Tau lepton reconstruction using the impact parameter
DESY Summer Student Report

Eline Maaike de Weerd
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School of Physics & Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, UK
DESY, Notkestraße 85, 22607 Hamburg, Germany

Abstract

The International Linear Collider is a future lepton collider, with one of its physics goals being detailed studies of the properties of the Higgs boson and related particles. For precision measurements, good reconstruction algorithms are essential. This project focuses on the hadronic decay of tau leptons. Studies of the Higgs self-coupling is one of the cases for which hadronic tau decay is relevant. In order to do more precise studies of the Higgs self-coupling, an algorithm to reconstruct these hadronic tau decays is needed, as so far reconstruction methods are optimised only for electrons and muons. This study describes a first attempt at the development of such a hadronic tau reconstruction algorithm, using the tau decay lifetime and the impact parameter. It is shown that the philosophy behind the algorithm seems promising. However, there is no accurate way to determine the actual tau lifetime as of now, and hence the algorithm is not yet applicable to more realistic situations. It is found that being able to accurately estimate the tau lifetime is essential for this reconstruction algorithm.

Supervisor: Dr. Shin-ichi Kawada

Contents

1 Introduction

1.1 The International Linear Collider

The International Linear Collider (ILC) is a future electron-positron (lepton) collider, of 200-500 GeV (extendable to 1 TeV) centre-of-mass energy. The ILC has been designed as a 'precision machine'; to carry out more detailed studies of the Higgs boson and particles related to it, and thus give insight in various properties of the Higgs field as postulated in the Standard Model. A schematic overview of the ILC is shown in figure 1. [1]

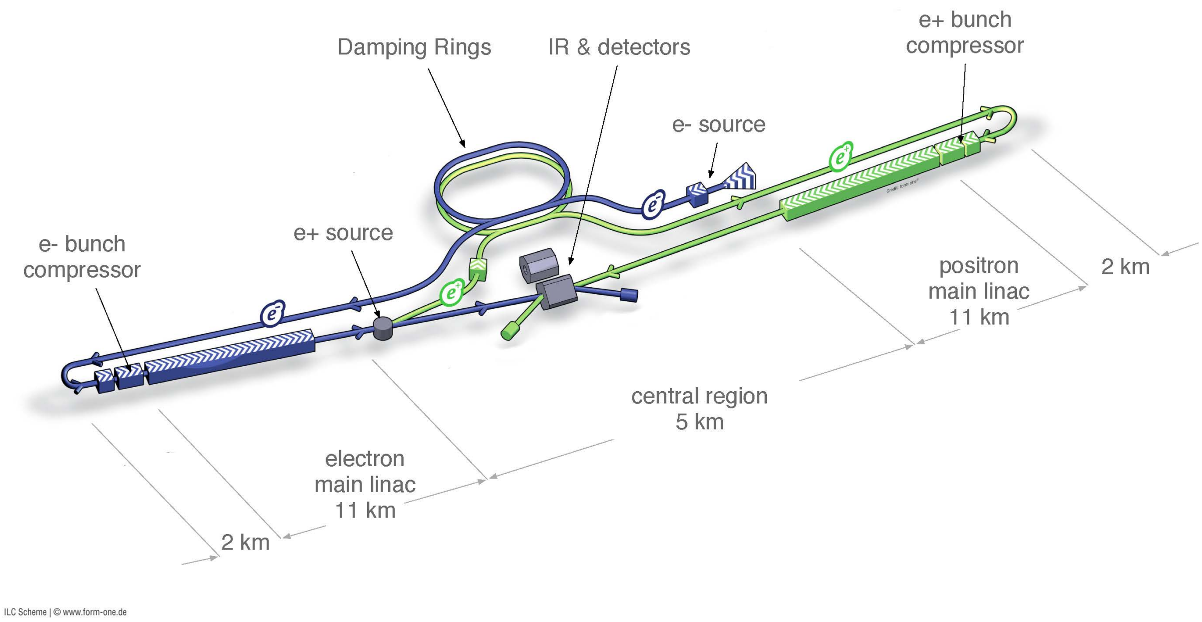

Figure 1: A schematic overview of the ILC [2].

