Second order correction for beam functions in Soft-Collinear Effective Theory

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Abstract

The report shows what I was learning and calculating during the DESY Summer Student Program 2013. This is basically idea of effective field theory on example of Soft-Collinear Effective Field Theory and calculation of second order correction to quark beam function which is proportional to the number of quark flavours. Report show necessary ingredients that one needs to learn to perform those calculations and get a basic insight in physics which it describes.

Contents

1 Introduction

I would like to mention at the beginning that all discussion is presenting the problems from my point of view. It is not necessarily the same as widely presented. I would like to also mention that I only recently started learning about effective field theories and factorization theorems in QCD. Actually getting familiar with this totally new for me concepts was a significant part of my activities on DESY Summer Student Program and practically all introductory things I am describing I learned about on the program.

1.1 Effective (field) theories

Basically an effective theory is simplification of the full theory which neglects some irrelevant degrees of freedom or interactions. For example if we want to study hydrogen atom, the Schrödinger equation with standard $1/r$ potential is an effective theory of the Standard Model, where we neglect a bunch of effects. For example neglecting proton structure basically means that we neglect momenta corresponding via uncertainty principle to scales shorter than a proton radius. Another example is a weak interaction which corresponds to energy scales of order of W boson mass m_W, which we are also neglecting in Schrödinger equation approximation. Effective field theories are connected with power expansions of full theory. For example a Schrödinger equation is obtained from a Dirac equation when we expand it in parameter $(\frac{1}{m_e})$, where m_e is electron mass. Actually the Dirac equation coupled to an external potential is already an effective field theory, but this simplification from Dirac equation to Schrödinger equation already explains a lot of concepts of Effective theories. When we go to constructing effective theories in quantum fields we simply apply the same principles as above. We expand the existing theories(possibly already effective) in some parameters which are small in our problems like $(\frac{Q^2}{M_W^2})$, where Q^2 is square of exchanged momenta between scattering particles. Actually this is how we obtain Fermi electroweak effective field theory.

1.2 Factorization of cross sections

There is well known inconvenience of QCD that there is no free quarks and gluons, hence the whole theory is non-perturbative. One could think that because of that there is little or no use of such theory. That is not the case due to asymptotic freedom. One can believe that because of that effect the high energy processes could be somehow described by perturbative QCD. Actually this is the case. It is rather obvious that bound states can't be described using Feynamn diagram technique which assumes existence of asymptotically free states. One can heuristically and intuitively argue that when we deal with hard processes, states similar to asymptotically free occur during some part of interaction. This is actually basic idea on which factorization theorems are build. Typically we have to deal with some scattering processes of QCD bound states such as protons or pions with other particles. What we interested in such processes is some differential cross section.We are interested in processes, where the factorization theorems of such cross sections hold. More precisely we can state for two body scattering (Explanation of constituents is below equation):

$$\frac{d\sigma}{dz} = \int \Bigg(\prod_{n=1}^{N} dx_k \prod_{m=1}^{M} dy_m \prod_{l} SX_l(x,\mu) \prod_{m} SY_k(y,\mu) \cdot H_{lk}(x,y,z,\mu) \cdot Soft(x,y,z,\mu) \Bigg) \left[1 + O\left(\frac{\Lambda}{Q}\right) \right] \tag{1}$$

$x = (x_1, x_2..., x_N)$ - variables describing structure of particle 1
$y = (y_1, y_2..., y_M)$ - variables describing structure of particle 2
$z = (Q, z_1, z_2.., z_L)$
$Soft(x, y, z, \mu)$ - soft part of interaction representing low energy processes during the interaction
μ - renormalization scale
SX,SY - functions describing non-perturbative structure of first and second particle respectively,they oftentimes are products of functions of separate variables(this is the case in processes described by SCET, where they decouple to soft and collinear part)
H_{lm} - perturbative function describing interaction between constituents l and m
Λ - dimensional scale, typically Λ_{QCD}
Q - energy scale of a process - the precise meaning is defined by an explanation of factorization
Actually it may seem at first glance that factorization is useless, especially the functions SX and SY may differ between processes. It seems like we are simply fitting some functions to results. What actually make factorization useful is that the structure functions are the same for wide range of processes in question, so we can actually check theory. It is also true that typically the dependence of SX and SY from only one variable with

