



Preliminary studies on $\Upsilon(1S)$ Visible and Invisible decays at the Belle II experiment

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Abstract

Belle II is a high precision experiment with a wide physics programme. The feasibility of study of the invisible decays of $\Upsilon(1s)$ is studied in channel $e^+e^- \rightarrow \Upsilon(4S) \rightarrow \pi^+\pi^-\Upsilon(1S)$; $\Upsilon(1S) \rightarrow \text{invisible}$ using Belle II simulation framework. The control channel $e^+e^- \rightarrow \Upsilon(4S) \rightarrow \pi^+\pi^-\Upsilon(1S)$; $\Upsilon(1S) \rightarrow \mu^+\mu^-$ is used to ensure that simulations for visible and invisible channels are consistent. Detector acceptance and reconstruction and trigger efficiency are studied in order to asset whether Belle II is able to search for the Dark matter in the described channel.

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1 Introduction

Belle II is a precision frontier experiment which is currently in final the stage of its construction in Tsukuba, Japan. It will continue and significantly extend the physics program of its predecessor experiment Belle. Namely, Belle II offers possibility to study rare processes which could possibly shade light on new physics which is needed to answer many open questions of modern physics. One example is the decay of $\Upsilon(1S)$ into invisible studied in this work. In fact, it is crucial to understand how many of these decays will be recorded by Belle II before the physics analysis is conducted since Belle II trigger system is optimized for CP violation studies which are the main goal of the experiment and can possibly reject important events containing the studied decays. In order to perform the feasibility study for $\Upsilon(1S)$ into invisible decays, the advanced Monte Carlo simulation framework is employed and trigger efficiency for different scenarios are examined in this report.

2 Motivation for Dark Matter search

The motivation for searches of a new kind of matter called dark matter (DM) comes from various astrophysical observations. Namely, observations of its visible gravitational effects on the ordinary matter. Measurements of stellar orbits inside galaxies, gravitational lensing and motion of hot clouds of gas inside galaxy clusters provide estimates of the mass distribution inside galaxies. It became obvious the baryonic matter could not explain the anomaly of rotational curves and there should have been a large amount of missing mass [1]. There are many theories attempting to solve puzzling experimental results by extension beyond the Standard Model of particle physics (BSM) because the Standard Model itself fails to explain interactions between Standard model particles and DM candidates. Supersymmetric models (SUSY) proposing supersymmetrizing partners to standard model particles are very popular yet there is no experimental evidence confirming them. Therefore, other theories such as those predicting existence of the Dark-Sector are getting more attention. For example, one of those is Asymmetric Dark Matter model (ADM) named after the observed asymmetry in the amount of DM over ordinary baryonic matter (see Figure 1) as measured by the WMAP experiment for Cosmic Microwave Background radiation [2]:

$$\frac{\Omega_{DM}}{\Omega_{BM}} \sim 5.$$

where the parameter Ω represents the cosmological abundance calculated at the value of critic density ρc .

BSM and ASM share following requirements on the DM candidate:

1. *cold* (non-relativistic), from the moment of its thermal production in the early universe.
2. *dark*, because it does not visible to any standard detection.

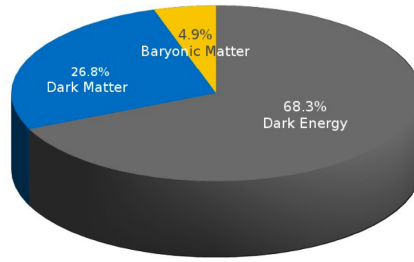


Figure 1: Representation of the relative abundances in our Universe: baryonic ordinary matter represent just a tiny 4.6%, if compared to the amount of Dark Matter ($\sim 24\%$) or Dark Energy ($\sim 72\%$), which is ultimately responsible for the acceleration process in the universe expansion [2].

3. *stable* on cosmological scale, since its effects are still observable nowadays

Some of these models also predict that there is so-called *dark photon* which is a new light boson. It is supposed to correspond to an extra symmetry $U(1)'$ [3, 4]. Theoretical calculations are in favour of the hypothesis that there is a mixing between proposed dark photon and ordinary SM photons. This would mean that looking for example for any enhancement in the invisible decay channels of SM particles can provide invaluable information about the dark sector [5].

