# Monte Carlo simulations of energy lost due to random voltage spread in an XFEL RF unit

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## Abstract

*A computer simulation was written and used in oreder to calculate the energy lost due to voltage spread in the cavities affecting an XFEL RF unit. Two different approaches were applied to simulate the outcome of the cavities performance under specific initial conditions.*

## 1 Introduction

The main target of XFEL is to produce X rays pulses extremely bright, ultrashort and with spatial coherence with very reduced spectrum. These pulses will be used in multidisciplinary reasearch experiments.

The heart of the XFEL complex is the L-band (1.3 GHz) electron linear accelerator with a nominal design energy of 17.5 GeV, operating at an accelerating gradient of 23.6 MV/m. It uses an advanced superconducting radio frequency (SRF) technology developed by the international TESLA Collaboration.[?, ?]

The SRF cavities are hollow devices with axial symmetries (Figure 1) , powered by microwaves and made of superconducting material.

An important figure of merit of RF cavities is the quality factor. The quality factor or Q factor is a dimensionless parameter that characterizes a resonator's bandwidth relative to its center frequency. The higher the Q factor, the narrower the bandwidth and therefore the higher the amplitudes that can be reached ( nevertheless SRF cavities are fundamentally limited by the critical magnetic field). But it is more difficult to tune because it is more sensitive to mechanical tolerances (even a slight change in helium pressure and vibrations can cause real problems).

For resonant frequencies in the range 100 to 1500 MHz, typical values for the quality factor are 10,000 to 50,000 for normal conducting copper cavities and $10^8$ to $10^{10}$ for superconducting cavities.

Figure 1: SRF cavity used in the XFEL project

In the XFEL 32 superconducting cavities (4 cryomodules with 8 cavities per module) are connected to one 10 MW multibeam klystron through a waveguide distribution system (Figure ??).

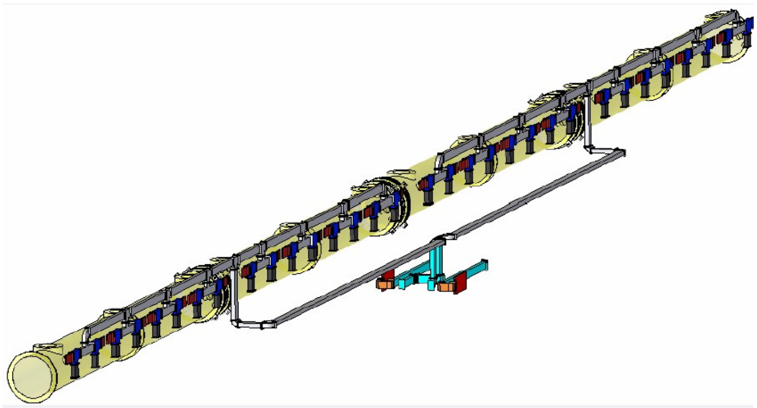

Figure 2: Waveguide distribution of XFEL powered by a 10 MW klystron.

The XFEL tunnel has limited space for the waveguide system and therefore a more compact waveguide distribution has been developed. The waveguide distribution is based on a binary cell which consists of two circulators connected to a shunt tee with integrated phase shifters. Four binary cells are combined by three asymmetric pretunable shunt tees. The asymmetric shunt tees allow to change the RF power for each pair of cavities and to reach the maximum cryomodule gradient.[?]

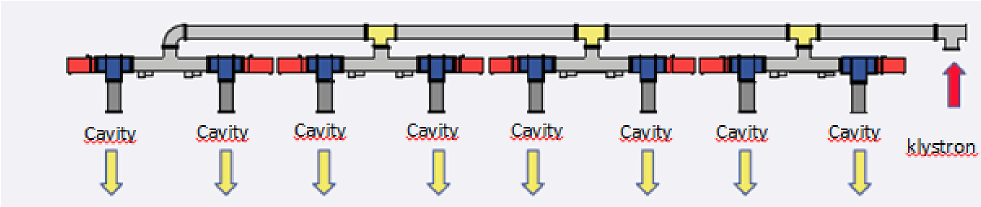


Figure 3: XFEL klystron wave-guide distribution for a RF unit (32 cavities)

A scheme of one of the four cryomodules is shown in figure **??**. This figure shown the eight cavities that conform a cryomodule and the klystron power input.