1.2 Motivation for Tau Reconstruction

The tau lepton is, with a mass of 1776.86 MeV, the heaviest lepton, and the only lepton that can decay hadronically. Overall, the tau lepton decays are complicated. The tau has a mean lifetime of $2.9 \times 10^{-13}s$. [3]

In previous studies, the tau reconstruction was treated in an inclusive manner [4] [5]. Furthermore, Higgs self-coupling at ILC has been studied, and in $e^+e^- \to Zhh$ where $Z \to l^+l^-$, an isolated lepton finder has been optimised for electrons and muons, but not for (hadronic) tau decays [6]. Hence, an algorithm to find hadronic tau decays can improve selection and the physics results. In this study, a tau reconstruction method using its lifetime and impact parameter is tried.

2 Method

2.1 Calculation

In order to be able to calculate anything at all, a formula for p_τ in terms of only known quantities is needed. Though this method is more generally applicable, for the purpose of this project only the hadronic tau decay is considered ($\tau \rightarrow X\nu$, X denoting any hadron), more specifically $\tau \rightarrow \pi\nu$. Using simple conservation of four-momentum, $E_\tau^2 = M_\tau^2 - |\vec{p_\tau}|^2 = (E_X + E_\nu)^2 - (|\vec{p_X} + \vec{p_\nu}|)^2$ holds. Rearranging this gives $M_\tau^2 = 2p_\perp^2 + 2E_X\sqrt{p_{\nu,L}^2 + p_\perp^2} - 2p_{X,L}p_{\nu,L}$, where $\perp$ and L denote the directions perpendicular and parallel to the tau momentum direction respectively, and $|\vec{p}| = p$ for brevity.

This expression contains only known things and $p_{\nu,L}$, which is hence determined. Now introducing the decay length of the tau l and the impact parameter x, which is the distance of closest approach of the interaction point (primary vertex) and the trajectory of X. Expressing $p_{X,\perp}$ and $p_{X,L}$ in terms of these quantities gives $p_{X,\perp} = p_\perp = E_X\frac{x}{l}$ and $p_{X,L} = E_X\sqrt{1-(\frac{x}{l})^2}$. Using this, $p_{\nu,L}$ can be expressed as a function of the decay length l and known quantities. Also, since $p_\tau = p_{\nu,L} + p_{X,L}$, it follows that p_τ is a function of l and known quantities.

The decay length l is related to the tau decay time in the laboratory frame, which is $l = vt'$, where $v = \beta c$, and the decay time t' is related to the proper decay time t by $t' = \gamma t$, from which follows that $l = \gamma\beta ct$. Using that $\gamma\beta = p_\tau/M_\tau$, the expression for the decay length can be rewritten as $l = \dfrac{tcp_\tau}{M_\tau}$. The lifetime of tau, t, is a known exponential distribution with an average lifetime $t_{av} = 2.9 \times 10^{-19}s$. Hence, an expression for p_τ in terms of only known quantities can be found. The full calculation is omitted here, as it is rather lengthy. The resulting expression is $p_\tau = \pm\sqrt{\left(\dfrac{E_X x^2}{t^2c^2} + \dfrac{M_\tau^2}{4E_X}\right)^2 + E_X^2 + 2\left(\dfrac{E_X x^2}{t^2c^2} - \dfrac{M_\tau^2}{4E_X}\right)E_X}$.

The verification of this formula is done by starting from the rest frame of the tau. In the rest frame, the neutrino and the hadron (X) are emitted back-to-back. Since the tau is at rest, its energy is simply $E_\tau = M_\tau$, and conservation of linear momentum implies that $\vec{p}_X + \vec{p}_\nu = 0$, or $|\vec{p}_x| = |\vec{p}_\nu|$ Using a simple planar coordinate system, the momentum $\vec{p}_X$ can be decomposed into lontigudinal and perpendicular components, $p_{X,L} = p_X\cos\theta$ and $p_{X,\perp} = p_X\sin\theta$, θ being the angle between the track of X and the L direction.

Now, under the assumption that the mass of X is negligibly small compared to the tau mass (this is of course already valid for the neutrino mass), it can be said that $p_X = E_X$ and $p_\nu = E_\nu$. Energy conservation then implies that $E_X = E_\nu = \dfrac{M_\tau}{2}$, and the components of the hadron momentum become $p_{X,L} = \frac{M_\tau}{2}\cos\theta$ and $p_{X,\perp} = \frac{M_\tau}{2}\sin\theta$.