2.1 Dark Matter detection

The solid proof of the existence of the DM needs to be obtained from DM interaction with SM particles. There are three main divisions of complementary DM searches depending on DM models and detection techniques (see Figure 2):

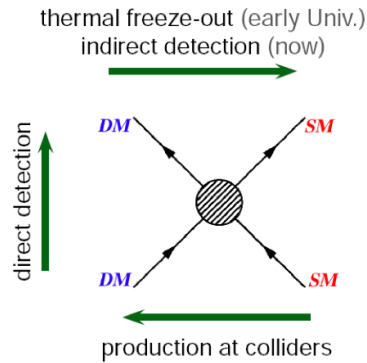


Figure 2: Synthetic representation of the three DM detection techniques.

1. Direct searches: studies of elastic scattering of DM-target particles in experiments examining the recoil kinematics. It is assumed that a DM wind [6] is coming

constantly from the centre of the Sun and our galaxy. Therefore, detectors with huge fiducial volume of interaction are build to attempt to study DM. Typical energy sensitivity should reach the order of 1-100 keV.

2. Indirect searches: studies of excess in the incoming flux of anti-matter, nuclei or neutrinos as they are produced by DM annihilation processes. There are successful experimes such as PAMELA and AMS02, that have recently measured an observed excess in the flux of positrons and anti-protons [7, 8].
3. Collider searches: studies of DM produced in particle collisions providing possibility of precise measurements (unlike astrophysics measurements). In order to discover DM particle particle collisions are studied at the high energy measurements at the LHC and high precision measurements at the flavour-factories (Belle II) [9].

Recently, the flavour factories are drawing scientific attention because they provide the ideal environment for potential discovery of the dark matter and dark forces.

3 Experimental overview

Results obtained in this work are based on simulated Monte Carlo collisions at the Belle II detector located at the accelerator SuperKEKB [10] at the High Energy Accelerator Research Organization (KEK) in Tsukuba, Japan.

3.1 KEKB and SuperKEKB accelerators

During its operation between 1999 and 2010 the KEKB accelerator complex, illustrated in Figure, 3, was used in order to deliver asymmetric electron positron collisions (8 GeV and 3.5 GeV respectively). Particles travel from the source through several pre-accelerators followed by a linear accelerator (LINAC), where the particles are accelerated to their corresponding collision energy. Then they are injected in two main storage rings: one for electrons and one for positrons [11]. Approximately 5000 particle bunches circu-

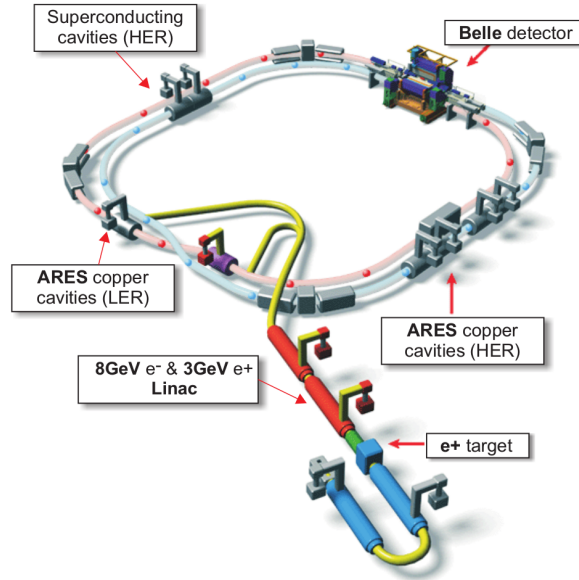


Figure 3: The KEKB accelerator complex. It shows the particle source, linear accelerator and two storage rings and the approximate position of the Belle detector in the complex [12].

late in each of the 3 km long storage rings and are brought to the interaction region at a crossing angle of ± 11 mrad. The used collision configuration ensures efficient separation of incoming and outgoing beams and hence, minimizes the amount of pile-up collisions [11].

The collider produces $\Upsilon(4S)$ resonances with center of mass energy $E_{CMS}=10.58$ GeV, and a Lorentz boost of $\beta \approx 0.425$ along z -axis in direction of the electron beam. SuperKEKB is an upgrade of KEKB accelerator that is going to deliver instantaneous luminosity $8 \cdot 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ what is about 40 times more than KEKB accelerator [13].

3.2 Belle and Belle II detectors

The Belle detector is a large-solid-angle magnetic spectrometer as shown in Figure 4. The double-wall beryllium beam pile is located in the center of the transversal plane of the detector and it is surrounded by the tracking system. Its innermost part is Silicon Vertex Detector (SVD), which is crucial for measurements of the vertex positions of B hadrons. From SVD one measures charge distributions obtained on orthogonally segmented strips and therefore, measure the two dimensional hit positions from which the particle track is reconstructed.

