Numerical Simulation of a Balanced Optical Microwave Phase Detector (BOMPD) in CppSim

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Abstract

A report on a project done at the DESY summer school during the summer of 2014 is given. The simulation of a Balanced Optical Microwave Phase Detector (BOM-PD) was done in a custom system simulation package CppSim. A simple but robust model was made allowing for qualitative simulation of BOMPD properties and noise performance with noise sources coming from the VCO, photo-detection proccess and from the optical input. The model is easily extensible so a precise simulation of current experimental results is possible.

Contents

1 Introduction

In this report I describe the project I did during the DESY student summer programme in Hamburg. The project was concerned with a simulation of a Balanced Optical Microwave Phase Detector (BOMPD) used for low noise synchronization of a tunable microwave signal source to a mode-locked loser.

Ultra-stable microwave signals are needed in next generation X-ray FELs, such is the European XFEL being built in DESY. These large-scale X-ray FEL facilities require extremely stringent timing precision and accuracy on large scales. This is done by using an ultra-low-jitter mode-locked laser as a master oscillator and syncrhonizing it with different sections of the system with timing-stabilized links. Figure 1. shows the schematic layot of the timing syncrhonization

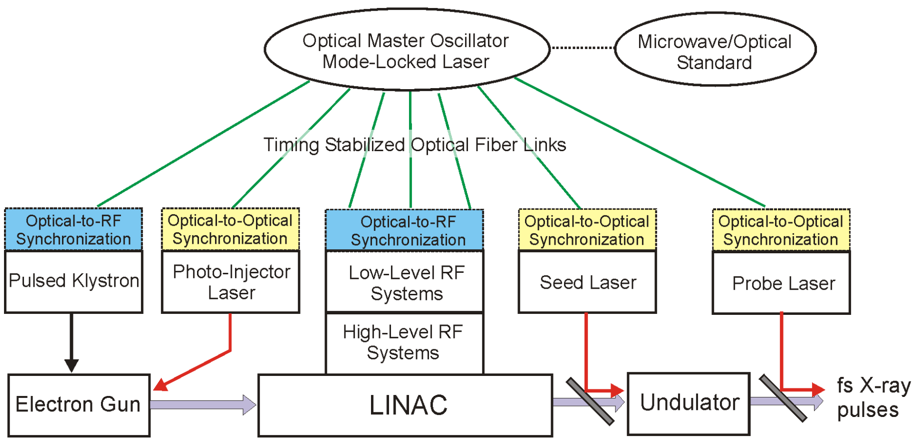

Figure 1: A schematic of a large-scale Free Electron Laser (FEL) system. The master oscillator mode-locked laser is used as a timing reference for the different parts of the system. Optical-to-microwave synchronization is used to produce a highly stable RF signal used to drive the electron beams through accelerator sections.

system that is be used for large-scale X-ray FELs. The starting point of the system is the mode-locked laser locked to an optical or a microwave standard which serves as the optical master oscillator of the system. Different lasers used in the FEL can be synchronized by optical-to-optical synchronization with the master laser. Optical-to-microwave stabilization is needed to syncrhonize the RF sources which drive the accelerator sections. The electron-beam dynamics is controlled by the microwave fields in the accelerator cavities and therefore a highly stable microwave signal, tightly synchronized between different accelerator sections is needed.

In Section 2. of this report I present an introduction to Microwave-signal synthesis from mode-locked lasers. In Section 3. I describe the operation of the Balanced Optical Microwave Phase Detector (BOMPD). In Section 4. I present methods and tools used to write an accurate simulation of a BOMPD. In Section 5. I describe the results obtained using this simulation and finally in Section 6. a conclusion on the project is given.

2 Microwave-signal synthesis from mode-locked lasers

Ultra-short optical pulse generation from mode-locked lasers has been long used for studies of physical phenomena on picosecond and femtosecond time scales. In the frequency domain, the optical pulse train from a mode-locked laser looks like a series of equaly spaced lines, which is called a frequency comb. The distance between these lines is set by the repetition rate of the pulses in the time domain, f_{rep}. Ultra-low timing jitter of optical wave trains can be used for the high-precision generation, distribution, measurement, and synhronization of optical and microwave signals. Optical pulse trains generated from standard, passively mode-locked Er-fiber lasers can easily achieve sub-10-fs and sub-fs timing jitter [1], while TiSa lasers can achieve even lower jitters.

2.1 Methods of microwave-signal synthesis from mode-locked lasers

Transfer of timing stability from the optical domain to the electronic domain is a challenging task [1]. The simplest way to generate a microwave signal from an optical pulse is direct photodetection using a photo diode which output is filtered to a desired frequency. This method suffers from excess phase noise (as given in [2]), which originates from aplitude-to-phase conversion in the photodetectors and microwave mixers. Then power drifts of the optical pulse train converts into excess timing jitter and drifts, and therefore a long term stability of the signal is degraded. Amplitude-to-phase conversion factor of photodiodes measured in [2] ranges from 1 to 10ps/mW, depending on the photodiode used. For and Er-doped fiber liser with $0.03\%_{rms}$ relative intensity noise this lead to 5-fs excess jitter when 10 mW of power is applied to the photodiode. But it is possible to limit the short-term jitter to 1-fs levels by a supression of excess noise sources. Still long-term jitter is large and for example measured to be 56ms in 100s as shown in [2].