Then, the entire system is boosted into the lab frame, in which tau is moving. Using superscript asterix to denote the value in the lab frame, the Lorentz factors are found to be $\gamma = \frac{E_\tau^*}{M_\tau}$ and $\gamma\beta = \frac{p_\tau^*}{M_\tau}$. Hence, the hadron energy transforms as $E_X^* = \dfrac{E_\tau^*}{2} + \dfrac{p_\tau^*}{2}\cos\theta$. Using the geometrical property of the similarity of triangles, the ratio $\frac{x}{ct}$ is found to be $\dfrac{x}{ct} = \dfrac{p_\tau^* \cdot p_{X,\perp}}{M_\tau \cdot E_X^*}$. Using these expressions, the calculated tau momentum corresponding to any 'true' tau momentum and angle can be found. This is done for both constant angle, varied true tau momentum (illustrated in figure 2) as well as a fixed true momentum and varied angle (shown in figure 3).

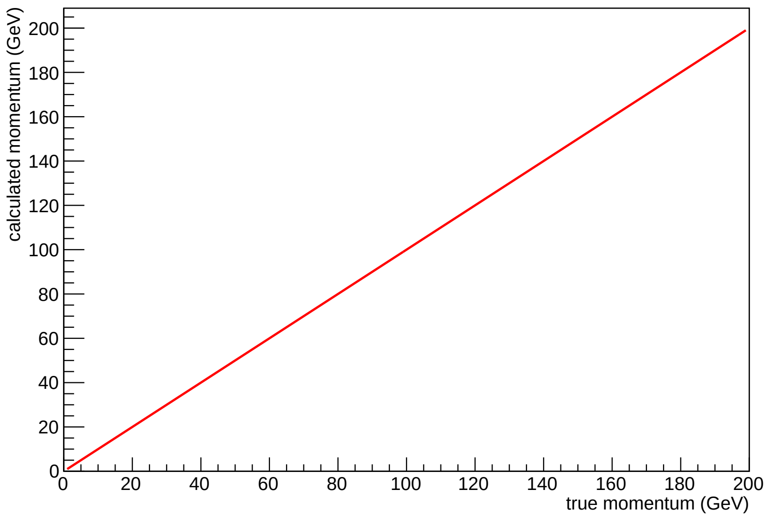

Figure 2: True versus calculated momentum for an input angle of 54 degrees.

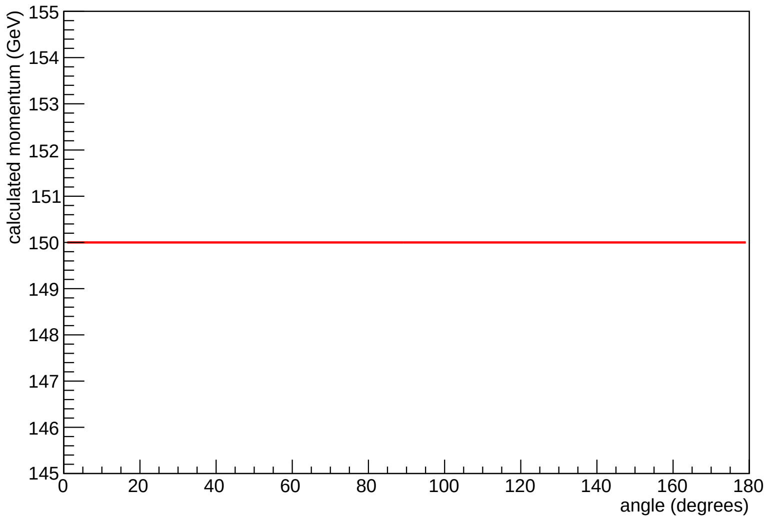

Figure 3: Calculated tau momentum, from a true tau momentum of 150 GeV.

2.2 Processor

After the formula is verified in the simple theoretical case, it is implemented in code. More specifically, a Marlin processor is developed to reconstruct the tau momentum from known (generator-level) information [7]. Monte Carlo (MC) samples simulation e^+e^- collisions with a centre-of-mass energy of 250 GeV are used, and only the $\tau \to \pi\nu$ process is considered. Information about the tracks of the particles is readily available, and using the available track parameters, the impact parameter x is calculated. In the framework, various track parameters are available, three of which are used in this studies. In the xy-plane, d_0 denotes the shortest distance between the reference point (see figure 4). Track parameter ϕ_0 is the angle between the x-direction and the direction of the momentum of the particle. In a coordinate system where the track of the particle is a straight line, and the track is parametrised by the s direction (distance along the track) and the z direction, the track parameter z_0 denotes the distance of closest approach between the reference point and the track (as shown in figure 5). The other track parameters are used to denote the curvature and slope of the track, all together determining the direction and curvature of the track.

