



# **Optimization of Induced-phase Modulation for Sub-fs Optical Waveform Generation**

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## **Abstract**

This report describes what I have learned and did in this summer program. The content is about dispersion and some nonlinear effects such as self phase modulation and induced phase modulation when ultrashort pulses propagate through the hollow fiber. An improved simulation Matlab program with graphical user interface was finished and some simulation results and conclusions are presented in this report.

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# 1 Theory of pulse propagation in fibers

Nonlinear fiber optics is becoming an important field of science and engineering, because there are lots of significant advances related to this. Most amazing effects are associated with changes of the refractive index when the pulse propagates through certain materials, which leads to the changes of phase, amplitude and the frequency.

## 1.1 Maxwell equations

As all the other phenomena in electromagnetic field, the propagation of the optical fields in fibers is governed by Maxwell equations, which are as follows:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{Eq. 1-1-1}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{Eq. 1-1-2}$$

$$\nabla \cdot \mathbf{D} = \rho_f \quad \text{Eq. 1-1-3}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{Eq. 1-1-4}$$

and the constitutive equations:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad \text{Eq. 1-1-5}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} \quad \text{Eq. 1-1-6}$$

where  $\mathbf{E}$  and  $\mathbf{H}$  are electric and magnetic vector fields, respectively, and  $\mathbf{D}$  and  $\mathbf{B}$  are corresponding electric and magnetic flux densities. The current density vector  $\mathbf{J}$  and the charge density  $\rho_f$  represent the sources for the electromagnetic field. In the

absence of free charges in a medium as the optical fiber, we have  $\mathbf{J} = \sigma \mathbf{E}$  and  $\rho_f = 0$

and  $\sigma$  is the electrical conductivity.

Taking the curl of Eq. 1-1-1 and using Eq. 1-1-2, Eq. 1-1-5 and Eq. 1-1-6, one can eliminate  $\mathbf{B}$  and  $\mathbf{D}$  in favor of  $\mathbf{E}$  and  $\mathbf{P}$  and obtain

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad \text{Eq. 1-1-7}$$

Where  $c$  is the speed of light in vacuum and  $\mu_0 \varepsilon_0 = 1/c^2$ . To complete the description, a relation between the induced polarization  $\mathbf{P}$  and the electric field  $\mathbf{E}$  is needed. Considering that isotropic nonlinear medium such as Argon and Neon gases used in an optical hollow fiber, the second-order nonlinear polarization  $\mathbf{P}$  can be ignored and the order higher than three is also ignored. So  $\mathbf{P}$  is written as Eq. 1-1-8.

$$\mathbf{P}(\mathbf{r}, t) = \mathbf{P}_L(\mathbf{r}, t) + \mathbf{P}_{NL}(\mathbf{r}, t) \quad \text{Eq. 1-1-8}$$

To simplify Eq. 1-1-7 before solving it, Eq. 1-1-9 is used.

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E} \quad \text{Eq. 1-1-9}$$

Then the wave propagation equation can be written as

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{\partial^2 \mathbf{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2} + \mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t} \quad \text{Eq. 1-1-10}$$

To solve Eq. 1-1-10 three assumptions are necessary. First, as the nonlinear changes in the refractive index are  $< 10^{-6}$ ,  $\mathbf{P}_{NL}$  is treated as a small perturbation to  $\mathbf{P}_L$ .

Second, the optical field is assumed to maintain its polarization linearly along the x-direction perpendicular to the fiber length so that a scalar approach is valid. Third, the optical field is assumed to be quasi-monochromatic, that is the pulse spectrum, centered at  $\omega_0$ , is assumed to have a spectral width  $\Delta\omega_0$  such that  $\Delta\omega_0/\omega_0 \ll 1$ . In the slowly varying envelope approximation (SVEA) adopted here, slowly varying amplitude part and the rapidly varying carrier-wave part of the electric field can be separated in the following form:

$$E(r, t) = \frac{1}{2} E_a(r, t) \exp(-i\omega_0 t) + c.c \quad \text{Eq. 1-1-11}$$

$E_a(r, t)$  is electric field amplitude. It is more convenient to work in the Fourier domain to solve Eq. 1-1-10. By using the method of separation of variable, slowly varying function can be written as:

$$\tilde{E}_a(\mathbf{r}, \omega - \omega_0) = F(x, y) \tilde{A}(z, \omega - \omega_0) \exp(i\beta_0 z) \quad \text{Eq. 1-1-12}$$

$\tilde{A}(z, \omega - \omega_0)$  is a slowly varying function of the propagation distance  $z$  and  $\beta_0$  is a wave number. The Eq. 1-1-10 in the Fourier domain of lead to be the following two differential equations under the SVEA:

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + [\varepsilon(\omega - \omega_0) k_0^2 - \tilde{\beta}^2] F = 0 \quad \text{Eq. 1-1-13}$$

$$2i\beta_0 \frac{\partial A}{\partial z} + (\tilde{\beta}^2 - \beta_0^2) A = 0 \quad \text{Eq. 1-1-14}$$

The wave number  $\tilde{\beta}$  is determined by solving the Eq. 1-1-13 using first-order perturbation theory, in which it is separated as:

$$\tilde{\beta} = \beta(\omega) + \Delta\beta(\omega) \quad \text{Eq. 1-1-15}$$

$$\Delta\beta = \frac{k_0 \iint_{-\infty}^{\infty} \Delta n |F(x, y)|^2 dx dy}{\iint_{-\infty}^{\infty} |F(x, y)|^2 dx dy} \quad \text{Eq. 1-1-16}$$

$\Delta n$  is a small perturbation given with the absorption coefficient  $\tilde{\alpha} = 2\mu_0 \sigma c$ .

$$\Delta n = \frac{n_2 |E|^2}{2} + \frac{i\tilde{\alpha}}{2k_0} \quad \text{Eq. 1-1-17}$$