SVD is installed inside the Central Drift Chamber (CDC), which is a large volume gas filled detector placed in the uniform magnetic field. The CDC is used in order to efficiently reconstruct the particles tracks, determine particle momenta with high precision and energy losses of charged particles by ionisation which is needed for particle identification. To fulfil this task, CDC is equipped with 50 detection layers and has 8400 drift cells [14], [12].

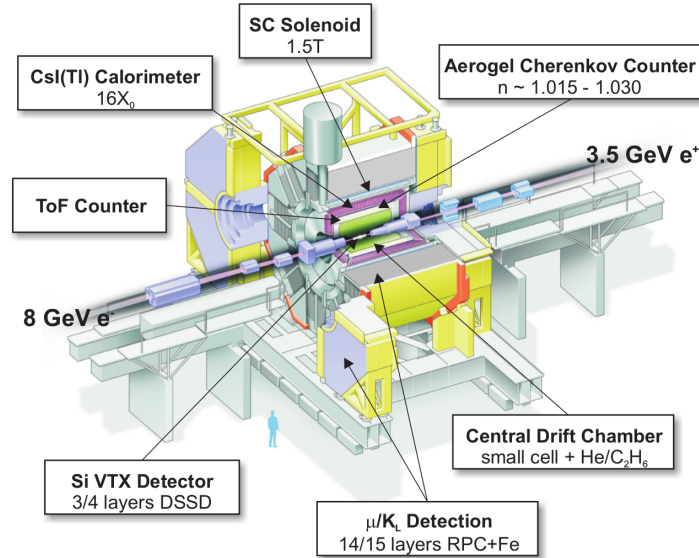


Figure 4: The overview of the Belle detector important components [12].

The next important component is the Belle particle identification system (PID). There is an array of threshold silica aerogel Cherenkov Counters (ACC) and it provides an efficient way how to separates charged Kaons and Pions using the Cherenkov emission [14], [12]. ACC is followed by the Time of Flight (TOF) counter layer, which measures the difference between the time of a collision at the interaction point and the time of interaction of a particle with TOF layer. This information is used to separate detected kaons, pions and photons [14], [12].

Another essential part of Belle detector is the Electromagnetic Calorimeter (ECL), which is designed to detect photons coming from B decays with high efficiency and good resolution in energy. In addition it also provides needed information for particle identification

system to improve separation of electrons and positrons from other particles using particle shower shapes. ECL neighbours the Superconducting Solenoid (SC) which provides magnetic field of 1.5 T in the detector [14].

The outermost part of the Belle detector is K_L^0 and Muon System (KLM). It used to detect and distinguish K_L^0 and muons, thanks to the fact that these particles leave different patterns in the detector [14], [12].

Detector Belle II represents the update of the detector Belle. In case of Belle II, the redesigned data-acquisition system with network of optical fibres is used and trigger electronics is replaced by a new system. Moreover, a pixel detector is added, providing better resolution for particle tracking; a larger solid angle will be covered by the new silicon vertex detector. In addition, some other parts are newly built, such as the central tracking chamber and an aerogel ring-imaging Cherenkov detector. The physics run of Belle II detector is expected to start in 2017 after some tests on the beam physics.

3.3 Simulations and reconstruction in Belle II

Thanks to the Belle II collaboration wide efforts, there is a competitive Monte Carlo (MC) simulation framework available. Simulations and reconstruction of collisions in the Belle II detector are conducted in several steps:

1. The combination of PYTHIA [17] - EVTGEN [16] is employed in order to generate files that contain information on the particle level, such as 4-momenta, decay trees, about the particles produced in the electron-positron collisions as well as consequent decays.
2. The GEANT4 software package [18] is used to simulate the propagation of the generated particles through a very detailed description of the detector geometry and materials, and to estimate their corresponding energy deposits. Furthermore, it is used to simulate the decays of long-lived, unstable particles such as long-lived hadrons, for example Λ^0 , Kaons etc. The result is a file containing the information about energy depositions in sensitive parts of the Belle II detector (detector hits).
3. Belle II reconstruction software reads from the detector hits and uses them in order to reconstruct the particle tracks in the detectors and eventually attempts to reconstructs the decay trees and identify the particles present in the detector.

Belle MC simulation framework provides a highly automatised way for each part of the simulations with wide options for specifications. This framework has been used to obtain results presented in this work.

