A study of double parton distributions within the framework of chiral perturbation theory

Thanat Sangkhakrit

Suranaree University of Technology, Thailand

Supervisor : Dr.Markus Diehl

DESY Theory Group

August 22, 2015

Abstract

In QCD, the coupling constant becomes large in the low energy regime, such that the usual perturbative approach using quark and gluon fields is not effective. In that case one needs to resort to other approaches – one of which is Chiral Perturbation Theory (ChPT). This is an effective theory of QCD applicable at low energies, constructed in accordance with the approximate chiral symmetry of QCD. In this work, we make use of ChPT to investigate the pion matrix elements of double-parton distribution (DPD)-type operators. DPDs are an important ingredient to calculate double parton scattering, the process in which two distinct hard parton interactions occur simultaneously in a hadron-hadron collision. This process can be relevant as a background and signal process at the LHC.

Contents

1 Introduction

In this work, we investigate the use of Chiral Perturbation Theory (ChPT), an effective theory of QCD applicable at low energies, to calculate the behavior of DPD-type matrix elements in the pion.

1.1 Main Goals of this work

- Use ChPT to calculate DPD-type operator(s) matrix elements of pion(s) in 4 cases including scalar, pseudoscalar, vector and axial-vector currents to describe double parton scattering in the processes involving pion(s).
- Explain each matrix elements through the corresponding Feynman diagrams, each diagram reveals the momentum conservation in which the corresponding external fields to each type of currents have to be satisfied.

1.2 Convention and definition

These formulas are borrowed from [3, 4]. The Fourier transformation are given by

$$f(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} f(\tilde{k}) \text{ and } \tilde{f}(k) = \int d^4x e^{ik\cdot x} f(x) \tag{1}$$

The 4-dimensional Dirac delta function is

$$\delta^{(4)}(k) = \frac{1}{(2\pi)^4} \int d^4x e^{ik\cdot x} \tag{2}$$

The free field expansion is given by

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(a_{\vec{p}} e^{-ip\cdot x} + a^\dagger_{\vec{p}} e^{ip\cdot x} \right) = \phi^+(x) + \phi^-(x) \tag{3}$$

where we defined

$$\phi^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a_{\vec{p}} e^{-ip\cdot x} \tag{4}$$

$$\phi^-(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a^\dagger_{\vec{p}} e^{ip\cdot x} \tag{5}$$

The partial derivative for spacetime coordinates is given by

$$\partial_\mu = \left(\frac{\partial}{\partial x^0}, \vec{\nabla} \right), \partial^\mu = \left(\frac{\partial}{\partial x^0}, -\vec{\nabla} \right) \tag{6}$$

From the previous equation, we obtain $\partial^\mu e^{ip\cdot x} = ip^\mu e^{ip\cdot x}$. The creation and annihilation operators is given through

$$|p_a\rangle = \sqrt{2E_{\vec{p_a}}} a^\dagger_{\vec{p_a}} |0\rangle, \langle p_a| = \langle 0| a_{\vec{p_a}} \sqrt{2E_{\vec{p_a}}} \tag{7}$$

The commutation between creation and annihilation operators of Klein-Gordon field is given by

$$\left[a^a_{\vec{k}}, a^{b\dagger}_{\vec{p}}\right] = (2\pi)^3\delta^{(3)}(\vec{k} - \vec{p})\delta^{ab} \tag{8}$$

The Feynman propagator is defined through

$$\langle 0| T(\phi(x)\phi(y)) |0\rangle = D_F(x - y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)} \tag{9}$$

The time order product of n-fields (where $\phi_a = \phi(x_a)$) can be resolved into a combination between normal order of those fields and all possible contractions between them via Wick's Theorem,

$$T(\phi_1\phi_2\phi_3...\phi_n) = N(\phi_1\phi_2\phi_3...\phi_n) + \text{all possible contractions} \tag{10}$$

2 Basic elements of this project

2.1 Double parton scattering

Double parton scattering is the process in which two distinct hard parton-parton interactions occur simultaneously in a hadron-hadron collision[1]. It can be an important background to rare single-scattering processes at the LHC, and is interesting to study in its own right as it reveals new information concerning the structure of the proton. An important quantity needed to calculate double parton scattering cross sections is the double parton distribution (DPD), which roughly speaking quantifies the number density of a pair of partons in the proton. Rather little is known about this quantity, and its value at any particular scale cannot be calculated using normal perturbative QCD. In analogy to the operators matrix elements, we can represent the double parton distributions for the proton-proton collisions as

$$F(x_1, x_2, \mathbf{y}; \mu) \sim \langle p| O_1(0; \mu)O_2(\mathbf{y}; \mu) |p\rangle \tag{11}$$

and

$$F(x; \mu) \sim \langle p| O(0; \mu) |p\rangle \tag{12}$$

In which the term μ stands for the renormalization scale. From the above definition, we are going to calculate the matrix elements in this form for the sub-processes including pion field(s). In which we will make use of chiral perturbation theory, the effective field theory of QCD using chiral symmetry as the approximate symmetry of the underlying theory to work out the matrix elements. The benefits from these matrix elements are

1. These matrix elements are equivalent to double parton distributions, so we can use them to explain the sub-processes occur between two pairs of partons inside the proton-proton collisions.

2. These matrix elements calculated from chiral perturbation theory would be a good compliment for a study of double parton distributions in the context of other QCD studies at low energies.

3. In the double parton scattering, double parton distributions is the one of important puzzles to work out the scattering cross-section for hadron-hadron collisions. So we can make use of those matrix elements to calculate the cross-section and other physical observables relate to proton-proton collisions.

4. The matrix elements that we're going to calculate are use for a practical training. Since the matrix elements for 2 vector currents between 2 protons would takes sometime to be finished.

