THRUST DISTRIBUTION OF $e^+e^- \to$ QUARK AND GLUON JETS

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Abstract

In this report, $e^+e^- \to$ quark and gluon jets are studied. The framework used for this is SCET, an EFT suited for jet physics. The event shape thrust of the quark and gluon jets are calculated and investigated. The thrust shapes of the quark and gluon jets are plotted for the singular part of NLO, LL and NLL level.

Contents

1 Introduction

In collider experiments high energetic collimated hadrons (jets) are commonly observed. The distribution and shape of jets can be used to gain information about the high energy processes and analysis of jet cross sections provides testing of quantumchromodynamics (QCD) [1]. In this report we look into jet physics, using an EFT approach called SCET, and study the event shape thrust of $e^+e^- \to$ quark and gluon jets.

In LHC the signals for example New Physics are in general from quark jets and the background signal from gluon jets. It is thus of importance to be able to distinguish quark and gluon jets. In this report we look into $e^+e^- \to$ quark and gluon jets, because this provides a clean environment for theoretically high-order perturbative computations [1]. We consider the process $e^+e^- \to$ quarks via an s-channel with a photon or Z-boson. The $e^+e^- \to$ gluons process considered is via a Higgs with a topquark loop. This process has a very small amplitude because of the small interaction of the Higgs with the low mass electrons and positrons. It is thus not a relevant process to study it for the process itself, but it can still be used to study how the thrust distribution of gluon jets compare with quark jets.

2 SCET

Soft-Collinear Effective Theory (SCET) is an effective theory suitable to describe jet physics with interactions of soft and collinear particles in the presence of a hard interaction. SCET is thus studied and used in this project. SCET will eventually allow us to factorize cross sections and resum Sudakov logarithms [2].

In SCET, particles are not necessarily integrated out, but modes are separated according to how their momentum components scale. In collider processes, we call the the momentum scale corresponding to the hard interaction the hard scale, Q. Collinear degrees of freedom are particles which move in or near a preferred direction (jet axes). Soft degrees of freedom are particles which have no preferred direction and have momenta much lower than Q. We will have different momentum regions and separate particles accordingly. In SCET we can thus have different fields which would represent one field in the full QCD theorem. For example, we could have collinear quarks, soft quarks, collinear gluons and soft gluons.

We use coordinates which make the different scalings in momentum components more transparent. These will be the lightcone coordinates, defined by the vectors $n^\mu = (1, \vec{n})$ and $\bar{n}^\mu = (1, -\vec{n})$, which satisfy $n^2 = 0 = \bar{n}^2$ and $n \cdot \bar{n} = 2$. Any vector p^μ can be decomposed in the lightcone basis:

$$p^\mu = \frac{n^\mu}{2}\bar{n} \cdot p + \frac{\bar{n}^\mu}{2} n \cdot p + p_\perp^\mu \equiv (p^+, p^-, p_\perp), \tag{1}$$

where $p^+ = n \cdot p$ and $p^- = \bar{n} \cdot p$. Hard momenta then scale as $p^\mu \sim Q(1, 1, 1)$. Collinear momenta scale as $p^\mu \sim Q(\lambda^2, 1, \lambda)$, where $\lambda << 1$ is a small dimensionless parameter. Soft momenta scale as $p^\mu \sim Q(\lambda, \lambda, \lambda)$. It is also possible to have ultrasoft degrees of freedom which scale as $p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$. These are SCET II theories [3], but in this report we only consider SCET I theories with soft momenta.

3 Thrust

In this report we look into the process $e^+e^- \to$ jets. Kinematically dominant is the production of two jets, but it is also possible that more jets are produced. The event shape variable thrust can be used to distinguish dijet events from events with more than two jets. Thrust is defined as [4]:

$$T = \max_{\vec{n_T}} \frac{\sum_i |\vec{p}|_i \cdot \vec{n}_T}{\sum_i |\vec{p}_i|}, \tag{2}$$

where i runs over all the final state particles and $\vec{n_T}$ is the thrust axis. Collinear (or anti-collinear) particles have a large projection onto the thrust axis, giving T near 1. Events with T near 1 are thus 2-jet like, while T going away from 1 means more than two jets production. It is more convenient to use the thrust variable $\tau = 1 - T$, which will be used from now on. This now means that in the situation $\tau \to 0$ we have dijets. In this project we will look into the thrust distributions of quark and gluon jets and compare them.