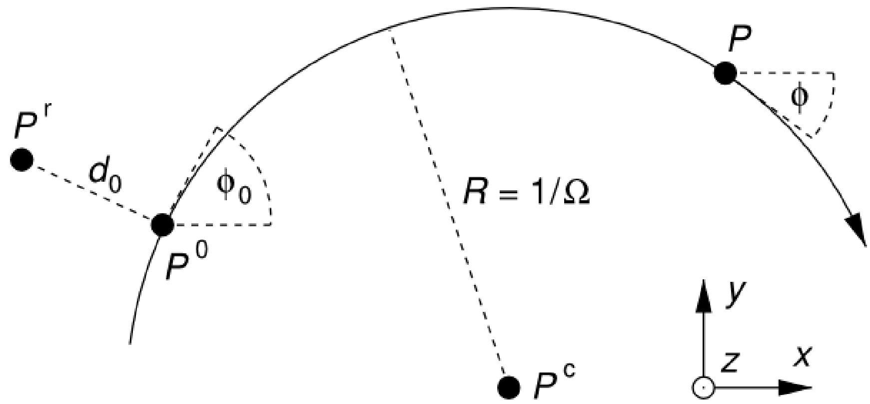

Figure 4: The projection of the track in the x-y plane. The track parameter d_0 denotes the distance of closest approach between the track and the reference point. [8]

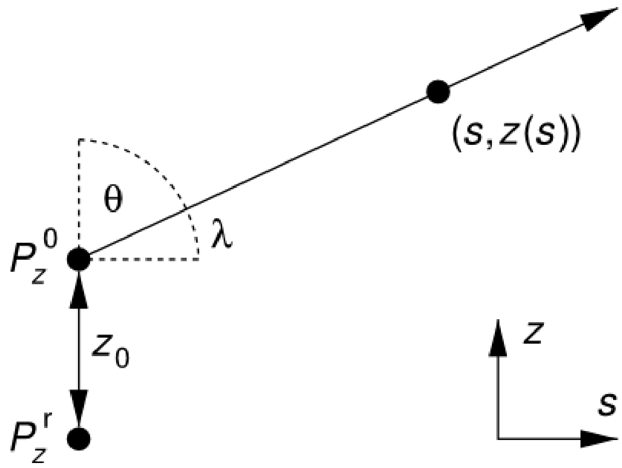

Figure 5: The projection of the track in the coordinate frame where the track is a straight line. The track parameter z_0 denotes the distance of closest approach between the track and the reference point in this projection. [8]

Track parameter d_0 is decomposed into x and y directions using the angle ϕ_0 as $\vec{d_0} = -d_0 \sin\phi_0 \vec{\hat{x}} + d_0 \cos\phi_0 \vec{\hat{y}}$. In three dimensions, the vector pointing to the point on the track of the pion denoted by d_0 and z_0 in the xy-plane and sz-plane respectively is $\vec{w} = -d_0 \sin\phi_0 \vec{\hat{x}} + d_0 \cos\phi_0 \vec{\hat{y}} + z_0 \vec{\hat{z}}$. To find the shortest distance to the track of the pion in three dimensions (i.e. the impact parameter x), the vector $\vec{w}$, which is known to be on the pion track, is dotted into the normalised momentum vector of the pion; $\frac{\vec{w} \cdot \vec{p}}{|\vec{p}|}$, which is the projection of the w-vector onto the direction of the track (calling this $\vec{v}$), thus reducing the problem to a two-dimensional one. The impact parameter x is then trivially calculated using Pythagoras; $x = \sqrt{\vec{w} \cdot \vec{w} - \vec{v} \cdot \vec{v}}$. [8]

The tau mass and constant c are well-known, and the energy of the pion is readily obtained from the input data. The remaining unknown is the tau lifetime, which is known to be an exponential distribution. In this project various values are taken for this tau lifetime, in order to study its effect on the accuracy of the reconstruction. As a simple start the average tau lifetime of $2.9 \times 10^{-13} s$ is taken, and as a second step the actual lifetime is calculated using $t = \frac{lM_\tau}{cp_\tau}$, which is a more accurate estimation, but unfortunately using information about the tau momentum, which is known in this case but not under realistic conditions. As a final step, the lifetime calculated using tau momentum information is plotted against the impact parameter x, and a simple linear fit is done to obtain a rough relationship between the lifetime and impact parameter. This linear function is implemented in the processor, to calculate the lifetime corresponding to any given value of the impact parameter.

