -\int d^3k_1 d^3k_2 \delta_D(k-k_{12})\beta(k_1,k_2)\tilde{\theta}(k_1,\tau)\tilde{\theta}(k_2,\tau)$$

Where

$$\alpha(k_1,k_2) \equiv \frac{k_{12}\cdot k_1}{{k_1}^2};\quad \beta(k_1,k_2) \equiv \frac{{k_{12}}^2(k_1\cdot k_2)}{2{k_1}^2{k_2}^2}$$

One can formally solve these equations under the assumption of a Einstein De Sitter Universe using a perturbative expansion:

$$\tilde{\delta}(k,\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(k)\,;\quad \tilde{\theta}(k,\tau) = -H(\tau)\sum_{n=1}^{\infty} a^n(\tau)\theta_n(k)$$

The equations of motions give:

$$\delta_n(k) = \int d^3q_1 \dots \int d^3q_n \delta_D(k - q_{1..n}) F_n(q_1, \dots, q_n) \delta_1(q_1) \dots \delta_1(q_n);$$

$$\theta_n(k) = \int d^3q_1 \dots \int d^3q_n \delta_D(k - q_{1..n}) G_n(q_1, \dots, q_n) \delta_1(q_1) \dots \delta_1(q_n)$$

And the kernels F_n and G_n given as:

$$F_n(q_1, \dots, q_n) = \sum_{m=1}^{n-1} \frac{G_m(q_1 \dots, q_m)}{(2n+3)(n-1)} [(2n+1)\alpha(k_1, k_2) F_{n-m}(q_{m+1}, \dots, q_n) + 2\beta(k_1, k_2) G_{n-m}(q_{m+1}, \dots, q_n)]$$

$$G_n(q_1, \dots, q_n) = \sum_{m=1}^{n-1} \frac{G_m(q_1, \dots, q_m)}{(2n+3)(n-1)} [3\,\alpha(k_1, k_2) F_{n-m}(q_{m+1}, \dots, q_n) + 2n\beta(k_1, k_2) G_{n-m}(q_{m+1}, \dots, q_n)]$$

Then the symmetrized kernels can be shown to be:

$$F_n^s(q_1, \dots, q_n) = \frac{1}{n!} \sum_{\pi} F_n(q_{\pi(1)}, \dots, q_{\pi(n)})$$

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One can now define the power spectrum as up to loop corrections:

$$P(k, \eta) = P^{(0)}(k, \eta) + P^{(1)}(k, \eta) + \cdots$$

Where the tree level power spectrum is given by the linear solution to the equations of motion:

$$P^{(0)}(k) = [\mathrm{D}^{(+)}]^2 P_L(k)$$

Where $\mathrm{D}^{(+)} = e^{\eta - \eta 0}$ is the time dependence of the growing mode of the fluid system in a matter dominated universe. The symmetrized kernels are then used to compute the 1 loop power spectrum characterized as:

$$P^{(1)}(k, \eta) = {\mathrm{D}^{(+)}}^4(\eta)(P_{13}(k) + P_{22}(k))$$

Where:

$$P_{13}(k) = 6 \int d^3q \; P_L(q) P_L(k) \; F_3^s(k, q, -q)$$

$$P_{22}(k) = 2 \int d^3q \; P_L(|\mathrm{k} - \mathrm{q}|) P_L(q) \; [F_2^s\,(k - q, q)]^2$$

2. CURRENT RESEARCH

When evaluated numerically, we find that the linear and 1 loop power spectrum were both modelled correctly in accordance to [1] . As well, figure 1 shows the linear and 1 loop power spectrums which reproduce the plots in [1]