other variables fixed, is sufficient to determine the dependence of SX and SY with respect to other variables, for example for PDF's that are functions of μ ans $x(f(x,\mu))$ we know differential equations with respect to μ that must be obeyed, so knowing value $f(x_0,\mu_0) = v_0$ for fixed x_0,μ_0 we can calculate $f(x_0,\mu)$ for any given μ. What I am going to need in the following parts of the report is two families of structure functions: already mentioned above parton distribution functions(PDF's) and beam functions.

1.2.1 Parton distribution function

The hadrons are bound states of quarks and gluons. Parton distribution function describes the probability of finding specified constituent inside the hadron, which carry specified momentum fraction of a hadron. Let us consider an example of two hadron scattering. Denote the PDFs of first hadron F as $f_j(x,\mu)$ and PDFs of second hadron G as $g_k(y,\mu)$.Here j and k denote the specified gluon or quark. We will be interested in processes $F + G \to Y + X$, where Y is product of hard scattering. It may be also state which is not asymptotically free but will become some hadrons and other particles later on. Lets not consider complicated case of Y going into hadron jets and instead consider Drell-Yan process $Y = l^+l^-$ - pair of lepton, anti-lepton. Then Q is momenta exchanged between initial hard colliding partons and final state particles. So the natural scale of the process is Q^2. Intuitively from that we conclude that our error will be of order $\frac{\Lambda_{QCD}^2}{Q^2}$, where Λ_{QCD} is the scale of non-perturbative interactions forming hadrons. We can state it more precisely in terms of cross section factorization:

$$\frac{d\sigma\left(F+G \to l^+l^- + X\right)}{dQ^2} = \\ \sum_{jk} \int \left(dxdy \quad f_j\left(x, \sqrt{Q^2}\right) \cdot g_k\left(y, \sqrt{Q^2}\right) \cdot \frac{d\sigma\left(j+k \to l^+l^-\right)}{dQ^2} \right) \left[1 + O\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)\right] \tag{2}$$

1.2.2 Beam Function

The following discussion is based by reference [1]. Beam functions are generalization of PDFs to describe initial jets that hadron is made from. The beam functions are based on the idea that partons inside hadrons are forming incoming jet radiations. The beam function $B_i(t,x,\mu)$ is precisely defined by the parton distribution functions $f_j(\xi,\mu)$ and jet functions $I_{ij}(x,t,\mu)$ by following equation [1]:

$$B_i(t,x,\mu) = \sum_j \int_x^1 \left[\frac{d\xi}{\xi} I_{ij}\left(\frac{x}{\xi}, t, \mu\right) f_j(\xi,\mu) \right] \tag{3}$$

Similarly to PDFs there are also factorization theorems for beam functions for some processes more exclusive than described by PDFs.

2 Soft-Collinear Effective Theory(SCET)

This section is based on the lecture notes [2] as well as [1]. I will use terms soft and collinear degrees of freedom. Collinear degrees of freedom are those connected with energies of order of interaction scale Q. Collinear degrees of freedom are placed along direction defined by momentum directions of initial hadrons in the centre of mass frame(For decay processes some other directions are specified). Soft degrees of freedom correspond to emissions and interactions that have no preferred direction in space and all momentum components small with respect to Q. SCET is an effective field theory used to describe interaction between soft and collinear degrees of freedom "during" the hard interaction process. There is always some typical momentum scale of interaction Q, to which SCET is referring. It is always the case that $Q \gg \Lambda_{QCD}$ and collinear momenta $|\vec{p_c}| \sim Q$ so as soft momenta $|\vec{p}_{soft}| \ll Q$. Sometimes soft momenta are perturbative $|\vec{p}_{soft}| \gg \Lambda_{QCD}$ and sometimes not $|\vec{p}_{soft}| \sim \Lambda_{QCD}$. There is a lot features of SCET that makes it interesting as an effective field theory:

- In SCET we usually refer to power counting parameter as λ, which is not necessarily $\frac{\Lambda_{QCD}}{Q}$
- There are normally two fields for every particle representing soft and collinear degrees of freedom which lead to transparent separation of scales.
- 1-loop calculation in SCET leads to renormalization group equations which sum up Sudakov double logarithms $\sum_k a_k \left(\alpha_s \log \frac{p}{Q}\right)^k$.
- Wilson lines appearing in SCET are related with various symmetries.
- There is integration of energy scales without integrating out entire degrees of freedom as opposite to classical effective quantum field theories.

2.1 SCET ingredients

In SCET we use the other then usual coordinates in Minkowsky space. To define them we use two null-vectors n and $\bar{n}$. The n vector is parallel to the direction of collinear particles. Vectors n and $\bar{n}$ obey relations:

$$n^2 = \bar{n}^2 = 0 \qquad n \cdot \bar{n} = 2 \tag{4}$$

They define a decomposition of every four vector p to light-cone coordinates as following:

$$p^+ = \bar{n} \cdot p \qquad p^- = n \cdot p \tag{5}$$

$$p = \frac{np^-}{2} + \frac{\bar{n}p^+}{2} + p_\perp \tag{6}$$

It is straightforward to see that $p_\perp \cdot n = p_\perp \cdot \bar{n} = 0$. We define also two projection operators that decompose the whole space of dirac spinors.

$$P_n = \frac{\not{n}\not{\bar{n}}}{4} \qquad P_{\bar{n}} = \frac{\not{\bar{n}}\not{n}}{4} \tag{7}$$

They are used to define collinear and soft fields from the quark field ψ in the following way

$$\hat{\xi}_n = P_n\psi \qquad \phi_{\bar{n}} = P_{\bar{n}}\psi \tag{8}$$

In our basis we can write each momenta us $p = (p^+, p^-, p_\perp)$ In SCET we can uniquely decompose momenta p to label p_l and residual p_r component which scales according to rules:

$$p = p_l + p_r \qquad p_l \sim Q(0, 1, \lambda) \qquad p_r \sim Q(\lambda^2, \lambda^2, \lambda^2) \tag{9}$$

Now our field $\hat{\xi}_n(x)$ can be written as discrete fourier transformation, because label momenta are in fact discrete (we are excluding $p_l = 0$, because than it is not collinear mode).

$$\hat{\xi}_n(x) = \sum_{p_l \neq 0} \xi_{n,p_l}(x) \tag{10}$$

Where ξ_{n,p_l} create mode with label momentum p_l. Now we define new operators and fields as follows:

$$P\xi_{n,p_l}(x) = p_l\xi_{n,p_l}(x) \qquad \bar{P} = \bar{n} \cdot P \qquad \xi_n = \sum_{p_l \neq 0} \xi_{n,p_l}(x) = e^{iP\cdot x}\hat{\xi}_n \tag{11}$$

The newly defined fields ξ_n are fields used in SCET. (Actually there is another field redefinition connected to the opposite sign in exponent of anti-particle operators in fourier expansion [2]). We also do the same operations for gluon fields. The Feynman Rules that will be used further are in references [2] and [3].

2.2 Wilson Lines in SCET

There are two types of Wilson lines in SCET. The first is collinear Wilson line and the second is ultrasoft Wilson line. Lets consider collinear Wilson lines first. They can be expressed as ([1],[2]):

$$W_n(x) = \left[\sum_{perms} \left(\frac{-g}{\bar{P}} \right) \bar{n} \cdot A_n(x) \right] \tag{12}$$

Where sum over permutations is understood as including all permutations of indices in QCD matrices products such as $\sum_{perms} T^aT^b = T^aT^b + T^bT^a$. This collinear Wilson lines appear in SCET lagrangian. It is possible because scaling of gluon field is $A_n(x) \sim Q$ and appear beacuse of integrating out degrees of freedom with significant offshellness. To each collinear Wilson line there are corresponding Feynman rules on every order of perturbation expansion.