Taking inverse Fourier transform and substituting Eq. 1-1-15, Eq. 1-1-16 and Eq. 1-1-17, the Eq. 1-1-14 becomes:

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2} A + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} = i\gamma \left( |A|^2 A + \frac{i}{\omega_0} \frac{\partial}{\partial T} (|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial T} \right) \quad \text{Eq. 1-1-18}$$

$$T = t - z/v_g = t - \beta_1 z \quad \text{Eq. 1-1-19}$$

$\beta_2$  and  $\beta_3$  represents group velocity dispersion (GVD) and third order dispersion (TOD) effect.  $T_R$  is the first moment of the nonlinear response function and  $\gamma$  is the nonlinear parameter, defined as

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}} \quad \text{Eq. 1-1-20}$$

$$A_{eff} = \frac{\left( \iint_{\infty} \Delta n |F(x, y)|^2 dx dy \right)^2}{\iint_{\infty} |F(x, y)|^4 dx dy} \quad \text{Eq. 1-1-21}$$

In Eq. 1-1-18 the third and fourth term in the left side express the dispersion effect and the first term in the right hand is the self-phase modulation effect and the other two is self-steeping effect and delayed Raman response, which is not the point of this report. Eq. 1-1-18 is called generalized (or extended) NLS equation, because it resembles the Schrodinger equation with a nonlinear potential term.

To understand more about the dispersion and SPM effect, solving Eq. 1-1-18 is a must. However this nonlinear partial differential equation does not generally lend itself to analytic solutions, and in this report it is solved with numerical methods which will be described later.

## 1.2 Dispersion effect

Dispersion is an important effect in pulse propagating through a hollow fiber. When an electromagnetic wave interacts with bound electrons of a dielectric, the medium response, in general, depends on the optical frequency, because pulses at different frequencies propagate at different speed through a fiber. This feature broadens pulses even when the nonlinear effects are not important, but the spectrum is still unchanged.

Mathematically, the effects of fiber dispersion are accounted for expanding the mode-propagation constant  $\beta$  in a Taylor series about the frequency  $\omega_0$  where the pulse spectrum is centered:

$$\beta(\omega) = n(\omega) \frac{\omega}{c} = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2} \beta_2(\omega - \omega_0)^2 + \dots \text{Eq. 1-2-1}$$

$$\beta_1 = \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c} \left( n + \omega \frac{dn}{d\omega} \right) \text{Eq. 1-2-2}$$

$$\beta_2 = \frac{1}{c} \left( 2 \frac{dn}{d\omega} + \omega \frac{d^2n}{d\omega^2} \right) \text{Eq. 1-2-3}$$

$n_g$  is the group index and  $v_g$  is the group velocity. Physically speaking, the envelop of an optical pulse moves at the group velocity and parameter  $\beta_2$  represents dispersion of the group velocity and is responsible for pulse broadening, which is called group-velocity dispersion (GVD). For ultrashort optical pulses, third-order dispersion (TOD) related to parameter  $\beta_3$  should also be considered and TOD is the reason for the asymmetric of the intensity profile.

Take a simple example of Gaussian pulse, defining a normalized amplitude  $U$  as

$$A(z, \tau) = \sqrt{P_0} \exp(-\alpha z/2) U(z, \tau) \text{Eq. 1-2-4}$$

The incident field is

$$U(0, T) = \exp\left(-\frac{T^2}{2T_0^2}\right) \text{Eq. 1-2-5}$$

Where  $T_0$  is the half-width (at  $1/e$ -intensity point) and in practice, it is customary to use the full width at half maximum (FWHM) in place of it and their relation is:

$$T_{FWHM} = 2(\ln 2)^{1/2} T_0 \approx 1.665 T_0 \text{Eq. 1-2-6}$$

By solving the simplified version of Eq. 1-1-18 which only has the term of dispersion effect, the amplitude  $U(z, T)$  can be deducted as well as the pulse width

$T_1$ :

$$T_1(z) = T_0 \left[ 1 + (z/L_D)^2 \right]^{1/2} \text{Eq. 1-2-7}$$

$L_D = T_0^2 / |\beta_2|$  is the dispersion length. For a given fiber length, short pulses broaden more attributed to a smaller  $L_D$ .

### 1.3 Self-phase modulation (SPM) effect

For intense electromagnetic fields, the response of any dielectric to light becomes

nonlinear, including optical fibers, so in Eq. 1-1-8,  $\mathbf{P}_{NL}$  is considered. As mentioned before, in  $\mathbf{P}_{NL}$  only the third-order is included and this originates from the third order susceptibility  $\chi^{(3)}$  and leads to the nonlinear refraction, a phenomenon referring to the intensity dependence of the refractive index:

$$\tilde{n}(\omega, |E|^2) = n(\omega) + n_2 |E|^2 \quad \text{Eq. 1-3-1}$$

The intensity dependence of the refractive index leads to a large number of interesting nonlinear effects, self-phase modulation (SPM) and induced-phase modulation (IPM) included.

SPM refers to the self-induced phase shift experienced by an optical field during its propagation in optical fibers and is responsible for spectral broadening of ultrashort pulses.

Back to Eq. 1-1-18, if only the SPM term is accounted, the general solution in form of normalized amplitude is obtained:

$$U(L, T) = U(0, T) \exp(-i\phi_{NL}(L, T)) \quad \text{Eq. 1-3-2}$$

$$\phi_{NL}(L, T) = |U(0, T)|^2 (L_{eff} / L_{NL}) \quad \text{Eq. 1-3-3}$$

Eq. 1-3-2 shows that SPM gives rise to an intensity-dependent phase shift but the pulse shape remains unaffected. Time dependence of  $\phi_{NL}$  -----temporally varying phase, implies that the instantaneous optical frequency differs across the pulse from its central value  $\omega_0$  and the time dependence of the difference  $\delta\omega$  is known as frequency chirping. With the pulse propagating down the fiber, new frequency components are generated continuously as the chirp induced by SPM increases. Thus, SPM can broaden the spectrum considerably especially with other nonlinear effects, creating even the supercontinuum generation.

