Applicability studies of SASE FEL pulse shape retrieval algorithm

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Abstract

X-ray Free electron lasers (XFELs) provide short radiation pulses with brightness significantly higher than that generated in traditional synchrotron radiation sources.

Aside of generation, it is necessary to measure the duration and preferably the shape of the radiation pulse. We are improving a method, based on the connection between the time and frequency domains. In this paper, we focus on its applicability studies while the method itself will be published elsewhere.

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1. Introduction

In an FEL the role of the active laser medium and the energy pump are both taken over by the relativistic electron beam. An optical cavity is no longer possible for wavelengths below 100 μm, because the reflectivity of metals and other mirror coatings drops quickly to zero at normal incidence. Here one has to rely on the principle of Self Amplified Spontaneous Emission (SASE) where the laser gain is achieved in a single passage of a very long undulator magnet. One big advantage of an FEL in comparison with a conventional laser is the free tunability of the wavelength by simply changing the electron energy [1].

SASE FELs can be described as narrow-band amplifiers of random input signal – shot noise in electron beam. In this case the output signal of such devices has properties of Gaussian random process: the real and imaginary parts of the slowly varying complex electric field amplitude of electric vector of the amplified radiation have Gaussian distribution [2].

The method of retrieval of SASE FEL pulse duration was proposed in [3] and studied further in [4]. It works under two assumptions: Gaussian statistics of analyzed radiation and quasistationarity.

Statistics of Gaussian random process strictly specking holds only in the linear amplification regime. Therefore, we try to verify up to which point we can apply this beyond linear regime. We want to verify the applicability of the relation $g_2 = |g_1|^2 + 1$, where g_2 and g_1 are the second and first order normalized correlation functions in frequency. We will show that, the applicability is surprisingly good also at saturation, though not justified by strict mathematics.

Quasistationarity is a process in a limited system that spreads within the system so quickly that in the time required for it to expand to the limits of the system its state does not have time to change. In examining a quasi-stationary process it is possible therefore to disregard the time required for it to spread through the system.

We want to verify lowest number of spikes where this statistics also conserves. So will show interesting and surprising result that quasistationarity requirement is not necessary for a proper pulse shape retrieval (reducing pulse duration to several spikes does not qualitatively change its shape).

Moreover, we want to study the influence of the spectrometer detector noise.

2. Theoretical background

2.1 Wiener-Khinchine theorem.

"Given a single known time function u(t), which may be one sample function of a random process, the *time autocorrelation function* of u(t) is defined by:

$$\widetilde{\Gamma_U}(\tau) \triangleq \overline{u(t+\tau)u(t)} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} u(t+\tau)u(t)dt$$

Closely related, but a property of an entire random process *U(t)*, is the *statistical autocorrelation function*, defined by

$$\Gamma_U(t_2, t_1) \triangleq< u(t_2)u(t_1) >= \iint_{-\infty}^{\infty} u_2 u_1 p_u(u_1, u_2; t_2 t_1) du_1 du_2$$

From a physical point of view, the autocorrelation function measures the structural similarity of *u(t)* and *u(t+τ)*, averaged over all time, whereas the statistical autocorrelation function measures statistical similarity of $u(t_1)$ and $u(t_2)$ over the ensemble.

For a random process with at least wide-sense stationarity, Γ_U is a function only of the time difference $\tau = t_2 - t_1$. For the more restrictive class of ergodic random process, the time autocorrelation functions of all sample functions are equal to the statistical autocorrelation function. For ergodic process, therefore,

$$\widetilde{\Gamma_U}(\tau) = \Gamma_U(\tau) \text{ (All sample functions)}$$

It is thus pointless to distinguish between the two types of autocorrelation function for such processes." [5]

So, Wiener-Khinchine theorem convince that, for a process that is at least wide-sense stationary, the autocorrelation function and power spectral density form a Fourier transform pair,

$$\Gamma_U(\tau) = \int_{-\infty}^{\infty} \zeta_U(\nu) e^{-i2\pi\nu\tau}\, d\nu$$

$$\zeta_U(\nu) = \int_{-\infty}^{\infty} \Gamma_U(\tau) e^{i2\pi\nu\tau}\, d\tau$$

2.2 The Siegert relation

"The time-dependent autocorrelation function of the fluctuations of the scattered field $g_1(\tau) = \frac{\langle E(t+\tau)E^*(t)\rangle}{\langle |E(t)|^2\rangle}$, and the corresponding normalized autocorrelation function of the intensity fluctuations $g_2(\tau) = \frac{\langle I(t+\tau)I(t)\rangle}{\langle I(t)^2\rangle}$ are related by the Siegert relation:

$$g_2(\tau) = 1 + \beta|g_1(\tau)|^2$$

Where the coefficient β is determined by the detection conditions; For ideal detection conditions β = 1 (in particular in the case of using a receiver with an aperture much smaller than the characteristic size of the correlation region of spatial fluctuations of the scattered field related to the mean speckle size). The Siegert relation is satisfied for optical fields with Gaussian statistics and a zero mean value of the field amplitude generated by multiply scattered disordered media consisting of a large number of non-interacting scatterers." [6]

2.3 The Gaussian Random Process

Perhaps the most important continuous random process in systems is the Gaussian random process.

So, we know that the statistics of the Gaussian random process are fulfilled under condition linear gain mode. In our case, the averaged spectrum in the time domain will look like a rectangle (flattop). Accordingly, the autocorrelation will have the form of a triangle. Also we assume that we work in quasistationarity requirement. Our task is to check this statistics in case of failure of these two conditions.