The soft Wilson lines are used in SCET to decouple soft degrees of freedom from collinear ones by another field redefinition:

$$\xi_{n,p}(x) = Y_n(x)\xi_{n,p}^{(0)} \qquad A_{n,p}(x) = Y_n(x)A_{n,p}^{(0)}(x)Y_n^\dagger(x) \tag{13}$$

This new fields $\xi_{n,p}^{(0)}$ and $A_{n,p}^{(0)}$ are the fields that we use in calculating PDFs and Beam functions and I will drop the superscript later on.

2.3 SCET PDF definition

To define PDF in terms of SCET It is necessary to define new fields [1]

$$\chi_n = W_n^\dagger(x)\xi_n(x) \qquad B_{n\perp} = \frac{1}{g}\left[W_n^\dagger(x)iD_{n\perp}W_n(x)\right] \tag{14}$$

where $D_{n\perp} = P_\perp + gA_{n\perp}$. We also denote a state of hadron with momentum p as $|Pt(p)\rangle$ (p is fully collinear momentum $p = np^-/2$). Then We define three bare operators [1]:

$$\begin{aligned}
Q_q^b(\omega) &= \Theta(\omega)\bar{\chi}_n(0)\frac{\bar{\not{n}}}{2}\left[\delta(\omega - \bar{P}_n)\chi_n(0)\right], \\
Q_{\bar{q}}^b(\omega) &= \Theta(\omega)tr\left\{\frac{\bar{\not{n}}}{2}\bar{\chi}_n(0)\left[\delta(\omega - \bar{P}_n)\chi_n(0)\right]\right\}, \\
Q_g^b(\omega) &= -\omega\Theta(\omega)B_{n\perp\mu}^c(0)\left[\delta(\omega - \bar{P}_n)B_{n\perp}^{\mu c}(0)\right]
\end{aligned} \tag{15}$$

Now PDF is defined as:

$$f_i(\omega/p^-, \mu) = \langle Pt(p)|\, Q_i(\omega,\mu)\, |Pt(p)\rangle \tag{16}$$

Where matrix elements will always be averaged over hadron spins which are suppressed in the notation. Obviously we don't know the hadron state, so we only can calculate renormalization group equations for PDFs by inserting highly unphysical gluon or quark free states instead of hadron state in the PDF definition. We actually do that in calculations. Its not hard to see that operators Q_i^b are time ordered $T\left\{Q_i^b(\omega)\right\} = Q_i^b(\omega)$, so we can use usual Feynman rules to calculate the renormalization group equations of PDFs in SCET.

2.4 SCET beam function definition

For beam functions we define similar operators and have analogous definition of beam functions for hadrons. For example quark operator and its fourier transform are(y^- corresponds to fourier transform of p^+ component):

$$\begin{aligned}
\tilde{O}_q^b(y^-, \omega) &= e^{-i\hat{p}^+y^-/2}\bar{\chi}_n\left(y^-\frac{n}{2}\right)\frac{\bar{\not{n}}}{2}\left[\delta(\omega - \bar{P}_n)\chi_n(0)\right] \\
O_q^b(|\omega|b^+, \omega) &= \bar{\chi}_n(0)\delta(\omega b^+ - \omega\hat{p}^+)\frac{\bar{\not{n}}}{2}\left[\delta(\omega - \bar{P}_n)\chi_n(0)\right]
\end{aligned} \tag{17}$$

Now the definition of Beam function is:

$$B_i(t, x = \omega/p^-, \mu) = \langle Pt(p)| \Theta(\omega) O_i(t, \omega, \mu) |Pt(p)\rangle \tag{18}$$