2.2 The running of strong coupling constant

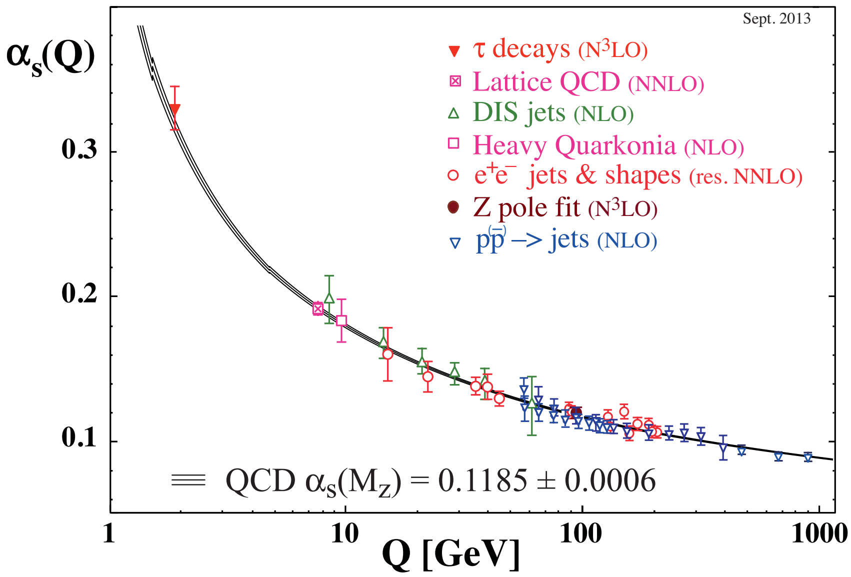


Figure 1: The plot of strong coupling constant α_s versus the respective energy scale Q [6].

The theory which describes the strong nuclear force is **Quantum Chromodynamics (QCD)**, containing 6 flavors of quark and 8 types of gluon as a fundamental objects. The interaction between quark fields can be represented by the exchange of gluon, the force carrier (or gauge field) of the strong interaction. One of the main interesting point in QCD is the coupling constant of the theory, It depends strongly on the energy scale. From the figure 1, the coupling constant is obviously low at the high energy scale so that one can applies the perturbation theory to work out the observables for such interesting processes without getting a trouble. In contrast, this coupling constant becomes large at a small energy scale so that the perturbation theory is no longer available, so the trouble has come at this point. This is the starting point of finding a new extension to explain the dynamics of a QCD at the low energies regime, later the effective field theory is taken into account to solve this problem systematically. In our present work, the effective field theory that we're going to use as a basic tool is so-called Chiral Perturbation Theory, It's going to be explained in the next section.

2.3 Chiral perturbation theory : ChPT

ChPT is an effective field theory used to describe QCD at the low energies using the approximate symmetry of QCD as a fundamental principle. ChPT was first proposed by Weinberg, later Gasser and Leutwyler formulated ChPT for the light mesons when external momenta are small relative to the chiral symmetry-breaking scale, $\Lambda \approx 1GeV$ [2]. The approximate symmetry I mentioned is so-called a **chiral symmetry**. Chiral symmetry (SU(N)$_L$ $\times$ SU(N)$_R$) is a symmetry which held by QCD in a limit that the quarks are massless. When the chiral symmetry is said to be spontaneous broken by the ground state, it gives rise to the Goldstone bosons. In ChPT, we want to construct the SU(N)$_L$ $\times$ SU(N)$_R$ Lagrangian describing the dynamics of the $N^2 - 1$ Goldstone bosons. In order to see how symmetry breaking generates Goldstone bosons, we start with the QCD Lagrangian by following the procedure from [5].

$$\mathcal{L} = \sum_{j} \bar{q}_j i \not{D} q_j + \sum_{jk} \bar{q}_j \left(m_q\right)_{jk} q_k \tag{13}$$

where $q_1 = u$, $q_2 = d$, and $q_3 = s$ respectively. The term m_q stands for a light quark mass. The light quark mass can be described via a matrix

$$\left(m_q\right)_{jk} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = diag\left[m_u, m_d, m_s\right] \tag{14}$$

The covariant derivative is defined through

$$D_\mu = \partial_\mu + i g_s A_\mu^b T^b, b = 1, 2, ..., 8 \tag{15}$$

The first term on the right side represents a regular partial derivative with respect to a spacetime coordinates. A term g_s stands for a strong coupling constant, A_μ^ν is a color gauge field, and T^b is a SU(3) color generator. In fact, the chiral symmetry is held for the Lagrangian involves the quark fields in the limit $m_q \to 0$ but It seems to be wrong for a chiral transformation of a quark bilinear inside a vacuum expectation value. Unfortunately the chiral symmetry $SU(3)_L \times SU(3)_R$ is said to be spontaneous broken to the vector transformation of a group $SU(3)_{L=R}$, technically says $SU(3)_V$, where the $3^2 - 1 = 8$ Goldstone bosons are therefore generated. In order to make a clearer concepts of the spontaneous symmetry breaking, a left handed and a right handed quark fields are defined through

$$q_b^L = \frac{1}{2}(1-\gamma_5)q_b = P^L q_b \tag{16}$$

$$q_b^R = \frac{1}{2}(1+\gamma_5)q_b = P^R q_b \tag{17}$$

Note that the transformation matrix P^L belongs to $\mathrm{SU}(3)_L$ while P^R belongs to $\mathrm{SU}(3)_R$ respectively. These 2 fields are connected with each other by a proper Lorentz transformation. In term of these left handed and right handed quark fields, the Lagrangian can be rewritten as

$$\mathcal{L} = \sum_j \bar{q}_j{}^L i\not{D} q_j^L + \sum_j \bar{q}_j{}^R i\not{D} q_j^R + \sum_{jk}\left[\bar{q}_j{}^L \left(m_q\right)_{jk} q_k^R + \bar{q}_j{}^R \left(m_q\right)_{jk} q_k^L\right] \tag{18}$$

In the limit $m_q \to 0$, the Lagrangian can be reduced to

$$\mathcal{L} = \sum_j \bar{q}_j{}^L i\not{D} q_j^L + \sum_j \bar{q}_j{}^R i\not{D} q_j^R \tag{19}$$

nsThis Lagrangian is invariant under a global chiral transformation. So that the left handed and the right handed quark fields are transform under $SU(3)_L$ and $SU(3)_R$ transformation respectively. The transformation of a quark bilinear under a chiral transformation can be seen through

$$\langle 0| \bar{q}_j{}^R q_k^L |0\rangle \to \langle 0| \bar{q}_i{}^R P_{ji}^{R*} P_{kl}^L q_l^L |0\rangle = v\left(P^L P^{R\dagger}\right)_{kj} \tag{20}$$

In case that $P^L = P^R$, the chiral transformation becomes a vector transformation of $SU(3)_V$ group providing vacuum invariant. Consequently the chiral symmetry group is spontaneous braking into a vector subgroup in order to provide the vacuum invariant of the quark bilinear under a transformation. In QCD, the Goldstone bosons can be interpreted through 3×3 unitary matrix. In other words, those Goldstone bosons are said to be arisen from the transformation of the quark bilinear away from its vacuum expectation value. For example, the quark bilinear in case of low-energy excitation can be written as

$$\bar{q}_j{}^R q_k^L \simeq v\Sigma_{kj} \tag{21}$$

with the transformation according to the chiral symmetry group

$$\Sigma \to P^L \Sigma P^{R\dagger} \tag{22}$$

The explicit form of Σ is

$$\Sigma = \exp\left(i\frac{2M}{f}\right) \tag{23}$$

where

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \tag{24}$$

The constant f appeared in the previous equation is a constant with a dimension of mass. The $SU(3)_V$ transformation implies that

$$M \to VMV^\dagger \tag{25}$$

Thus one says that π, K, and η are transform as an $SU(3)_V$ octet.