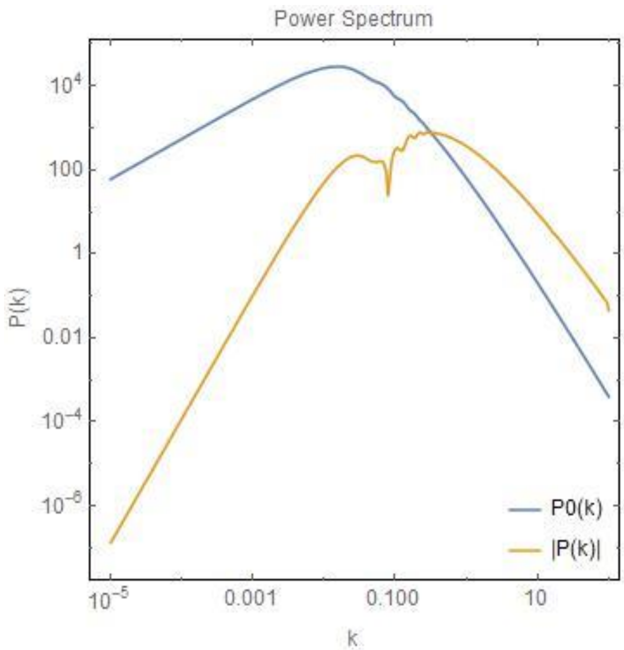

Figure 1: The linear power spectrum plotted with the 1-loop power spectrum as a function of the momenta k

We then create and check an unequal time correlated power spectrum model and make sure that it matches supporting data. We used plots given [2] to cross check our code and power spectrum models. The unequal time correlated power spectrum can be described using SPT as:

$$P_{\mathrm{UETC}}(k,\eta_1,\eta_2) = e^{\eta 1+\eta 2}\left[P_L(k) + e^{\eta 1+\eta 2}P_{22}(k) + \frac{[e^{2\eta 1} + e^{2\eta 2}]}{2}P_{13}(k)\right]$$

And the equal time correlated power spectrum as:

$$P_{\mathrm{ETC}}(k,\eta) = P_{\mathrm{UETC}}(k,\eta,\eta) = e^{2\eta}[P_L(k) + e^{2\eta}[P_{13}(k) + P_{22}(k)]]$$

Using a geometric mean interpretation of the equal time correlators, we can represent unequal time correlators such that:

$$P_{\mathrm{GUETC}} = (P_{\mathrm{ETC}}(k,\eta_1)P_{\mathrm{ETC}}(k,\eta_2))^{1/2}$$

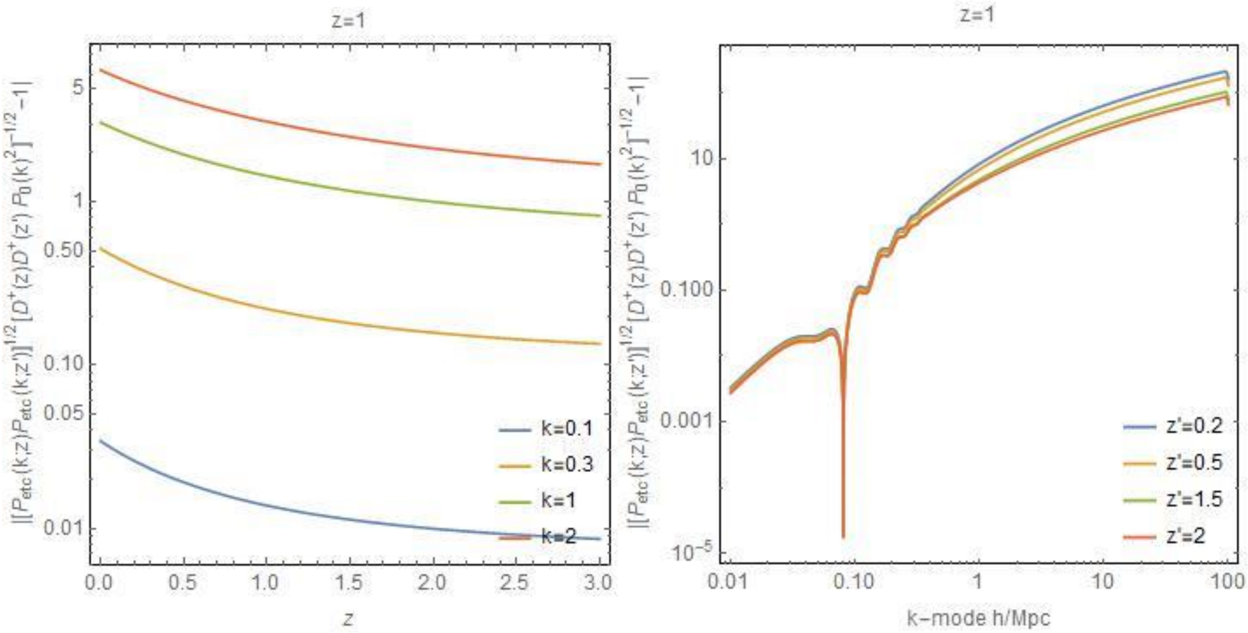

Figure 2: Geometric mean generated power spectra compared with geometric mean linear power spectra as functions of z (left) and k-modes (right)