SRF cavities are limited by their quench limit (transition to normal state). The quench limit of the cavities is not the same for all the caivites, this number varies due to the random effects that happen during the fabrication. There is a statistical spread in the maximum cavity performance. The ideal case of running all cavities with equal power would then limit the overal performance of the RF unit to the weakest performing cavity, and a significant loss in total achievable voltage would result.

In principle the solution for the maximum performance is achieved by adjusting the forward power to the cavity ($P_{for}$), the fill time of the cavity ($t_{fill}$) and the cavity coupling, which we will discuss in terms of the external Q ($Q_{ext}$). The solution is unique for a given beam current $I_b$. In principle if we had control over all these parameters, we can match the system to any choice of beam current and cavity voltages for a given RF unit. However, the XFEL does not have adjustable power control on individual cavities: the power distribution ratios are fixed for the zero current case during installation and cannot be changed. Therefore, in general, when we accelerate beam we will a loose a little gradient, the amount of which is a function of the beam current.

## 2 Methodology

Since the maximum voltage is not the same for all the cavities, it is not possible that the waveguide distribution system from klystron to cavities divides the power equally between the 32 cavities. Therefore, the power distribution system will be tailored to deliver a forward power to the individual cavities which matches their performance limits. Thus we can in principle achieve the maximum energy gain from the RF unit. Unfortunately this is compromised by beam loading.

Since in XFEL the power ratio can only be fixed per each two cavities , the cavities have to be paired and the lowest value of the voltage has to be chosen.

During this study the resistance of the SRF cavities has not been taken into account.

During the $t_{fill}$ the evolution of the voltage in the cavity ($V_c$) is given by:

$$V_c(t) = V_g(1 - e^{-t/\tau}) \tag{1}$$

Where $V_g$ is the voltage driven by the 'klystron' and $\tau$ is the time constant of the charging/ discharging of the cavity, it is given by: $\tau = \frac{Q_{ext}}{\pi fo}$.

At t=t$_{fill}$, once the voltage desired is reached, the constant beam current,$I_b$, starts. The effects of the beam in the voltage are described by:

$$V_b(t) = -I_b R_{ext}(1 - e^{\frac{-(t-t_{fill})}{\tau}}) \tag{2}$$

Where the $R_{ext} = \frac{r}{Q} Q_{ext}$

The steady-state voltage in a cavity in the presence of beam is

$$V_a = V_g - I_b \frac{r}{Q} Q_{ext} \tag{3}$$

The beam effects can be added to our electric circuit where :

- $V_a$ is the required accelerating voltage for that cavity, which we want to make as close to the quench limit as possible.
- $V_g$ is the voltage driven by the "klystron"
- $\frac{r}{Q}$ is the characteristic impedence of the cavity (a constant for the cavities) and is equal to 1036 Ω
- $Q_{ext}$ is the external Q, which is adjustable via a motor on the power coupler in the range of $1 - 10 \cdot 10^6$. In this simulation the value of $Q_{ext}$ has been setted constant, equal to $4.8 \cdot 10^6$

The necessary forward microwave power from the following equation is given by equation

$$P_{for} = \frac{1}{4} \frac{V_g^2}{\frac{r}{Q} Q_{ext}} \tag{4}$$

The remaining parameter we need to consider is the fill time $t_{fill}$. This is the time it takes at the beginning of the RF pulse to charge up the cavity to the required voltage $V_a$ at which point the beam pulse (beam current) arrives. The fill time can be calculated as:

$$t_{fill} = -ln\left(1 - \frac{V_a}{V_g}\right) \frac{Q_{ext}}{\pi f_0} \tag{5}$$

A consequence of driving 32 cavities with a single RF source (10MW klystron) is that the fill time must be the same for all cavities, which further constrains the problem.