To by-pass the amplitude-to-phase conversion an optoelectronic phase-locked loop (PLL) can be used synchronizing the optical pulse train with a high-quality voltage-controlled oscillator (VCO). The key issue here becomes a development of a drift-free, low-jitter phase detector to extract the phase difference between the input optical wave train and the output microwave signal in the optical domain. Thus a timing error information is obtained in the optical domain and then fed into a VCO by a photo detection proccess which avoids the timing/phase drift of the photodetection and amplification stages. The schematic representation of a Opto-electronic Phase-Locked Loop (PLL) is given in Figure 2.

One of the methods used is the microwave synchronization using balanced intensity detector. In this case the phase error is converted into intensity imbalance between two optical pulses

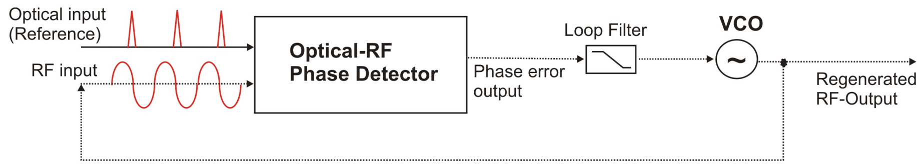

Figure 2: A schematic representation of an Opto-electronic Phase-Locked Loop.

from two differentially modulated intensity modulators and this information is used to close the loop. This was first described in [3].

3 BOMPD

The synchronization of microwave signals with optical pulse trains based on the balanced-intensity detection provides a very simple interface between the optical and electronic domains but it is difficult to scale the noise performance because of the limited optical power applicable to the photodiode and the drift in beam combining and splitting [1]. The Balanced Optical Microwave-Phase Detector (BOMPD) avoids these problems by using a differentially biased Sagnac fiber loop first described and implemented in [4].

3.1 Description of the BOMPD

A schematic of a BOMPD is given in Figure 3. Inputs to the BOM-PD are the optical pulse train at a frequency f_r and the microwave signal with a frequency f_0 generated by a VCO. In the locked state the frequency of the VCO is locked to a comb-line in the frequency spectrum of the mode locked laser such that $f_0 = Nf_r$ where N is an odd integer. As a black-box the BOMPD produces a voltage signal ΔV proportional to the phase difference θ_e between the optical pulse train and the microwave signal when the phase difference is small, $\Delta V = K_{PD} \cdot \theta_e$. This error signal is fed into a feedback Proportional-Integral controler (PI controler) which drives the VCO input and this closes the PLL operation.

The Sagnac-loop fiber integral in the BOMPD consits of a 50:50 coupler which splits the input optical signal and produces two counter-propagating pulses, fiber loop, and an unidirectional phase modulator which phase-shifts only clockwise propagating pulse. These pulses interfere on the output of the Sagnac loop, and the output optical power is given as $P_{out} = P_{in} \sin^2(\Delta\phi/2)$ ([5]), where $\Delta\phi$ is the net phase shift between two propagating pulses and is proportional to the voltage applied to the phase modulator. When no RF signal is applied to the phase modulator, the output optical power is 0 i.e. the pulses interfere destructively.

A reference signal is isolated from the input optical pulse train, by photo-detection and filtering the signal at the odd-harmonic of the optical pulse train ($Nf_r = (2n+1)f_r$). This signal is frequency divided by a factor of two and and then applied to the phase modulator. The clockwise sub-pulse will obtain alternating phase shifts because it is aligned with maxima/minima in the voltage signal alteratingly, but the output optical power will still be zero as evident from the Sagnac loop transmission function (Figure 4.).

Next step is to apply a sum of the reference signal and the VCO outpur to the phase modulator. Then the output optical pulse train is modulated by a half-repetition frequency $f_r/2$ because the phase modulation by the VCO signal will have the same polarity on adjecent pulses which moves the biasing points to left or right in the Sagnac loop transmission function (Figure 4.) depending on whether θ_e is smaller or larger than zero. When phase error is small then the modulation depth of the output optical signal is proportional to θ_e as derived in [5]. In this step the phase/time mismatch was converted to amplitude modulation in the optical domain, and

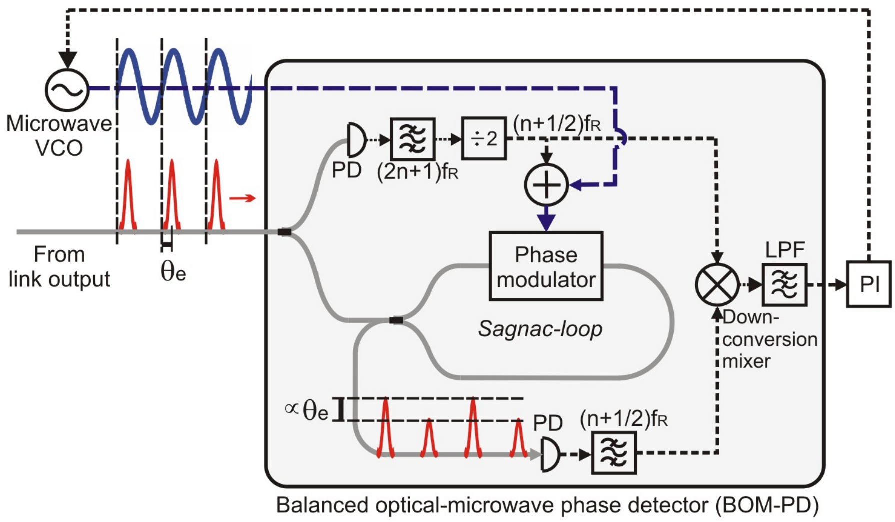

Figure 3: Schematic representation of a Balanced Optical Microwave Phase detector.