4 $\Upsilon(1S)$ invisible decays feasibility study

4.1 Dark matter studies at the Belle II experiment

There many low-mass (bellow $10\text{GeV}/c^2$) dark matter searches mostly based on direct detection. Even though they have impressive performance, the irreducible background

of neutrinos coming from the Sun embodies the limitation for their searches and final estimations as shown in Figure 5. However, this region is will be accessible to collider

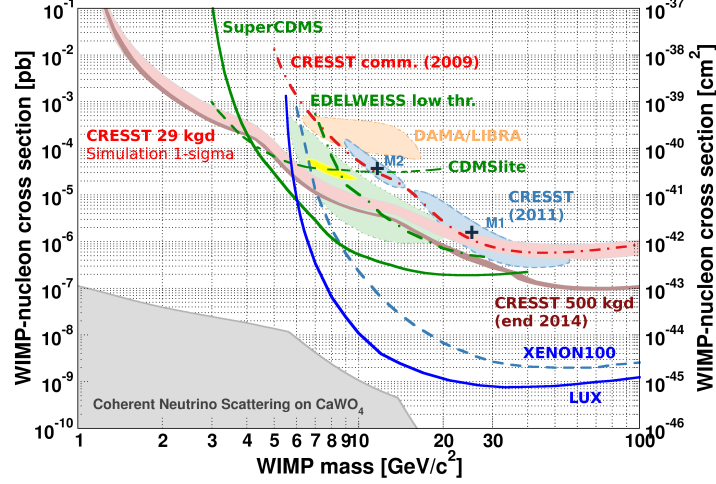


Figure 5: The limits for DM searches set by cryogenic experiments. Lower left corner shows the area with irreducible background caused by coherent neutrino scattering from the Sun. The Belle II experiment is capable of measuring the in the region with DM mass lower than 5 GeV and between 10^{-40} and 10^{-45} cm^2 .

experiments with low energy as in case of Belle II direct dark matter production. Bound states of b quark and \bar{b} -antiquark $\Upsilon(ns)$ are of particular interest thanks to the allowed dark matter production in the phase space of two low mass DM particles [20] yet they are generally strongly suppressed. For example,

$$\frac{Br(\Upsilon(1S) \rightarrow \nu\bar{\nu})}{Br(\Upsilon(1S) \rightarrow e^+e^-)} \sim 4.14 \times 10^{-4}$$

The main aim of this study is to investigate feasibility of rare decay

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow \pi^+\pi^-\Upsilon(1S) \\ \Upsilon(1S) \rightarrow \chi\bar{\chi}$$

at Belle II experiment, where χ and $\bar{\chi}$ represent invisible decay products. This study is new and one of the kind since other experiments, unlike Belle II, do not collect sufficient amount of data.

In fact, in order to measure $\Upsilon(1S)$ invisible decays one must study the kinematic of pion pair present in the decay which is the only source of information about the decay. Any new signal would result in an excess of events in the recoil mass distribution which is equivalent to the mass of the meson $\Upsilon(1S) = 9.460 \text{ GeV}/c^2$. Recoil mass is calculated using equation:

$$M_{\text{rec}}^2 = s + M_{\pi^+\pi^-}^2 - 2E_{\pi^+\pi^-}^{\text{cms}}\sqrt{s} \quad (1)$$

where $M_{\pi^+\pi^-}$ is mass of the pion system, $E_{\pi^+\pi^-}^{\text{cms}}$ is energy of pion system in the center of mass frame and \sqrt{s} is the collision energy.

4.2 Sample Characteristic

In our study, we use MC samples generated using Belle II framework (see 3.3). Data samples for both visible and invisible channels described in 4.1 consist of 10^5 Belle II collisions events in which described decays corresponding to mentioned channels are present. The visible channel simulations are used as a control sample to ensure that software is configured correctly and well understood as well as to ensure that obtained distributions for selected quantities are in agreement with expectations based on physics ruling the studied processes. Figure 6 shows the distribution of the number of decay candidates as a function of flight direction of π^- expressed by $\cos \theta$. As expected, there is more events in which we observe pion in forward and backwards direction. Another distribution to consider is the distribution of total momentum of a pion shown in Figure 7 and it is also in agreement with theoretical expectations.

