Estimation of Negative Limits on the Effective Charge Radius of Quark

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Abstract

Extended analysis of possible contributions of Beyond Standard Model processes in the framework of modified cross sections with quark charge radius t-factor was made in this paper using H1 and ZEUS combined data sets of inclusive deep inelastic cross sections in neutral and charge current ep scattering. The research focuses on the estimation of the lower limit on quark radius. The developed approach results the 95% C.L. value of the squared electroweak charge radius of quark $-((0.489 \pm 0.051) \cdot 10^{-16}\ cm)^2$

Contents

1 Introduction

Precision measurements of deep inelastic $e^{\pm}p$ scattering (DIS) cross sections in the domain of negative four-momentum-transfer squared, Q^2, above about 10^5GeV^2 allow searches for Beyond Standard Model (BSM) contributions, if such contributions are linked to processes setting in at scale of around one TeV. The search for deviations from the SM model predictions reaches beyond the center-of-mass energy of the $e^{\pm}p$ interactions, because any such BSM processes can modify the scattering cross sections via virtual effects. The cross sections would be affected by new kinds of interactions where BSM particles would be exchanged. This could be leptoquarks for which $e^{\pm}p$ scattering is particularly sensitive, because of possibly direct couplings. Another possibility are graviton exchanges in models with large extra dimensions. Many other exotic exchanges have been proposed. As we assume to be far below the scale of the actual new physics, we can approximate all such BSM interactions as contact interactions, CI. The fact that $eeqq$ CI is not renormalisable is not relevant in our kinematic domain.

The cross sections would also be influenced by a finite radius of the quarks. In all cases, one searches for a deviation of the observed from the predicted cross section for ep scattering at the highest available Q^2. The result on CI is then interpreted with respect to the model to be tested. The predictions are calculated from Parton Distribution Functions, PDFs. If those are derived from the same data which is used to search for CI, special care has to be taken. In an analysis of 1994-2000 $e^{\pm}p$ data [1], the ZEUS collaboration searched for CI using basically independent PDFs. The limit on the quark charge radius, in the classical form factor approximation, was $0.85 \cdot 10^{-16}$ cm.

In our research we used another procedure to set the limits on the BSM contributions. The main difference from the previous studies is that BSM contributions and QCD evolution are fitted simultaneously. In the analysis [2] this approach was successfully used and the upper 95% C.L. limit on the quark radius was set. Research presented here is concentrated on setting the lower 95% C.L. limit of quark radius.

2 Data samples and HERAFitter framework

2.1 Data combining

The H1 and ZEUS detectors were general purpose instruments which consisted of tracking systems surrounded by electromagnetic and hadronic calorimeters and muon detectors, ensuring close to 4π coverage of the ep interaction point. The presented study is based on the inclusive NC and CC cross-section measurements for unpolarised $e^{\pm}p$ scattering at HERA, resulting from the combination [3] of the all available data from H1 and ZEUS experiments.

The data was taken with several E_p (proton beam energy) values and the double-differential cross sections were published by the two experiments for different reference $\sqrt{s}$ and (x_{Bj}, Q^2) grids. In order to average a set of data points, the points had to be translated to common $\sqrt{s_{com}}$ values and common (x_{Bj}, Q^2) grids.

The averaging of the data pints was performed using the HERAverager [4] tool which is based on a χ^2 minimization method. The 2927 published cross-section values were combined to 1307 averaged cross-section measurements. For the resulting 1620 degrees of freedom, a $\chi^2_{\text{min}} = 1687$ was obtained, demonstrating very good consistency of all considered data sets.