2.4 Application of effective Lagrangian : A Pion-Pion scattering

The Feynman Rules for a $\pi\pi$ interaction can be derived by using the effective Lagrangian from ChPT. To begin the derivation, we start with the effective Lagrangian satisfying $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry

$$\mathcal{L}_2 = \frac{F^2}{4} Tr[D_\mu U (D^\mu U)^\dagger] + \frac{F^2}{4} Tr[\chi U^\dagger + U\chi^\dagger] \tag{26}$$

The constant F is a pion decay constant in the chiral limit. The SU(2) matrix U contains the Goldstone bosons fields inside while the term χ is relates to the singlet scalar quark condensate through a constant B. Moreover the covariant derivative is defined through

$$D_\mu A \equiv \partial_\mu A + iAl_\mu - ir_\mu A \tag{27}$$

After we set the external fields equal to zero, the effective Lagrangian becomes

$$\mathcal{L}_2 = \frac{F^2}{4} Tr[\partial_\mu U (\partial^\mu U)^\dagger] + \frac{F^2}{4} Tr[\chi U^\dagger + U\chi^\dagger] \tag{28}$$

where

$$\chi = 2B\mathcal{M} = 2B \begin{pmatrix} \hat{m} & 0 \\ 0 & \hat{m} \end{pmatrix},$$

$$U = exp\left(i\frac{\phi}{F}\right), \phi = \sum_{j=1}^{3} \phi_j \tau_j = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

This Lagrangian is invariant under parity transformation, therefore It turns out that the including terms must contain an even powers of field ϕ. Therefore this Lagrangian can be expressed by

$$\mathcal{L}_2 = \mathcal{L}_2^{2\phi} + \mathcal{L}_2^{4\phi} + ... \tag{29}$$

From the previous expression, we may obtain the interacting Lagrangian which contains 4 fields as

$$\mathcal{L}_2^{4\phi} = \frac{1}{6F^2}\left(\phi_i \partial^\mu \phi_i \partial_\mu \phi_j \phi_j - \phi_i \phi_i \partial_\mu \phi_j \partial^\mu \phi_j\right) + \frac{M^2}{24F^2}\left(\phi_i \phi_i \phi_j \phi_j\right) \tag{30}$$

where $M^2 = 2B\hat{m}$. The Feynman rule for the interacting vertex as shown in the figure 2 is proven by using the expression for the invariant amplitude from quantum field theory by expanding the scattering operator (S) through Dyson series, then work out

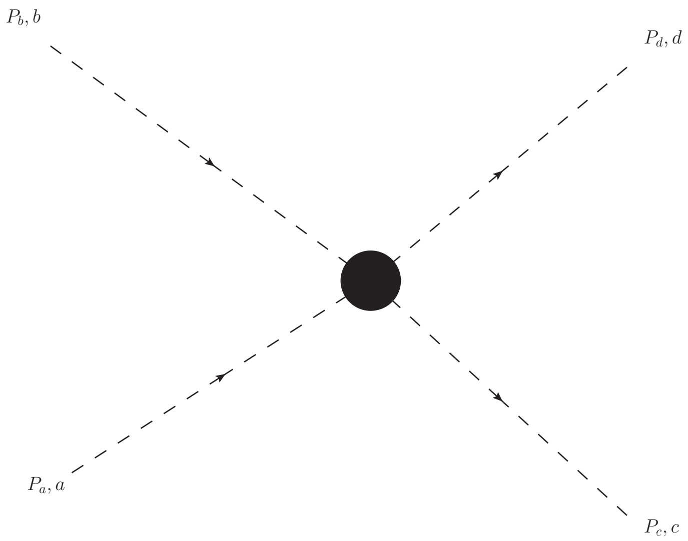


Figure 2: $\pi\pi$ scattering diagram

the matrix element contains the order of S we're interested in. After a calculation is done with contraction of the fields, we get

$$\mathcal{M} = i\left[\delta_{ab}\delta_{cd}\frac{s-M^2}{F^2} + \delta_{ac}\delta_{bd}\frac{t-M^2}{F^2} + \delta_{ad}\delta_{bc}\frac{u-M^2}{F^2}\right] - \frac{i}{3F^2}\left(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}\right)\left(\Lambda_a + \Lambda_b + \Lambda_c + \Lambda_d\right) \tag{31}$$

where $\Lambda_a = p_a^2 - M^2$. The other 3 parameters s,t and u represent the mandelstam variables,

$$s = (p_a + p_b)^2, t = (p_a - p_c)^2, u = (p_a - p_d)^2 \tag{32}$$

3 Calculation of matrix elements

In this section, we may apply the ideas from ChPT to work out the matrix elements of 4 types of current to understand what may happens to the double parton scattering within the pp collisions. We begin with the effective Lagrangian in $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ sector

$$\mathcal{L}_2 = \frac{F^2}{4}Tr[D_\mu U(D^\mu U)^\dagger] + \frac{F^2}{4}Tr[\chi U^\dagger + U\chi^\dagger] \tag{33}$$

Where $\chi = 2B(s+ip)$. The covariant derivative and the SU(2) matrix U are similar to those in the previous section. In quantum field theory, the interacting Lagrangian can be expressed through

$$\mathcal{L}_{int}(x) = J(x)\phi_{ext}(x) \tag{34}$$

Where $\phi_{ext}(x)$ represents the external field which coupling to the corresponding current density. In order to obtain the explicit form of the current density, we need to represent each part of the effective Lagrangian as a multiplication between external field and current as in the previous equation. The first term in the effective Lagrangian includes the vector and axial vector fields while another term contains a scalar and pseudoscalar fields respectively. Anyway It is convenient to rewrite the previous Lagrangian as

$$\mathcal{L}_2 = \mathcal{L}_2^{v,a} + \mathcal{L}_2^{s,p} \tag{35}$$

where $\mathcal{L}_2^{v,a} = \frac{F^2}{4} Tr[D_\mu U (D^\mu U)^\dagger]$ and $\mathcal{L}_2^{s,p} = \frac{F^2}{4} Tr[\chi U^\dagger + U \chi^\dagger]$

