DESY Summer Students Programme
Report

Studying the process $e^+e^- \to t\,\bar{t}$ at LO, NLO and with POWHEG matching using WHIZARD

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Abstract:
In this report, I compare predictions at LO, NLO, NLO-inclusive and with the POWHEG method of event generation. I discuss these results to better understand how these methods work and what their differences are. I will also briefly discuss the relationship between NLO-inclusive results and the matrix element method at NLO.

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1 Introduction

My project was concerned with the simulation of top pair production in electron-positron scattering. I performed collider physics simulations. My main task was to understand what the matrix element method (MEM) at next-to-leading order(NLO) [1, 2] is and to generate unweighted and weighted events using the programs WHIZARD [4] and Rivet [3] to plot distributions of various observables.

WHIZARD is a program for efficient calculations of multi-particle scattering cross sections and simulated event samples. It is operated by the SINDARIN language. One of the tools that WHIZARD uses is OpenLoops [5], which is an implementation of the Open Loop algorithm [5]. WHIZARD uses it for fast numerical evaluation of loop matrix elements. The Rivet program, which I use for analysis of events generated by WHIZARD, is a multi-purpose data analysis C++ framework. I use it to obtain histograms of the the events generated by WHIZARD. It also allows to automatically generate plots of the filled histograms.

My study is focused on top-quark pair production. The top-quark is the heaviest quark, $m_t = 173.2\,\text{GeV}$ considered one of the most important particles of the Standard Model. Top quark events can form a significant contribution in Higgs boson searches or new physics studies. Currently they are abundantly produced at the LHC. Top pair production could be studied in even more detail at a possible future high energy lepton collider such as ILC or CLIC. One mainly requires a sufficiently high center-of-mass energy (it equals $\sqrt{s} = 500\,\text{GeV}$ in my pseudo-experiments). To date, the greatest lepton accelerator (Large Electron-Positron Collider at CERN) could collide leptons at an energy of 209 GeV which is far below the threshold $2m_t = 346.4\,\text{GeV}$. Future lepton colliders will be able to produce top pairs not only at threshold but also above in the continuum.

2 Theory

The standard approach to compile scattering cross sections is perturbation theory. Perturbation theory is applicable if the problem can be expanded in a small parameter. This parameter is α_S for fixed-order QCD computations.

Summing up the higher order terms, we can obtain more and more exact solutions. The terms that I use in my project are leading-order (LO) and next-to-leading order (NLO). In a LO calculation we only keep the diagrams necessary to obtain the desired final state. In most cases these are only tree-level diagrams. The total cross section for such an event is equal to:

$$\sigma = \int B d\Phi_n, \tag{1}$$

where B is defined as $|M_n^{(0)}|^2$ and $M_n^{(0)}$ is the lowest order matrix element for the particles of interest. Φ_n stands for the phase space built of final-state particles. If we want to obtain more precise results we need to consider NLO. It can be obtained by including virtual and real corrections (virtual and real gluon emission). The total cross section is

$$\sigma = \int d\Phi_n \left((B + V) + \int d\Phi_{rad} R \right), \tag{2}$$

where $d\Phi_{rad}$ stands for radiation phase space which contains the additional gluon momentum. V stands for virtual corrections, $V = 2Re|M_n^{(0)*} M_n^{(1)}|$. It represents one-loop Born interference term. R is a tree level matrix element with one additional parton and

$R = |M_{n+1}^{(0)}|^2$, called the real contribution. POWHEG is another method used in this project. It allows to supply NLO predictions with leading log accuracy, by resumming ratios of $\frac{R}{B}$. We can define a differential cross section:

$$\sigma = \int \left(B + V + \int d\Phi_{rad} R \right) d\Phi_n \left(\Delta(p_t^{min}) + d\Phi_{rad} \Delta(k_T(\Phi_{rad})) \frac{R(\Phi_{rad})}{B} \right) , \qquad (3)$$