#### 1.4 Induced-phase modulation (IPM) effect

When two optical pulses at different wavelengths co-propagate inside a single mode fiber, induced-phase effect is not avoidable to consider, because it will broaden the spectrum between two input frequencies. However, to generate the two-octave continuum, two pulse are supposed to be generated from one common optical source such as fundamental wave and its second harmonic wave so that they can be synthesized.

Like the SPM effect, as the optical field propagates inside the fiber, it acquires an intensity-dependent nonlinear phase shift:

$$\phi_j^{NL}(z, \omega_j) = n_2(\omega_j) \cdot \omega_j / c \left( |E_j(\omega_j)|^2 + 2 |E_{3-j}(\omega_{3-j})|^2 \right) z \quad \text{Eq. 1-4-1}$$

Where  $j = 1, 2$ . The first term is SPM and the second results from phase modulation of one wave by another co-propagating wave and is responsible for IPM. The factor 2 on the right-hand side of Eq. 1-4-1 shows that IPM is twice as effective as IPM for the same intensity. However, two pulse propagates at different group velocities and this mismatch plays an important role as it limits IPM interaction as pulses walk off from each other.

Adding the IPM effect and ignore self-steeping and delayed Raman response, Eq. 1-1-18 can be written as:

$$\frac{\partial A_1}{\partial z} + \frac{\alpha_1}{2} A_1 + i \frac{\beta_{21}}{2} \frac{\partial^2 A_1}{\partial T^2} - \frac{\beta_{31}}{6} \frac{\partial^3 A_1}{\partial T^3} = i \gamma_1 \left( |A_1|^2 + 2|A_2|^2 \right) A_1 \text{ Eq. 1-4-2}$$

$$\frac{\partial A_2}{\partial z} + \frac{\alpha_2}{2} A_2 + i \frac{\beta_{22}}{2} \frac{\partial^2 A_2}{\partial T^2} - \frac{\beta_{32}}{6} \frac{\partial^3 A_2}{\partial T^3} = i \gamma_2 \left( |A_2|^2 + 2|A_1|^2 \right) A_2 \text{ Eq. 1-4-3}$$

Where:

$$\gamma_j = \frac{n_2 \omega_j}{c A_{eff}}, \quad j=1,2 \text{ Eq. 1-4-4}$$

The definition of  $A_{eff}$  is given in Eq. 1-1-21 and this parameter is known as the effective mode area, depending on fiber parameters such as the core radius and the core mode. It is usually given by comparing theoretical and experimental results, and in my simulation, it is set to be  $A_{eff} = 0.6\pi a^2$ ,  $a$  is the radius of the fiber.

$n_2$  is the nonlinear refractive index given by  $n_2 = K \cdot \bar{n}_2$ ,  $\bar{n}_2$  is its value for  $\lambda = 790nm$  at  $p_0 = 1atm$  and  $T_0 = 0^\circ C$  and  $\bar{n}_{2Ar} = 9.8 \times 10^{-24} m^2 / W$ ,  $\bar{n}_{2Ne} = 0.74 \times 10^{-24} m^2 / W$ . The calculation of ratio  $K$  can be seen in Reference 8.

For the dominant propagation mode  $EH_{11}$  the mode-propagation constant  $\beta$  and the loss constant are given by Reference 9:

$$\beta = \frac{2\pi}{\lambda} \left[ 1 - \frac{1}{2} \left( \frac{2.405\lambda}{2\pi a} \right)^2 \right] \text{ Eq. 1-4-5}$$

$$\frac{\alpha}{2} = \left( \frac{2.405}{2\pi} \right)^2 \frac{\lambda^2}{2a^3} \frac{v^2 + 1}{(v^2 - 1)^{1/2}} \text{ Eq. 1-4-6}$$

$v$  is the ratio of the linear refractive index of the external medium to that of the internal media and is assumed to be 1.45. To obtain  $\beta_2$  and  $\beta_3$ , it is necessary to calculate the derivatives of  $\beta$ .

Physically, because of optical nonlinear effect, the intense of pulse electric field which is time dependent, causes an increase in the refractive index, leading to a time dependent phase change. Then new frequency will be generated because it is related to the derivative of this phase.

## 1.5 Pressure gradient

Ultrashort laser pulses find their applications in diverse fields, most of which expected the pulse to be shorter and with higher energy. It has long been recognized as a powerful technique to realize the pulse compression using gas-filled hollow fibers, which are small tubes like filters and are not the common fiber technically.

Good quality spectrum and phase of the output pulses are necessary in order to further compressed while unfortunately, due to the potential occurrence of self-focusing and ionization, pulse energy delivered into a hollow fiber in practical applications is limited. This can be improved by the pressure gradient method, delaying self-focusing and maintaining phase modulation.

The threshold of the input pulse energy should be below or at the level where ionization starts to occur and the threshold is given as:

$$P_c = \frac{\lambda^2}{2\pi n_2} \text{ Eq. 1-5-1}$$

Where  $\lambda$  is the central wavelength of the pulse and  $n_2$  is the nonlinear refractive index at  $p$  and  $T$  :

$$n_2 = K \cdot \bar{n}_2 \cdot \frac{p(z)T_0}{p_0T} \text{ Eq. 1-5-2}$$

The pressure of the hollow fiber is set to be  $p_b$  (nearly zero) at the optical input side to a maximum  $p_e$  at the output side. The distribution of the gas pressure flowing inside the hollow fiber is given by:

$$p(z) = \sqrt{p_b^2 + \frac{z}{L}(p_e^2 - p_b^2)} \text{ Eq. 1-5-3}$$

From the equations, it is obvious that the beginning  $n_2$  is small so the critical power is higher, permitting the entrance of higher pulse energy.