2.2 HERAFitter

HERAFitter [5] is an open-source package that provides a framework for the determination of the parton distribution functions (PDFs) of the proton and for many different kinds of analyses in Quantum Chromodynamics (QCD). It encodes results from a wide range of experimental measurements in lepton-proton deep inelastic scattering and proton-proton (proton-antiproton) collisions at hadron colliders. These are complemented with a variety of theoretical options for calculating PDF-dependent cross section predictions corresponding to the measurements. The framework covers a large number of the existing methods and schemes used for PDF determination. The data and theoretical predictions are brought together through numerous methodological options for carrying out PDF fits and plotting tools to help visualise the results. While primarily based on the approach of collinear factorisation, HERAFitter also provides facilities for fits of dipole models and transverse-momentum dependent PDFs. The package can be used to study the impact of new precise measurements from hadron colliders.

Figure 1 gives a schematic overview of the HERAFitter structure. In our search we used modified HERAFitter trunk that allows to make QCD fits and C.I. searches.

3 Theory

3.1 General discription of QCD analysis

In the following, we briefly describe the framework used in the perturbative QCD (pQCD) analysis of the combined data, as used in the HERAPDF2.0 study [3]. Only cross sections for Q^2 starting from $Q^2_{\text{min}} = 3.5\,\text{GeV}^2$ were used in the analysis. A fit to the data, resulting in the set of PDFs, was obtained by solving the DGLAP evolution equations at NLO in the $\overline{\text{MS}}$ scheme. This was done using the program QCDNUM within the HERAFitter framework [5].

In this approach, the PDFs of the proton, xf, are generically parameterized at the starting scale $\mu^2_{\text{f}_0}$ as

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2) \quad , \tag{1}$$

where x is the fraction of the proton's momentum taken by the struck parton in the infinite momentum frame. The PDFs parameterized are the gluon distribution, xg, the valence-quark distributions, xu_v, xd_v, and the u-type and d-type anti-quark distributions, $x\bar{U}$, $x\bar{D}$. The PDFs parameter values were fit to the data using a χ^2 method,

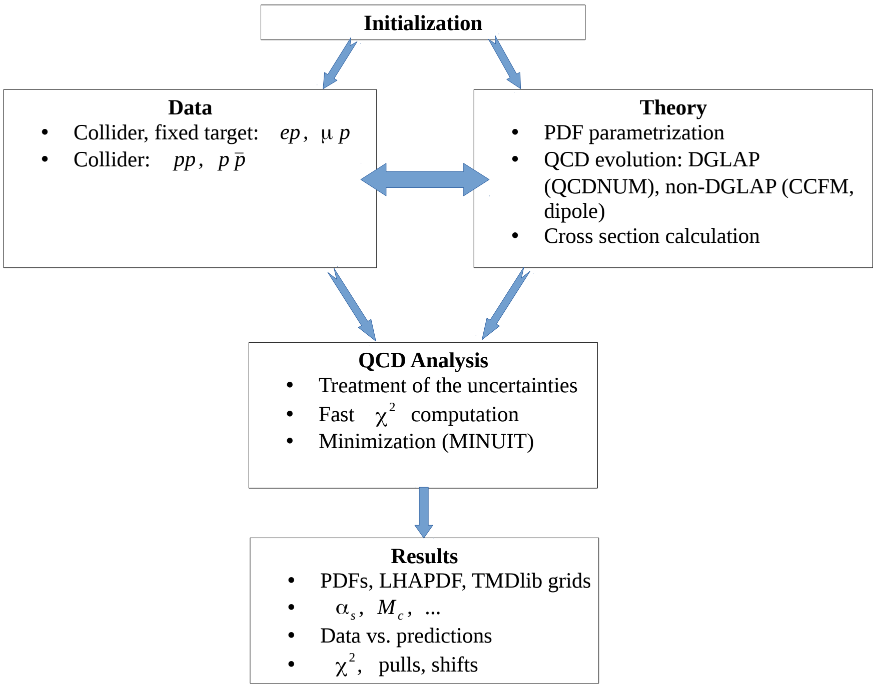


Figure 1: Schematic overview of HERAFitter program

taking into account statistical uncertainties, as well as uncorrelated and correlated systematic uncertainties of the input experimental data.