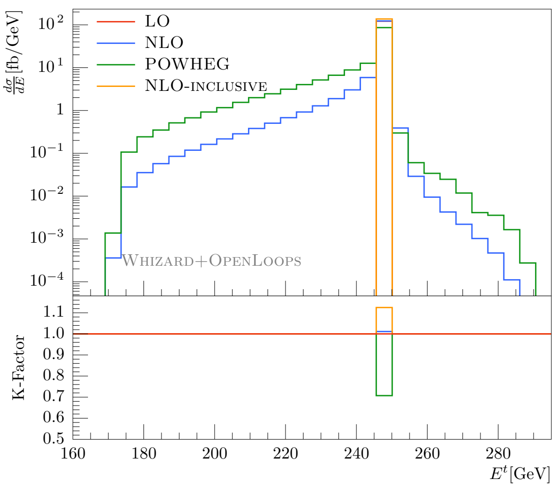
where Φ_{rad} is the radiation phase space and $\Delta(p_T)$ stands for Sudakov form factor, which describes the probability that no emissions occurs between p_T and $p_{T_{max}}$. It can be defined by

$$\Delta(p_T) = \exp \left[- \int d\Phi_{rad} \frac{R(\Phi_{rad})}{B} \theta \left(k_T^2(\Phi_{rad}) - p_T^2 \right) \right] . \qquad (4)$$

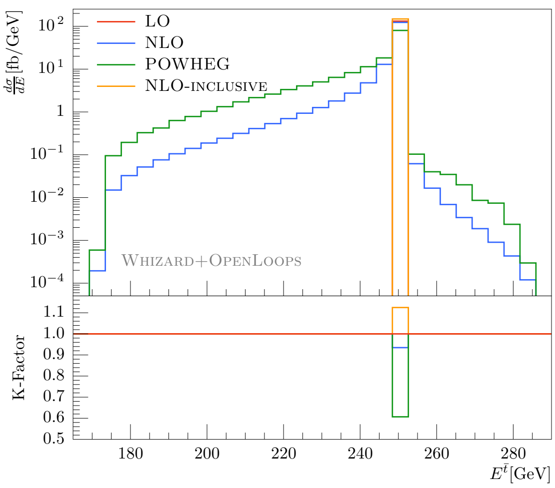
A benefit of the POWHEG method is that negative events only occurs in regions where perturbation theory is not applicable.
We modified POWHEG event generation in WHIZARD to optionally skip gluon emissions, corresponding to setting the second bracket in eq.(3) to one. We call this "NLO-inclusive", as the weights correspond to the NLO cross section, but projected on the Born phase space $d\Phi_n$.

3 Study of top pair production

As the top quark is a short-lived unstable particle, experimentally we do observe only jets (and leptons) from its decay.



(a) t quark



(b) $\bar{t}$ quark

Figure 1: Energy distributions of t and $\bar{t}$ quarks.

As expected, Figure 1 presents the distributions of energy of t and $\bar{t}$ quarks. They are almost the same with little differences for events above 240 GeV. The differences are likely due to insufficient statistics. I can also observe a big peak at 250 GeV. Furthermore, histograms for LO and NLO-inclusive consist only of the peak at this position, as there is no gluon produced for these event. The only value of energy of all these events is 250 GeV because of conservation of energy:

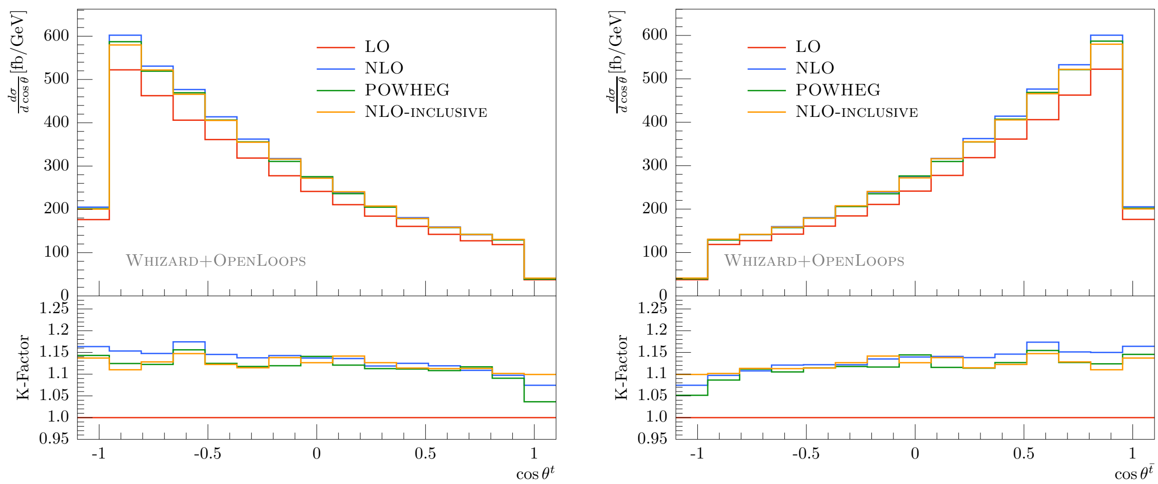
$$p_t = (E_t, \vec{p_t}); \quad p_{\bar{t}} = (E_{\bar{t}}, \vec{p_{\bar{t}}}) = (E_t, -\vec{p_t}),$$