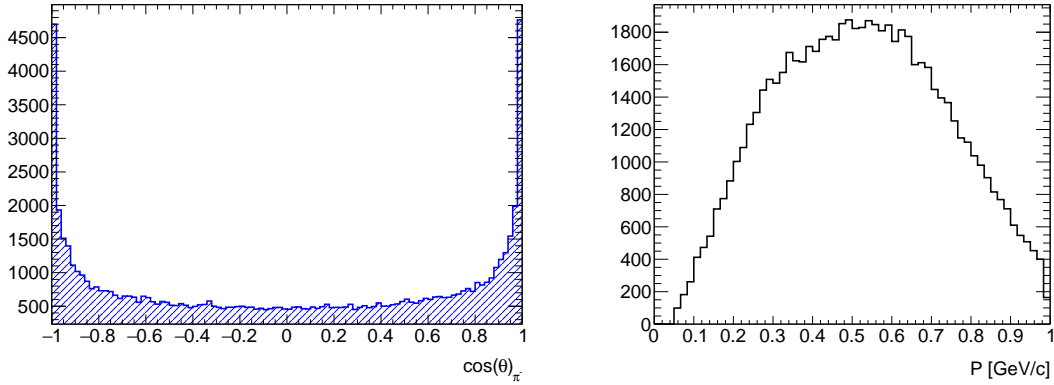


Figure 6: The distribution of the number of events as a function of flight direction angle $\cos \theta$ for π^- .

Figure 7: The distribution of the number of events as a function of the total momenta of a pion.

The correlation between momentum components of pions is also examined and one example is presented in Figure 8. One more important quantity to consider is the opening angle between pions present in the decay shown in the Figure 9, where we observe that number events is increasing with opening angle between pions.

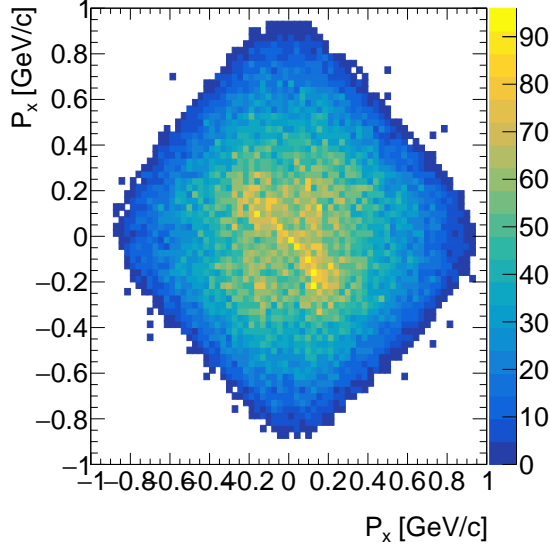


Figure 8: The distribution of correlation between x -components of momenta of pions.

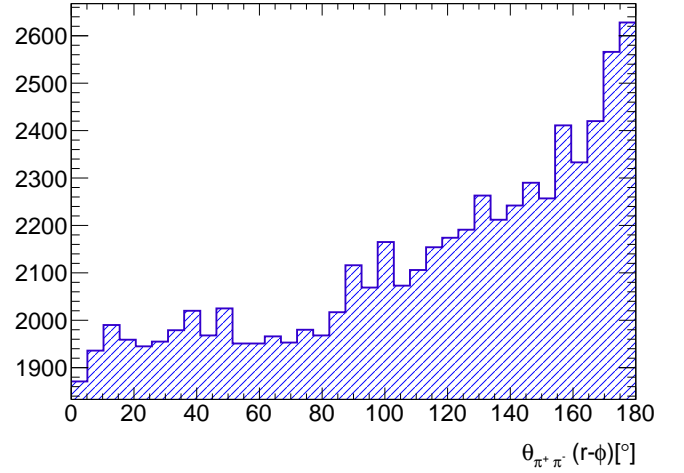


Figure 9: The distribution of the number of events as a function of the opening angle between pions.

These distributions are compared with distributions obtained from simulations of invisible channels and they are in good agreement with each other, which confirms that correctness of the performed simulations. Another studied quantity is ratio between transversal momentum of pions in visible and invisible channels as a function of transversal momentum shown in Figure 10. As we can see in the ratio distribution, the ratio is mostly close to value 1 in entire examined range which is another confirmation the good performance of the simulation framework.