4 Factorization

Using the factorization theorem, the thrust distribution can be factorized in a hard function part, jet functions part and soft function part [4]:

$$\frac{d\sigma}{d\tau} = \sigma_B H(Q, \mu) \int d\tau_n d\tau_{\bar{n}} d\tau_s \delta(\tau - \tau_n - \tau_{\bar{n}} - \tau_s) J_n(\tau_n, \mu) J_{\bar{n}}(\tau_{\bar{n}}, \mu) S(\tau_s, \mu), \tag{3}$$

where σ_B is the Born cross section.

The hard function is obtained by taking the absolute square of the Wilson coefficient [4]:

$$H(Q, \mu) = \left|C(Q, \mu)\right|^2 = 1 + \frac{\alpha_s C_F}{2\pi} \left(-4 \log^2 \frac{Q}{\mu} + 6 \log \frac{Q}{\mu} - 8 + \frac{7\pi^2}{6} \right). \tag{4}$$

The quark jet and soft functions can be calculated as the vacuum matrix element of a

2-point collinear function. The jet function is given by [4]:

$$J(\tau,\mu) = \delta(\tau) + \frac{\alpha_s C_F}{4\pi}\left[\left(2\log^2\frac{Q\xi}{\mu^2} - 3\log\frac{Q\xi}{\mu^2} + 7 - \pi^2\right)\delta(\tau) + \left(4\log\frac{Q\xi}{\mu^2} - 3\right)\frac{1}{\xi}\mathcal{L}_0\left(\frac{\tau}{\xi}\right) + 4\frac{1}{\xi}\mathcal{L}_1\left(\frac{\tau}{\xi}\right)\right] \tag{5}$$

and the quark soft function by [4]:

$$S(\tau,\mu) = \delta(\tau) + \frac{\alpha_s C_F}{4\pi}\left[\left(-8\log^2\frac{\xi}{\mu} + \frac{\pi^2}{3}\right)\delta(\tau) - 16\log\frac{\xi}{\mu}\frac{1}{\xi}\mathcal{L}_0\left(\frac{\tau}{\xi}\right) - 16\frac{1}{\xi}\mathcal{L}_1\left(\frac{\tau}{\xi}\right)\right] \tag{6}$$

Here ξ is a dimensionful dummy variable and the $\mathcal{L}_i$ are the plus distributions [5].

The hard, jet and soft functions each contain logarithms, which may grow large and be problematic. But because each of these functions depend on a single scale, we can evaluate the hard, jet and soft functions at respectively the scales $\mu_H \sim Q, \mu_J \sim \sqrt{Q\tau}, \mu_S \sim \tau$, so that the large logarithms in the functions disappear. Then we use the renormalization group evolution to evolve each function to a common scale μ.

5 Renormalization Group Evolution

In QCD hadron jet production in e^+e^- collisions happen via an s-channel exchange of a photon or Z-boson. We will first look into e^+e^- to quarks. In SCET the current then involves collinear quarks and we have: $(\bar{\xi}_{\bar{n}} W_{\bar{n}})\Gamma_i (W_n^\dagger \xi_n)$, where W_n are Wilson lines needed to make the quark fields collinearly gauge-invariant. $\chi_n \equiv W_n^\dagger \xi_n$ is thus the quark jet field [4]. Calculating the matrix elements and matching the renormalized matrix elements from SCET to QCD then gives the Wilson coefficient of the effective SCET operator:

$$C(Q,\mu) = 1 + \frac{C_F\alpha_s(\mu)}{4\pi}\left[-\log^2\left(\frac{-Q^2 - i0}{\mu^2}\right) + 3\log\left(\frac{-Q^2 - i0}{\mu^2}\right) - 8 + \frac{\pi^2}{6}\right] \tag{7}$$