Situation 1 : Finding $V_a^\mu(x)$ and $A_a^\mu(x)$

The vector and axial vector currents are defined through

$$V_a^\mu = R_a^\mu + L_a^\mu \tag{36}$$

$$A_a^\mu = R_a^\mu - L_a^\mu \tag{37}$$

First, we need to find the expressions for L_a^μ and R_a^μ. In case of L_a^μ, we set $r_\mu = 0$ so that the $\mathcal{L}_2^{v,a}$ takes the form

$$\mathcal{L}_2^{v,a} = \frac{iF^2}{2} Tr[l_\mu \partial^\mu U^\dagger U] + ... \tag{38}$$

In which only the linear term in left handed field is shown, The left handed field l_μ can be paremeterized by

$$l_\mu = \frac{1}{2}\left(l_{\mu a}\tau_a + l_{\mu 0} I\right) \tag{39}$$

Then we expand U inside the trace of our Lagrangian up to the third order and substitute the expression for l_μ. After the traces are evaluated, we get

$$L_a^\mu = \frac{F}{2}\partial^\mu \phi_a - \frac{\epsilon_{abc}}{4}(\partial^\mu \phi_b \phi_c - \phi_b \partial^\mu \phi_c) + \frac{1}{6F}\phi_b(\phi_a \partial^\mu \phi_b - \phi_b \partial^\mu \phi_a) + O(\phi^4) \tag{40}$$

In the same way as L_a^μ, we set $l_\mu = 0$ and follow the previous steps, we get the expression for R_a^μ as

$$R_a^\mu = -\frac{F}{2}\partial^\mu \phi_a - \frac{\epsilon_{abc}}{4}(\partial^\mu \phi_b \phi_c - \phi_b \partial^\mu \phi_c) + \frac{1}{6F}\phi_b(\phi_a \partial^\mu \phi_b - \phi_b \partial^\mu \phi_a) + O(\phi^4) \tag{41}$$

So that we obtain

$$V_a^\mu(x) = \frac{\epsilon_{abc}}{2}\left(\phi_b \partial^\mu \phi_c - \partial^\mu \phi_b \phi_c\right) + O(\phi^4) \tag{42}$$

$$A_a^\mu(x) = -F\partial^\mu \phi_a - \frac{1}{3F}\phi_b\left(\phi_a \partial^\mu \phi_b - \partial^\mu \phi_a \phi_b\right) + O(\phi^5) \tag{43}$$

Situation 2 : Finding $S(x)$ and $P_a(x)$

We begin our calculation by using the expression for $\mathcal{L}_2^{s,p}$. We rewrite this Lagrangian in terms of scalar and pseudoscalar fields as

$$\mathcal{L}_2^{s,p} = \frac{BF^2}{2} Tr[(s + ip)U^\dagger + U(s - ip)] \tag{44}$$

The scalar and pseudoscalar fields can be parameterized by

$$s = s_a\tau_a + s_0 I \tag{45}$$

$$p = p_b\tau_b + p_0 I \tag{46}$$

After plugged the expansion of s and p into the Lagrangian and expanded U up to the third order, we end up with the expression for scalar and pseudoscalar current densities,

$$S(x) = 2BF^2 - B\phi_a\phi_a + O(\phi^4) \tag{47}$$

$$P_a(x) = 2BF\phi_a + \frac{B}{3F}\phi_a\phi_b\phi_b + O(\phi^5) \tag{48}$$

We've derived the expressions for four types of current density. After this, we'll make use of them to calculate the matrix elements and work out the Feynman rules for the particular vertices involving pion field.

3.1 Case I : Matrix elements for the scalar current

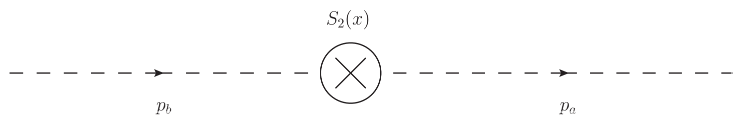
For one incoming pion and one outgoing pion, the matrix element that those two pions coupling with the single scalar current is

$$\begin{aligned}
\langle\pi^a| S_2(x) \left|\pi^b\right\rangle &= \delta_{df} \langle 0| \sqrt{2E_{\vec{p_a}}} a_a(\vec{p_a})(-B\phi_d^-(x)\phi_f^+(x))\sqrt{2E_{\vec{p_b}}} a_b^\dagger(\vec{p_b}) |0\rangle \\
&= -B\delta_{df} \langle 0| \sqrt{2E_{\vec{p_a}}} a_a(\vec{p_a})(-B\phi_d^-(x)\phi_f^+(x))\sqrt{2E_{\vec{p_b}}} a_b^\dagger(\vec{p_b}) |0\rangle \\
&= -B\delta_{df} \int\int \frac{d^3p_d d^3p_f}{(2\pi)^6}\sqrt{\frac{E_{\vec{p_a}}E_{\vec{p_b}}}{E_{\vec{p_d}}E_{\vec{p_f}}}} e^{ip_d\cdot x}e^{-ip_f\cdot x} \langle 0| a_a(\vec{p_a})a_d^\dagger(\vec{p_d})a_f(\vec{p_f})a_b^\dagger(\vec{p_b}) |0\rangle \\
&= -Be^{i(p_a-p_b)\cdot x}\delta^{ab}
\end{aligned} \tag{49}$$

The Fourier transformation of this matrix element is

$$\langle\pi^a| \tilde{S}_2(p) \left|\pi^b\right\rangle = -B\delta^{ab}(2\pi)^4\delta^{(4)}(p + p_a - p_b) \tag{50}$$

This matrix element can be expressed by



Another interesting case is the matrix element for a one incoming pion and one outgoing pion coupling to two scalar currents,

$$\langle \pi^a | S(x)S(y) \left| \pi^b \right\rangle = \delta^{cf}\delta^{de} \langle \pi^a | (2BF^2 - B\phi_c(x)\phi_c(x))(2BF^2 - B\phi_d(y)\phi_d(y)) \left| \pi^b \right\rangle \quad (51)$$