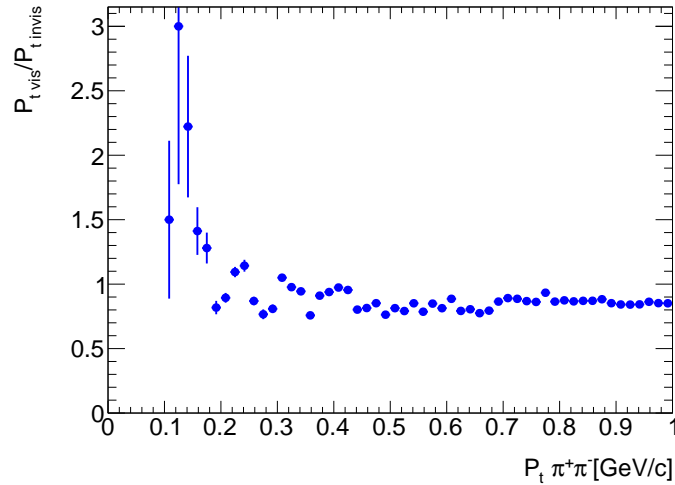


Figure 10: The ration between number of pions in visible and invisible channel as a function of the pion transversal momentum.

The last performed check on control sample is the reconstruction of the invariant mass of the $\Upsilon(4S)$ and $\Upsilon(1S)$ shown in the Figure 11. As we see in the mass distributions, the reconstructed mass is in agreement with expected values $m_{\Upsilon(4S)} = 10579.4 \pm 1.2 \text{ MeV}$ and $m_{\Upsilon(1S)} = 9460.30 \pm 0.26 \text{ MeV}$ [19].

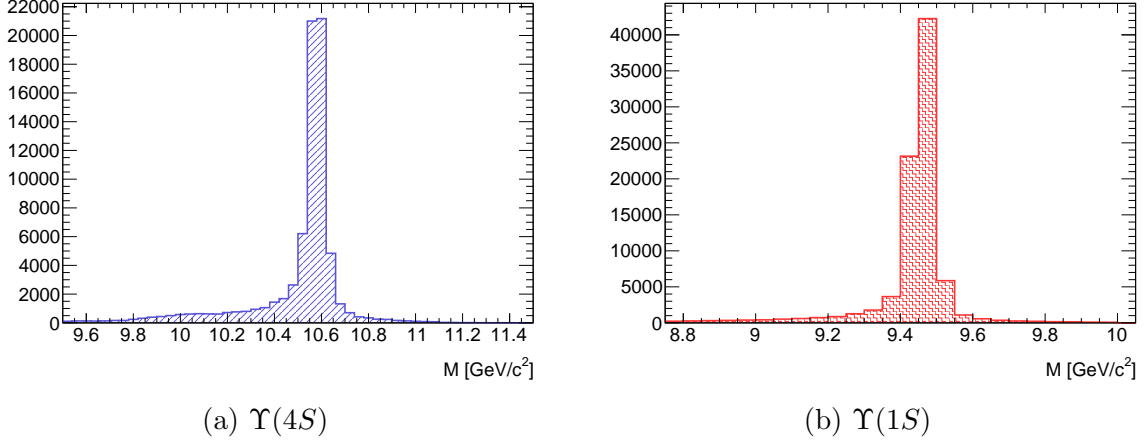


Figure 11: Reconstructed mass distributions for $\Upsilon(4S)$ and $\Upsilon(1S)$.

4.3 Signal and background

To study the rare decays in described channel in Belle 2 experiment we consider both signal and background events. The signal event consists of two charged particle tracks which are curved in magnetic field 1.5 T as shown in Figure (12). In case of a signal event, the tracks correspond to charged pions which are created in $\Upsilon(4S)$ decay. By studying these two pions recoil kinematics, we can infer information about the $\Upsilon(1S)$ invisible decays. Unfortunately, there are many situations where background events are very similar to signal. Apart from QED combinatorial background, there is also the peaking background originating from $\Upsilon(1S)$ decays into charged leptons pair which is emitted in forward and background region which is outside of detector acceptance. Neuron networks are used to minimize effect of the QED background but peaking background is irreducible and requires the precise estimation of number of expected background events and corresponding error.

4.4 Total efficiency estimation for invisible channel

In order to assess whether the study of the described channel in Belle II experiment is feasible, we examine the trigger procedures in Belle II. Namely, it is of our interest to know whether the standard trigger could be used to collect events with two *long tracks* which refers in this case to two pion tracks reaching detector systems outside from CDC, as well as, to estimate whether this technique is leading to acceptable total efficiency estimates.