Uncertainties of the PDFs resulting from the experimental uncertainties were determined with the criterion $\Delta\chi^2 = 1$ and verified using the Monte Carlo method based on analyzing a large number of pseudo data sets called replicas. The two approaches gave consistent estimates of experimental errors. The uncertainties on HERAPDF2.0 due to the choice of model settings and the form of the parameterization were also evaluated, and the total PDF uncertainty was obtained by adding in quadrature the experimental, the model and the parameterization uncertainties. For more details, see [3].

3.2 Quark form factor

One of the possible methods to search for deviations from SM predictions in *ep* scattering is to assign a finite size for the radius of the electroweak charges of electrons and/or quarks while assuming the SM gauge bosons remain point-like and their couplings are unchanged. The expected suppression of the SM cross section can be described using a semi-classical form factor approach. If the expected deviations are small, the SM predictions for the cross sections are modified, approximately, to:

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{SM}}{dQ^2}\left(1 - \frac{R_e^2}{6}Q^2\right)^2\left(1 - \frac{R_q^2}{6}Q^2\right)^2 , \tag{2}$$

where R_e and R_q are the root-mean-square radii of the electroweak charge of the electron and the quark, respectively. Figure 2 can explain how will the fits change with introducing this additional parameter (R_q^2).

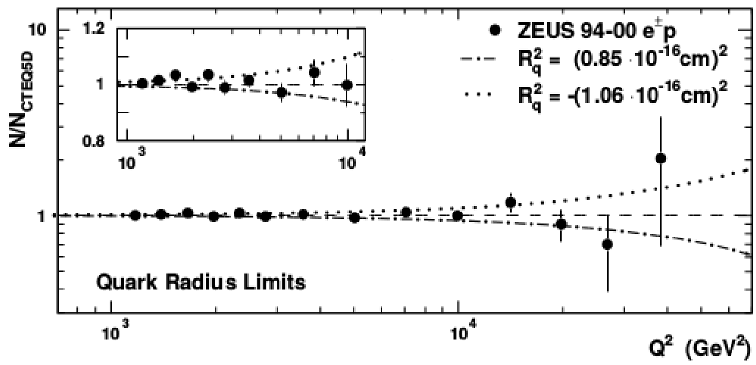


Figure 2: Combined 1994-2000 data compared with 95% C.L. exclusion limits for the effective mean-square radius of the electroweak charge of the quark. Results are normalized to the Standard Model expectations calculated using the CTEQ5D parton distributions. The insets show the comparison in the $Q^2 < 10^4$ GeV^2 region, with a linear ordinate scale [1].

In the present analysis only the possible finite spatial distribution of the quark charge is considered and the electron is assumed to be point-like ($R_e \equiv 0$).

The QCD analysis described in the previous subsection is extended by introducing R_q^2 as additional model parameter and modifying all $e^{\pm}p$ DIS cross-section predictions according to formula (2), to take into account possible finite R_q. The estimate of the quark radius squared resulting from the simultaneous fit of R_q^2 and the PDF parameters to the data is

$$R_q^{2\ Data} = -0.511\ (\pm 3.06) \cdot 10^{-6}\ \mathrm{GeV}^{-2} ,$$

in good agreement with SM expectations. However, as the resulting value is negative and at high Q^2 the statistical fluctuations are large we decided to estimate lower limit of the radius.

4 Estimation of R_q^2 limits

We used probability scan as the main approach in estimating the R_q^2 limits. Its feature is generation of large amount of Monte Carlo replicas. Then we compare the most likely value of the quark radius squared, determined from the χ^2 minimization, for the actual input data and for a large number of equivalent replicas.

Excluded at the C.L. 95% are R_q^2 values which, in more than 95% of the replicas, result in the fitted radius squared value, $R_q^{2\,Fit}$, lower than that obtained for the data, $R_q^{2\,Data}$.