For example, the non-trivial matrix elements which possibly describe the physical process are $\langle \pi^a | \phi_c(x)\phi_c(x)\phi_d(y)\phi_d(y) \left| \pi^b \right\rangle$ and $\langle \pi^a | \phi_c(x)\phi_c(x)\phi_d(y)\phi_d(y) \left| \pi^b \right\rangle$. These 2 matrix elements are equivalent so that we choose from one of them to do the calculation. We get

$$\begin{aligned} \langle \pi^a | \phi_c(x)\phi_c(x)\phi_d(y)\phi_d(y) \left| \pi^b \right\rangle &= \langle \pi^a | \phi_c(x) D_F^{cd}(x-y)\phi_d(y) \left| \pi^b \right\rangle \\ &= D_F^{cd}(x-y) \left[\langle \pi^a | \phi_c^-(x)\phi_d^+(y) \left| \pi^b \right\rangle + \langle \pi^a | \phi_d^-(y)\phi_c^+(x) \mid \pi^b \rangle \right] \end{aligned} \quad (52)$$

In our notation $D_F^{cd}(x-y) = D_F(x-y)\delta^{cd}$. After the contractions inside the vacuum expectation values have made, the results we're looking for are

$$\langle \pi^a | \phi_c^-(x)\phi_d^+(y) \left| \pi^b \right\rangle = \delta^{ac}\delta^{bd} e^{i(p_a \cdot x - p_b \cdot y)}$$

$$\langle \pi^a | \phi_d^-(y)\phi_c^+(x) \left| \pi^b \right\rangle = \delta^{ac}\delta^{bd} e^{i(p_a \cdot y - p_b \cdot x)}$$

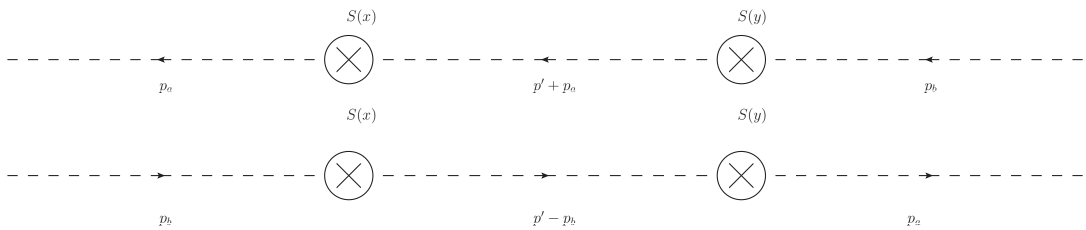
So that the matrix element included a single propagator with the vertex factor in front is

$$\langle \pi^a | S(x)S(y) \left| \pi^b \right\rangle_{1p} = B^2 D_F(x-y)\delta^{ab} \left[e^{i(p_a \cdot x - p_b \cdot y)} + e^{i(p_a \cdot y - p_b \cdot x)} \right] \quad (53)$$

The Fourier transformation of this matrix element can be represented by

$$\langle \pi^a | \tilde{S}(p')\tilde{S}(k) \left| \pi^b \right\rangle_{1p} = iB^2(2\pi)^4\delta^{ab} \left[\frac{\delta^{(4)}(p' + p_a + k - p_b)}{(p' + p_a)^2 - m^2 + i\epsilon} + \frac{\delta^{(4)}(k + p' - p_b + p_a)}{(p' - p_b)^2 - m^2 + i\epsilon} \right] \quad (54)$$

This matrix element can be expressed by



3.2 Case II : Matrix elements for the pseudoscalar current

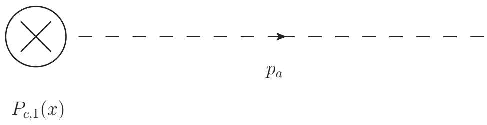
First of all, we'll consider the matrix element for one outgoing pion coupling to the single pseudoscalar current

$$\langle \pi^a | P_{c,1}(x) \left| 0 \right\rangle = \langle \pi^a | 2BF\phi_c \left| 0 \right\rangle = 2BF \int \frac{d^3 p_c}{(2\pi)^3} \sqrt{\frac{E_{\vec{p_a}}}{E_{\vec{p_c}}}} e^{ip_c \cdot x} \langle 0 | a_a(\vec{p_a}) a_c^\dagger(\vec{p_c}) \left| 0 \right\rangle \quad (55)$$

After the integration is evaluated, the result is

$$\langle\pi^a|\, P_{c,1}(x)\,|0\rangle = 2BF\delta^{ac}e^{ip_a\cdot x} \tag{56}$$

This matrix element can be expressed by



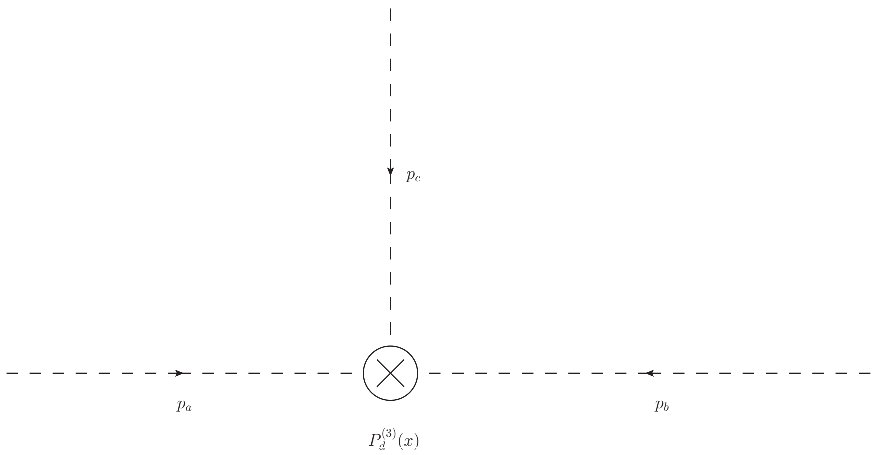
In case of 3 incoming pions, we might calculate

$$\begin{aligned}\langle 0|\, P_d^{(3)}(x)\,\left|\pi^a\pi^b\pi^c\right\rangle &= \langle 0|\,\frac{B}{3F}\phi_d(x)\phi_e(x)\phi_e(x)a_a^\dagger(\vec{p_a})a_b^\dagger(\vec{p_b})a_c^\dagger(\vec{p_c})2\sqrt{2E_{\vec{p_a}}E_{\vec{p_b}}E_{\vec{p_c}}}\,|0\rangle\\ &= \frac{B\delta^{ef}}{3F}\int\int\int\frac{d^3p_d d^3p_e d^3p_f}{(2\pi)^9}e^{-i(p_d+p_e+p_f)\cdot x}\sqrt{\frac{E_{\vec{p_a}}E_{\vec{p_b}}E_{\vec{p_c}}}{E_{\vec{p_d}}E_{\vec{p_e}}E_{\vec{p_f}}}}\zeta\\ &= \frac{2B}{3F}e^{-i(p_a+p_b+p_c)\cdot x}(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc})\end{aligned} \tag{57}$$