We demand the usual renormalization scale independence equation; the bare coefficient should not depend on the renormalization scale:

$$0 = \mu\frac{d}{d\mu}C^{bare} = \mu\frac{d}{d\mu}[Z_C(\mu)C(\mu)] = \mu C\frac{d}{d\mu}Z_C + \mu Z_C\frac{d}{d\mu}C \tag{8}$$

allowing us to calculate the anomalous dimension of the Wilson coefficient:

$$\mu\frac{d}{d\mu}C = -\frac{1}{Z_C}\mu\frac{dZ_C}{d\mu}C \equiv \gamma_C C \tag{9}$$

We find:

$$\gamma_C = -\frac{\alpha_s}{4\pi}\left[4C_F \log\left(\frac{\mu^2}{-Q^2 - i0}\right) + 6C_F\right] \tag{10}$$

In this process the anomalous dimension can always be written in the following form [4]:

$$\gamma_C(\mu,\omega) = -a_C \Gamma_{cusp}[\alpha_s(\mu)] \log\left(\frac{\mu}{\omega_C}\right) - \gamma_C[\alpha_s(\mu)], \tag{11}$$

where the anomalous dimension is now separated in a cusp Γ_{cusp} and non-cusp γ_C part, and the constant a_C and dimensionful variable ω_C depend on the current. This formula holds to all orders in α_s:

$$\Gamma_i(\alpha_s) = \sum_{n=0}^{\infty} \Gamma_{in} \left(\frac{\alpha_s}{4\pi}\right)^{n+1}, \quad \gamma_i(\alpha_s) = \sum_{n=0}^{\infty} \gamma_{in} \left(\frac{\alpha_s}{4\pi}\right)^{n+1}, \tag{12}$$

where the subscript $i = q, g$ for quarks and gluons respectively. We need to solve Eq. (9) and for later convenience we want to write the solution (for the hard function, which is the absolute square of the Wilson coefficient) as:

$$H(Q,\mu) = H(Q,\mu_0) U_H(\mu,\mu_0), \tag{13}$$

where $U_H(\mu,\mu_0)$ is the evolution factor that runs the hard function from μ_0 to any arbitrary scale μ. Eq. (9) can be solved by integrating it from μ_0 to μ using a change of variables to α_s: $d\log\mu = \frac{d\alpha_s}{\beta[\alpha_s]}$. The running of α_s also needs to be accounted for: $\mu \frac{d}{d\mu}\alpha_s(\mu) = \beta(\alpha_s)$, where we use the expansion of the β-function in powers of α_s:

$$\beta(\alpha_s) = -2\alpha_s \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} \tag{14}$$

We then find:

$$\log\left[\frac{H(Q,\mu)}{H(Q,\mu_0)}\right] = -\omega \log\left(\frac{\mu_0}{Q}\right) - K_\Gamma + K_\gamma, \tag{15}$$

where ω, K_Γ and K_γ are defined as [4]:

$$\begin{aligned} \omega(\mu,\mu_0) &= \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta[\alpha]} \Gamma_{cusp}[\alpha] \\ K_\Gamma(\mu,\mu_0) &= \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta[\alpha]} \Gamma_{cusp}[\alpha] \int_{\alpha_s(\mu_0)}^{\alpha} \frac{d\alpha'}{\beta[\alpha']} \\ K_\gamma(\mu,\mu_0) &= \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta[\alpha]} \gamma[\alpha] \end{aligned} \tag{16}$$

Exponentiating Eq. (15) now gives the solution is the desired form with the evolution factor of the hard function:

$$U_{Hi}(Q,\mu,\mu_0) = \left| e^{-K_{\Gamma_i}(\mu,\mu_0) + K_{\gamma_i}(\mu,\mu_0)} \left(\frac{\mu_0^2}{Q^2}\right)^{-\frac{1}{2}\omega_i(\mu,\mu_0)} \right|^2 \tag{17}$$