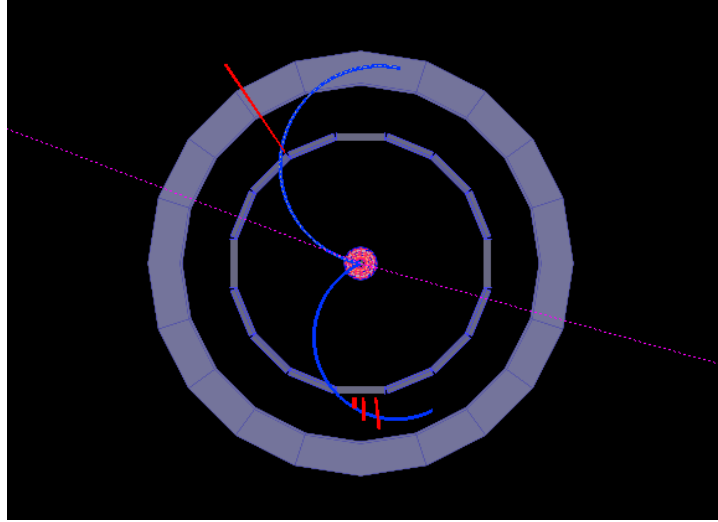


Figure 12: Illustration of a signal event inside Belle II detector. Blue lines represent reconstructed tracks of pions and red lines show the corresponding energy depositions.

Generally, mentioned tracks correspond to low-momentum pions, which are usually turning inside the drift chamber before their absorption. Pions share maximum energy 1119.1 MeV provided from invariant mass difference between decaying $\Upsilon(4S)$ and created $\Upsilon(1S)$.

Figure 13 shows the distribution of the invariant mass of the pion system in the studied channel and it represents a new result since this distribution has not been published so far. We can also extract the mass of the recoiling system against di-pion system which reproduces $\Upsilon(1S)$ invariant mass shown in Figure 14, which is a valuable confirmation that the invisible channel decays are reconstructed correctly.

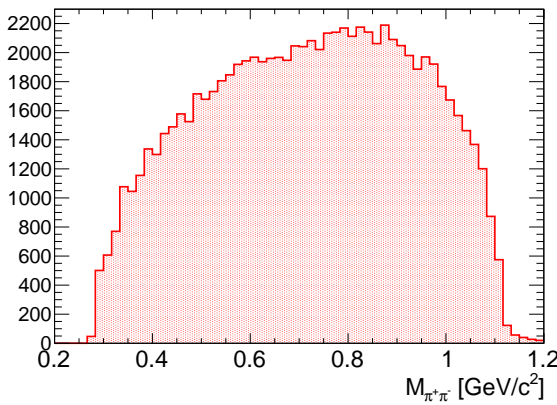


Figure 13: Distribution of the invariant mass of the pion system in the studied channel.

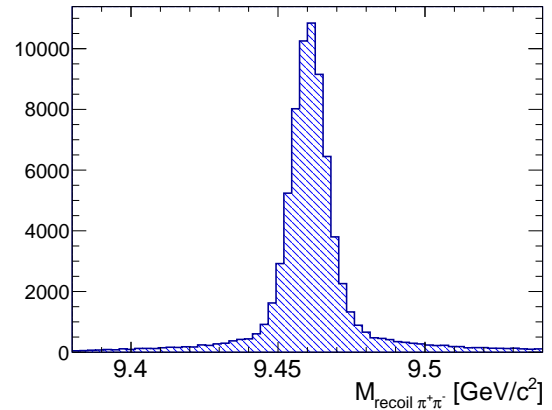


Figure 14: Distribution of the mass of the recoiling system against di-pion system peaking at the invariant mass of $\Upsilon(1S)$ meson.

The total efficiency is obtained with requirements:

1. Trigger requirement: two long tracks
2. Momentum requirements: each of 3-momentum components must be in the interval $[-1,1]$ GeV
3. Particle identification requirements

The obtained distributions are shown in Figure 15, where we observe differences requiring momenta cuts and: 0 or more tracks outside CDS, 1 or more tracks outside CDS and 2 track outside CDS with PID.