To set the limit, distribution of $R_q^{2\,Fit}$ values had to be reconstructed from QCD fits to multiple replicas, for different values of the assumed true radius, $R_q^{2\,True}$.

The PDFs with fixed $R_q = 0$ were used to generate multiple data replicas for given $R_q^{2\,True}$.

Replica data-sets were created by taking the reduced cross sections calculated from PDFs and fluctuating their values randomly within given statistical and systematic uncertainties taking into account correlations. All uncertainties were assumed to follow the Gaussian distribution. For each replica, the generated value of the cross section at the point i was calculated as:

$$\mu^i = \left[m_0^i + \sqrt{\delta^2_{i,\mathrm{stat}} + \delta^2_{i,\mathrm{uncor}}} \cdot D_i \cdot r_i\right] \cdot (1 + \sum_j \gamma_j^i \cdot r_j) \quad , \tag{3}$$

where D_i is the measured cross section value at the point i and m_0^i is the expected cross section at this point for the considered R_q^{true} value. γ_j^i, $\delta_{i,\mathrm{stat}}$ and $\delta_{i,\mathrm{uncor}}$ are the relative correlated systematic, relative statistical and relative uncorrelated systematic uncertainties of the input data, respectively. r_i and r_j are random numbers generated from normal distribution for each data point i and for each source of correlated systematic uncertainty j, respectively.

The probability scan procedure is divided into two approaches. The first way is called "R_q only" scan. We used PDFs fit on data with fixed value of $R_q = 0$ as an input for fit with fixed PDFs parameters and free R_q^2 parameter. As a result the $R_q^{2\,Data}$ was found.

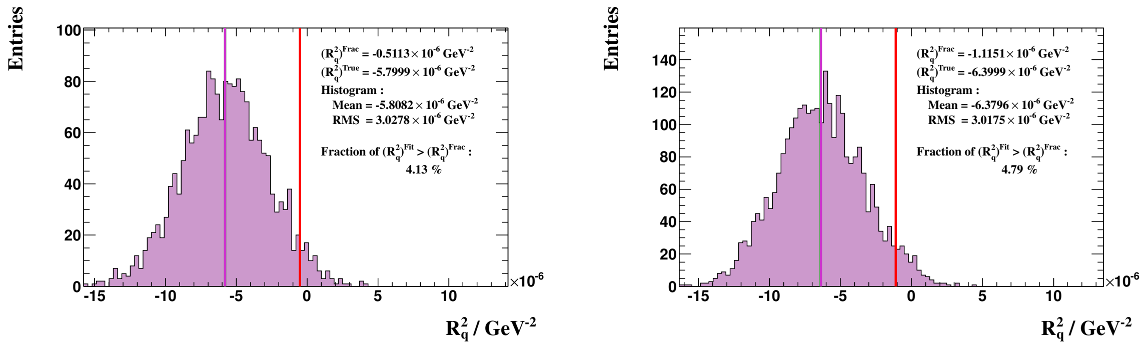
Then we made R_q^2 fit with fixed PDFs for each replica. We've got several distributions of $R_q^{2\,Fit}$ for central fit and variations (See figure 3) and evaluated the fraction of $R_q^{2\,Fit} > R_q^{2\,Frac}$, where $R_q^{2\,Frac}$ is the value of R_q^2 calculated from "$R_q only$" fits for fixed PDFs.