The forward power can be different during the beam pulse compared to the fill time (no beam). If we call the ratio of fill time power to beam pulse power r, then the evolution of the voltage in the cavity is given by:

$$V_a(t) = \sqrt{4P_{for}R_{ext}}((1-\sqrt{r})e^{-(t-t_{fill})/\tau} + \sqrt{r} - e^{-t/\tau}) - I_b R_{ext}(1 - e^{(-t+t_{fill})/\tau}) \tag{6}$$

with $R_{ext} = (\frac{r}{Q})Q_{ext}$ and $r = \frac{Pfill}{Pbeam}$

We assume that the individual cavity performance of each cavity can vary between 20 MV an 30 MV. This range of values have been chosen for this simulation, nevertheless, will not be the real values use in the XFEL ( specifications of ILC). The XFEL approach will be to tailor the waveguide distribution such that the cavities can be driven close to their limits as possible. Due to it is not possible to control each cavity separately, the cavities are driven in “adjacent pairs” with equal forward power. The weakest cavity in each pair will define the forward power for that pair. Therefore using an algorithm that gives pseudo-random numbers we are able to make a distribution of the cavities in a unit (32 cavities for the XFEL experiment).Then we pair then (see Figure **??**) . The weakest cavity in each pair will define the forward power of that par.After pairing the cavities , we normalize each value , with the purpose of having the mean of this distributio fixed to 25 MV.

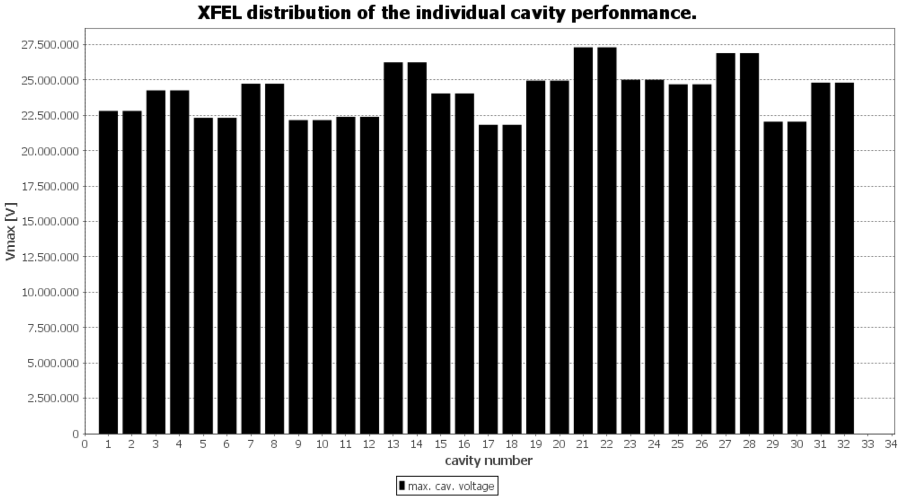


Figure 4: Pairing the cavities

The next step is to calculate the required forward power to each of these pairs running at the operational gradient. For XFEL, the value of $t_{fill}$ is of 820 $\mu s$, with all cavities adjusted to a fixed $Q_{ext} = 4.8 \cdot 10^6$. First, the forward power required for each cavity to achieve its voltage in the required $t_{fill}$ has to be calculated. The first step for this is to calculate $V_c$ from eq **??** and its forward power from eq. **??**. The klystron power $P_k$ is the sum of each individually cavities.