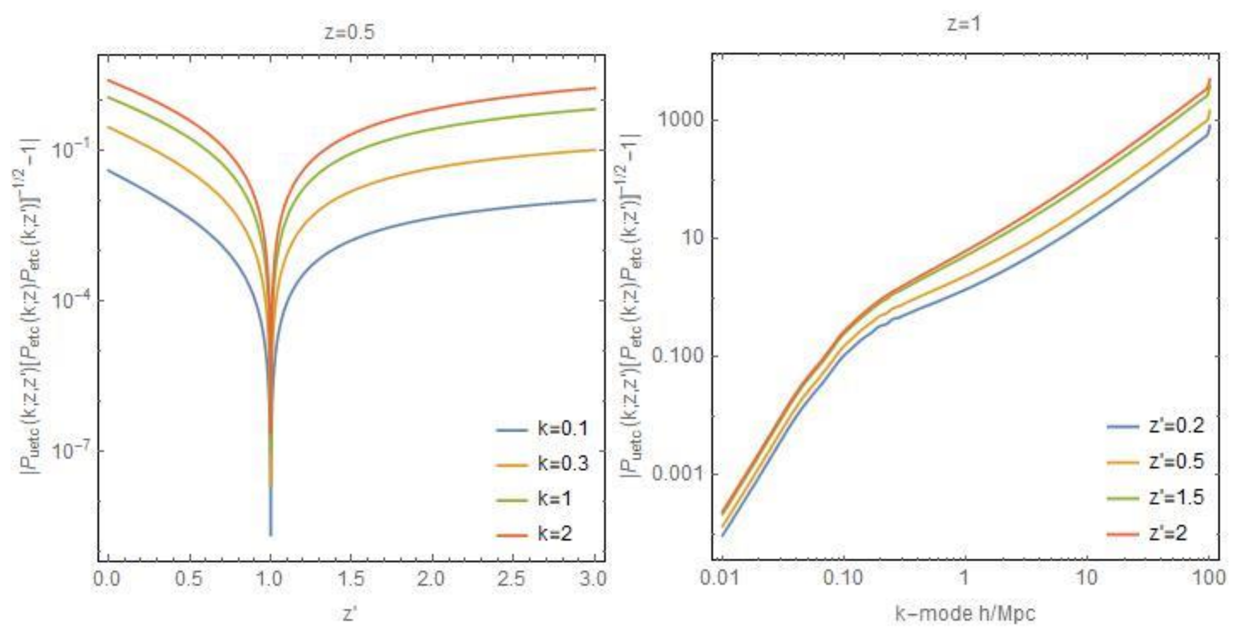

Figure 3: The unequal time correlator power spectra (SPT) compared with the geometric mean generated power spectra as functions of z (left) and k-modes(right)

The Eikonal Approximation:

In the eikonal method, we sum over soft contributions of the Fourier modes to allow for a better prediction of the power spectrum. The density contrast is given a new definition where:

$$\delta_{\mathrm{eik}}(k) \equiv \gamma(k,\eta)\delta(k)$$

$$\gamma(k,\eta) \equiv e^{\int_0^{\eta} \Gamma(\eta\prime)\mathrm{d}\eta\prime}$$

$$\Gamma(k,\eta) \equiv \int_S d^3q \frac{k\cdot q}{q^2}\theta(q,\eta)$$

Then the eikonal power spectrum can be defined as:

$$< \delta_{\mathrm{eik}}(k,et)\delta_{\mathrm{eik}}(k',et') >= \delta_D(k+k')P^{(0)}(k) < \gamma(k,\eta)\,\gamma(-k,\eta') >$$

The last correlator can be evaluated using the cumulants of the linear fields:

$$< \gamma(k,\eta)\gamma(-k,\eta') > \;=\; < e^{\int_0^{\eta} \Gamma(\eta\prime)\mathrm{d}\eta\prime} e^{\int_0^{\eta\prime} \Gamma(\eta")\mathrm{d}\eta"} > \;=\; e^{-k^2\sigma^2(e^{\eta}-e^{\eta\prime})^2}$$