The χ^2 formula used for fitting R_q and PDF parameters, and possible correlated systematic shifts in the input data is:

$$\chi^2(\boldsymbol{m}, \boldsymbol{s}) = \sum_i \frac{\left[m^i + \sum_j \gamma_j^i m^i s_j - \mu^i\right]^2}{\left(\delta^2_{i,\mathrm{stat}} + \delta^2_{i,\mathrm{uncor}}\right)^2 D_i^2} + \sum_j s_j^2 \quad , \tag{4}$$

where the vector $\boldsymbol{m}$ represents the set of cross section predictions m_i and the components s_j of the vector $\boldsymbol{s}$ represent correlated shifts of the cross sections in units of sigma of the respective correlated systematic uncertainties; the summations over j extend over all correlated systematic uncertainties. It was checked that the usage of fixed statistical and uncorrelated systematic uncertainties in the denominator of the first r.h.s term of eq. (4,) taking the values from the data, minimizes the biases in the fit results.

The last step is to find several fractions that correspond to different values of $R_q^{2\,True}$ which lay between 3% and 8% and calculate which one refers to 5% value and it will be our limit on R_q^2 of 95% C.L. Examples of final plots are shown in figure 4.



(a) Distribution of fitted Monte Carlo replicas for central fit of "QCD+R_q" scan

(b) Distribution of fitted Monte Carlo replicas for D_{U_v} variation fit of "QCD+R_q" scan

Figure 3: Generated Monte Carlo distributions of $R_q^{2\,Fit}$ with the calculated fraction of $R_q^{2\,Fit} > R_q^{2\,Frac}$

Similar approach was made for "QCD+R_q" scan. To find $R_q^{2\,Data}$ we fitted PDFs and R_q^2 parameter simultaneously. Also for each replica we made R_q^2 and PDFs fit. Then analogically estimated R_q^2 limits.

As the cross check of Probability scan we used χ^2 scan. We had also two ways of doing it ("R_q-only" and "QCD+R_q" like in Probability scan). One is to find the minimum value of χ^2 from Standard Model QCD fits (including central fit, model and parametrization variation fits, see table 1) for different fixed values of R_q^2. Then to subtract the value of minimum from all the data points and look for negative one that

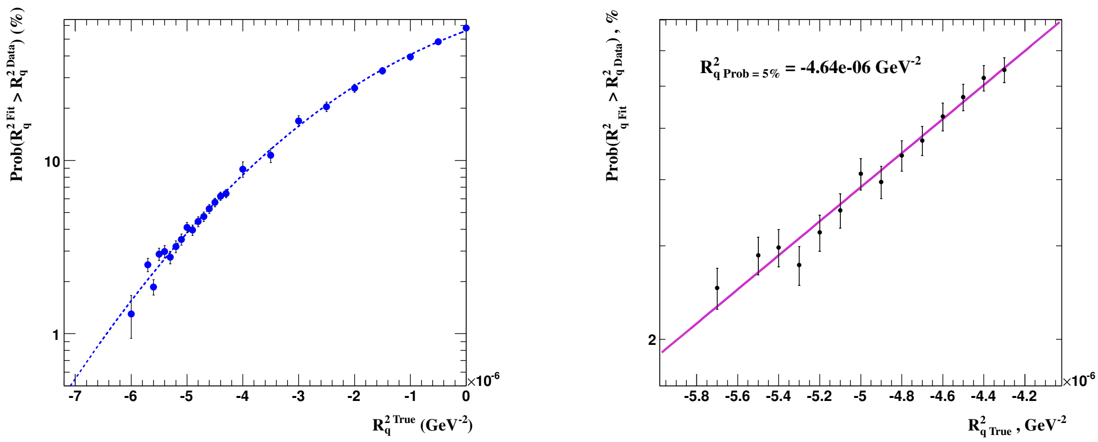
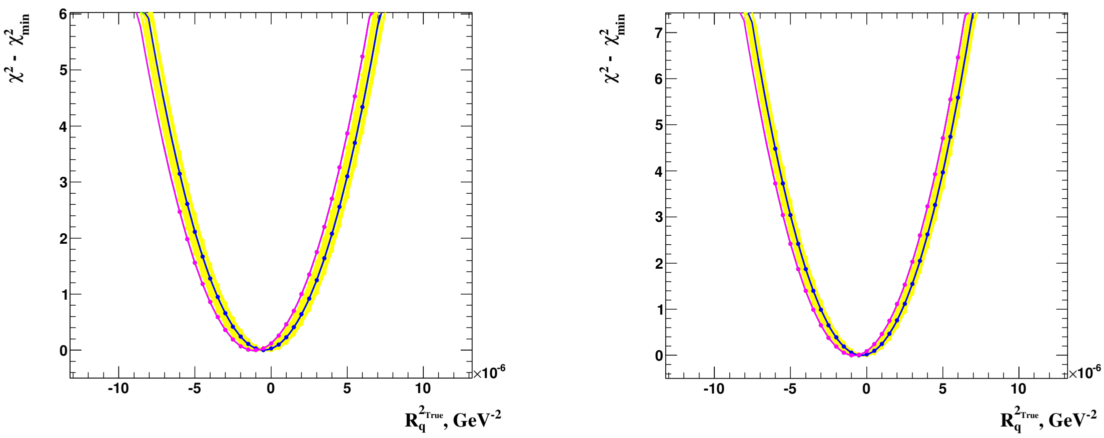