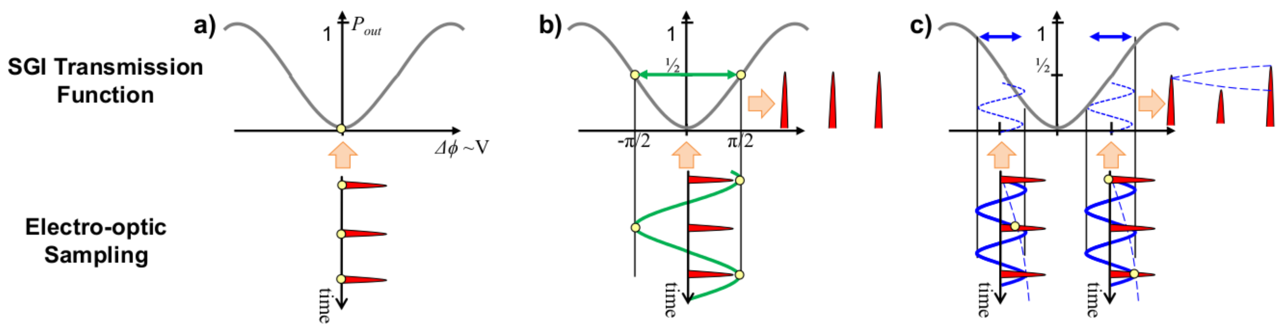

Figure 4: Sampling of the input optical signal by a Sagnac-loob fiber. On the top of each picture a graph of Sagnac-loop transmission function $P_{out} = P_{in} \sin^2(\Delta\phi/2)$ is shown. a) no applied signal to the phase modulator and the output optical power is zero. b) reference signal of frequency $(n + 1/2)f_r$ is applied producing no modulation c) added VCO signal to reference produces amplitude modulation proportional to the phase error.

therefore it is not susceptible to the amplitude-to-phase conversion in the photodetection process. To get the error signal in the electronic domain to drive the VCO, the photodetected output from the Sagnac loop is first band filtered to the reference frequency and then downconverted by mixing it with the reference signal and then passing it through a low-pass filter. This error is input to the PI controler which drives the VCO, and this completes the PLL operation.

3.2 Past experimental results

As first reported in [4] and in [5], the BOMPD shows excelent noise performance, with a short term jitter of 12.8 fs and a long-time jitter of 48fs with measurement times up to 4 hours. Later results as reported in [6] were that short-term jitter is 4.4 fs integrated from 1 Hz to 1 MHz, which is higher than the best direct photodetection result. On the other hand the long-term measurement shows a sub 10 fs jitter which corresponds to a relative stability of 10^{-19}. The newest results, [7], bring these results to less than 2 fs RMS jitter in short term and less than 1 fs RMS jitter in the long term. As shown in [5] the fundamental noise floor to the BOMPD is limited by the shot noise in the photodiodes but in the real experiments this is usually determined by other noise sources and non-linearities such as those in amplifiers.

4 Method

The primary motivation for the simulation of the BOMPD was to make a workable model which could be used in future experiments as a benchamrk on what could be achieved in the experiment. The simulation package used is CppSim, a free package for system simulation written by Michael H. Perrott. It is particulary suited for simulation of PLLs using special techniques for accurate simulations of PLLs described in [9]. The underlying code is written in C++ allowing for fast execution times. CppSim provides a large library of C++ classes useful for system simulation and signal analysis. Several classes included provide easy analysis of simulated signals, elaborate control over input and output parameters, sampling of the output signals and edge detection for jitter calculations.

4.1 CppSim

CppSim package includes a GUI program Sue2, a signal viewing program CppSimView and a set of C++ classes which describe the basic elements that can be used in CppSim simulations. The Sue2 program is a CAD-like program for graphicaly constructing a system using pre-defined or user-defined blocks. The schematic made is then converted to a "netlist" format, which describes the included blocks in a hierarchical manner. Then on running the simulation CppSim converts this file into C++ code using the definitions of primitives given by a library of C++ classes. Common system blocks are included like VCOs, amplifiers and filters which enable easy building of more complex objects. The internal approach in CppSim to simulating systems is to use nodal analysis with output at a node depending on the input and the internal state of the node. At each simulation time-step the states of nodes are calculated sequentaly.

4.2 Implementation of the BOMPD

The modules used in the BOMPD simulation were mostly already implemented in the CppSim, with some modules implemented manually. First step was to make a perfect model of a BOMPD without any noise to prove that it is possible to simulate the BOMPD in CppSim. Next imperfections were added such as the VCO noise, photo-diode noise and optical source noise. The main modules included were:

- Optical source element is producing a train of single point pulses at the repetition frequency f_r. It was taken from the example library OpticalPLL_Example included with CppSim. The power of the optical source is a parameter of the element. It uses the signal_source element which can generate sine, square or pulse outputs. It also allows for simulating noise by inputing the phase imput into the "phase" input of the element.

- Sagnac loop was implemented as a simple element with two inputs; optical one which is modulated through a perfect mixer by the $\sin^2$ of the half of the electrical input, which replicates the transmission function of the Sagnac loop.

- The band-pass filter and low-pass filter were already implemented in CppSim.

- VCO element used was one already implemented in CppSim, both noisless and noisy version. The photodiodes were simple gain elements in the perfect model.

- PI controler is implemented as an element adding the integrated and proportional signals. The gain parameter is K_p or the proportional parameter, and the integrator parameter is K_i or the integration parameter of the PI controler.