3 Results

As a first step, a plot is made using the approximation that the tau lifetime is constant and always takes on its average value of $2.9 \times 10^{-13} s$. As mentioned in section 2.2, 250 GeV MC samples are used in this analysis. In figure 6, it is shown that while the reconstruction is not hugely inaccurate for low momentum, it is seen to be very poor for momenta around 125 GeV; where a sharp peak in the true momentum values is observed, whereas the calculated values are monotonically decreasing.

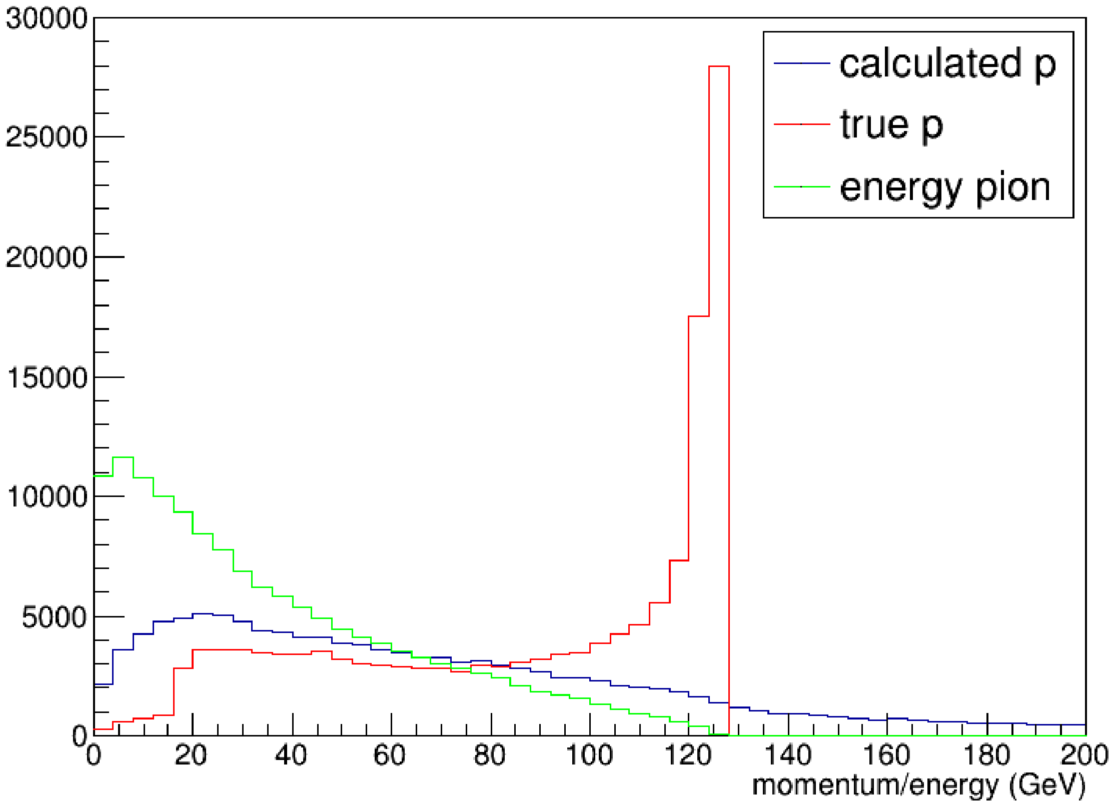

Figure 6: The calculated tau momentum (blue line), true tau momentum (red line) and the energy of the pion (green line) for the assumption that the tau lifetime is constant using $2.9 \times 10^{-13}s$.

In order to verify the main idea of this reconstruction algorithm, in the next step the true lifetime of the tau is calculated, following the formula mentioned in section 2.2, $t = \frac{lM_\tau}{cp_\tau}$. The results of this analysis are displayed in figure 7, in which a clear overlap of the true and calculated tau momentum histograms is visible. The similarity in shape of the two histograms indicates possible correctness of the main philosophy of this type of reconstruction.