In this report a constant and equal phase for all the cavities is assumed, so therefore, the vector sum only involves the amplitudes of the voltages of each cavities. The parameter r has to be modified in a such way that the vector sum remains flat during all the beam loading time. The vector sum is given by equation **??**.

$$\sum V(t) = \sum_{i=1}^{N} \sqrt{4P_{for}R_{ext}}(1-\sqrt{r})e^{\frac{-(t-t_{fill})}{\tau}} + \sqrt{r} - e^{-t/\tau}) - I_b R_{ext}(1 - e^{\frac{-t+t_{fill}}{\tau}}) \quad (7)$$

The vector sum feedback will adjust both, P and r, to achieve a flat slope for the voltage. The voltage must be remains constant since the acceleration has to be the same for all the beam. The value of r tha allow us to have a flat vector sum is:

$$r = (1 - e^{t_{fill}/\tau}) \cdot (1 + \frac{N_c \cdot I_b 1036 Q_{ext}}{\sum V}) \tag{8}$$

Where $N_c$ is the number of cavities.

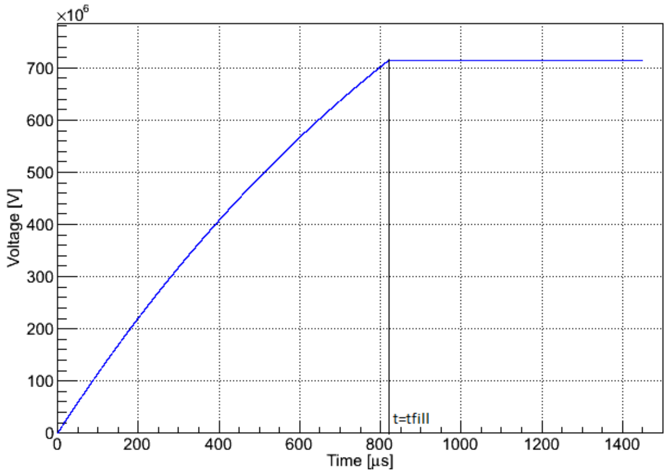


Figure 5: Evolution of the voltage (vector sum) during tfill and tbeam.

The Figure **??** shows the behaivour of the voltage desired. We can see that, although we have arranged for the vector sum to be flat at the required value; the individual cavities all have different slopes but the average is flat. So we proceed to normalise the individual cavity voltages to their quench limits as shown in Figure **??**.

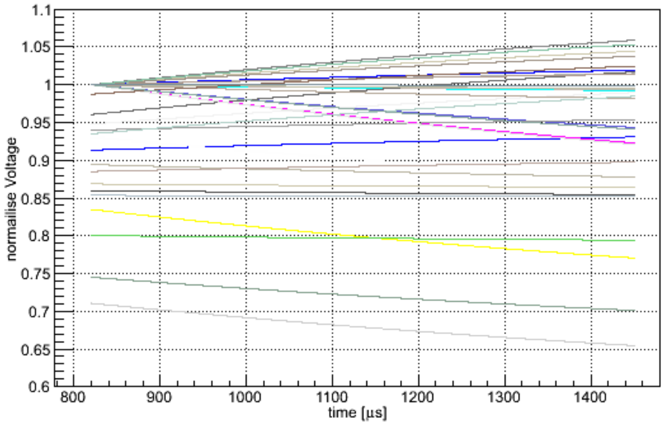


Figure 6: Accelerating voltage of each cavity divided by its maximum determined value in order to measure its quenching

Several of the cavities continue to charge up after the fill time and exceed their quench limits (i.e. they go above 1 in Figure **??**).

No cavity should be allowed to exceed its quench limit. In order to avoid the quench, we can proceed in two different ways:

- Reducing the total value of the power generated by the klystron.
- Reducing the value of the accelerating voltage of the pair that is quenching.

## 2.1 Total klystron power reduction

With this method we verify which cavity quenched then, for each cavity that quenched we reduce the klystron forward power (and therefore the vector sum) until no cavity exceeds its quench level.