where we note that in the second line, $\zeta = \langle 0|\, a_d(\vec{p_d})a_e(\vec{p_e})a_f(\vec{p_f})a_a^\dagger(\vec{p_a})a_b^\dagger(\vec{p_b})a_c^\dagger(\vec{p_c})\,|0\rangle$. The Fourier transformation of this matrix element is

$$\langle 0|\,\tilde{P}_d^{(3)}(k)\,\left|\pi^a\pi^b\pi^c\right\rangle = \frac{2B}{3F}(2\pi)^4\delta^{(4)}(k-p_a-p_b-p_c)(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc}) \tag{58}$$

This matrix element is therefore represented by



What we are going to compute for the next one is the pion matrix element coupling to 2 pseudoscalar currents in the form

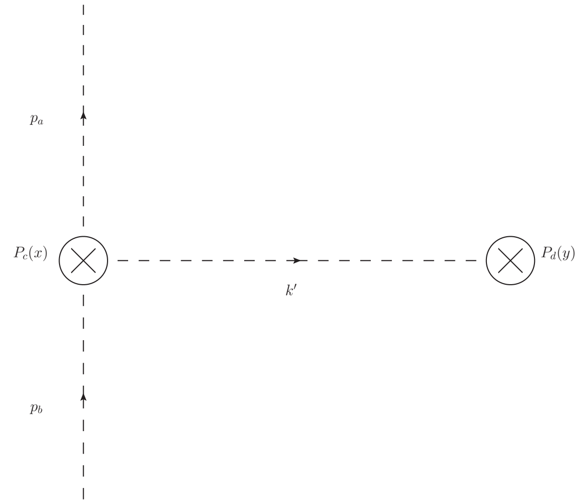
$$\langle\pi^a|\, P_c(x)P_d(y)\,\left|\pi^b\right\rangle = \left\langle\pi^b\right|(2BF\phi_c(x)+\frac{B}{3F}\phi_c(x)\phi_e(x)\phi_e(x))(2BF\phi_d+\frac{B}{3F}\phi_d(y)\phi_f(y)\phi_f(y))\left|\pi^b\right\rangle \tag{59}$$

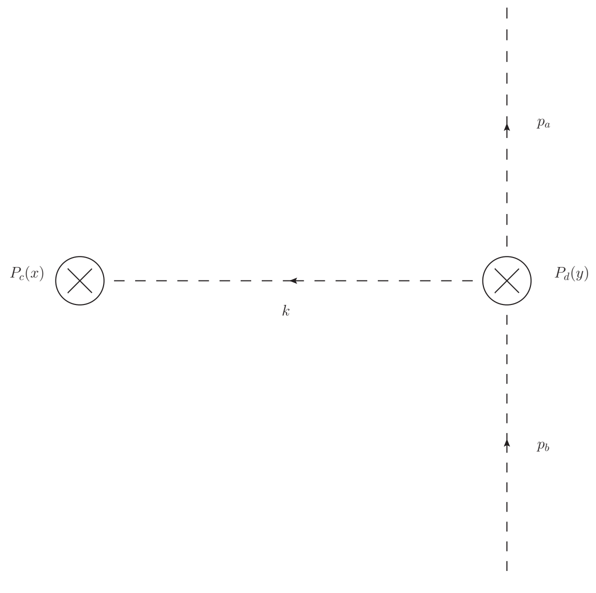
The calculation for this matrix elements is pretty long, so that the calculation will not be shown here. The fully connected matrix elements we got from the calculation for a no loop cases take the form

$$\frac{2B^2}{3}\left\langle\pi^a\right| D_F^{cd}(x-y)\phi_e^-(x)\phi_e^+(x)\left|\pi^b\right\rangle = \frac{2B^2}{3}D_F(x-y)e^{i(p_a-p_b)\cdot x}\delta^{ab}\delta^{cd} \tag{60}$$

$$\frac{2B^2}{3}\left\langle\pi^a\right| D_F^{cd}(x-y)\phi_f^-(y)\phi_f^+(y)\left|\pi^b\right\rangle = \frac{2B^2}{3}D_F(x-y)e^{i(p_a-p_b)\cdot y}\delta^{ab}\delta^{cd} \tag{61}$$

where another 4 terms can be worked out by exchanging the isospin indices. These 2 matrix elements can be represented by