The view of the Sue2 modeling program and the ideal model of the BOMPD is presented in Fig. 5.

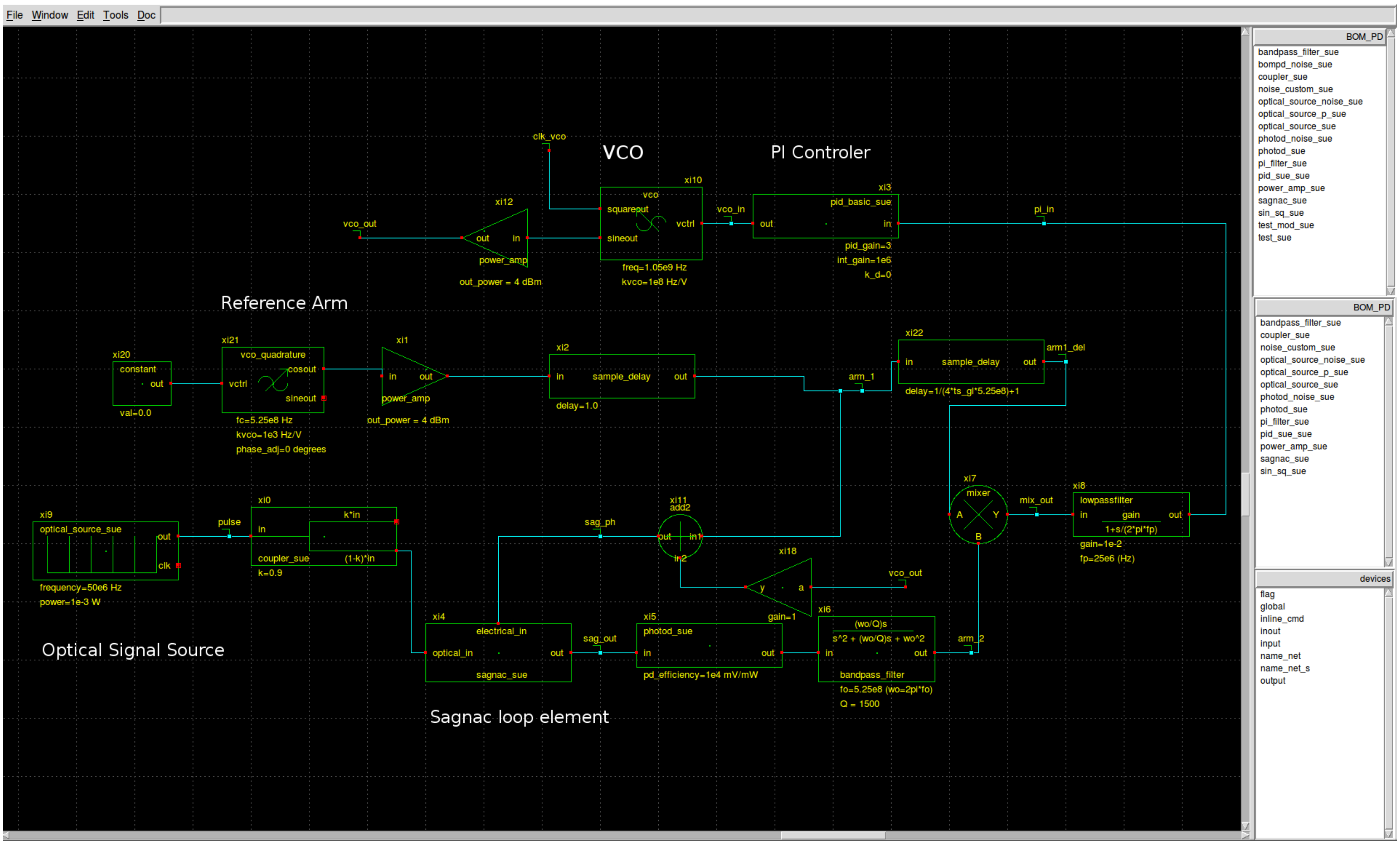

Figure 5: The view of the Sue2 modeling program belonging to the CppSim package. In the central view is a perfect model of a Balanced Optical Microwave-Phase Detector (BOMPD) consisting of multiple elements representing optical and electrical components of a real BOMPD.

Main parts of the model are the reference arm, the Sagnac loop arm and then the phase error output of the BOMPD which is input into the PI controler and then into the VCO. In the experimental implementation of a BOMPD the reference arm uses a digital frequency divider on a filtered input optical signal. In our simplified model this was deemed unnessecery to implement, so a reference was provided by a VCO at the desired reference frequency. It was made sure that signals in the simulation have a sensical physical units. The optical signals were adjusted by the optical source power. The electrical signals output from the main VCO and the reference VCO were adjusted by a power_amp element in units of dBm. The conversion coefficient of the photo-diode was given in units of mV/mW of input optical power. Band-pass and low-pass filters transfer function were unit-less.

4.3 Implementation of noise in the BOMPD

CppSim provides an implementation of classes and elements which provide random sequences with white and 1/f noise spectral power densities. The parameters specified in the white noise element are offset frequency f_{off} and the noise at the offset frequency, n_{off} in dBc/Hz. The output of the is noise varation multiplied by a random Gaussian variable with a mean of 0 and a variance of 1. The 1/f noise is implemented in the class `OneOverfPlusWhiteNoise` [10]. It produces a random Gaussian sequence that includes white noise and 1/f noise in the low frequencies. The 1/f noise is obtained by filtering the white noise with 6 pole-zero pairs each separated 7.5 decades apart. Corner frequency f_{corner} defines the frequency at which the power of the white noise component is the same as the power of the 1/f noise.