As the pressure changes with  $z$ , other parameters as  $\beta_2$ ,  $\beta_3$  and  $n_0$  also changes. However, these dependences is extremely slow compared to not only the spatial variation of the carrier wave with respect to  $z$  but also compared to amplitude, therefore Eq. 1-4-2 and Eq. 1-4-3 are still reasonable.

## 2 Simulation method

### 2.1 Split-step Fourier method

The NLS equation (Eq. 1- 4-2 or Eq. 1- 4-3) is a nonlinear partial differential equation that does not generally lend itself to analytic solutions so a numerical approach is therefore often necessary. Split-step Fourier method has been used extensively to solve the pulse-propagation problem in nonlinear dispersive media and so does this simulation.

Using split-step Fourier method, it is necessary to write Eq. 1- 4-2 in the form:

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A \text{ Eq. 2- 1-1}$$

$\hat{D}$  is a differential operator that accounts for dispersion and losses within a linear medium and  $\hat{N}$  is a nonlinear operator that governs the effect of fiber nonlinearities.

$$\hat{D}_1 = -\frac{i\beta_{21}}{2} \frac{\partial^2}{\partial T^2} + \frac{\beta_{31}}{6} \frac{\partial^3}{\partial T^3} - \frac{\alpha_1}{2} \text{ Eq. 2- 1-2}$$

$$\hat{N}_1 = i\gamma_1 (|A_1|^2 + 2|A_2|^2) \text{ Eq. 2- 1-3}$$

$\hat{D}_2$  and  $\hat{N}_2$  have the similar expression.

Though in general, dispersion and nonlinearity act together, the split-step Fourier method obtains an approximate solution by assuming that they act independently over a small distance  $h$ :

$$A(z+h, T) \approx \exp(h\hat{D}) \exp(h\hat{N}) A(z, T) \text{ Eq. 2- 1-4}$$

In Fourier domain, operator  $\partial/\partial T$  is replaced by  $-i\omega$ . As  $\hat{D}(i\omega)$  is just a number in the Fourier space, the evaluation of equation is straight forward.

### 2.2 Graphical User Interface (GUI)

My project in programming is based on part of the elementary program supervisor gave me. The original version already has some basic structure of the simulation but it is quite slow and does not make full use of Matlab's advantages in matrix calculation. During the summer program, I made some progress in the following aspects:

- Write a graphical user interface, making the program more friendly to other users. With the GUI, one can easily do simulations with different pulse width, initial delay time and energy. The results are presented in form of graphs and also saved in files. The outlooks of it are as follows:

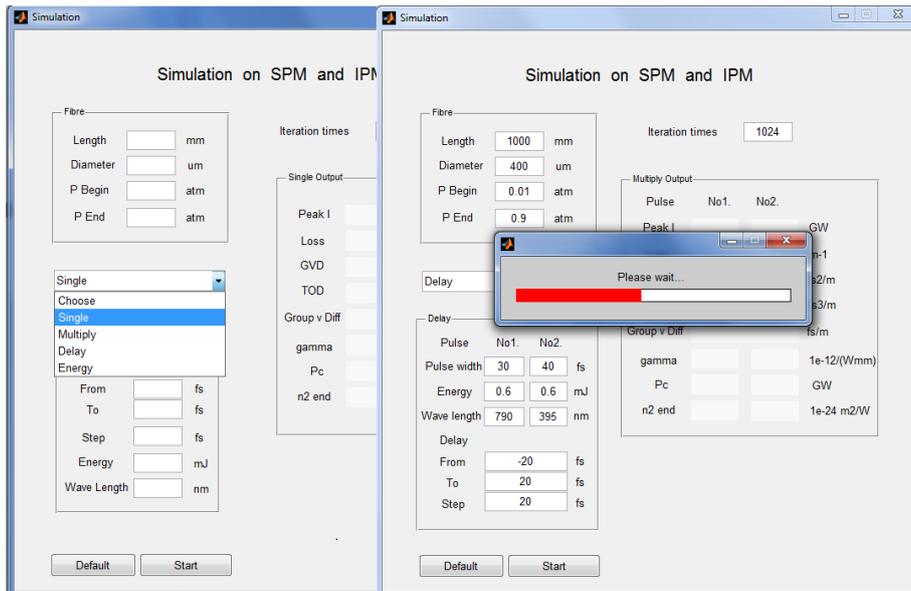


Fig.2-1 Graphical User Interface I

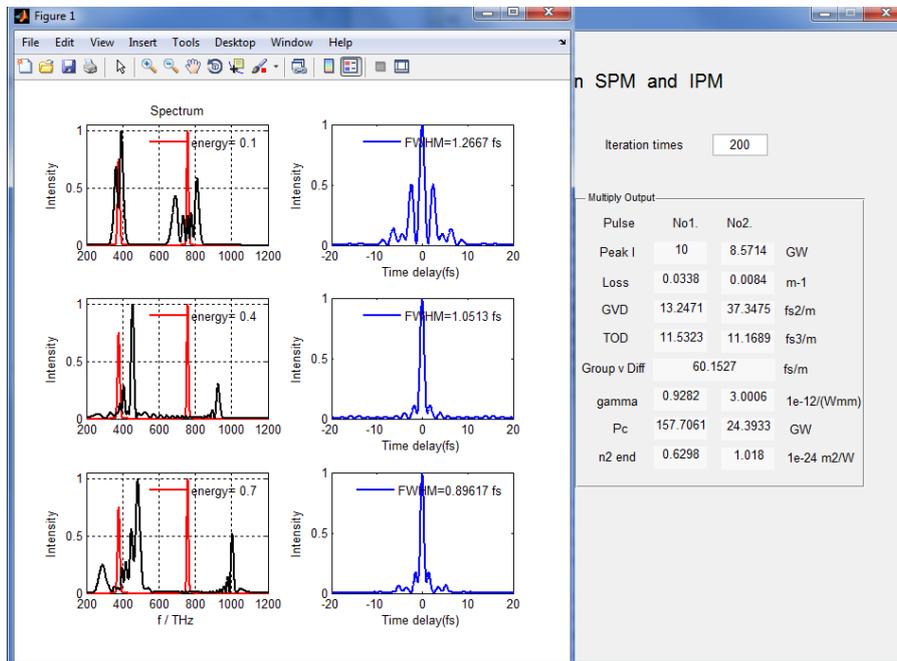


Fig.2-2 Graphical User Interface II

Fig.2-1 shows the interface waiting for input parameters and the one when program is running. Fig.2-2 shows the interface when running is finished, some important results are displays in the picture form and some are given in numbers in the right side boxes. Generally, it is quite easy to use.