Figure 4: Probability scan for central fit of "R_q only". On the first plot full probability graph is shown. On the second one the part of previous plot is illustrated from 2% to 8% in logarithmic scale

corresponds to $\Delta\chi^2 = (1.64)^2$. This value refers to 95% C.L. The Results of χ^2 scans are illustrated in figure 5. The other way of making χ^2 scan is to do QCD fit on data using different fixed values of R_q^2. Similarly we've got results for $QCD + R_q^2$ scans.



(a) χ^2 scan for QCD fits with additional R_q^2 parameter

(b) χ^2 scan for R_q^2 fits only. Standard Model QCD fits were used

Figure 5: Result of χ^2 scan. Blue curve shows $\chi^2 - \chi^2_{min}$ distribution for central fit. Yellow curves refer to model variations and green - to parametrization variations. Pink one corresponds to D_{U_v} variation

5 Results

Estimation procedure described above was set for different model and parametrization variations. In figures 6 and 7 the comparison between two methods is shown.

From the figures above the lowest value of 95% C.L. limit can be found. It corresponds to D_{u_v} parametrization variation. And it is fair for R_q-only and also for QCD+R_q procedures. Therefore, the final result of Probability scan is:

$$R_q - only: \; R_q^{2\,Limit} = -((0.447 \pm 0.069) \cdot 10^{-16}\, cm)^2$$

$$QCD + R_q: \; R_q^{2\,Limit} = -((0.489 \pm 0.051) \cdot 10^{-16}\, cm)^2$$

The results of χ^2 scan are explained below. All plots of χ^2 distributions are shown on figure 5. The value that we are interested in (D_{u_v} parametrization variation) is illustrated on figure 8.

So the results of cross check via χ^2 scan converted to cm^2 units are:

$$R_q - only: \; R_q^{2\,Limit} = -(0.451 \cdot 10^{-16}\, cm)^2$$

$$QCD + R_q: \; R_q^{2\,Limit} = -(0.491 \cdot 10^{-16}\, cm)^2$$

Table 1: Model and parametrization variations

Model variations			
Variation	Standard value	Lower limit	Upper limit
Q^2_{min} [GeV^2]	3.5	2.5	5.0
M_c(NLO) [GeV]	1.47	1.41	1.53
M_b [GeV]	4.5	4.25	4.75
f_s	0.4	0.3	0.5
$f_{s\,HERMES}$	-	0.3	0.5
$\alpha_s\,(M_Z^2)$	0.1180	0.1146	0.1220
Parametrization variations			
Q^2_0 [GeV^2]	1.9	1.6	2.2 ($M_c = 1.53$ GeV)
D_{U_v}	-		+

6 Conclusions

The H1 and ZEUS combined measurement of inclusive deep inelastic cross sections in neutral and charged current $e^{\pm}p$ scattering, was used to set limit on the possible physics beyond SM in the classical quark form factor approximation.