The noise of the Optical Source element is provided by a twenty_db_rolloff_noise element implemented in the OpticalPLL_Example library, which gives out the integrated phase noise for the input phase of the singal source element. The noise of the photo diode was implemented by adding the white noise signal to the output of the photo-diode.

5 Results

Careful calibration of all important parameters in the system was needed to obtain the locking between the optical source and the VCO. In both [4], [5] a 44.26 MHz Er-doped fiber laser was used, while in later works ([6] and [7]) a higher pulse repetition frequency laser was used of 200MHz. In all works a VCO of center frequency of about 10GHz was used. Firstly for the proof of concept optical repetition frequency of 50MHz and the VCO of central frequency of about 10GHz were used in the simulation. Following [9], a time step T_s was such that there were from 50 to 200 simulation points in one VCO period which is the highest frequency present in the system. The time step was also set such that $1/T_s$ was a large even integer of the repetition frequency and so that a pulse exactly coincides with a time simulation point. Even in this case a floating point error was observed so phase of the optical signal drifted from its expected value during simulation which produced a pulse "spreading" to two simulation points. The code of `SigGen` class was manually changed to account for that error so the intensity from the optical source is nevertheless same. In all cases of simulation a trade-off between the length of simulation and the size of the output had to be made. For checking convergence and short-time simulations, number of simulation points n_{step} was from 5e6 to 1e7 points. In the case for checking long term convergence, a longer simulation times were needed, up to 1e10 points which took up to 6 hours on an Intel i5 machine.

5.1 Ideal model and convergence

The calibration process for parameters usually started with checking signal levels in all parts of the BOMPD. The power of signals input to the mixer was made to be approximately the same. The electrical input to the Sagnac loop was checked such that peak-to-peak values don't exceed $\pi/2$ (if the signal level was larger then π then the BOMPD could *jump* to a next minimum in the transmission function degrading the stability) and also that VCO output and reference arm signals have the same power. It was also made sure that the peaks of the reference signal were aligned with optical pulses which meant delaying the reference signal by one simulation point. Before mixing the reference signal was delayed by a $\pi/2$ phase, using a delay element with delay of $1/(4f_{reference}T_s) + 1$. The locking frequency at first was taken to be the 201^{st} harmonic of the optical pulse train so then $f_{reference} = 100.5f_r$. Then the bandwidth and gain of the low pass were adjusted such that high-frequency oscilltions are low. Finally the PI controler parameters were adjusted such that settling time and stability of the BOMPD output are on a desired level.

In Figure 6., a graph of the input to the VCO is given showing settling behaviour of the BOMPD. VCO central frequency was exactly set to the locking frequency so the VCO input settles to oscillattion around the zero point. If we zoom in to the settled region we see that the input to the VCO oscillates with reference frequency, in this case $201f_r = 10.05$GHz, which is

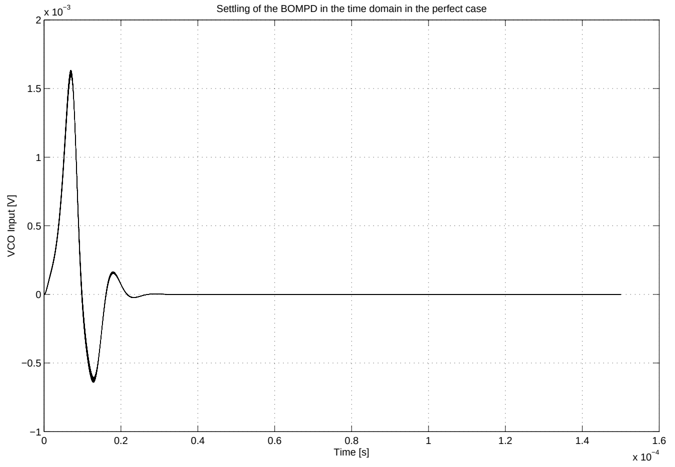

Figure 6: The input to the Voltage-Controlled Oscilaltor in the BOMPD. The settling behaviour can clearly be seen, and in the steady state it settles to zero because the VCO frequency, $f_{vco} = 10.05GHz$ was exactly set to be at a reference frequency.

due to the fact that in the perfect model low-pass filter attenuates high frequency signal but does not remove it. To check how the BOMPD locks the VCO signal to the input signal we can plot a graph of phase difference between the optical pulse input and the VCO output. The plot of phase difference or relative jitter between the optical input and VCO output is shown in Figure 7.

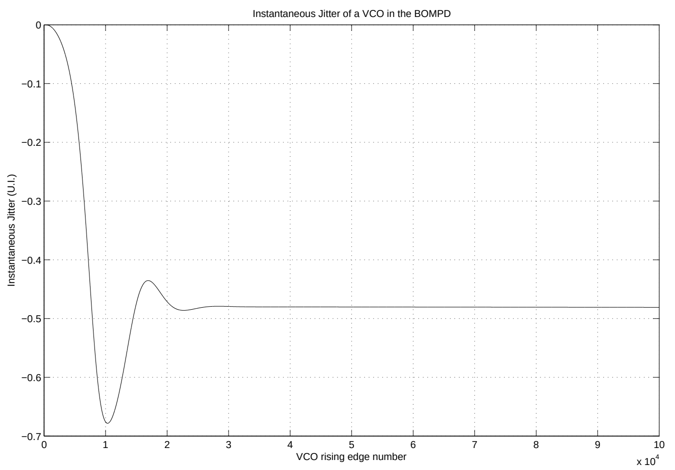

Figure 7: Instantaneous jitter between the input optical pulse and the output of a VCO. In the steady state jitter drifts by a small amount.