$$p_t + p_{\bar{t}} = (\sqrt{s}, \vec{0}) \,,$$

$$\begin{cases} E_t^2 - \vec{p_t}^2 = m_t \\ E_{\bar{t}}^2 - \vec{p_{\bar{t}}}^2 = m_t \end{cases} \Rightarrow E_t = E_{\bar{t}} = \frac{\sqrt{s}}{2}.$$

For NLO and POWHEG there is an additional gluon momentum in these equation and that is why the energy of the top quark momentum differs from $\frac{\sqrt{s}}{2}$. Since the number of LO events equals 0 for all energy values except one bin, the K-factor is different from 1 only for the peak bin.

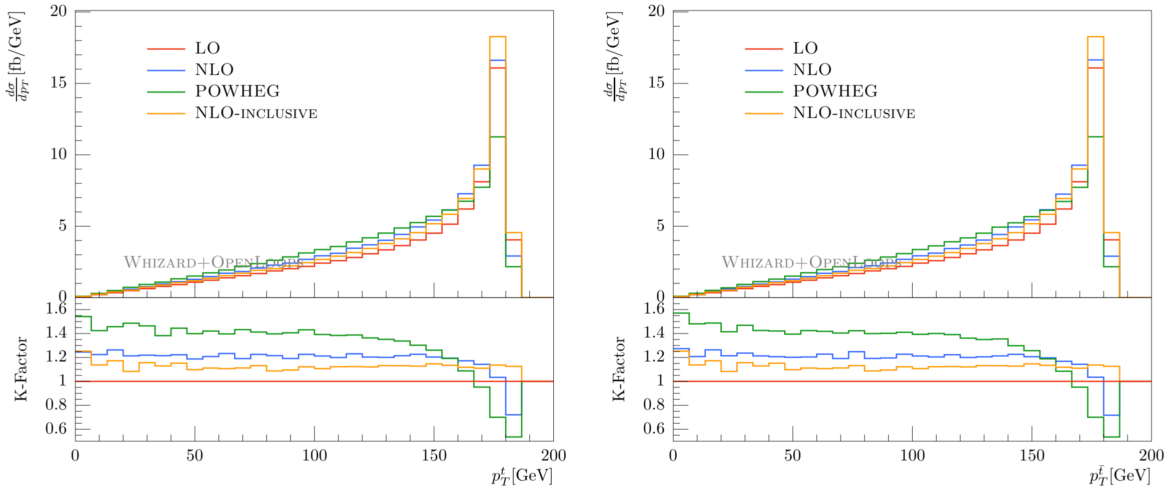
At first glance, the plots in figures 2(a) and 2(b) are mirrored. However, the peaks in both pictures are at opposite positions (for t quark at -0.73 and in the 0.87 for $\bar{t}$ quark), which means that the quark pairs are produced with high probability at these angles towards beam axis. For $\cos\theta = \pm 1$ the probability is smallest. All histograms are alike thus K-factors for all methods are approximately equal to 1 (between 1 and 1.2). Total cross section for LO is the least, thus the probability of this event is the smallest for every bin. In these graphs I can also observe a forward-backward asymmetry. This phenomenon originates from an interference of the vector and axial-vector $t\bar{t}$ production vertices at tree level in the γ/Z exchange.



(a) t quark

(b) $\bar{t}$ quark

Figure 2: $\cos\theta$ distributions of t and $\bar{t}$ quarks.



(a) t quark   (b) $\bar{t}$ quark

Figure 3: p_T distributions of t and $\bar{t}$ quarks.

In Figure 3 I can see two equivalent plots, which means that the produced quarks have the same transverse momentum. There is one peak at 180 GeV. Using this value in $E = \sqrt{p^2 + m^2}$, we obtain approximately the energy peak in figures 1(a) and 1(b). The POWHEG histograms are the most uniform of all four. This is due to the damping of soft emissions by the Sudakov exponent. The NLO-inclusive is the highest of all four, overall caused by the positive K-factor and Born phase space. The K-factor differs from 1 for all histograms. It takes values between 0.55 and 1.55. For values of momenta less than 153 GeV, the LO histogram has the smallest total cross section.