Each time the process is repeated the parameters of the equation (6) are calculated again, in order to maintain the vector sum constant for a given klystron power.

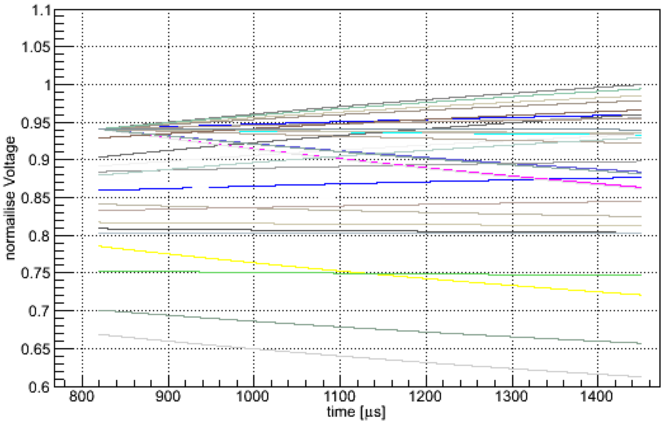


Figure 7: Voltage of each cavity divided by its maximum value in order to measure its quenching limit by reducing the klystron power, so the quench of the cavities is avoided.

As we can see in Figure **??** the quenching point of every cavity is never crossed therefore we ensure that each cavity runs at its highest possible accelerating voltage without any cavity quenching.

## 2.2 Voltage reduction on cavities

With this method, first we verify which cavity quenched. Then, to each cavity that quenched we reduce the initial voltage of that cavity and its pair. The klystron forward power is also reduced (and therefore the vector sum), but only by the amount due the pair that is being adjusted. All other cavities stay at constant power. This is the difference to the first method.

After that, we verify if with the new reduced initial voltage there is a cavity that quench, if so, we repeat the process as many times as necessary.

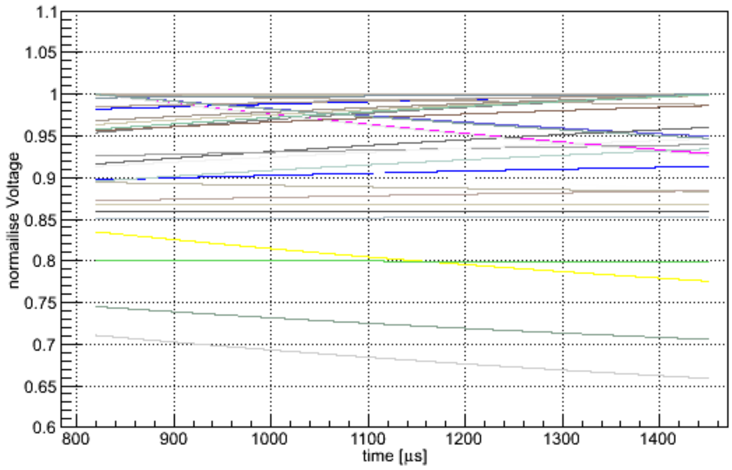


Figure 8: Voltage of each cavity divided by its maximum value in order to calculate its quenching limit by reducing the cavities initial voltage, so the quench of the cavities is avoided

# 3 Analysis and results

## 3.1 Power klystron reduction

A Monte Carlo study of the variation of the voltage with the current has been made. The current has been changed for 0 to 5mA in steps of 0.1 mA. For a given value of the current the simulation has been run with 100,000 RF units with random voltages taken from a flat distribution (around 25 MV with a ± 20%.)