- Change most of the loops into higher diversion in matrices and enable the similar calculations to run simultaneously.
- Combine all the parameter calculation files so that there is no need to change between different input conditions manually.
- Research on methods in calculating some parameters as to make the spectrum results in consistent with some papers.

The outline of the program structure can be seen in Fig.2-3.

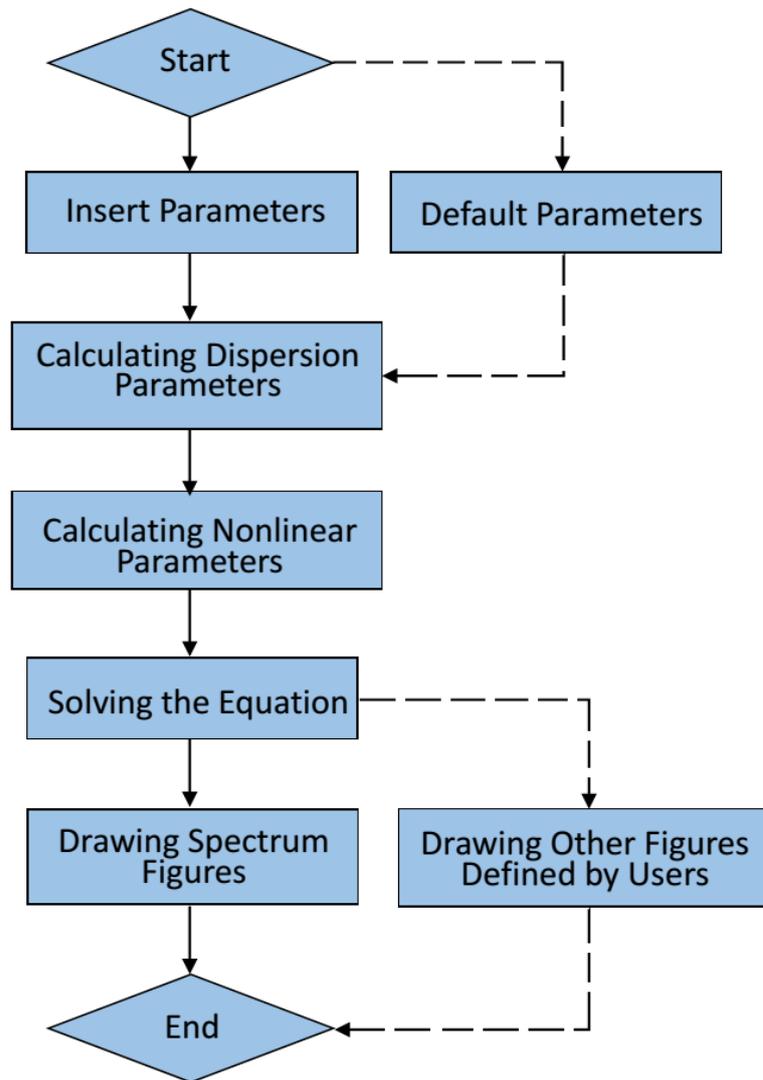


Fig.2-3 Structure of Simulation Program

### 3 Simulation results

Based on the matlab program described in last chapter, some related simulations are made in both single pulse and two pulses conditions with pressure gradient. Their input parameters are as follows:

Table 3-1 Parameters of Pulses

	Single pulse	Two pulses	
Wavelength /nm	800	790	395
Frequency /THz	374.7	379.5	759.0
Input pulse energy /mJ	0.6	0.6	0.6
Input pulse width /fs	30	30	40
Initial delay /fs	----	-20	

Parameters of the hollow fiber is in consistent with experimental conditions given

as:

Table 3-2 Parameters of Hollow Fiber

Length/m	Diameter/ $\mu\text{m}$	Input pressure/atm	Ending pressure/atm	Gas type
1	400	0.01	0.9	Argon

If parameters are not mentioned to be changed in the following results part, then they are the same as the table values above.

### 3.1 Energy dependence

Higher energy has always been a pursuit of the output pulses, so it is necessary to know if input energy is the higher the better. The results of the simulations are in Fig.3-1, left side is when there is only one input pulse and right side is when there are two. The red line represents the spectrum of the input pulses.

From Fig.3-1, it is clear that in case of single input pulse, higher energy brings better broadening effect but the spectrum become asymmetric, resulting from the third-order dispersion effect.

For two input pulses case, there is a huge gap between two pulses at 0.1mJ, but both of the respective spectrum is broadened, comparing to the red input ones, because of the SPM effect. Keeping increasing the energy, while the spectrum shifts blue, IPM can be seen. From 0.1mJ to 0.6mJ, the central part between two pulses becomes better but it goes down with the increase of energy due to some other nonlinear effects, resulting from the nonlinear refractive index changes brought by energy increasing.