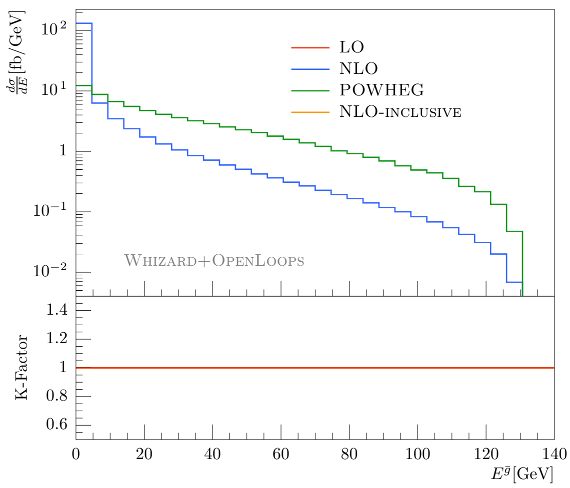


Figure 4: Gluon energy distribution.

In Fig. 4 I show the gluon energy distribution. There, only NLO and POWHEG contribute. The energy of gluons is on average lower than the energies of quarks. For NLO graphs there are more events for low energies, unlike POWHEG for which there are more events than NLO for higher energies.

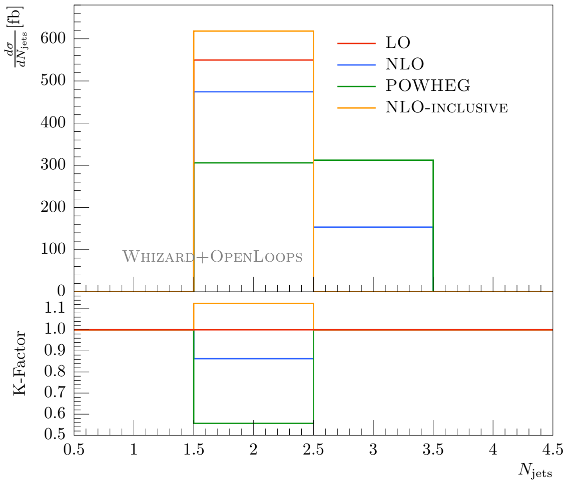
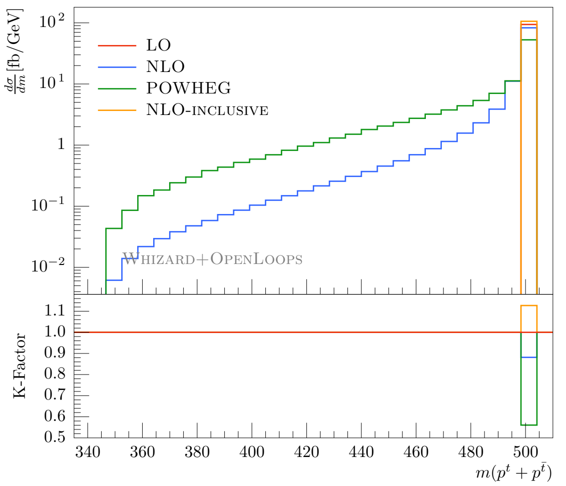


Figure 5: Distribution of number of jets.

Fig. 5 presents the distribution of $Njets$, the number of jets. For LO and NLO-inclusive events there are always two jets (for t and $\bar{t}$ quark), for NLO and POWHEG there can be 2 or 3 jets (one additional gluon jet). The picture shows that there are more observable 3-jet events for POWHEG than for NLO.



(a) top quark pair

Figure 6: Distribution of sum of $t\bar{t}$ momenta and leading and 2nd leading jet momenta.

Fig. 6 depicts the invariant mass of the top quark pair. We can observe a peak at 500 GeV. I can also see that all events for LO and NLO-inclusive are in one bin like in the other energy histograms. The CM-energy for this process was 500 GeV and that is why there is a peak at this value of energy of $t\bar{t}$.

4 Summary and Outlook

I presented different methods based on perturbation theory. My project as well as this report are mainly focused on studies of the obtained data. Finally, I want to mention that the NLO-inclusive method opens up new possibilities. Firstly, as the events are positive, unlike standard NLO events, one can generate unweighted event samples. This improves the statistical quality of the generated events.

References

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