2.4 Relation between Time and Frequency domains

The time domain and frequancy domain of the laser radiation are related by the Fourier transform:

$$E(\omega) = \mathcal{F}_t E(t) = \int_{-\infty}^{\infty} E(t)e^{i\omega t}dt \ .$$

An average number of maxima in both domains are equal. Moreover, $\Delta\omega \sim \tau_{coh}^{-1}$ and $\delta\omega \sim T^{-1}$, where $\Delta\omega$ – average bandwidth of radiation spectrum, $\delta\omega$ – bandwidth of a single spike, T – average duration of the radiation pulse, τ_{coh} - duration of a single temporal spike (coherence time).

2.5 Correlation functions

The stochastic nature of the SASE FEL radiation allows to introduce a radiation parameter, the correlation function of the field of laser radiation.

The normalized spectral correlation function of the first order is defined as:

$$g_1(\omega, \Delta\omega) = \frac{\langle E(\omega + \Delta\omega) E^*(\omega - \Delta\omega)\rangle}{\sqrt{\langle |E(\omega + \Delta\omega)|^2\rangle \langle |E(\omega - \Delta\omega)|^2\rangle}},$$

Where $E(\omega + \Delta\omega)$ – is the laser radiation field at frequency $\omega + \Delta\omega$ and $\langle |E(\omega)|^2\rangle$ denotes a radiation field squared and averaged over an ensemble.

The second-order normalized spectral correlation function is defined as:

$$g_2(\omega, \Delta\omega) = \frac{\langle |E(\omega + \Delta\omega)|^2 |E(\omega - \Delta\omega)|^2\rangle}{\langle |E(\omega + \Delta\omega)|^2\rangle \langle |E(\omega - \Delta\omega)|^2\rangle}.$$

Since the spectrometer can measure only the squared modulus of electric field in frequency domain, $|E(\omega)|^2 = I(\omega)$, we can rewrite the expression above as

$$g_2(\omega, \Delta\omega) = \frac{\langle I(\omega + \Delta\omega) I(\omega - \Delta\omega)\rangle}{\langle I(\omega + \Delta\omega)\rangle \langle I(\omega + \Delta\omega)\rangle},$$

The inverse Fourier transform of the non-normalized first order spectral correlation function $G_1(\omega, \Delta\omega) = \langle E(\omega + \Delta\omega) E^*(\omega - \Delta\omega)\rangle$ gives the power spectral density in the time domain:

$$\langle I(t)\rangle = \mathcal{F}^{-1}_{\Delta\omega}\{G_1(\omega, \Delta\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(\Delta\omega) e^{-i\Delta\omega t} d\Delta\omega$$

We can neglect dependence on ω, since we assume quasistationarity.

The Fourier transform from the second-order non-normalized spectral correlation function

$$G_2(\omega, \Delta\omega) = \langle |E(\omega + \Delta\omega)|^2 |E(\omega - \Delta\omega)|^2\rangle - \langle |E(\omega + \Delta\omega)|^2\rangle \langle |E(\omega - \Delta\omega)|^2\rangle,$$

according to Wiener-Khinchine theorem, gives:

$$\mathcal{F}_{\Delta\omega}\{G_2(\omega, \Delta\omega)\} = I(t) * I(t),$$

where symbol * denotes correlation and $I(t) = |E(t)|^2$.

The importance of the autocorrelation function is due to two circumstances. First, the autocorrelation of the signal can often be measured directly, which ultimately gives the opportunity to experimentally determine the spectral density of the signal. To find the frequency spectrum of power in digital or analog form, perform the Fourier transform of the experimentally measured autocorrelation function.

Secondly, the autocorrelation function often allows analytically to calculate the spectral power density for a random process model, which is described only statistically.

We can reconstruct the autocorrelation of the averaged radiation pulse by applying Fourier transform to second order correlation function $G_2(\Delta\omega)$:

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_2(\Delta\omega)e^{-i\Delta\omega\tau}\, d\Delta\omega$$

3. Reconstruction results

3.1 Linear regime and saturation

We consider the example of a 10µm-long flat-top SASE FEL pulse with 500eV photon energy. We study the shape of the reconstructed autocorrelation of the pulse shape both in the linear regime and saturation.