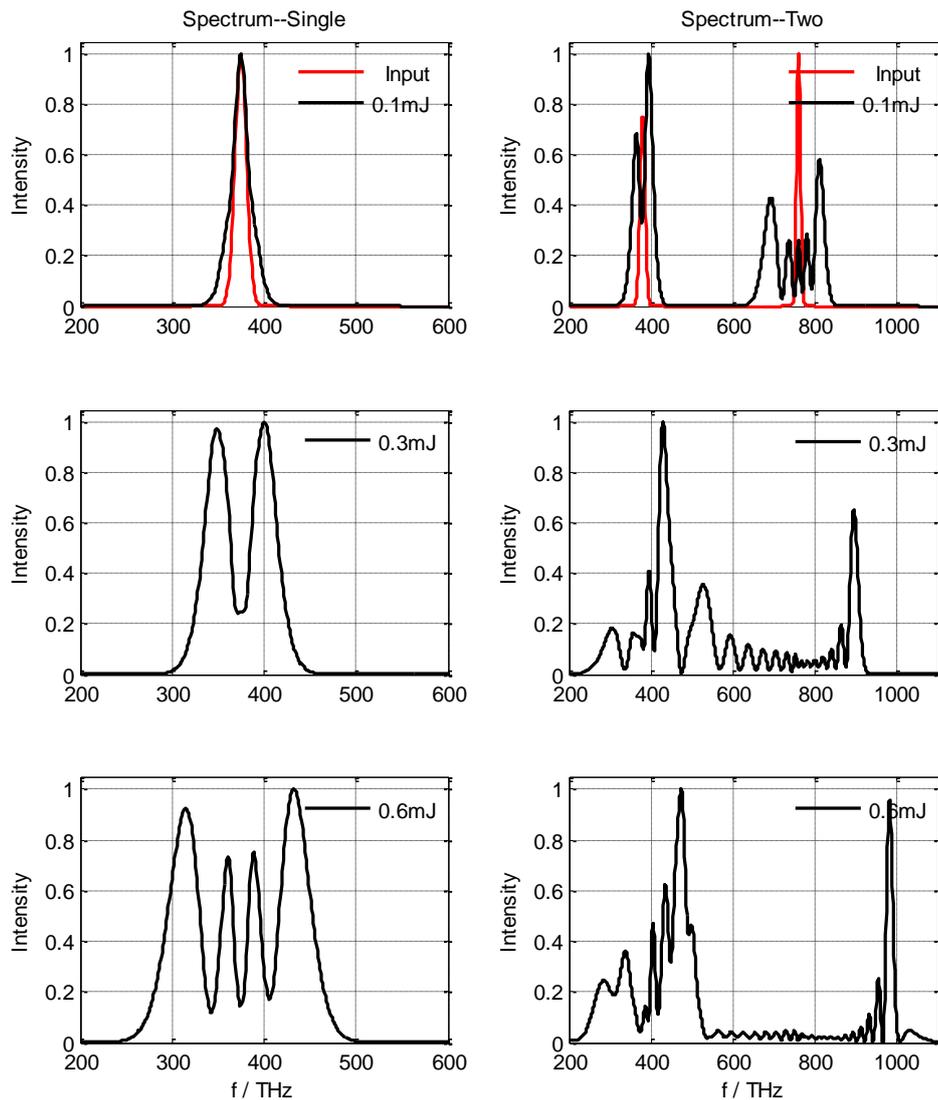


Fig.3-1 Energy Dependence

### 3.2 Ending pressure dependence

Keeping the input pressure to be 0.05 atm and changing the ending pressure from 0.1 atm to 0.9 atm, the simulation results are in Fig.3-2, where the left is for one and right is for two. Essentially, higher pressure means there are more gas molecules in the hollow fiber, which enables more interactions between pulses and gas. Thus, SPM and IPM will be strengthened, leading to broader spectrum. However, when the pressure keeps going up, ionization and other nonlinear effects should be considered and then IPM effect, which broadens between two pulses become worse.