Now if we calculate beam function and PDF for the same hadron state up to the same order ϵ^0 in regularization parameter ϵ, then by equation 3 we can derive the jet structure functions I_{ij} by matching our results. The equation 3 do not hold precisely for beam functions and PDFs defined in terms of SCET, instead one can prove satisfying analogue [1]:

$$B_i(t, x, \mu) = \sum_j \int_x^1 \left[\frac{d\xi}{\xi} I_{ij}\left(\frac{x}{\xi}, t, \mu\right) f_j(\xi, \mu) \right] \left[1 + O\left(\frac{\Lambda_{QCD}^2}{t}\right) \right] \tag{19}$$

Actually the main goal of my work was to calculate second order correction to quark beam function which is proportional to number of quark flavours n_f. To calculate beam functions we define a new operator $T_i = T\{O_i\}$ and then one can prove that [1]:

$$\begin{aligned}
&\langle Pt(p)| \Theta(\omega) O_i(t, \omega, \mu) |Pt(p)\rangle = \\
&\delta(t)\delta(\omega - p^-) \langle Pt(p)| \bar{\chi}_n(0) \frac{\not{\bar{n}}}{2} |Pt(p)\rangle_{connected} \langle Pt(p)| \chi_n(0) |Pt(p)\rangle_{connected} \\
&+ Disc_{t>0} \langle Pt(p)| T_q(t, \omega) |Pt(p)\rangle_{connected}
\end{aligned} \tag{20}$$

Where subscript connected stands for connected feynman diagrams, where disconnection in diagrams comes from separation of χ and $\bar{\chi}$ in position space in O_i operator (equation 17). Discontinuity of function g is defined as:

$$Disc_{x>x_0} g(x) = \lim_{B \to 0} \Theta(x - x_0) \left[g(x_0 + iB) - g(x_0 - iB) \right] \tag{21}$$

3 Two loop correction to beam function into PDF matching

All calculations are made using $\overline{MS}$ renormalization scheme with dimension $D = 4 - 2\epsilon$

3.1 Calculation of fermion loop correction to collinear gluon propagator

Figure 1 shows diagram corresponding to fermion loop correction of collinear gluon propagator. This correction should be the same as in full theory(QCD), because we have only one direction in calculation. I wanted to do some cross check but didn't had time to do it, so I will only state a bare result known from QCD [4]:

$$\Pi^{\mu\nu}(l, \mu_b) = 8 i n_f T_f \left(\frac{\mu_b^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \left(l^2 g^{\mu\nu} - l^\mu l^\nu\right) (-l^2)^{-\epsilon} \frac{-g^2}{(4\pi)^{d/2}} \int_0^1 dx\, (x(1-x))^{1-\epsilon}\, \Gamma(\epsilon) \tag{22}$$

In final calculation the counter term should be also added, but I haven't calculate the counterTerm part so I will only show the result without counterTerm subtracted .

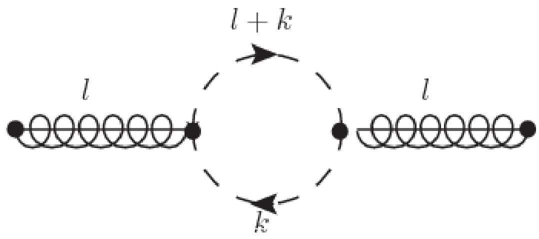

Figure 1: Fermion loop correction to gluon propagator

3.2 2 loop diagrams proportional to number of flavours

As was said before to do matching between PDFs and beam functions we can calculate them for artificial states of quarks and gluons. For matching we are supposed to use quark and gluon states with momenta proportional to n. I will use following variables:

$$p = (0, p^-, 0) \qquad t = \omega b^+ \qquad z = \frac{\omega}{p^-} \tag{23}$$

With this variables we can write $O_q(t, z, \mu_b)$ and $Q_q(z, \mu_b)$. For calculating diagrams we will use rule from equation 20. Lets first consider diagram 2a. Amplitude for this diagram after some simplifications can be expressed as:

$$Cor^a = -\left(\frac{\mu_b^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} g^2 C_F \frac{\theta(z)}{z}$$
$$\int \frac{d^d l}{(2\pi)^d} \frac{\frac{1}{2} tr\left\{V_n^{\mu}(p, p+l) V_n^{\nu}(p+l, p)\frac{\bar{\not n}\not n}{4}\right\} \Pi_{\mu\nu}(l, \mu_b)\,(p^- + l^-)^2}{(p+l)^2 \cdot (l^2)^2} \delta(\omega - (p+l)^-)\delta(b^+ + l^+) \tag{24}$$

Where

$$V_n^{\mu}(p, k) = n^{\mu} + \frac{\not k_{\perp}\gamma_{\perp}^{\mu}}{k^-} + \frac{\gamma_{\perp}^{\mu}\not p_{\perp}}{p^-} - \frac{\not k_{\perp}\not p_{\perp}}{p^-}\bar{n}^{\mu} \tag{25}$$

and it corresponds to SCET gluon fermion vertex $igT^a V_n^{\mu}\frac{\bar{\not n}}{2}$. Further calculation of this diagram goes in several steps:

- Changing integration variables to $l^+, l^-, l_{\perp}$ with Jacobian equal 1/2
- Integrating delta functions

- Introducing Feynman parameters for denominator(If we consider loop counter term there would be 0 instead of ϵ):

$$\frac{1}{(\vec{l}_\perp^2+t)^2\left[\vec{l}_\perp^2-t(1/z-1)\right]^{2+\epsilon}}=\int_0^1 dx(1-x)x^{1-\epsilon}\frac{\Gamma(4+\epsilon)}{\Gamma(2)\Gamma(2+\epsilon)}\frac{1}{\left[\vec{l}_\perp^2-t\left(\frac{x}{z}-1\right)\right]^{4+\epsilon}} \tag{26}$$

- Integrating terms $\frac{(\vec{l}_\perp^2)^2}{\left[\vec{l}_\perp^2-t\left(\frac{x}{z}-1\right)\right]^{4+\epsilon}}$, $\frac{\vec{l}_\perp^2}{\left[\vec{l}_\perp^2-t\left(\frac{x}{z}-1\right)\right]^{4+\epsilon}}$, $\frac{1}{\left[\vec{l}_\perp^2-t\left(\frac{x}{z}-1\right)\right]^{4+\epsilon}}$,
- Taking the discontinuity $Disc_{t>0}$ of an amplitude.
- Integrating over Feynamn parameter.

It is worth mention that using $\overline{MS}$ scheme of renormalization we are making use of analytical continuation of integration results from regions where results are converging. During calculation I weren't aware of that so I always wanted to have results converging for some ϵ from any neighbourhood of 0(<0 or >0 depending of the type of divergence). That wasn't the case for Feynamn parameter integral at the end .That was a reason why my calculations were a little bit more complicated, because I had to depending on which situation I was dealing, $\frac{1}{(\vec{l}_\perp^2+t)^2\left[\vec{l}_\perp^2-t(1/z-1)\right]^{2+\epsilon}}$ or $\frac{\vec{l}_\perp^2}{(\vec{l}_\perp^2+t)^2\left[\vec{l}_\perp^2-t(1/z-1)\right]^{2+\epsilon}}$ or $\frac{(\vec{l}_\perp^2)^2}{(\vec{l}_\perp^2+t)^2\left[\vec{l}_\perp^2-t(1/z-1)\right]^{2+\epsilon}}$ do once, twice or none decomposition of denominators:

$$\frac{1}{(l^2-A)(l^2-B)}=\frac{1}{A-B}\left(\frac{1}{l^2-B}-\frac{1}{l^2-A}\right)$$