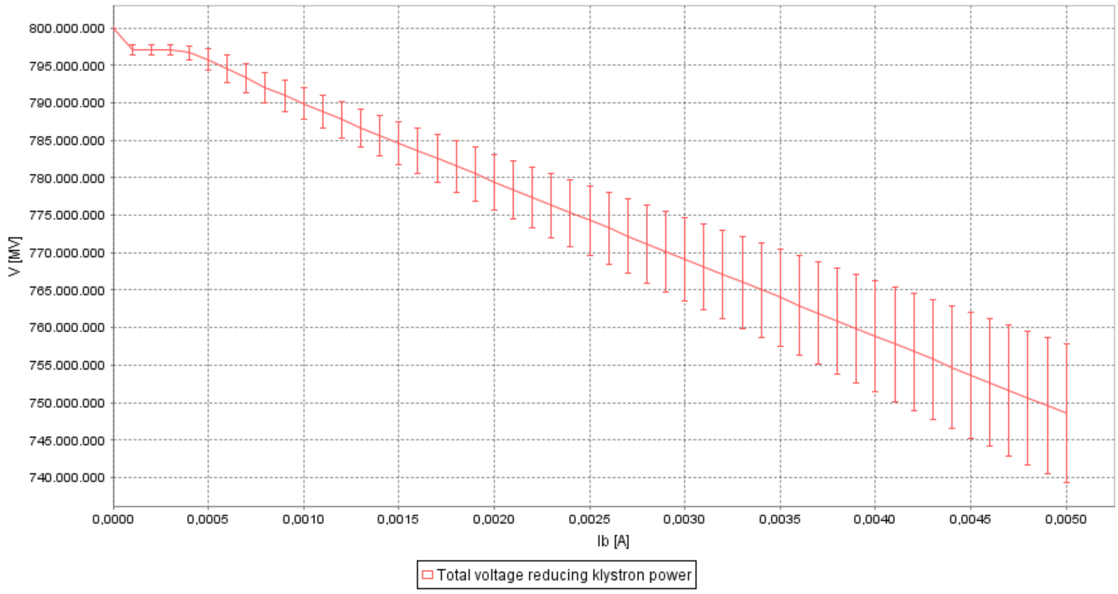


Figure 9: Evolution of the vector sum with the current Ib

The results obtained have been plotted in the Fig. **??**. As it can be seen in Fig. **??** when there is no current in the rf cavity the vector sum remains constant (no quenchs have been detected). Increasing the current, the number of cavities quenching also increase and therefore the voltage sum decrease (due to the power of the klystron has to be reduced).

It is also can be seen that the error (standart deviation) of the vector sum increase with the current. This fact can be explained in the following way; the higher the current, the higher the number of cavities that could quench during the beam loading. When no current is accelerated in the SRF cavity no quench is detected and so therefore, no reduction of the maximum voltage has to be done and all the vector sum of all the simulations will be the same and the desviation from the mean value will be almost zero. If the current is increased the cavities will start to quench during the beam loading. The number of cavities quenching varies from one simulation to another and therefore the vector sum will vary in evey simulation.

## 3.2 Power distribution ratio reduction

A Monte Carlo study has been done when working with the distribution ratio reduction, in order to calcultate the dependence of the vector sum with the voltage. This dependency is shown in figure **??**.

Accelerating with a value of 5 mA the vector sum of the cavities has been reduced in a 7.5% using the first method and a 1.44%.