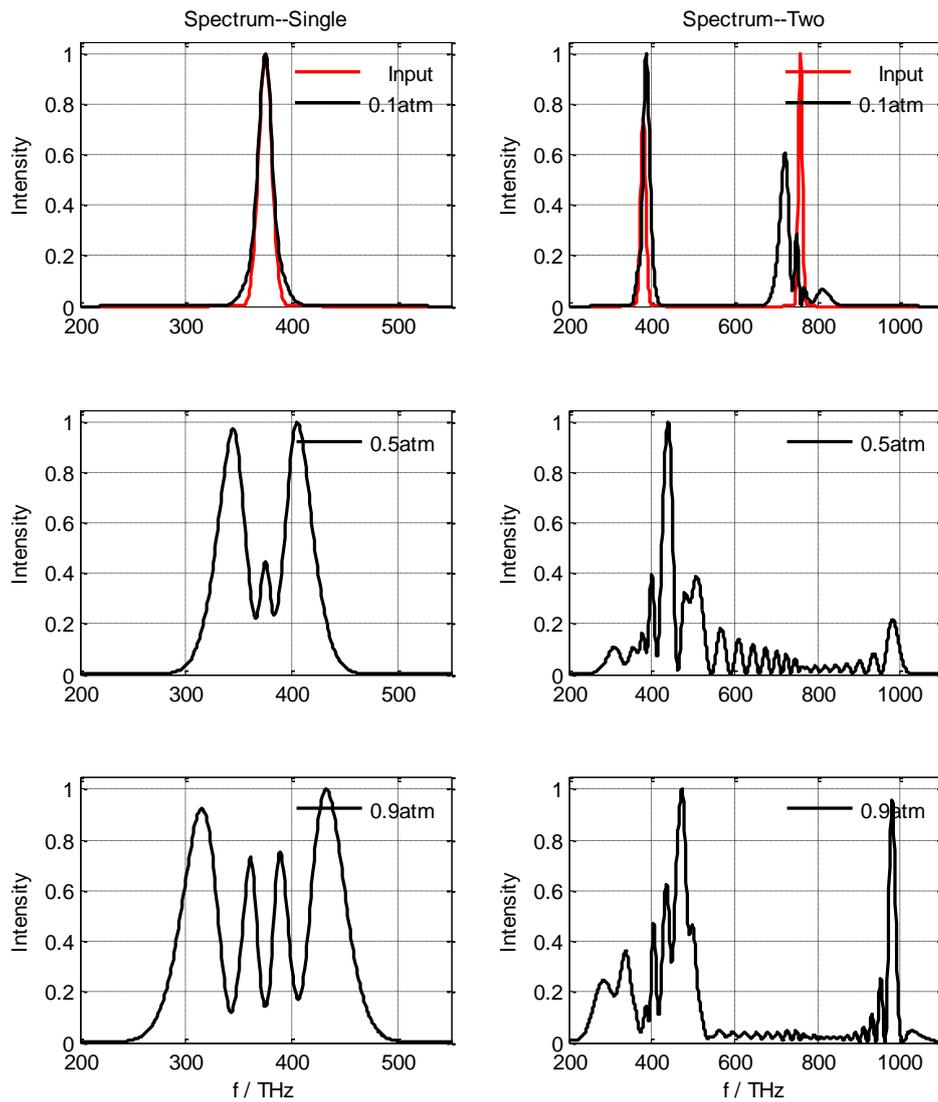


Fig.3-2 Pressure Dependence

However, if only SPM effect is considered which means the single pulse condition, the positive relation between broadening effect, in other way---bandwidth (length between 1% peak power on left and right side of the spectrum), and pressure, energy is quite good in experimental conditions, showed in Fig.3-3.

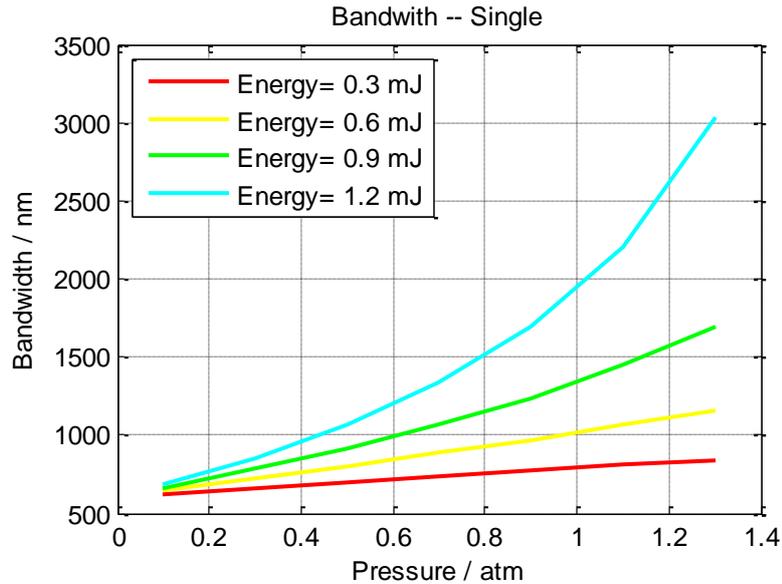


Fig.3-3 Bandwidth

To some extent, in the above conditions, the bad effects of high energy and high pressure have more impacts on IPM rather than SPM. This may be attributed to the coefficient 2 in Eq. 1- 4-2 and Eq. 1- 4-3 in front of the IPM term. Larger weight means larger influence.

### 3.3 Pulse width dependence

Initial pulse width is another variable. Three different kinds of pulse width are chosen and the results can be seen in Fig.3-4.

From the figure, it is quite clear that the spectrum becomes narrow when increasing the pulse width which is closely related to the peak power. The increasing of pulse width means the decreasing of peak power, because energies of each of them are set to be 0.6mJ, and the changes of the spectrum can find the explanation in the energy part.