The Final Result up to order $\frac{1}{\epsilon}$ was:

$$cor^a=\frac{\alpha^2 C_f n_f T_f\theta(z)}{32\pi^2\mu^2}\frac{\theta\left(t/\mu^2\right)}{\left(t/\mu^2\right)^{1+2\epsilon}}\left\{\frac{2(1-z)}{3\epsilon}+\frac{16}{9}\Big[-2z+6(z-1)log(1-z)-\right.$$
$$\left.9(z-1)log(z)+8\Big]+O(\epsilon)\right\} \tag{27}$$

Where we have to use expansion:

$$\frac{\theta(t/\mu^2)}{\left(t/\mu^2\right)^{1+2\epsilon}}=\frac{\delta\left(t/\mu^2\right)}{2\epsilon}+\left(\frac{1}{(t/\mu^2)}\right)_+-\epsilon\left(\frac{\ln\left(t/\mu^2\right)}{(t/\mu^2)}\right)_++\epsilon^2\left(\frac{\ln^2\left(t/\mu^2\right)}{(t/\mu^2)}\right)_+ \tag{28}$$

The whole result include also order ϵ^0, but I decided that it is not necessary to show that order, especially because it is really long term. It shouldn't be taken for granted that this result is correct, especially when you take into account, that I was finding a lot of errors in my calculations.