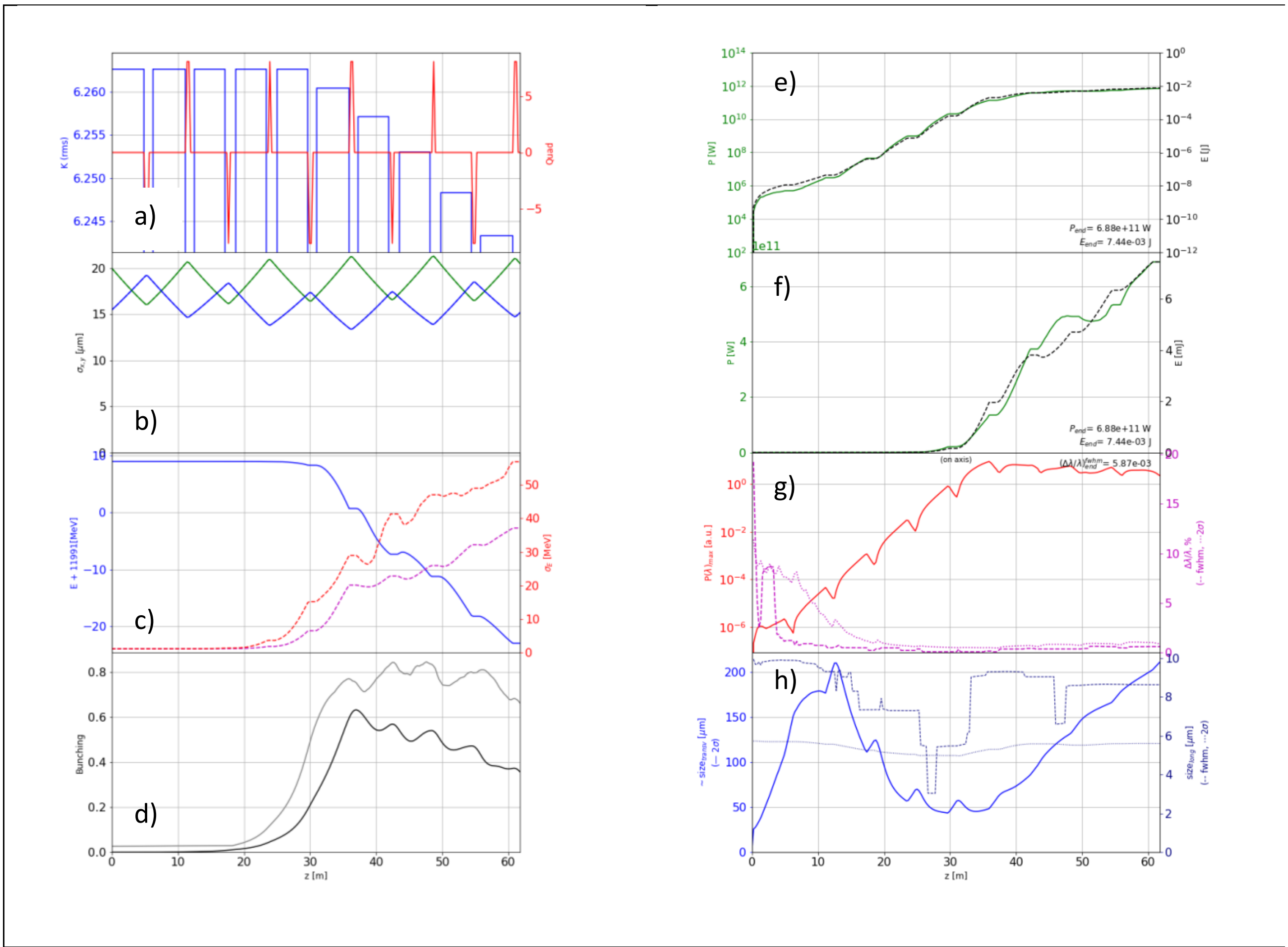


Fig.1 Electron beam (first column) and emitted radiation (second column) parameters as function of undulator length:

a) Undulator parameter K and quadrupole field strength; b) σ_{xy} – rms size of electron beam (blue and green for horizontal and vertical dimensions correspondingly); c) mean electron energy (blue) and energy spread (red and purple for peak and average values) d) Electron beam bunching (grey

– peak value, black – average); e) Peak radiation power (green) and pulse energy (black) in logarithmic scale; f) The same in linear scale; g) Spectral density of radiation (solid) and spectral bandwidth (doted and dashed); h) Cross section of radiation.

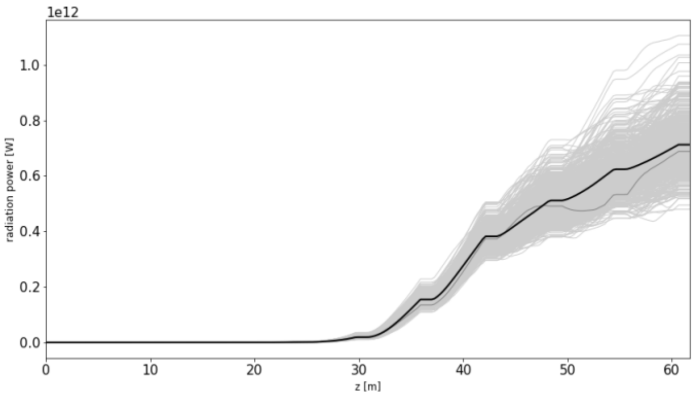


Fig. 2 Radiation peak power as a function of an undulator length. The light-grey lines refer to single-shot realizations. The darker gray line refers to one typical shot. Black line is the average.

Fig.1, 2 show that the linear mode ends around at a distance of 30 m.