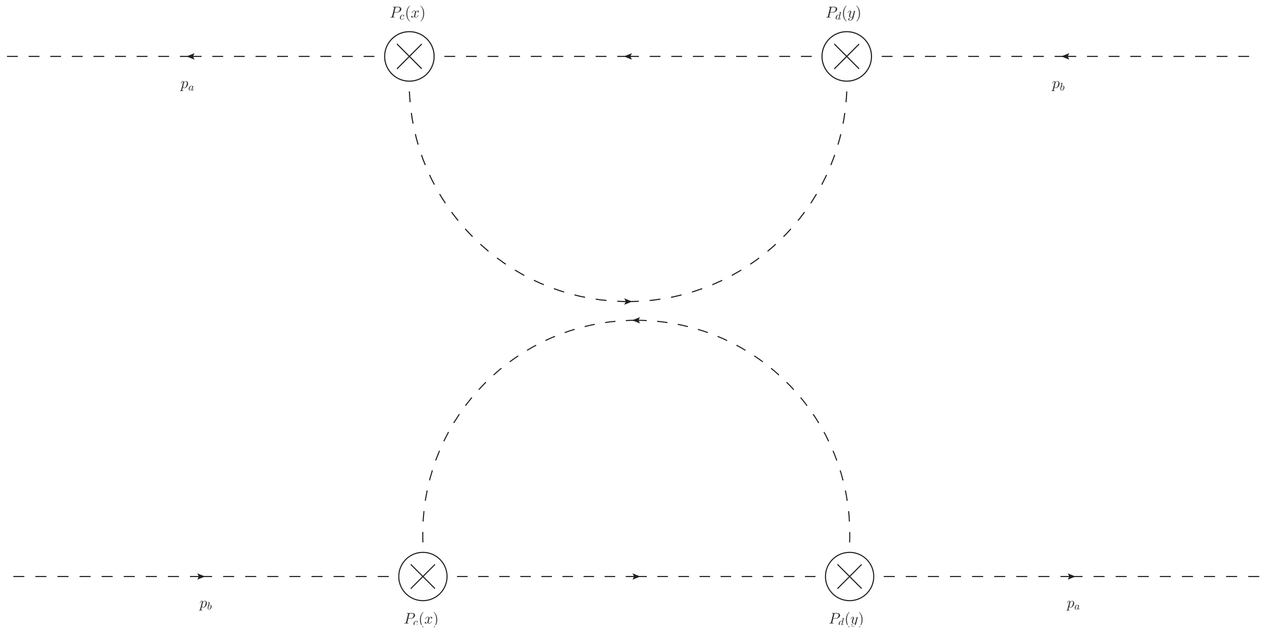


For a single-loop diagrams, the matrix elements take the form

$$\frac{B^2}{9F^2}\left\langle\pi^a\right| D_F^{cd}(x-y)D_F^{ef}(x-y)\phi_e^-(x)\phi_f^+(y)\left|\pi^b\right\rangle = \frac{B^2}{9F^2}D_F^2(x-y)e^{i(p_a\cdot x-p_b\cdot y)}\delta^{ab}\delta^{cd} \tag{62}$$

$$\frac{B^2}{9F^2}\left\langle\pi^a\right| D_F^{cf}(x-y)D_F^{ed}(x-y)\phi_f^-(y)\phi_e^+(x)\left|\pi^b\right\rangle = \frac{B^2}{9F^2}D_F^2(x-y)e^{i(p_a\cdot y-p_b\cdot x)}\delta^{ab}\delta^{cd} \tag{63}$$

by exchanging the isospin indices, another 4 matrix elements can be obtained. These 2 matrix elements can be represented by

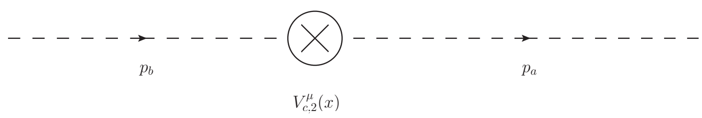


3.3 Case III : Matrix elements for the vector current

We consider in case of one incoming pion and one outgoing pion coupling to the single vector current, the matrix element takes the form

$$\begin{aligned}
&\langle \pi^a | V^\mu_{c,2}(x) \left| \pi^b \right\rangle = \langle 0 | \sqrt{2E_{\vec{p_a}}} a_a(\vec{p_a}) \frac{\epsilon_{cde}}{2} \left(\phi_d(x) \partial^\mu \phi_e - \partial^\mu \phi_d(x) \phi_e(x) \right) \sqrt{2E_{\vec{p_b}}} a^\dagger_b(\vec{p_b}) \left| 0 \right\rangle \\
&= \frac{\epsilon_{cde}}{2} (\langle 0 | \sqrt{2E_{\vec{p_a}}} a_a(\vec{p_a}) \phi_d(x) \partial^\mu \phi_e(x) \sqrt{2E_{\vec{p_b}}} a^\dagger_b(\vec{p_b}) \left| 0 \right\rangle - \langle 0 | \sqrt{2E_{\vec{p_a}}} a_a(\vec{p_a}) \partial^\mu \phi_d(x) \phi_e(x) \sqrt{2E_{\vec{p_b}}} a^\dagger_b(\vec{p_b}) \left| 0 \right\rangle) \\
&= e^{i(p_a - p_b)\cdot x} \frac{\epsilon_{cde}}{2} \left(-ip^\mu_b \delta^{adbe} + ip^\mu_a \delta^{ae} \delta^{bd} - ip^\mu_a \delta^{ad} \delta^{be} + ip^\mu_b \delta^{ae} \delta^{bd} \right) \\
&= \frac{i\epsilon_{cde}}{2} (p^\mu_a + p^\mu_b) e^{i(p_a - p_b)\cdot x} (\delta^{ae} \delta^{bd} - \delta^{ad} \delta^{be})
\end{aligned} \tag{64}$$

This matrix element can be represented by

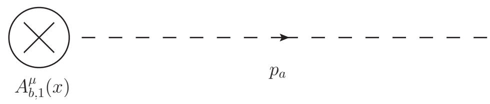


3.4 Case IV : Matrix elements for the axial vector current

The first simplest case is the matrix element for one outgoing pion couplings to the single axial vector current

$$\begin{aligned}\langle\pi^a| A^\mu_{b,1}(x)|0\rangle &= \langle 0|\sqrt{2E_{\vec{p_a}}}a_a(\vec{p_a})(-F\partial^\mu\phi^+_b)|0\rangle \\ &= -F\int\frac{d^3p_b}{(2\pi)^3}\sqrt{\frac{E_{\vec{p_a}}}{E_{\vec{p_b}}}}ip^\mu_b e^{ip_b\cdot x}\langle 0|a_a(\vec{p_a})a^\dagger_b(\vec{p_b})|0\rangle \\ &= -iFp^\mu_a e^{ip_a\cdot x}\delta^{ab}\end{aligned} \tag{65}$$

This matrix element can be represented by



Another interesting case is the matrix element with three incoming pions

$$\begin{aligned}\langle 0|A^\mu_{d,3}(x)\left|\pi^a\pi^b\pi^c\right\rangle &= \langle 0|A^\mu_{d,3}(x)a^\dagger_a(\vec{p_a})a^\dagger_b(\vec{p_b})a^\dagger_c(\vec{p_c})\sqrt{2E_{\vec{p_a}}}\sqrt{2E_{\vec{p_b}}}\sqrt{2E_{\vec{p_c}}}|0\rangle \\ &= \frac{\delta^{ef}}{3F}(c_1-c_2)\end{aligned}$$

where c_1 and c_2 are defined through

$$c_1 = \langle 0|\phi^+_e\partial^\mu\phi^+_d\phi^+_f a^\dagger_a(\vec{p_a})a^\dagger_b(\vec{p_b})a^\dagger_c(\vec{p_c})\sqrt{2E_{\vec{p_a}}}\sqrt{2E_{\vec{p_b}}}\sqrt{2E_{\vec{p_c}}}|0\rangle \tag{66}$$

$$c_2 = \langle 0|\phi^+_e\phi^+_d\partial^\mu\phi^+_f a^\dagger_a(\vec{p_a})a^\dagger_b(\vec{p_b})a^\dagger_c(\vec{p_c})\sqrt{2E_{\vec{p_a}}}\sqrt{2E_{\vec{p_b}}}\sqrt{2E_{\vec{p_c}}}|0\rangle \tag{67}$$

by working out the vacuum expectation values, we get the simpler forms of c_1 and c_2 as