For the jet functions and soft function the equations are similar. Instead of a multiplicative renormalization group equation we now have one with a convolution.

$$\mu\frac{d}{d\mu}F(t,\mu) = \int dt'\gamma_F(t-t')F(t',\mu), \tag{18}$$

where $F = J, \bar{J}, S$. This equation can be solved by going to Fourier space, and the solution of this equation is [8]:

$$F(t,\mu) = \int dt' F(t-t',\mu_F)U_F(s',\mu,\mu_F), \tag{19}$$

where $U_F(t',\mu,\mu_F)$ is the evolution kernel evolving the jet or soft function from μ_F to μ.

The evolution kernel of the jet function is given by [4]:

$$U_{Ji}(t,\mu,\mu_0) = \frac{e^{\gamma_E\omega_i(\mu,\mu_0)}e^{2K_{\Gamma_i}(\mu,\mu_0)+K_{\gamma_i}(\mu,\mu_0)}}{\Gamma(-\omega_i(\mu,\mu_0))}\left[\frac{1}{\mu_0^2}\mathcal{L}^{-\omega_i(\mu,\mu_0)}\left(\frac{t}{\mu_0^2}\right) - \frac{1}{\omega_i(\mu,\mu_0)}\delta(t)\right], \tag{20}$$

where $\mathcal{L}^a$ is the plus distribution defined as in [5].

The solutions of the integrals in Eq. (16) up to orders needed for NLL are [6]:

$$\omega_i(\Gamma_i) = -\frac{\Gamma_{i0}}{\beta_0}\left(\log(r) + \frac{\alpha_s(\mu_0)}{4\pi}\left(\frac{\Gamma_{i1}}{\Gamma_{i0}}\right)(r-1)\right) \tag{21}$$

$$K_{\Gamma,i}(\Gamma_i) = -\frac{\Gamma_{i0}}{2\beta_0^2}\left(\frac{4\pi}{\alpha_s(\mu_0)}\left(1-\frac{1}{r}-\log(r)\right) + \left(\left(\frac{\Gamma_{i1}}{\Gamma_{i0}} - \frac{\beta_1}{\beta_0}\right)(1-r+\log(r)) + \frac{\beta_1}{2\beta_0}\log^2(r)\right)\right) \tag{22}$$

$$K_{\gamma,i}(\gamma_i) = -\frac{\gamma_{i0}}{2\beta_0}\log(r), \tag{23}$$

where $r = \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}$.

6 Gluon jets

We also look into the process of $e^+e^- \to$ gluon jets. This happens through a Higgs with a topquark loop as said before. As before, the gluon hard function is obtained by taking the absolute square of the Wilson coefficient, which is obtained by matching the SCET matrix element to the QCD one.

The gluon hard function is given by [6]:

$$H_g(Q,\mu) = \alpha_s^2\left(1 + \frac{\alpha_s}{4\pi}\left[-6\log^2\frac{Q^2}{\mu^2} + 22 + \pi^2\right]\right). \tag{24}$$

The gluon jet function is given by [7]:

$$J_g(s,\mu_J) = \delta(s) + \frac{\alpha_s}{2\pi}\left[\left(\left(\frac{2}{3}-\frac{\pi^2}{2}\right)C_A + \frac{5}{6}\beta_0\right)\delta(s) - \frac{\beta_0}{2\mu^2}\mathcal{L}_0\left(\frac{s}{\mu^2}\right) + \frac{2C_A}{\mu^2}\mathcal{L}_1\left(\frac{s}{\mu^2}\right)\right], \quad (25)$$

and the gluon soft function is the same as the quark one, but with $C_F \to C_A$:

$$S(\tau,\mu) = \delta(\tau) + \frac{\alpha_s C_A}{4\pi}\left[\left(-8\log^2\frac{\xi}{\mu} + \frac{\pi^2}{3}\right)\delta(\tau) - 16\log\frac{\xi}{\mu}\frac{1}{\xi}\mathcal{L}_0\left(\frac{\tau}{\xi}\right) - 16\frac{1}{\xi}\mathcal{L}_1\left(\frac{\tau}{\xi}\right)\right]. \quad (26)$$