The locking is good, but not perfect as a drift of relative jitter can be seen over long times. The source of this error is not evident but it is suspected that it comes from a floating-point error present in the simulation, more specifically when the simulation calculates the phase of the optical source and the VCO there could be a relative drift because another "reference" VCO is used to calculate the relative jitter.

A small table of parameters used in the simulation for obtaining convergence is given in a Table 1. for a reference for future users of this BOMPD simulation.

Table 1: Values of parameters used in the simulation of the BOMPD for obtaining convergence.

T_s	Ref. frequency f_r	Optical power	LPF f_b	PI controler K_i
1/(10e10)	5.025e9	1W	1e6 - 25e6	2e6-1e7
1/5e10	5.25e8	1W	25e6	2.5e5
1/5e10	5.25e8	1mW	25e6	1e5 - 2e6

5.2 Inclusion of noise

Firstly the noise in the VCO is included by including the VCO element with 1/f noise already present in the CppSim. The VCO (reference) frequency was reduced to 1.05GHz so that larger time steps could be used in the simulation and thus longer simulation times enabling examination of the noise performance in low frequency regions. The noise level of the VCO thatt was used for the presentation of results was 1/f noise with offset frequency $f_{off} = 1e6$ and the noise level at f_{off} was -125dBc/Hz. In Figure 8. the settling of the BOMPD in time domain is shown. This graph is very similar to Figure 6. but in the steady state region we observe random fluctuations around the settling point which is in this case again around zero. Jitter plot would show a similar trend as one shown in Figure 7. but with some random fluctuations around the steady point. To consider the effectivness and performance of the BOMPD the best thing to do is to

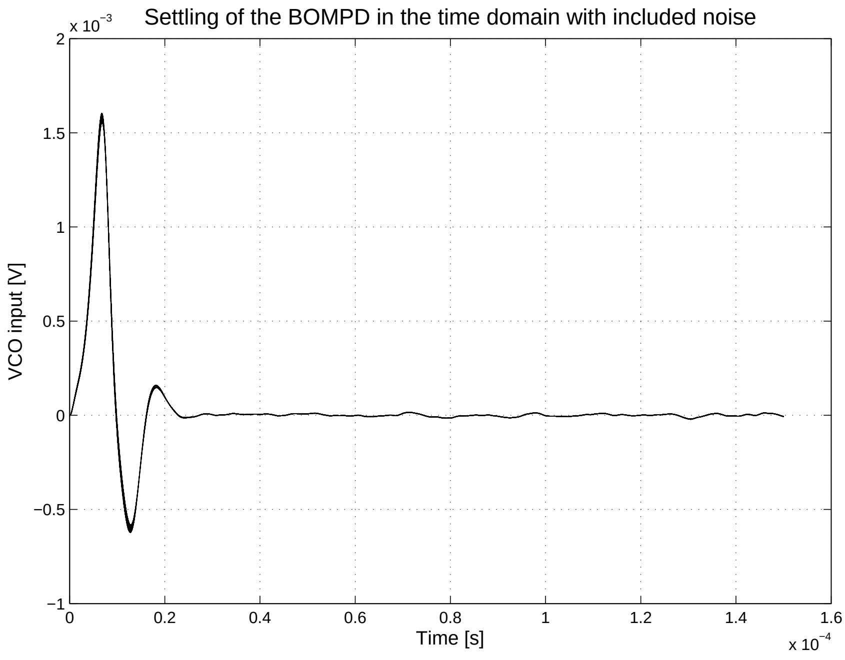

Figure 8: The input to the VCO in the BOMPD when the VCO is noisy. The settling behaviour can clearly be seen, and in the steady state it settles to zero because the VCO frequency was exactly set to be at a reference frequency.

plot the phase noise in frequency domain of a free-running VCO and of a VCO connected with a BOMPD in-loop. In figure 9, it is clearly seen that BOMPD reduces the phase-noise in the lower frequencies, in this case around 1e5Hz which is the effective bandwidth of the BOMPD in this case. From these phase-noise measurements integrated timing jitter can be calculated, in a range of frequencies. This was done by integrating the phase noise spectral density from a certain frequency and then converting the result into timing jitter by scaling it with the VCO frequency. The result of this calculation is shown in Figure 10, where the intgration was done from 10MHz