The eikonal expression is then given as

$$P_{\mathrm{eik}}(k,\eta_1,\eta_2) = e^{\eta_1+\eta_2-(e^{\eta_1}-e^{\eta_2})^2k^2\sigma^2}P^{(0)}(k)$$

As well, the 1 loop correction to the eikonal expression can be made to match the 1 loop from SPT and can be defined as:

$$P_{\mathrm{eik1loop}}(k,\eta_1,\eta_2) = e^{-(e^{\eta_1}-e^{\eta_2})^2k^2\sigma^2}[e^{\eta_1+\eta_2}\,P^{(0)}(k) + \Delta(k,\eta_1,\eta_2)]$$

Where:

$$\Delta(k,\eta_1,\eta_2) = \; P_{\mathrm{UETC}}(k,\eta_1,\eta_2) - e^{\eta_1+\eta_2}\,P_0(k)[1-[e^{\eta_1}-e^{\eta_2}]^2k^2\sigma^2]$$

These equations have not yet been produced in previous literature. We examine the effects of these methods in the figures below.

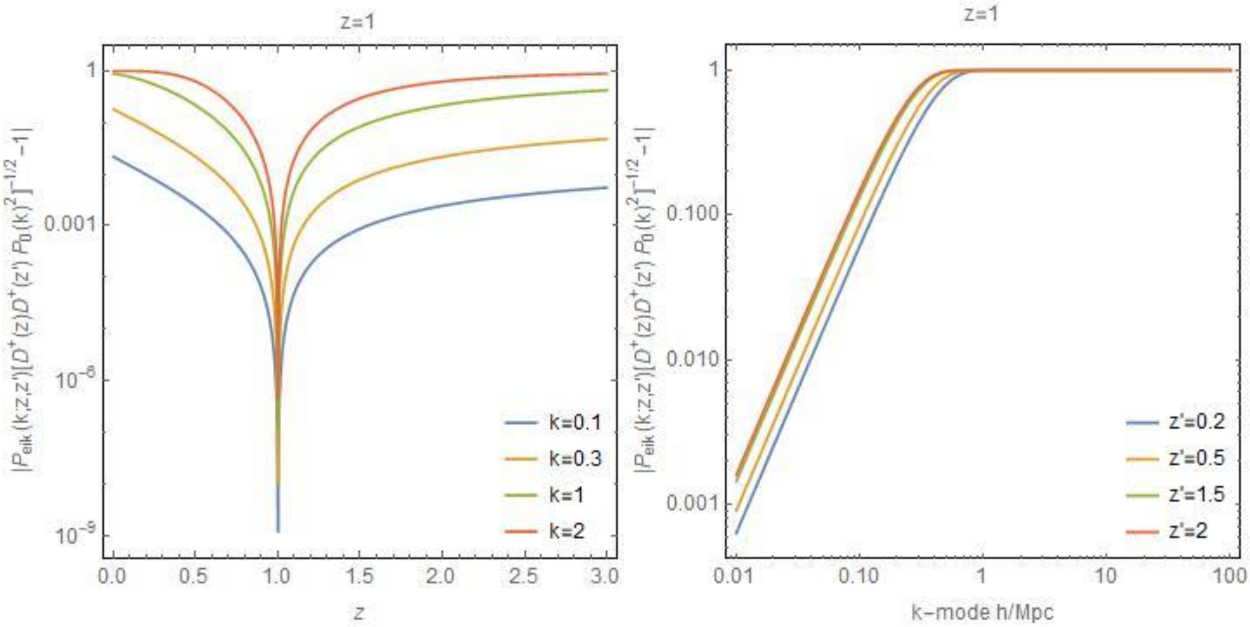

Figure 4: The eikonal expression compared with the linear power spectrum as functions of z(left) and k-modes(right)