This model, describing possible effects due to quark substructure or finite spatial distribution of the quark charge, was used as a test scenario to demonstrate the improved limit setting procedure.

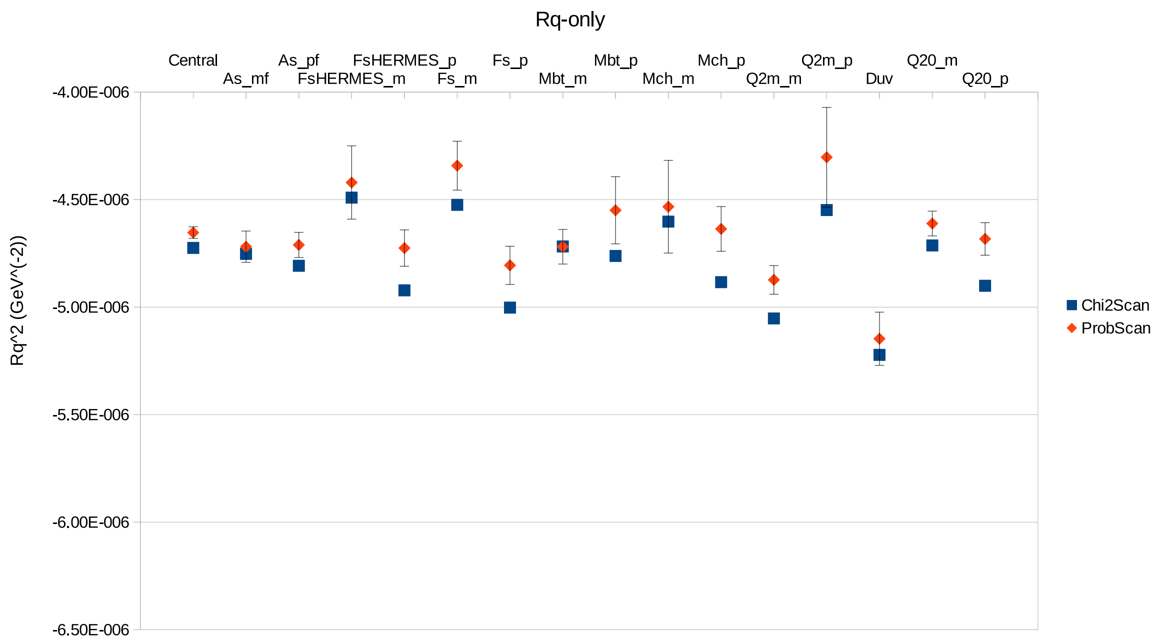


Figure 6: Comparison of $R_q^{2\,Limit}$ values between different variations for R_q-only procedure