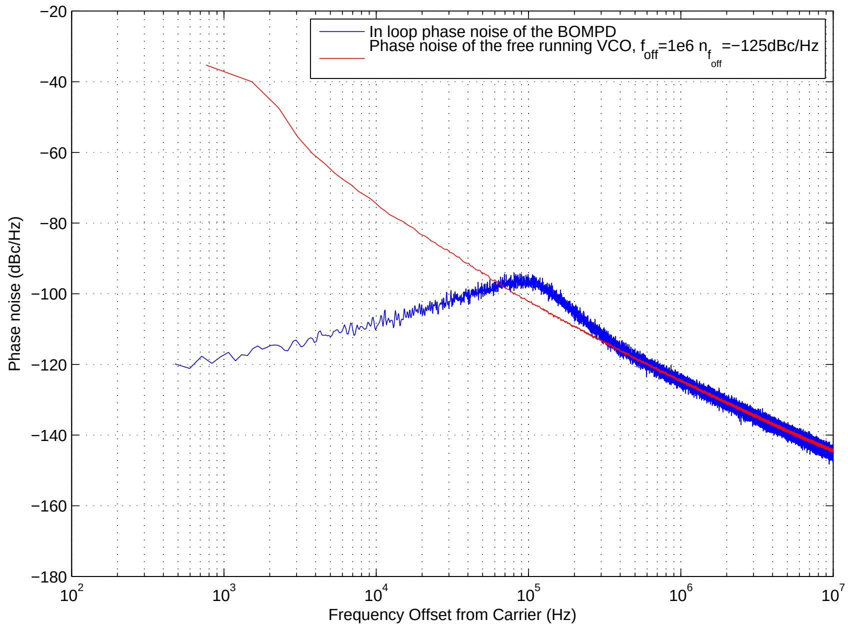

Figure 9: The red curve shows a phase noise of a free-running VCO with noise at f_{off} -125dBc/Hz. The blue curve is the phase noise of the VCO locked to a BOMPD. A clear reduction of phase noise in low frequencies is observed.

dow to about 1-10kHz. As evident from 9 at high frequencies the noise performances of the free-running VCO and the locked VCO are the same but then in the lower frequency region they behave differently. The blue curve shows the timing jitter of the locked VCO and at first it has a "bump" because of the PI controler operation. Then it settles to a constant value of about 1e-12s while free-running VCO jitter diverges (because the noise goes as 1/f in lower frequencies).

The effect of the photo-diode noise was also examined in the BOMPD operation. It was implemented as a simple addition of a white noise component to the photo-diode output. The noise variation was 4e-20 W^2/Hz which was increased on purpose to show the clear variation of the phase noise (value of about 4e-22 W^2/Hz was taken from a datasheet for a high speed EOT ET-3500 photodiode, at a wavelength $\lambda \approx 1.5\mu m$). The photodiode shot noise increases overall noise floor, as seen in Figure 11 which can be seen as the approximately constant rise of the noise in lower frequencies where the BOMPD operates.

Next and final imperfection considered in this model was the effect of the optical source noise on the performance of the BOMPD. Although the mode-locked lasers have much better noise performance than the commercially available VCOs, as theoretically shown in [1] and experimentally checked in [4] and [6] the optical source noise affects the VCO performance in the lower frequencies. This is because the BOMPD does exclude AM-to-PM noise in the system, but the phase noise from the optical source just "propagates" through the system. In the simulation this was implemented by using the noisy optical source provided in OpticalPLL_Example library and by using a noisy VCO with the same noise performance in generating the reference signal. The simulation of this case is shown in Figure 12. The blue curve is the noise of the locked VCO when the optical source has noise spectral power of -150dBc/Hz at the offset frequency f_{off}. The black line is the -20dB roll-off noise of the optical source. We see that in the region of BOMPD operation the VCO noise closely follows the optical source noise unlike the noise of the locked VCO without optical source noise (green curve). But it is noticable that the VCO noise is shifted by about $\approx$ 20dB up from the optical source signal and this is attributed to the fact that the VCO has 10.5 times the repetition frequency of the optical source.

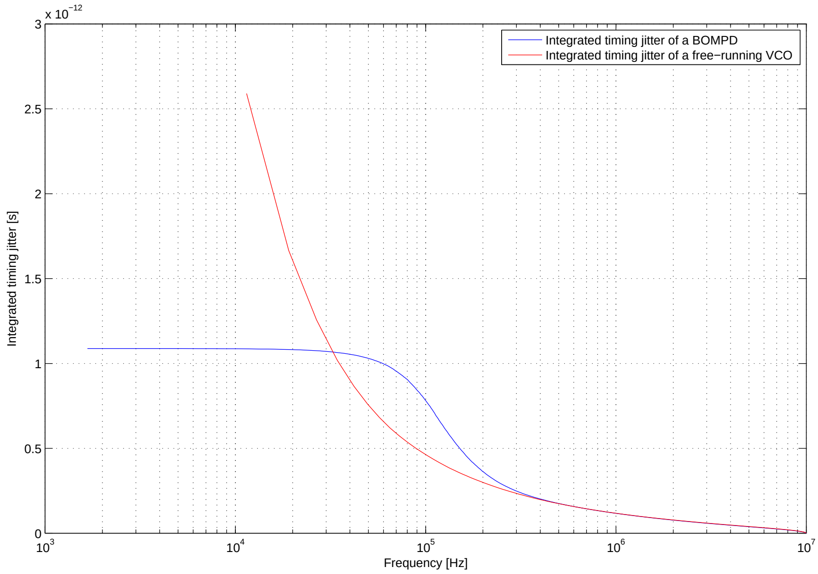

Figure 10: The red curve shows integrated timing jitter from 10Mhz of a free-running VCO with noise spectral denisty -125dBc/Hz at f_{off}. The blue curve is the integrated timing jitter of a VCO connected with a BOMPD.

6 Conclusion

In this report the results of a summer student programe done in DESY during the summer of 2014 was presented. An accurate and robust simulation of a Balanced Optical-Microwave Phase Detector was implemented in the systems simulation package CppSim. A simplified model of a BOMPD was implemented and in the perfect case without any noise sources it was shown that it locks the VCO output signal to the optical signal perfectly. Imperfections were introduced by considering the noise sources in the system. Main noise source in the system is the phase noise of a free running VCO. It was shown that this noise is successfuly attenuated in the low frequency region by the BOMPD, in this simple model. Then the effect of the shot noise in the photo-detection process and the effect of the optical source noise was considered and shown to be in agreement with qualitative considerations. The BOMPD simulation shows a *proof of concept* and it should be possible to extend the model to get quantitative results in agreement with the experiment. The model of the BOMPD is also easily extendible such that a complicated implementation can be done e.g. by including noise in the amplification stages or non-linearities in the Sagnac loop. This could enable a complete simulation of experimental results such as AM-to-PM supression ratio in [7] and could even help with new experimental results in the future.