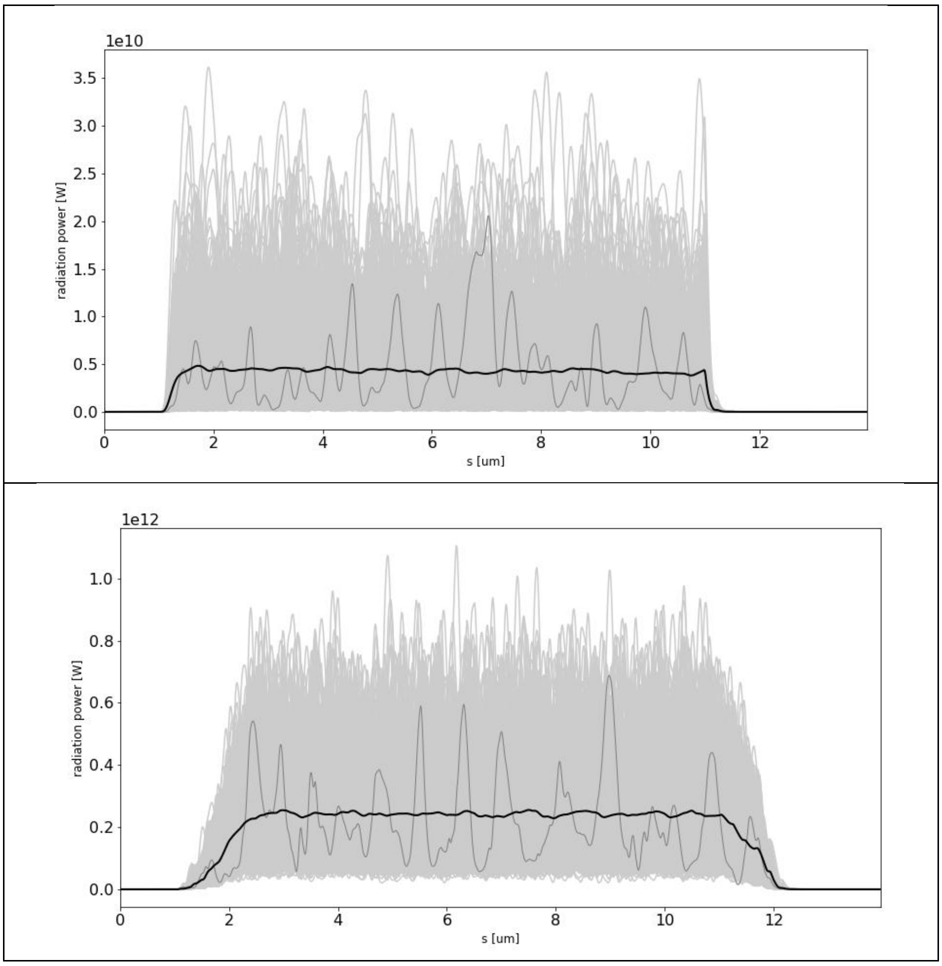


Fig. 3 Radiation power as a function of beam length. First figure shows radiation power on length of undulator at 30 meters (end of linear mode), second - radiation power on length of undulator at 70 meters (deep saturation).

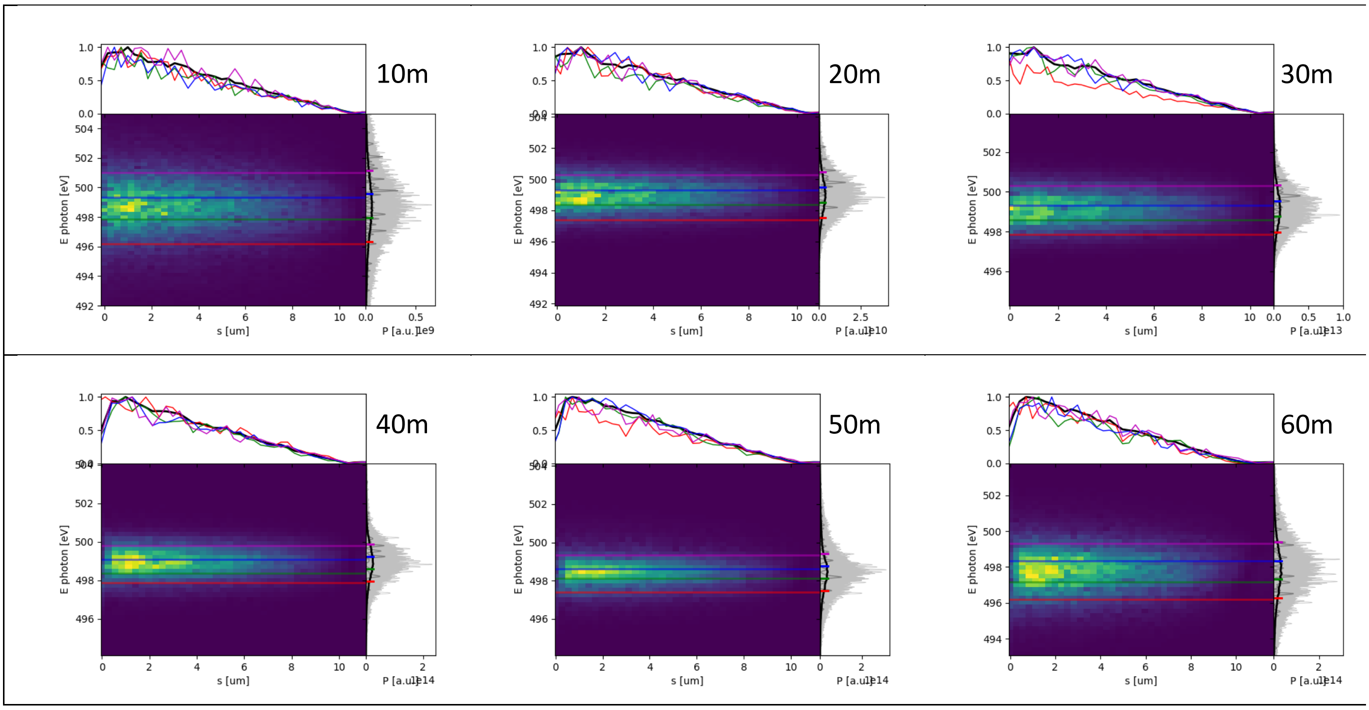


Fig. 4 The reconstructed autocorrelation of the 10μm flat-top SASE FEL radiation calculated from autocorrelation functions with different central frequencies (presented here as photon energies on vertical scale). Subplots on the right and the top present radiation spectra and autocorrelation line-offs for selected central frequencies. Black line represents an average autocorrelation. All lines on the top plot are normalized to unity as their peak value. Saturation starts at 40 meters of undulator length.