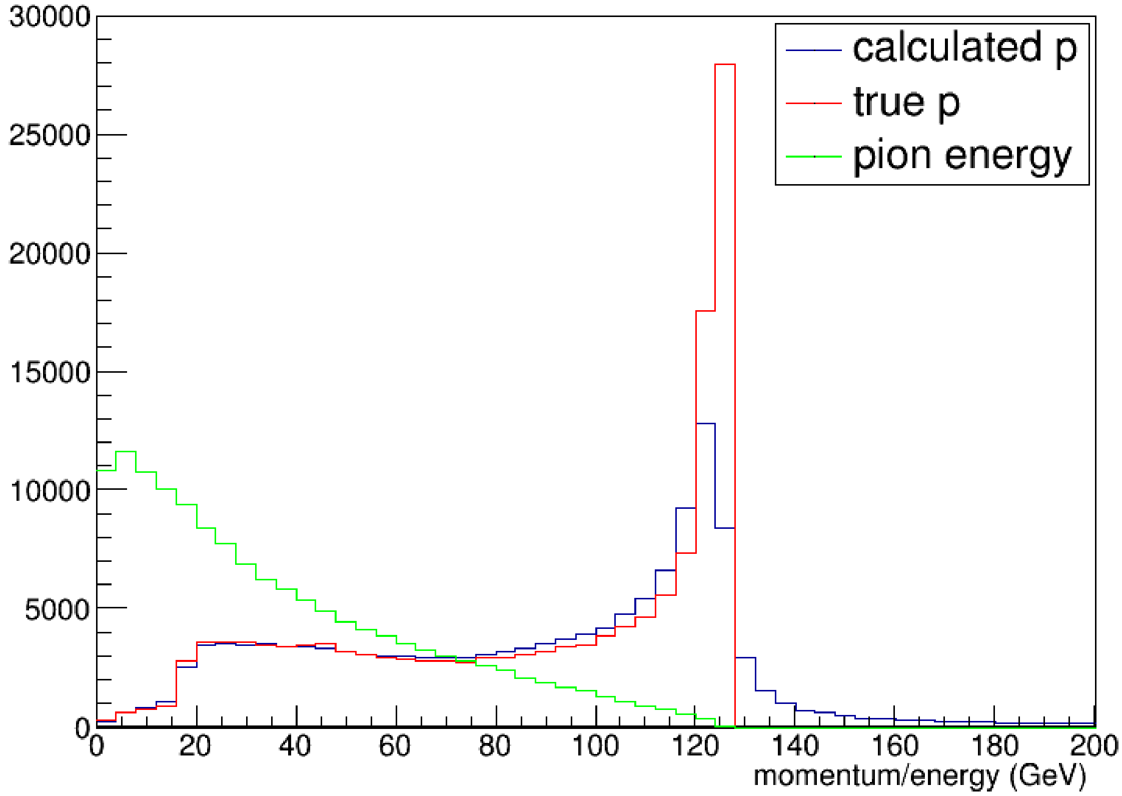

Figure 7: The calculated tau momentum (blue line), true tau momentum (red line) and pion energy (green line) for the tau lifetime calculated using the true tau momentum.

Next, a logarithmic plot of the impact parameter x against the lifetime calculated using the true tau momentum is made (shown in figure 8). A simple linear fit is done using ROOT [9], which yields the equation $\log_{10} t = 0.65514 \cdot \log_{10} x - 12.0136$, inverting the logarithm gives an expression to calculate a rough estimate of the lifetime corresponding to a particular value of the impact parameter, $t = 10^{0.65514 \cdot \log_{10} x - 12.0136}$.