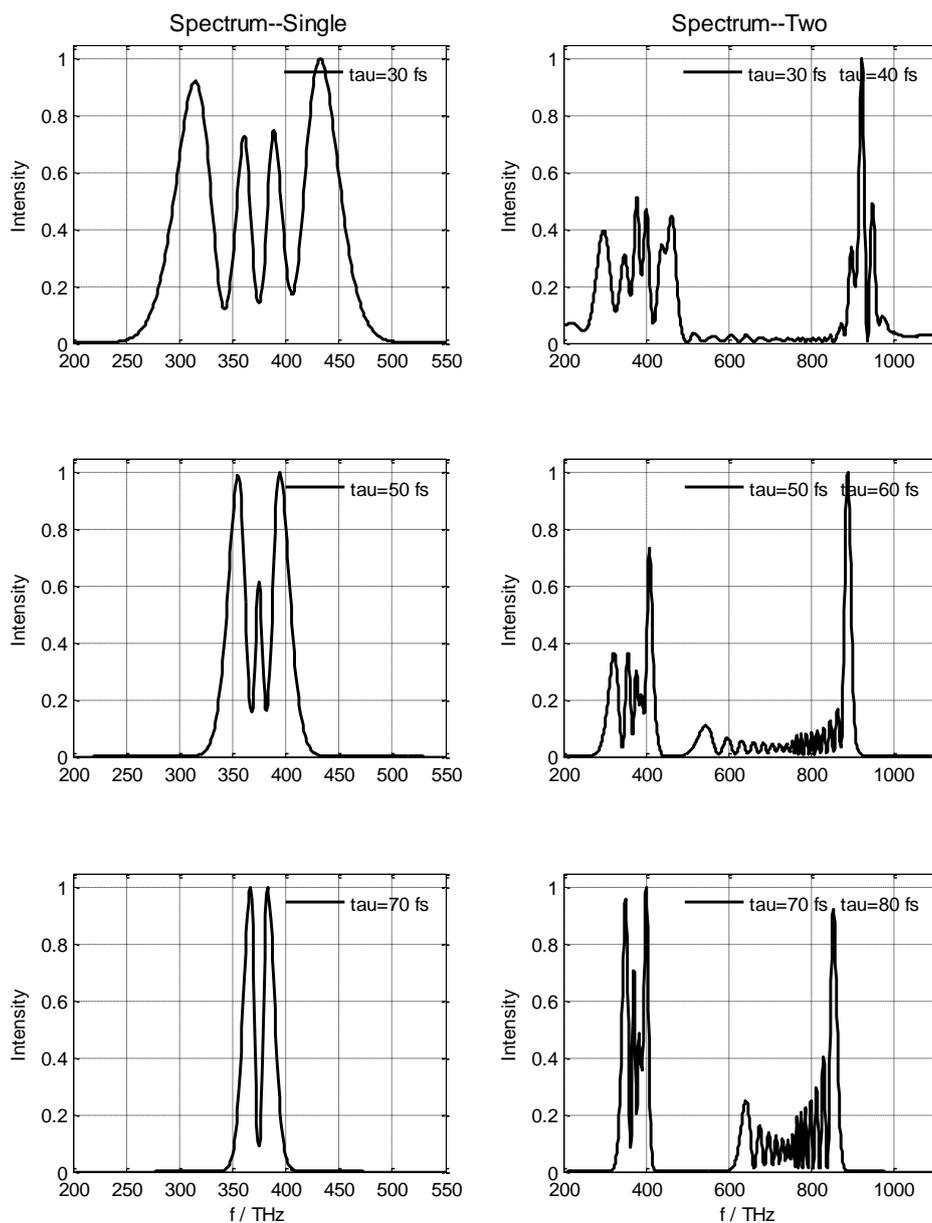


Fig.3-4 Pulse Width Dependence

### 3.4 Initial delay dependence

The initial delay time determines how the input two pulses interact with each other, because in case of IPM the frequency shift depends on the group-velocity difference between pulses and on the fiber length. When delay time is  $-20$ fs which means two pulses meet at the end of the hollow fiber (higher frequency one—pulse2 leads), the trailing edge of pulse2 interacts with the leading edge of pulse1. So because of the IPM effects, pulse2 is frequency-shifted slower and pulse1

higher, forming a larger overlap of the two pulses.

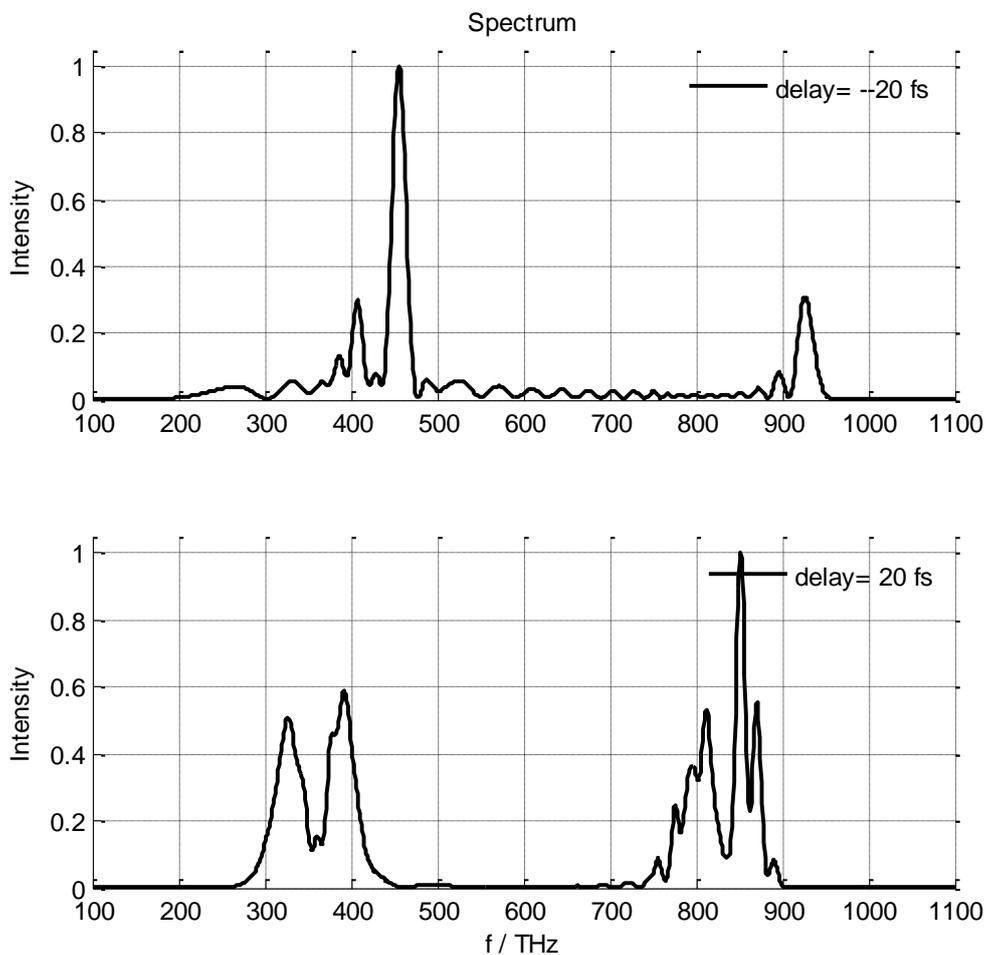


Fig.3-5 Delay Time Dependence

When initial delay time is 20fs, two pulses meet before the hollow fiber. A huge gap between the two pulses can be seen in Fig.3-5, because two pulses propagate independently in the fiber only within SPM effects respectively.

## 4 Summary

Numerical methods in nonlinear optical fields have been widely used with the development of nonlinear applications. From the program based on NLS equations of my project, some conclusions are also arrived.

Higher energy and ending pressure may be useful to broaden the spectrum in some ranges but it also has bad effects due to other nonlinear effects. Other parameters such as pulse width and initial delay time also impacts on broadening effects and they have relations to other variables, so finding a set of proper and better parameters of input pulse or pulses is quite essential.

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