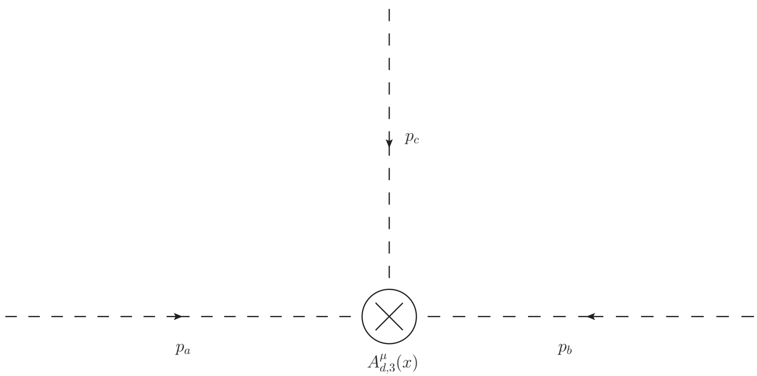
$$c_1 = -ie^{-i(p_a+p_b+p_c)\cdot x}[p^\mu_a(\delta^{ad}\delta^{bf}\delta^{ce}+\delta^{ad}\delta^{be}\delta^{cf})+p^\mu_b(\delta^{ae}\delta^{bd}\delta^{cf}+\delta^{af}\delta^{bd}\delta^{ce})+p^\mu_c(\delta^{ae}\delta^{bf}\delta^{cd}+\delta^{af}\delta^{be}\delta^{cd})]$$

$$c_2 = -ie^{-i(p_a+p_b+p_c)\cdot x}[p^\mu_a(\delta^{af}\delta^{be}\delta^{cd}+\delta^{af}\delta^{bd}\delta^{ce})+p^\mu_b(\delta^{ae}\delta^{bf}\delta^{cd}+\delta^{ad}\delta^{bf}\delta^{ce})+p^\mu_c(\delta^{ae}\delta^{bd}\delta^{cf}+\delta^{ad}\delta^{be}\delta^{cf})]$$

After substituted c_1 and c_2, what we get is

$$\begin{aligned}\langle 0|A^\mu_{d,3}(x)\left|\pi^a\pi^b\pi^c\right\rangle = \frac{i}{3F}e^{-i(p_a+p_b+p_c)\cdot x}(&\delta^{ad}\delta^{bc}(-2p^\mu_a+p^\mu_b+p^\mu_c) \\ &+\delta^{ac}\delta^{bd}(-2p^\mu_b+p^\mu_a+p^\mu_c)+\delta^{ab}\delta^{cd}(-2p^\mu_c+p^\mu_b+p^\mu_a))\end{aligned} \tag{68}$$

This matrix element can be represented by



4 Conclusion

We have obtained the matrix elements and the corresponding Feynman diagrams to describe the processes including pion(s). The result can be seen in the section 3. This work demonstrates that ChPT provides a good framework to work out the matrix elements including pion(s) from the spontaneous chiral symmetry breaking of chiral symmetry by the ground state. The one of the important matrix elements including pion(s) is the case that two vector currents coupling to 1 incoming pion and 1 outgoing pion, which eventually describing Compton Scattering between pion and photon. In summary, ChPT can be useful method to work out the matrix elements for various current densities coupling to pion(s) as inspired by the equivalence between DPD-type operator matrix elements and double parton distributions through Mellin moments. This methods can be applied to calculate the matrix elements in case of of 2 vector currents for pp collisions, which is a more realistic quantities to describe double parton scattering occurs in pp collisions at CERN.

5 Acknowledgement

First of all, I would like to thank to the National Science and Technology Development Agency (NSTDA) of Thailand and Synchrotron Light Research Institute (SLRI) for providing me a supports to participate this activity at DESY Hamburg. Thanks to Mrs.Pankamon Sornsuwan and Mrs.Reutai Chongsrid, and Mrs.Andrea Schrader. They always support, encourage and kindly answer me a questions about this summer student program during my preparation period before the start of this program. Thanks to my beloved project supervisor at DESY, Dr.Markus Diehl who always kindly encourages my work and kindly answers me even a stupid questions whenever I got stuck in any aspect of my summer student project, I might say that he always give me an absolutely happiness time and inspirations in working at DESY theory group. Thanks to my thesis supervisor, Professor Dr.Yupeng Yan and colleagues at Suranaree University of Technology, Thailand for encouraging me to participate this summer student program. Also big thanks to the beloved project supervisor, Assistant Professor Dr. Suphot Musiri who gave me a chance to work in a theoretical physics area when I was a bachelor student at Srinakharinwirot University. Thanks to Dr.Nopmanee Supanam and Dr.Kem Pumsaard from Srinakharinwirot University for the inspirations to work in high energy physics research area. Thanks to my colleagues at my office, Mr.Augustinas Malinauskas and Mr.Fernando Romero Lopez, they gave me a nice discussions and helped me in solving a plenty of problem. Also big thanks to another 3 Thai summer students who joined a trip with me, especially Mr.Chairote Piyakulworawat who always accompany me whenever I went to any place in Hamburg. Above all, thanks to my family for the encouragement and kindness supporting me all the time. Finally I wanna give a very big thanks to all summer students from around the world and the organising team for providing good activities that made me cross to another border of knowledge like this.

References

[1] Markus Diehl. *Multiparton interactions: Theory and experimental findings.* Proceedings of Science. arXiV:1306.6059v[hep-ph], 2013.

[2] Supanam N. , Fearing H.W. , Yan Y. *Baryon chiral perturbation theory with virtual photons and leptons.* Journal of High Energy Physics, vol. 2010, pp. 124 (2010).

[3] M.E. Peskins and D.V. Schroeder. *An Introduction to Quantum Field Theory.* Addison-Wesley, Reading, Massachusetts, 1993.

[4] Stefan Scherer and Matthias R. Schindler. *A Primer for Chiral Perturbation Theory.* Springer, 2012.

[5] Mark B. Wise. *Combining Chiral and Heavy Quark Symmetries.* Lectures given at the CCAST Symposium on Particle Physics at the Fermi Scale, 1993.

[6] Particle Data Group *Particle Physics Booklet.* 2014