Shape of autocorrelation function depends on undulator length and regime. Based on these results, one can conclude that the Gaussian random process statistics applies both in the linear amplification mode and in the saturation regime, that is, the relation $g_2(\omega, \Delta\omega) = 1 + |g_1(\omega, \Delta\omega)|^2$ is valid in both cases.

3.2 Quasistationarity requirement

Now we want to check impact of the quasistationarity assumption by analyzing the signal with duration comparable with coherence time ($N_{spikes} \sim 3$). The sample is a signal which has photon energy 500 eV and pulse duration 1 um.

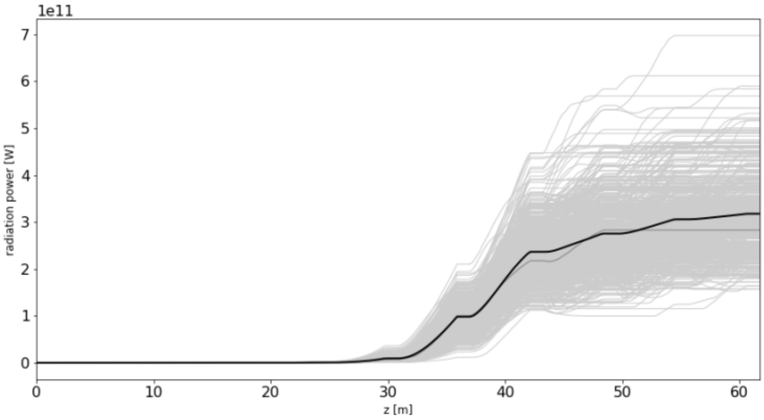


Fig. 5 Radiation power as a function of undulator length. The light grey lines represent 500 independent simulations. The darker gray line shows a single realization. Black line – average power.

The Fig. 5 shows that the linear mode ends approximately at a distance of 30 m.

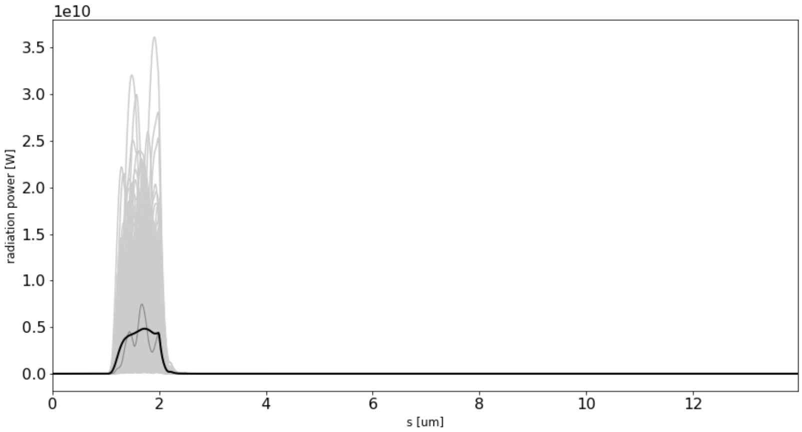


Fig. 6 Radiation power as a function of beam length.

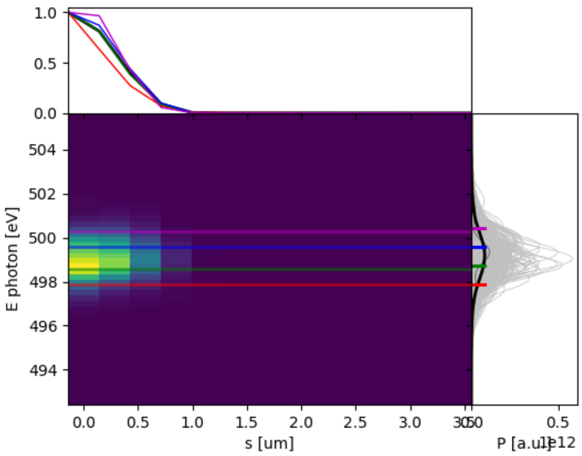


Fig. 7 Shape of the reconstructed autocorrelation function in case of short SASE FEL pulse at 30 meters.

One can see that if only several temporal modes are present in the radiation pulse, the calculated autocorrelation function still allows to estimate SASE FEL pulse duration.

3.3 Noise Effect

Now we want to check how the noise of detector affects the reconstruction of autocorrelation function. We assume 10μm-long pulse with 500eV photon energy obtained in the end of linear amplification regime (30 m). We add Gaussian noise with different rms values in terms of an average spectrum peak value.