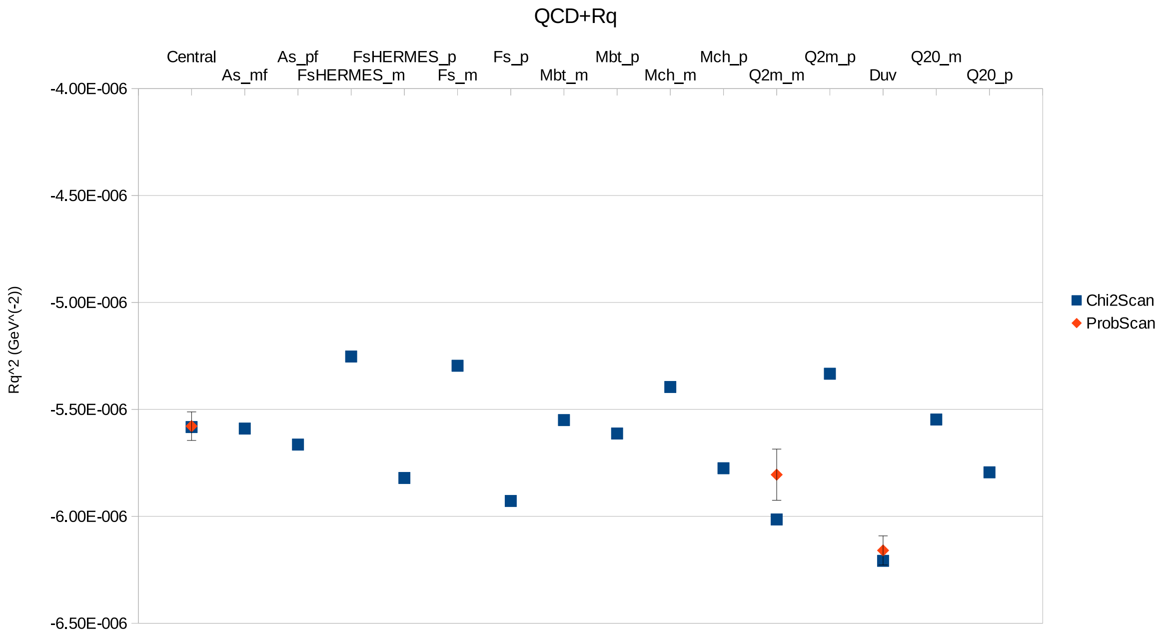


Figure 7: Comparison of $R_q^{2\,Limit}$ values between different variations for QCD+R_q procedure

The QCD analyses of HERA data, providing a set of parton distribution functions HERAPDF2.0, was extended to take into account possible cross section modification due to a finite quark charge radius.

As the same data are used to calculate PDFs and to set limit on BSM scenario, the limit setting procedure was based on a simultaneous fit of PDF parameters and R_q^2,

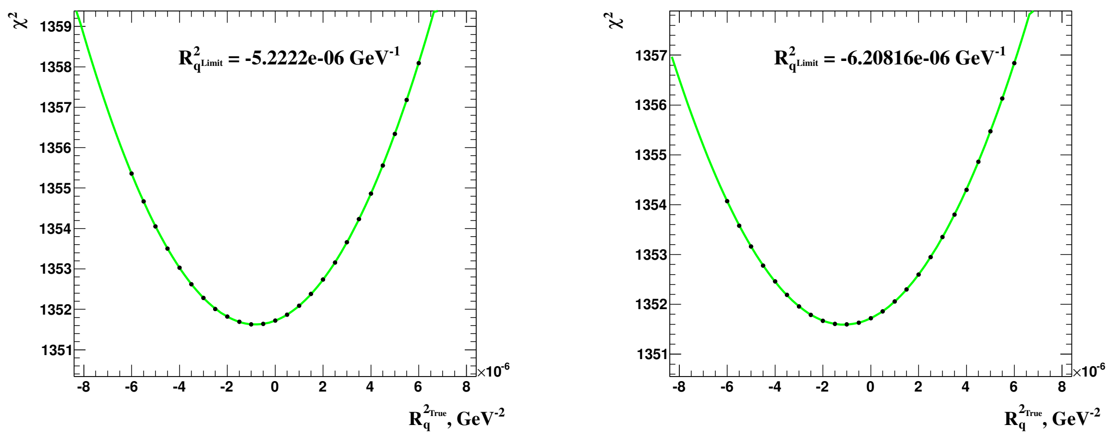


Figure 8: χ^2 scan made for D_{u_v} parametrization variation

properly taking into account possible contributions from the BSM processes in the QCD fit to the data.

In our research we successfully estimated the 95% C.L. value of the lower limit on the quark radius:

$$R_q^2 > -((0.489 \pm 0.051) \cdot 10^{-16}\ cm)^2$$

Another method, χ^2 scan, is fully consistent with the Probability scan. So it assures us that the procedure of estimating limits was set correctly.

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