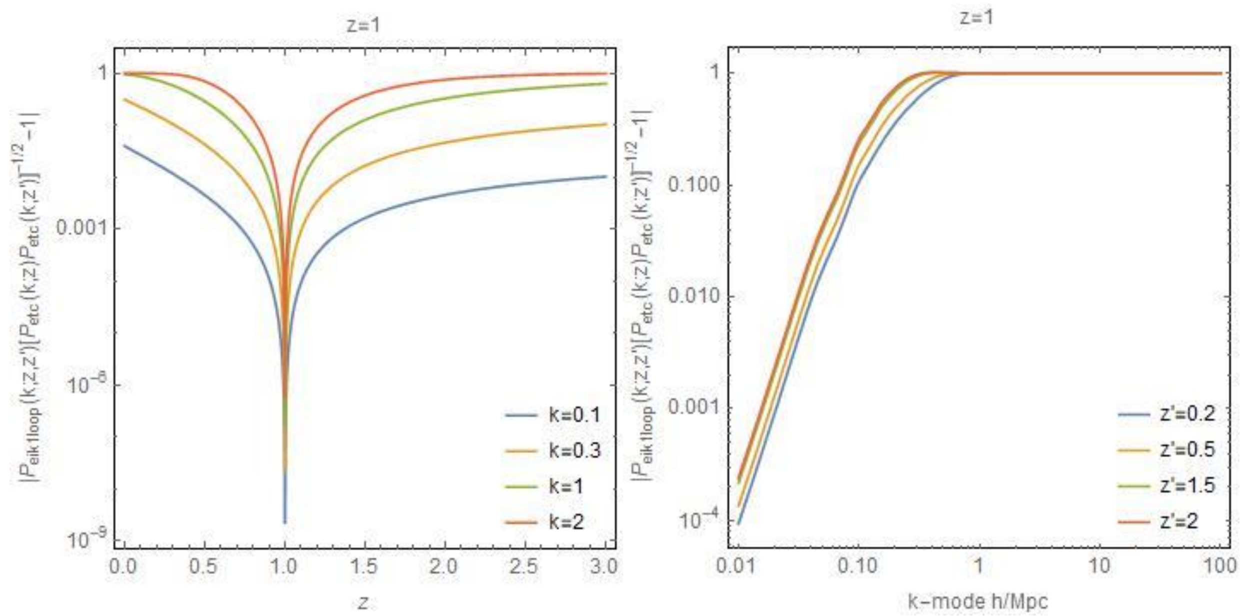

Figure 5: Eikonal expression with 1-loop correction compared with the equal time correlated power spectrum as functions of z (left) and k-modes(right)

3. CONCLUSIONS:

The power spectra of the eikonal expressions both do well at producing power spectra. Since the soft long-wavelength Fourier modes are summed over, the power spectra are characteristic. It allows for a more realistic interpretation of the power spectra over the range of momenta (k) and time-scales (z).

As well, the power spectra are all positive and restricted to a range that are all less than 1. The domain of momenta space and time space all represent natural power spectra that would not enter negative values or values of those that are greater than 1. This shows that the eikonal approximation does a much better job at representing the power spectra than those of SPT.

4. FUTURE RESEARCH

Bessel/Lomel Effects:

We can also try to develop spherical Bessel function representations of the power spectrum in unequal times. The power spectrums of unequal time and equal time can be respectively described by projected power spectra which can be measured experimentally. We can explore these functions:

$$C_l{}^{\delta\delta}(r_1, r_2) = \int \frac{2\mathrm{dk}k^2}{\pi} \mathrm{P}(k; r_1, r_2) j_l(kr_1) j_l(kr_2)$$

Where j_l are spherical Bessel functions of the second kind. The projected power spectra would be the integrals of these functions with their projection kernels:

$$C_l{}^{\mathrm{AB}}(r, r') = \int_0^r \mathrm{dr}_1 \int_0^{r'} \mathrm{dr}_2\ F_A(r, r_1)\ F_B(r', r_2)\ C_l{}^{\delta\delta}(r_1, r_2)$$

These functions seemed to have behaved wildly and were too oscillatory for our computations to work effectively. The projected power spectrum could not be computed due to time restraint and thus was left for a future project.

Lagrangian (Zel'dovich) Perturbation Theory

Another method to sum soft effects would be to use Lagrangian perturbation theory. It would be interesting to test the effects of these theories on the power spectra and compare them with our eikonal results. Further research is needed to continue exploring new methods of producing viable power spectra.

5. REFERENCES:

[1] Large-Scale Structure of the Universe and Cosmological Perturbation Theory
https://arxiv.org/abs/astro-ph/0112551

[2] Unequal-Time Correlators for Cosmology
https://arxiv.org/abs/1612.00770

[3] Resummed propagators in multi-component cosmic fluids with the eikonal approximation
https://arxiv.org/abs/1109.3400