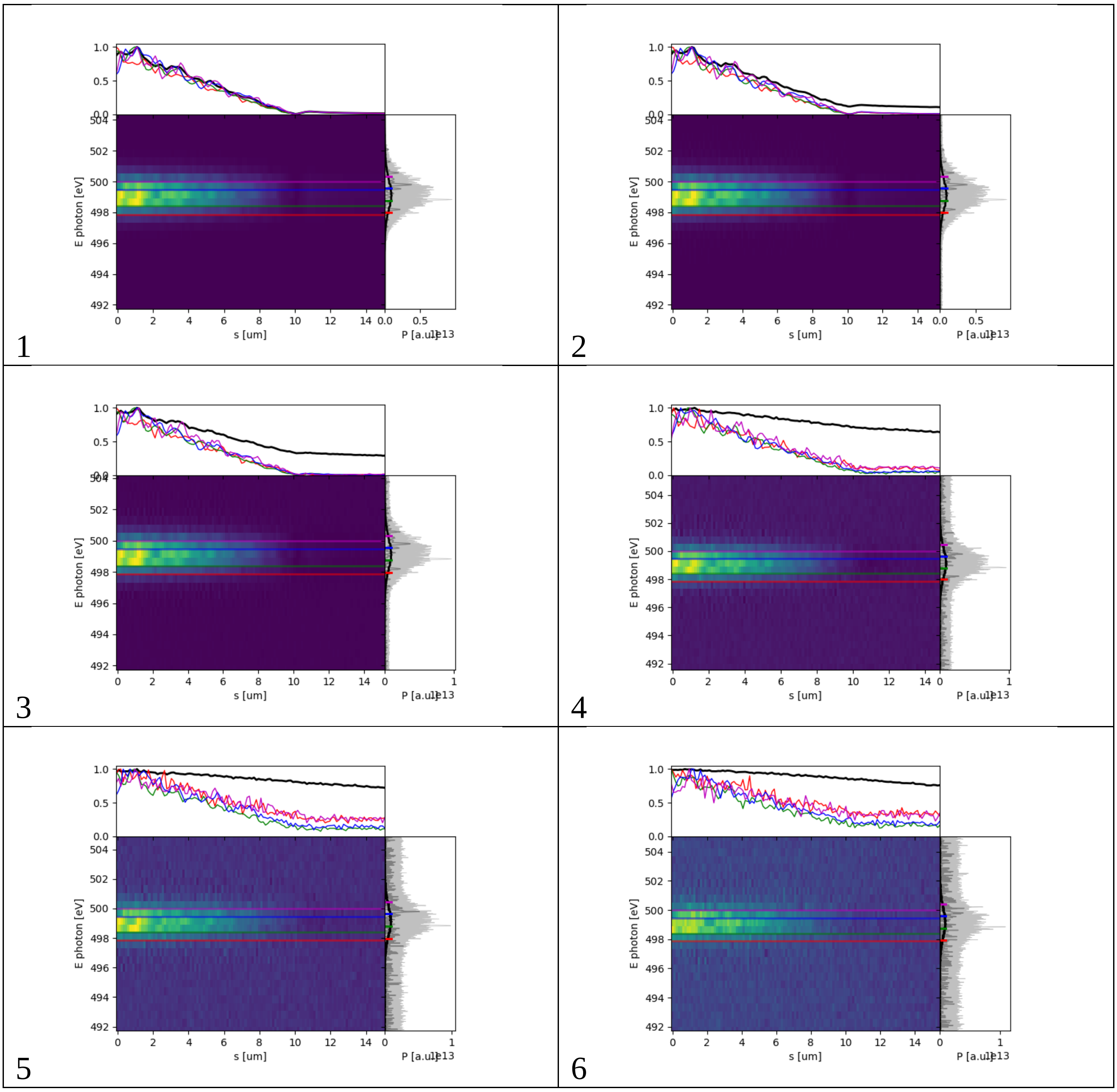


Fig. 9 Shape of the reconstructed autocorrelation function. We assume 10μm-long pulse with 500eV photon energy obtained at the end of linear amplification regime (30 m). With noise equal to:

1) without noise

2) 10% of the averaged spectrum value
3) 20% of the averaged spectrum value
4) 50% of the averaged spectrum value
5) 80% of the averaged spectrum value
6) 100% of the averaged spectrum value

If the noise does not exceed one tenth of the averaged signal value, the autocorrelation function is preserved. But if the noise does exceed a half of the average value of the signal, then this case starts to seriously spoil the form of the autocorrelation function.

So we can see that low levels of noise do not affect shape of reconstructed autocorrelation.

3.4 Effect of transverse coherence

FEL radiation has very good transverse coherence and up to now we were analyzing on-axis FEL spectra. Since spectrometer would provide spectra of transversely averaged radiation, the limited transverse coherence might affect the reconstructed pulse shape. Now we want to compare reconstructions based on the on-axis and transversely averaged spectra at the end of the linear mode and in saturation.

We consider the example of a 10µm-long flat-top SASE FEL pulse with 500eV photon energy.