7 Results

The needed loop order corrections for LL, NLL, NLL' and NNLL analyses are shown in Table 1. Cusp and non-cusp refer to the anomalous dimensions, and α_s up to three loops is given by [7]:

$$\frac{1}{\alpha_s(\mu)} = \frac{X}{\alpha_s(M_Z)} + \frac{\beta_1}{4\pi\beta_0}\log(X) + \frac{\alpha_s(M_Z)}{16\pi^2 X}\left(\left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right)(1-X) + \frac{\beta_1^2}{\beta_0^2}\log(X)\right) \quad (27)$$

where $X = 1 + \alpha_s(M_Z)\log\left(\frac{\mu}{M_Z}\right)\frac{\beta_0}{2\pi}$. The needed coefficients of the β-function and the quark and gluon anomalous dimensions are given in the appendix.

Table 1: Loop order corrections [1]

	Cusp	Non-cusp	$\beta(\alpha_s)$	matching	α_s
LL	1	-	1	tree	1
NLL	2	1	2	tree	2
NLL'	2	1	2	1	2
NNLL	3	2	3	1	3

The results of the calculation of the thrust distribution for quark jets and gluon jets are shown in this section. We divide Eq. (3) by the Born cross section, so that the hard functions of both the quark and gluons are 'normalized' to: $1 + \mathcal{O}(\alpha_s)$. We also use a dimensionful thrust variable τ.

7.1 NLO

For the NLO singular part of the calculations for quark and gluons (Figure 1 and Figure 2) we use a $Q = 125$ GeV. There is no RGE happening here, and the hard, jet and soft functions are also just evaluated at this momentum value.

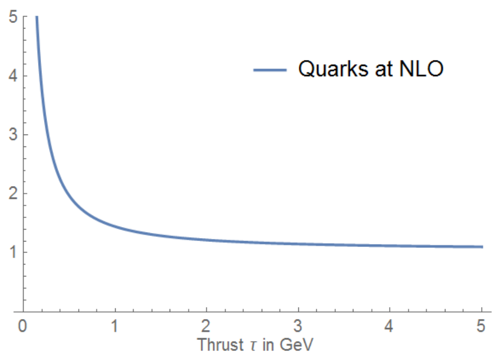

Figure 1: Thrust distribution of quark jets at NLO.

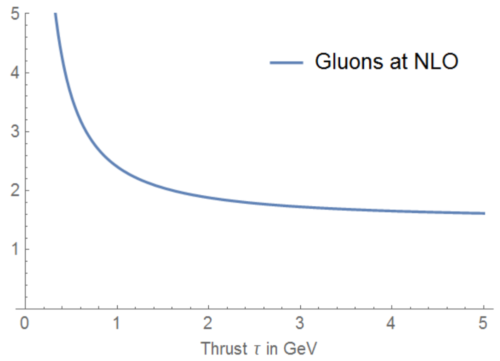

Figure 2: Thrust distribution of gluon jets at NLO.

7.2 RGE improved analysis

For the LL (Figure 3) and NLL (Figure 4) calculations , we use $Q = 125$ GeV. Now, the hard, jet and soft functions are evaluated at their canonical scales $\mu_H = Q, \mu_J = \sqrt{Q\tau}, \mu_s = \tau$, and then all functions are evolved to μ_S, using the evolution kernels Eq. (17) and Eq. (20).

In Figure 5 and Figure 6 the quarks and gluons are compared with each other at LL and NLL.

Varying the hard scale Q at the values $150, 200$ and 250 GeV, we also plot the thrust distribution at NLL for quarks (Figure 7) and gluons (Figure 8).