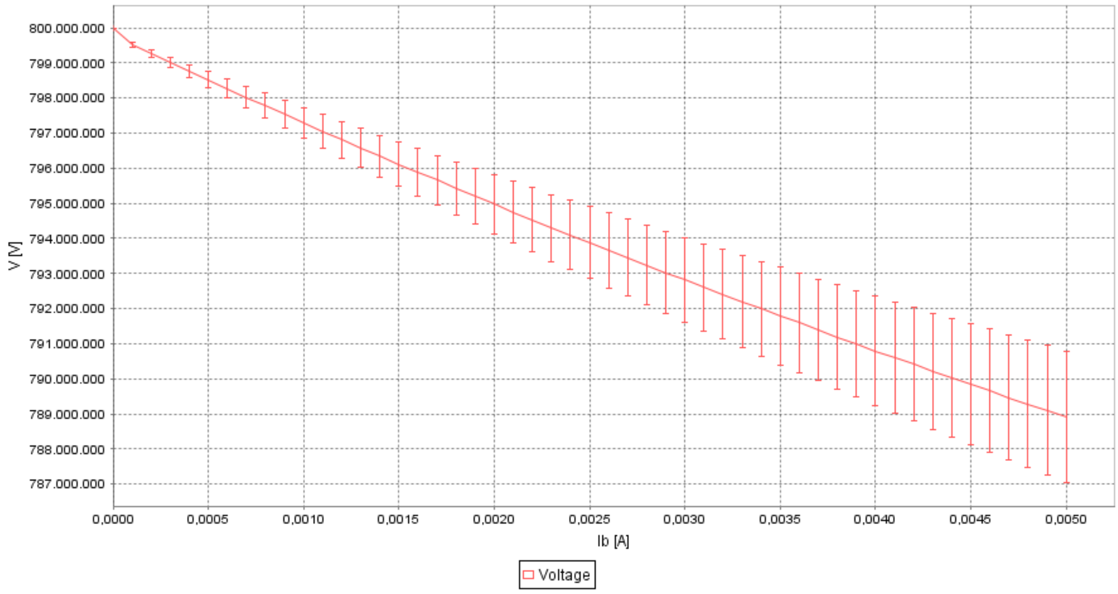


Figure 10: Evolution of the vector sum with the current Ib

## 4 Conclusions

During this study the the value of the $Q_{ext}$ has been fixed, in consequence, the conclusions of the results are only valid for assuming $Q_{ext}$=constant.

Using the first method the reduction of the vector sum is higher than using the second method. The slope of the first mehtod is 4.9 times greater than the slope of the second method. This is due to when using the first method the voltage of all cavities are reduced, even if these cavities are not quenching. Whereas in the second method, only the voltage of the cavity that is quenching is reduced.

The final value of the vector sum when using the first method (klystron reduction) is smaller than the final value of the vector sum using the reduction of the paired voltage. When working with a value of the current of Ib=5 mA the difference of the values of the vector sum of this two methods is approximately a 6.6%.

## 5 References

## References

[1] M.Atarelli et al. *"XFEL: The European X-Ray Free-Electron Laser: Technical design report"*, DESY Report DESY 2006-097, July 2006.

[2] V. Vogel, L.Butkowski, et al. *Results of Testing of multi-beam klystrons for the european XFEL*,Proceedings of LINAC2012, Tel-Aviv, Israel, 2012.

[3] V. Katalev, S. Choroba. *Results of Testing of multi-beam klystrons for the european XFEL*,Proceedings of PAC07, Albuquerque, New Mexico, USA

[4] Thomas P. Wangler *"RF Linear Accelerators"* 2nd. Edition. WILEY-VCH, 2008.

## 6 Apendix. JAVA programing code

Steps that have been taken when doing the simulation:

- Generating random numbers between 20MV and 30 MV. These values are generated in a double[] of the same length that the number of cavities
- Pairing the cavities and recalculate the value of the voltage to obtain a average of 25 MV.
- Calculating the forward power of the klystron , the power ratio distribution and the value of r ( flat voltage) $\rightarrow P_k eq.4$ , pdr[i] eq.6, r eq.8
- Calculate the evolution of the voltage (normalised divided by the maximum voltage.) during the beam pulse duration. If at the end of the beam time the voltage is greater than 1, the cavity has quenched. The evolution of the volage is calculated for each cavity at two moments: t=$t_{fill}$ and $t_{end}$
- Once the quench has been detected we can proceed in two different ways.
  1. Reducing the total power of the klystron in steps of 0.1% $\rightarrow Pk* = 0.999$
  2. Reducing the power ratio distribution in steps of 0.1% $\rightarrow pdr[i]* = 0.999$
- This reduction is included in a loop. $\rightarrow$ do while voltage $\geq$ 1.