I would like to thank my supervisor Aram Kalaydzhyan for a great help and guidance through this project and the group leader Franz Kärtner for hosting me Ultrafast Optics and X-Rays Division. I would also like to thank the whole crew of the DESY summer program 2014 as this summer wouldn't be as good and fun without them.

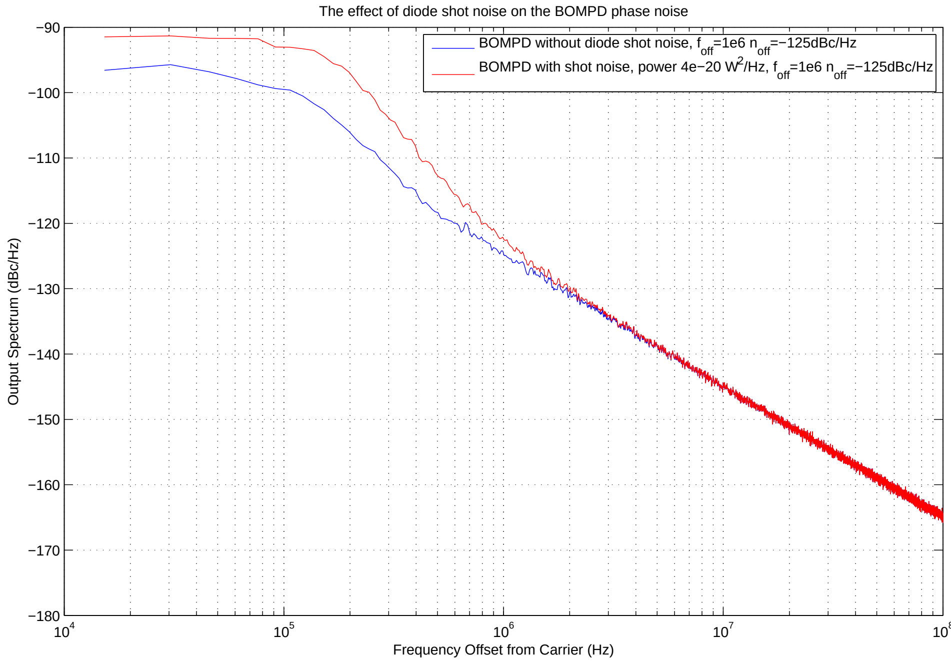

Figure 11: The blue curve is the phase noise of a BOMPD without shot-noise in the diode. Red curve is the phase noise of a BOMPD when a shot noise of variation 4e-20 W^2/Hz is applied to the photo-diode output.

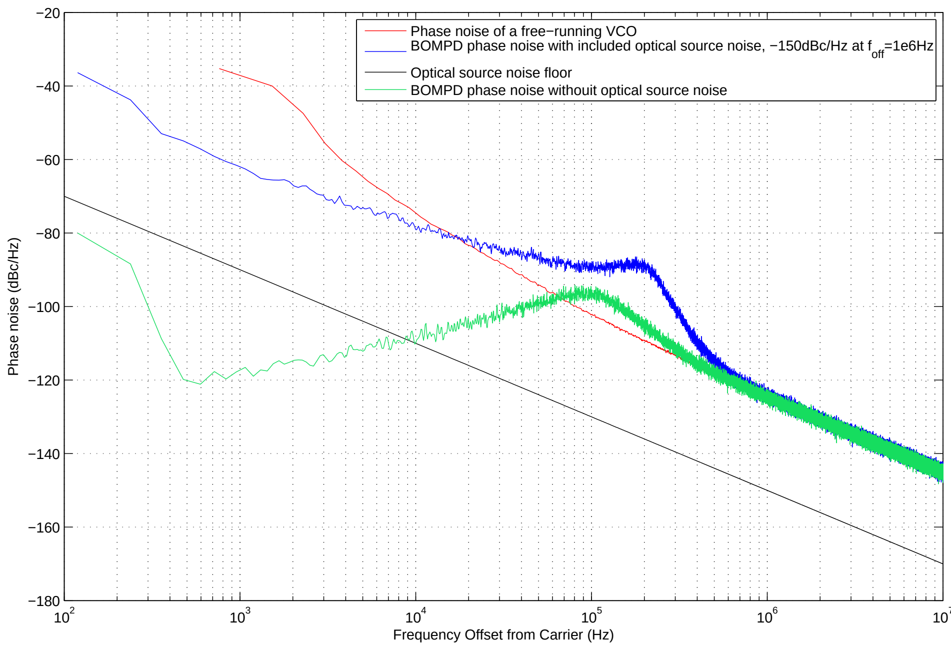

Figure 12: The red curve represents the phase noise of a free-running VCO with noise -125dBc/Hz at 1MHz. The blue green curve is the phase noise of a VCO with noise -125dBc/Hz at 1MHz locked to a BOMPD. The blue curve is the noise of a VCO locked to the BOMPD with noisy optical source, with noise -150dBc/Hz at 1Mhz. The black line is the phas noise of the optical source. We can clearly see that the noise of the BOMPD output follows the optical source noise at low frequencies but with a shift of $\approx$ 20dB which is atributed to 10.5 times higher frequency of the VCO to the optical source.

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