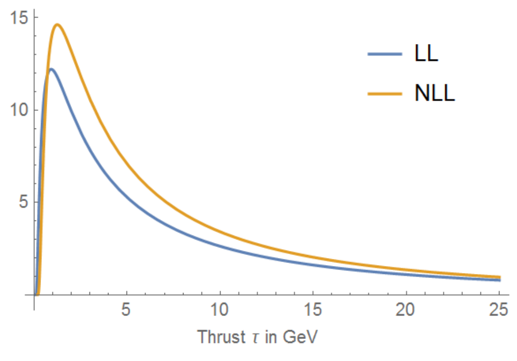

Figure 3: Thrust distribution of quark jets at LL and NLL. Here $Q = 125$ GeV.

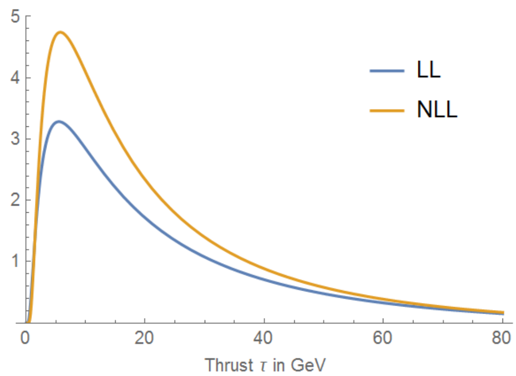

Figure 4: Thrust distribution of gluon jets at LL and NLL. Here $Q = 125$ GeV.

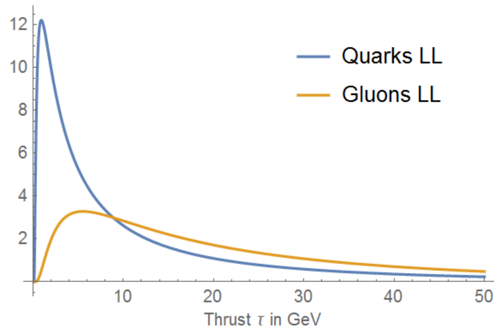

Figure 5: Thrust distribution of quark jets versus gluon jets at $Q = 125$ GeV and at LL.

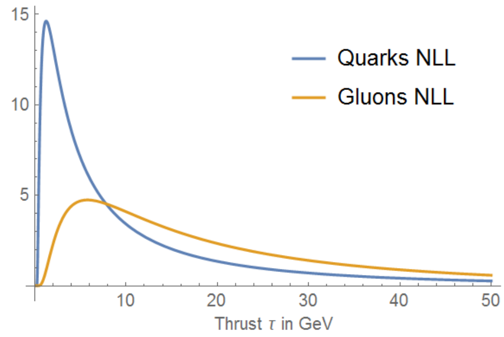

Figure 6: Thrust distribution of quark jets versus gluon jets at $Q = 125$ GeV and at NLL.

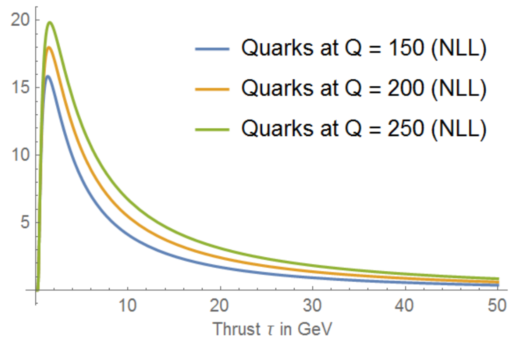

Figure 7: Thrust distribution of quark jets at NLL with $Q = 150, 200, 250$ GeV.

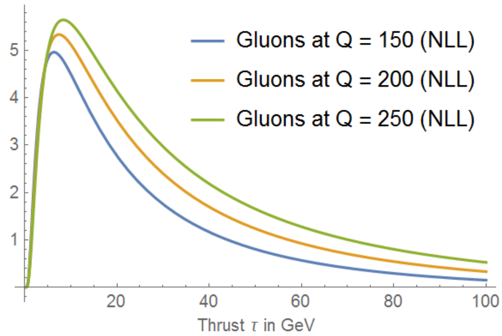

Figure 8: Thrust distribution of gluon jets at NLL with $Q = 150, 200, 250$ GeV.