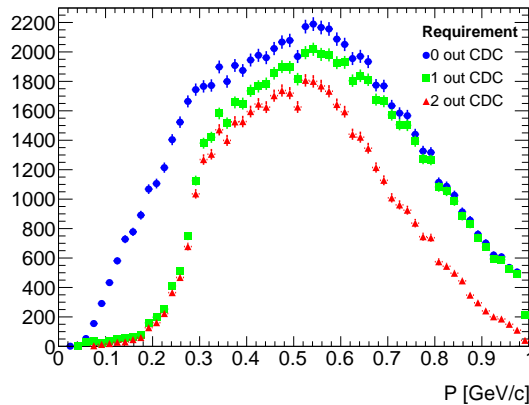


Figure 15: The distributions of the number of accepted pions in as a function of pion momenta for different requirements: 0 or more tracks outside CDS (blue), 1 or more tracks outside CDS (green) and 2 track outside CDS with PID (red).

It is favourable to select the detector regions where the described low-momentum pions physics analysis would be not considered as a significant background source. We define three different pion opening angle ranges in $r - \phi$ plane and for each of them we define efficiency:

$$\varepsilon = \frac{N_{\text{reconstructed}}(\theta_{\pi^+\pi^-})}{N_{\text{simulated}}} \quad (2)$$

where $N_{\text{reconstructed}}(\theta_{\pi^+\pi^-})$ is the number of reconstructed events as a function of opening angle and $N_{\text{simulated}}$ is the total number of simulated events. The obtained results are stated in Table 1.

From obtained results the estimation of the expected peaking background can be inferred. We consider the worst case scenario, which is one order of magnitude away from truth, we assume that each of $\Upsilon(1S)$ which is not reconstructed is related to the muons travelling outside the detector acceptance. If we combine this study with other channels of invisible decays of $\Upsilon(nS)$, we can impose the constrains on this decay. Namely, it is obtaining the upper limit which is at the level of $\sim 4.3 \times 10^{-5}$, which is approximately

Detector Acceptance	$\sim 78\%$
Trigger requirement: 2 tracks outside CDC + PID + Momentum cuts $\pi\pi$ opening angle	
(0° - 180°)	$\sim 49\%$
(100° - 180°)	$\sim 25\%$
(130° - 180°)	$\sim 18\%$

Table 1: Detector acceptance and detector efficiency for different opening angle intervals.

4.3 times higher than standard model prediction and roughly 8 times smaller than current upper limits obtained from BABAR which are set to 3×10^{-4} [21]. Therefore, we expect improvement of one order of magnitude.

On the other hand, we consider a more realistic scenario in which out of all the produced $\Upsilon(1S)$ mesons, 0.026% will have the signature of $\Upsilon(1S)$ invisible decays because of the products travelling through the detector outside the acceptance. In this scenario, the number of peaking background events and corresponding error can be estimated. If we set a constrain on the signal at 90% CL, Belle II will be capable of measuring this process for achieved efficiency exceeding value of $\sim 13\%$. As can be seen from the Table 1, this value is reached. More details need to be examined in order to improve and correct the final estimations. Nevertheless, the preliminary results presented in this work imply that the study of invisible $\Upsilon(1S)$ decays using di-pion transitions are possible to be conducted within the Belle II experiment. Furthermore, if recoil mass distribution exhibit any excess it may be a sign for the new physics.

5 Conclusions

The motivation for this work was to assess the feasibility of studying the decays of $\Upsilon(1S)$ mesons into invisible particles using Belle II detector. Decays of $\Upsilon(1S)$ mesons may represent a valuable source of knowledge in the dark matter searches. However the trigger system of Belle II detector is not optimized for measurements of these decays. Therefore, we study the decay channel $e^+e^- \rightarrow \Upsilon(4S) \rightarrow \pi^+\pi^-\Upsilon(1S); \Upsilon(1S) \rightarrow \text{invisible}$ using Belle II simulation framework in order to examine possibility of conducting the physics analysis based on this channel and we use control channel $e^+e^- \rightarrow \Upsilon(4S) \rightarrow \pi^+\pi^-\Upsilon(1S); \Upsilon(1S) \rightarrow \mu^+\mu^-$ to confirm that obtained results are consistent. Results obtained from Monte Carlo simulations of visible and invisible decay channel are in agreement with each other and with theoretical expectations. The detector acceptance in this channel is estimated to be $\sim 87\%$. Moreover, the efficiency for the trigger that requires detection of two tracks outside the CDC after applying particle identification and used cuts is $\sim 47\%$ for the pion opening angle interval (0° - 180°), $\sim 19\%$ for the pion opening angle interval (100° - 180°) and $\sim 14\%$ for opening angle interval (130° - 180°). Based on this observations we conclude that Belle 2 will be able to search for dark matter in invisible $\Upsilon(1S)$ decays using di-pion transitions.

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