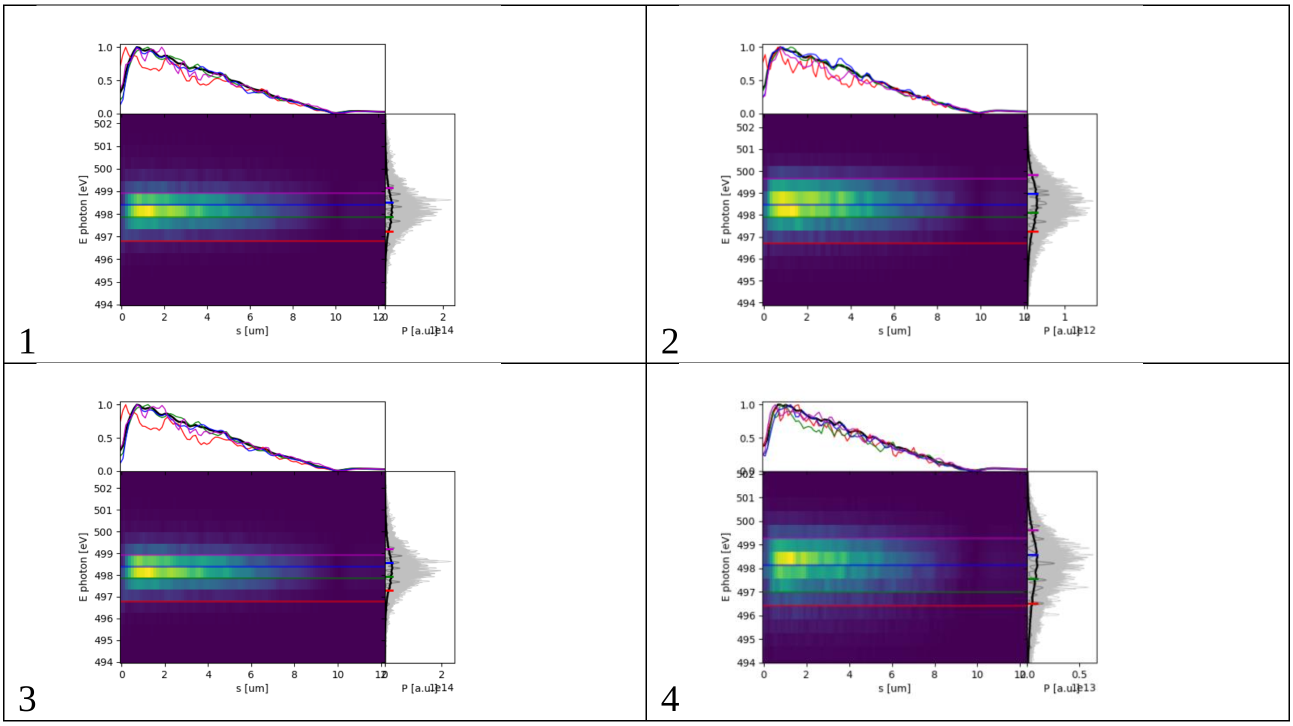


Fig. 10 Shape of the reconstructed autocorrelation function:

1) based on the on-axis at the end of linear regime (30m);
2) transversely averaged spectrum at the end of linear regime (30m);
3) based on the on-axis at the saturation (60m);
4) transversely averaged spectrum at saturation (60m).

3.5 Linear regime and saturation for Hard X-ray

We consider the example of a 1um-long flat-top SASE FEL Hard X-ray pulse with 8000eV photon energy. We study the shape of the reconstructed autocorrelation of the pulse shape both in the linear regime and saturation.

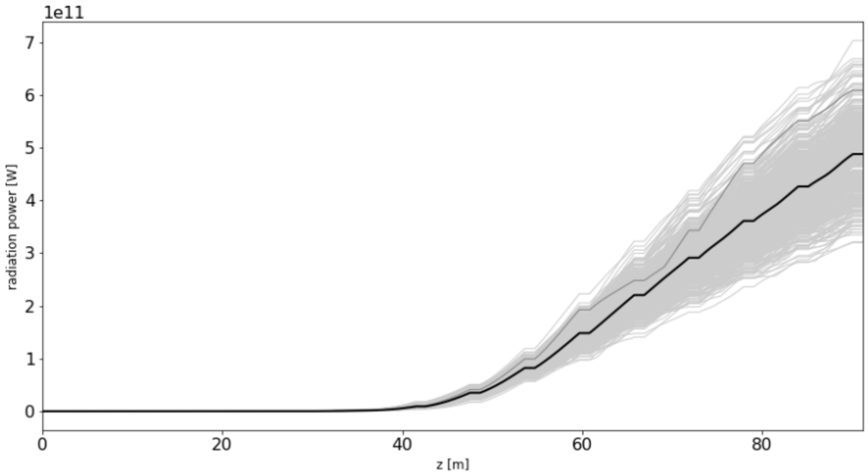
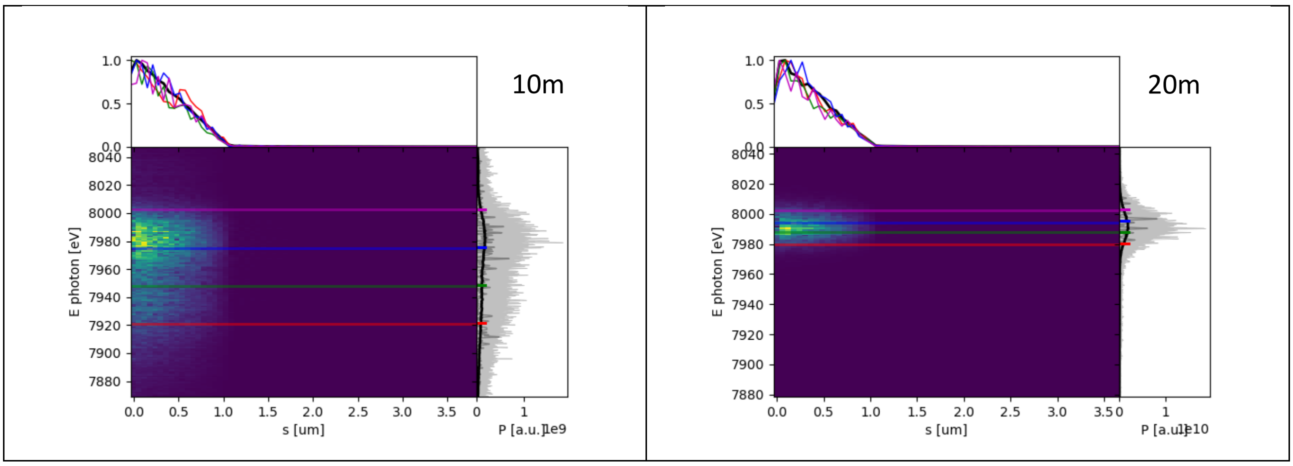


Fig. 11 Radiation power as a function of undulator length. The light grey lines represent 500 independent simulations. The darker gray line shows a single realization. Black line – average power.