8 Discussion

At NLO the thrust spectrum diverges at τ near to 0 for both quark and gluon jets. When we do renormalization group evolution, the divergences at small τ disappear. For quark jets at LL the peak is around 0.8 GeV. At NLL, the peak shifts to 1.2 GeV. For gluons we see that the peak is around 5 GeV and shifts to 5.5 GeV. Comparing the quarks with the gluons, we see that at both LL and NLL the peak for the quarks is located at smaller τ compared to the gluons. We also see in all the figures the expected qualitative difference between quark and gluon jets: the thrust shape is narrower for the quark jets and broader for the gluon jets.

In Figure 7 and Figure 8 the Q-values are varied from $150, 200$ and 250 GeV. The qualitative behaviour and comparisons of the quark and gluon jets are the same. The peak shifts slightly to higher τ.

9 Conclusion

In this project we have studied SCET and $e^+e^- \to$ quark and gluon jets. We have calculated and plotted the thrust distributions at NLO, LL and NLL. We have also looked at NLL' calculations and have the ingredients to continue the calculations to NNLL, but did not manage to finish these.

10 Acknowledgements

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11 Appendix: coefficients β-function and anomalous dimensions

All the used coefficients for the β-function and all anomalous dimensions used in this report are given in this section [4], [6].

The used coefficients for the β-function are:

$$\begin{aligned}
\beta_0 &= \frac{11}{3}C_A - \frac{4}{3}T_F n_f \\
\beta_1 &= \frac{34}{3}C_A^2 - \left(\frac{20}{3}C_A + 4C_F\right)T_F n_f \\
\beta_2 &= \frac{2857}{54}C_A^3 + 2T_F n_f\left(C_F^2 - \frac{205}{18}C_A C_F - \frac{1415}{54}C_A^2\right) + 4T_F^2 n_f^2\left(\frac{11}{9}C_F + \frac{79}{54}C_A\right)
\end{aligned}$$

Here, $n_f = 5$, since the top quark has been integrated out in our theory. The used cusp anomalous dimensions coefficients for quarks are:

$$\begin{aligned}
\Gamma_{q0} &= 4C_F \\
\Gamma_{q1} &= 4C_F\left(\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right)
\end{aligned} \tag{28}$$

and for gluons:

$$\begin{aligned}
\Gamma_{g0} &= 4C_A \\
\Gamma_{g1} &= 4C_A\left(\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right)
\end{aligned} \tag{29}$$

The non-cusp anomalous dimensions for the quark hard function are:

$$\begin{aligned}\gamma_{qH0} &= -6C_F \\ \gamma_{qH1} &= -C_F\left(\left(\frac{82}{9}-52\zeta_3\right)C_A+\left(3-4\pi^2+48\zeta_3\right)C_F+\left(\frac{65}{9}+\pi^2\right)\beta_0\right)\end{aligned} \tag{30}$$

and for the gluon hard function:

$$\begin{aligned}\gamma_{gH0} &= -2\beta_0 \\ \gamma_{gH1} &= \left(-\frac{118}{9}+4\zeta_3\right)C_A^2+\left(-\frac{38}{9}+\frac{\pi^2}{3}\right)C_A\beta_0-2\beta_1\end{aligned} \tag{31}$$

The quark jet function non-cusp anomalous dimensions are:

$$\begin{aligned}\gamma_{qJ0} &= 6C_F \\ \gamma_{qJ1} &= C_F\left(\left(\frac{146}{9}-80\zeta_3\right)C_A+\left(3-4\pi^2+48\zeta_3\right)C_F+\left(\frac{121}{9}+\frac{2\pi^2}{3}\right)\beta_0\right)\end{aligned} \tag{32}$$

and for the gluon jet function:

$$\begin{aligned}\gamma_{gJ0} &= 2\beta_0 \\ \gamma_{gJ1} &= \left(\frac{182}{9}-32\zeta_3\right)C_A^2+\left(\frac{94}{9}-\frac{2\pi^2}{3}\right)C_A\beta_0+2\beta_1\end{aligned} \tag{33}$$

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