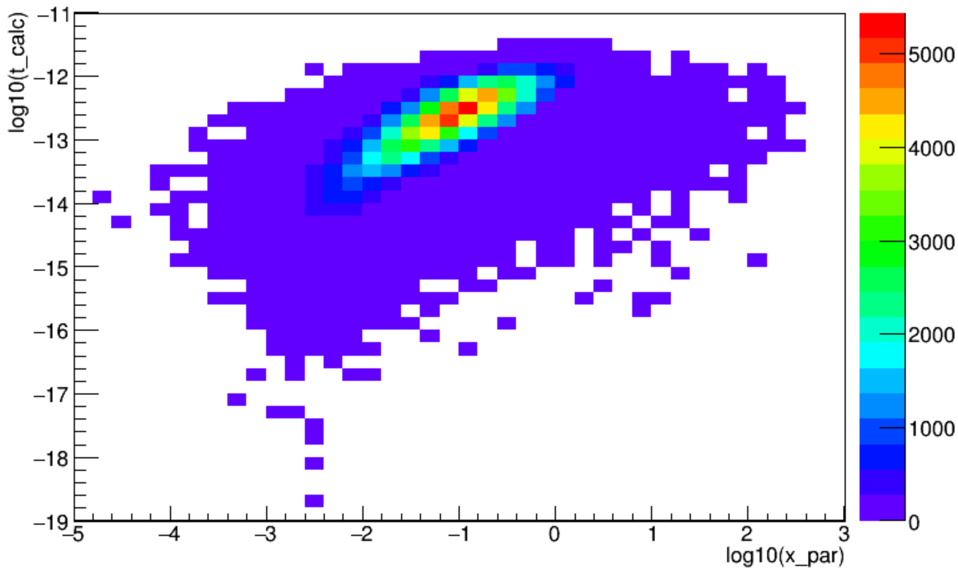

Figure 8: Logarithmic plot of impact parameter x against calculated lifetime of the tau.

Using this rough estimation to reconstruct the tau momentum, the results as shown in figure 9. This method of reconstruction shows a slightly more accurate reconstruction for low momentum compared with 6, but again gives very poor results for the momentum around 125 GeV.

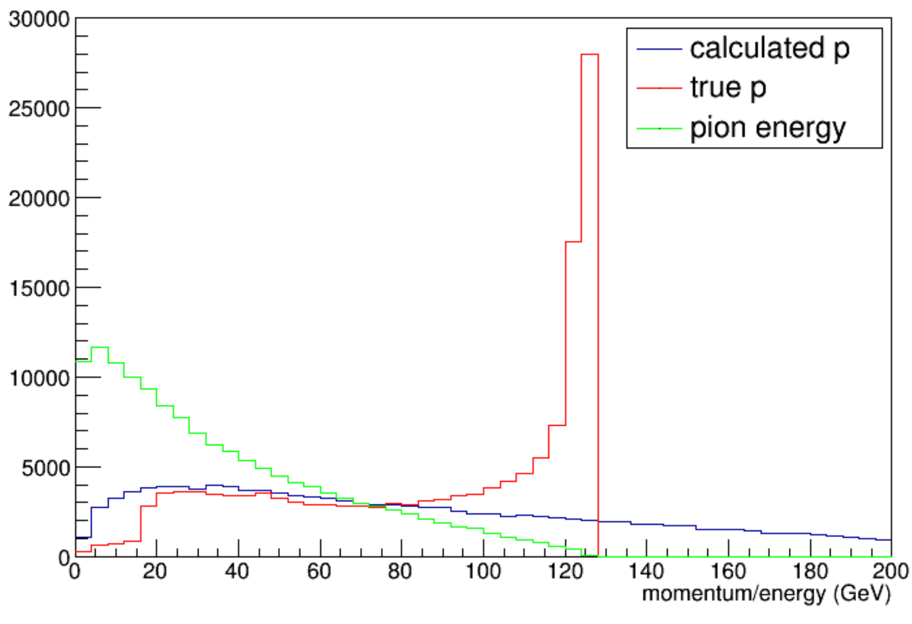

Figure 9: Calculated tau momentum (blue line), true tau momentum (red line) and pion energy (green) for a tau lifetime obtained from a linear fit on the impact parameter-lifetime plot.

4 Conclusion

4.1 Conclusion

From the analyses performed in this project, it seems that the main idea of reconstructing the tau momentum using this method is promising, since the results when using the true value for the lifetime are good. However, so far, no accurate way of determining a value for the tau lifetime has been found, and hence the reconstruction algorithm is not yet applicable to realistic cases. This study only dealt with a simplified case of reality, in which there is assumed to be no effect of a magnetic field, meaning the particles all move in straight lines.

4.2 Future Studies

Given the current endpoint of the project, further studies are needed before this algorithm can be implemented and used for reconstruction in realistic cases. As mentioned in section 4.1, improvement could be made should an accurate way of determining the lifetime of the tau be developed. Furthermore, taking into account the curvature of the tracks could lead to a higher level of precision.

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