The Fig. 11 shows that the linear mode ends approximately at a distance of 40 m.



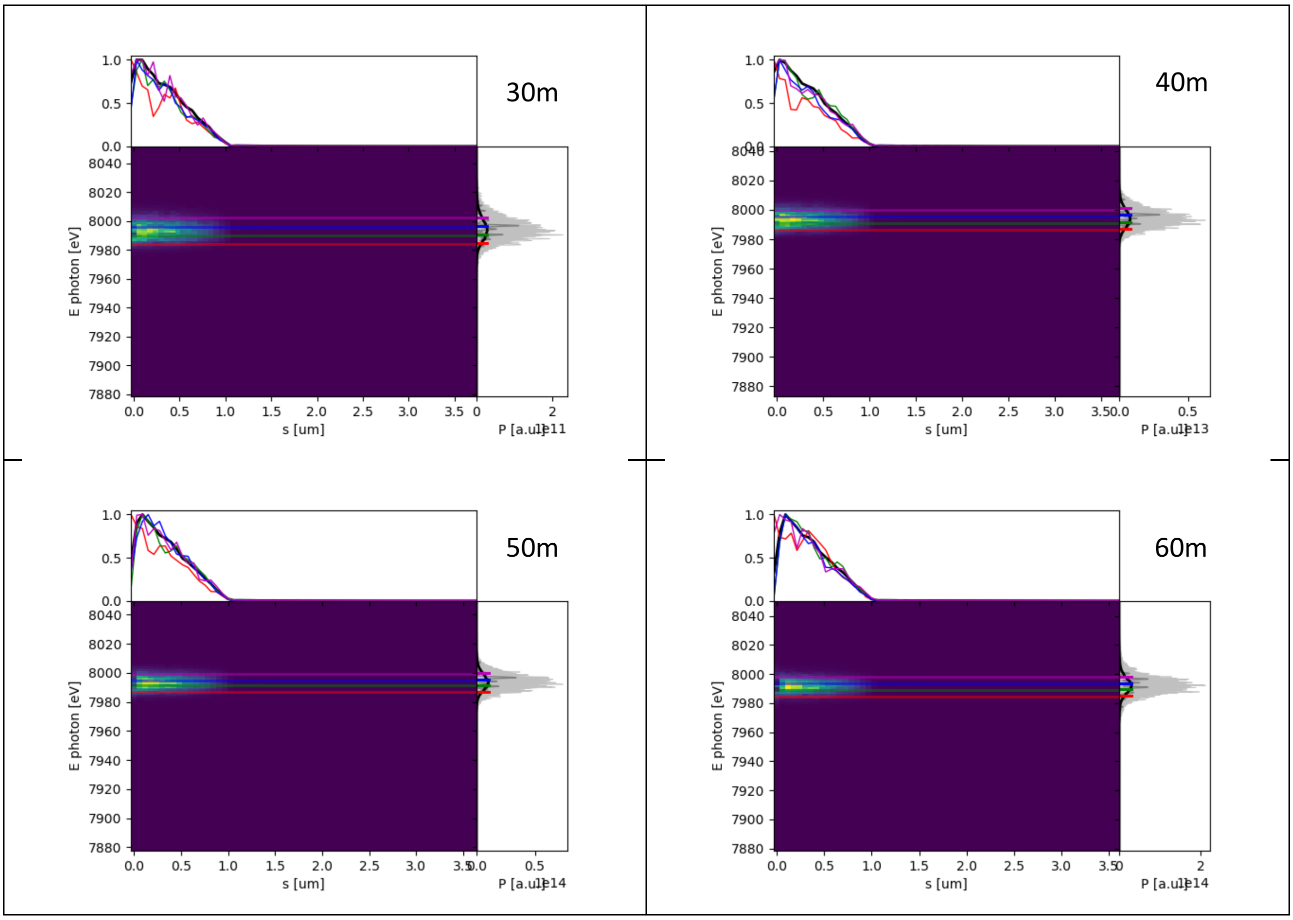


Fig. 12 The reconstructed autocorrelation of the 1μm flat-top SASE FEL radiation calculated from autocorrelation functions with different central frequencies (presented here as photon energies on vertical scale). Subplots on the right and the top present radiation spectra and autocorrelation line-offs for selected central frequencies. Black line represents an average autocorrelation. All lines on the top plot are normalized to unity as their peak value. Saturation starts at 40 meters of undulator length.

Summary and conclusions

We have shown that regardless of whether we have a linear mode or saturation mode Gaussian random process we can reconstruct the autocorrelation of the SASE FEL pulse. The reconstruction is also valid for a very short FEL pulses with a few modes in them. Spectrometer noise affects the reconstruction but at levels below around 10% of an average signal value this effect is negligible.

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