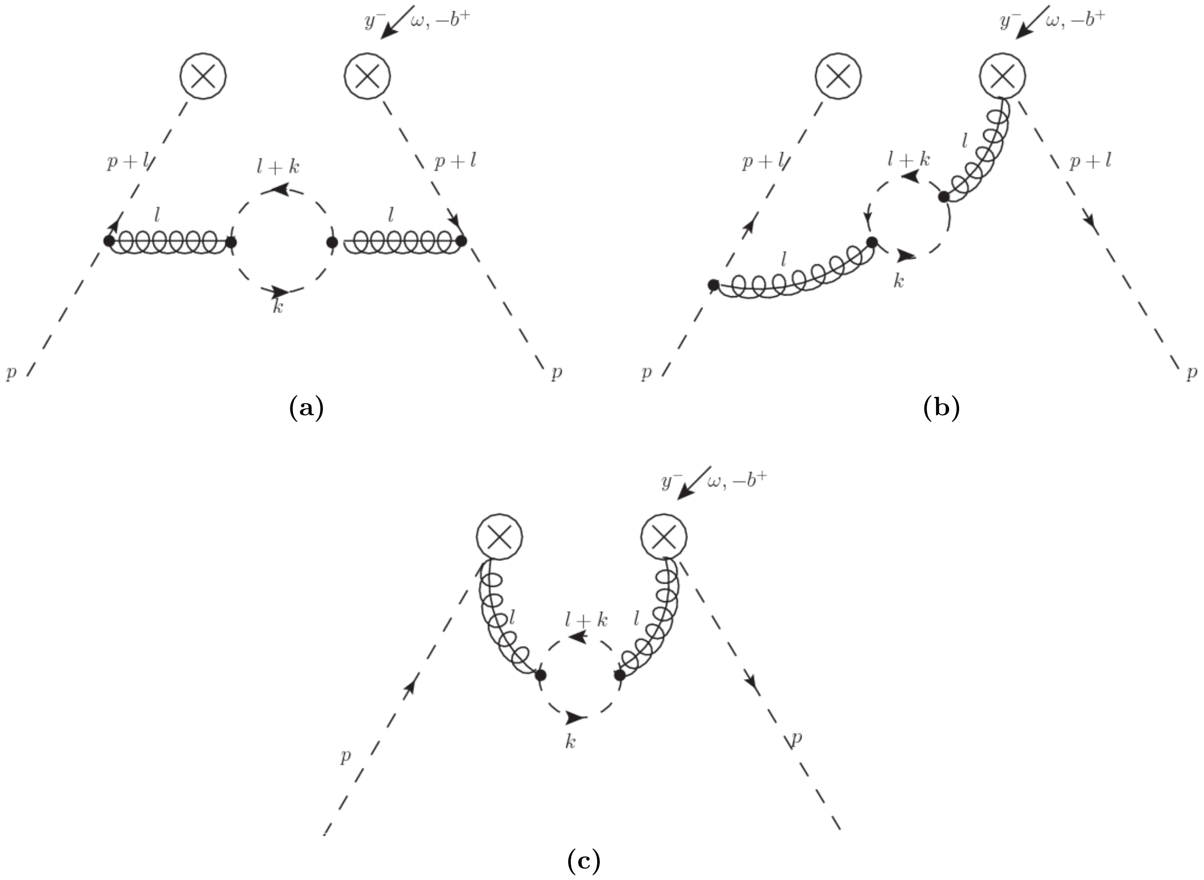

Figure 2: Diagrams corresponding to second order correction to beam functions proportional to n_f

Now lets consider the second diagram 2b. There is mirror graph which corresponds to exactly the same amplitude. The whole amplitude corresponding to this two diagrams may be written after some simplification as:

$$Cor^b = -2\left(\frac{\mu_b^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} g^2 C_F \frac{\theta(z)}{z}$$
$$\int \frac{d^d l}{(2\pi)^d} \frac{\frac{1}{2} tr\left\{V_n^\mu(p, p+l)\frac{\bar{n}\!\!\!/ n\!\!\!/}{4}\right\} \bar{n}^\nu \Pi_{\mu\nu}(l, \mu_b)\,(p^- + l^-)}{(p+l)^2 \cdot (l^2)^2} \delta(\omega - (p+l)^-)\delta(b^+ + l^+) \qquad (29)$$

Another again it is amplitude without fermion loop counter term. The calculation procedure of this amplitude is pretty much the same as for diagram 2a with one difference that denominator is now $\frac{1}{(\vec{l}_\perp^2+t)\left[\vec{l}_\perp^2-t(1/z-1)\right]^{2+\epsilon}}$. The calculation leads to result:

$$cor^B = \frac{-\alpha^2 C_f n_f T_f \theta(z)}{16\pi^2\mu^2}\frac{\theta\left(t/\mu^2\right)}{\left(t/\mu^2\right)^{1+2\epsilon}}\Bigg\{\frac{8z\delta(1-z)}{3\epsilon^2} + \frac{16\left(\delta(1-z) - 3z\left(\frac{1}{1-z}\right)_+\right)}{9\epsilon} -$$
$$\frac{4}{27}[\delta(1-z)(9\pi^2-44)12\left(\frac{1}{1-z}\right)_+(5z+9zlog(z)-3) - 72z\left(\frac{log(1-z)}{1-z}\right)_+] + O(\epsilon)\Bigg\} \quad (30)$$

Where you have to use the identity 28 for term $\frac{\theta(t)}{t^{1+2\epsilon}}$ It is another again to order of $\frac{1}{\epsilon}$ and is not full result in a sense that term proportional to ϵ^0 should also be fully included. Finally I only state the result for third diagram 2c:

$$cor^C = \frac{\alpha^2 C_f n_f T_f \theta(z)}{32\pi^2\mu^2}\frac{\theta\left(t/\mu^2\right)}{\left(t/\mu^2\right)^{1+2\epsilon}}\Bigg\{\frac{8\delta(1-z)}{3\epsilon} + \frac{16}{9}\Bigg[(\delta(1-z) - 3\left(\frac{1}{1-z}\right)_+\Bigg] - \frac{4}{27}\epsilon\Bigg[(\delta(1-z)(9\pi^2-44) + 24\left(\frac{1}{1-z}\right)_+(3\ln(z)+1) - 72\left(\frac{\ln(1-z)}{1-z}\right)_+\Bigg] + O\left(\epsilon^2\right)\Bigg\} \quad (31)$$

Where now it is a full result.

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