# Double Parton Distribution Functions

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**Abstract**

Double parton scattering may constitute a significant background to important processes such as Higgs boson production. This paper examines certain features of double parton distribution functions, which are required to fully calculate the cross-sections for double parton interactions. A code for carrying out the leading-order evolution of such PDFs is described and tested. A product of single PDFs is inputted and evolved, the results of which are compared to products of single PDFs at different evolution scales. It is observed that for many PDF sets they correspond closely for low values of the momentum fraction $x$. Polarized PDFs are also an important element of the total cross-section of double parton processes, and some of their features are examined here. The extent to which doubly polarized double PDFs are washed out by evolution is examined for the case of a low evolution starting scale, and it is observed that this suppression is slightly different than in the case of a higher starting scale. This suggests that the starting point is important when carrying out the evolution of polarized double PDFs. Finally, the results of rewriting the evolution code for the case where one parton is polarized and one is unpolarized are described. It is observed that the results are mainly consistent with an expected relationship between such PDFs and doubly polarized PDFs. It is hoped that studies of double PDFs like those in this paper will help clarify their structure, and enable accurate predictions to be made regarding an important background process at colliders like the LHC.

# Contents

# 1 Introduction

In my project, I focus on the theory of quantum chromodynamics (QCD), and in particular on double parton scattering between the parton constituents of two protons. Most scattering processes which take place at a proton-proton collider involve scattering of single partons within protons - quarks and gluons - off of one another. However, in many cases it can also happen that two partons within each proton scatter off each other in a single proton-proton collision. It has been shown that the contribution to background processes at the LHC and other colliders due to double parton collisions can be significant [1–3], and it is therefore desirable to have a fuller understanding of them. In this paper, I use a pre-existing evolution code described in [4], in order to generate and examine some of the features of *double parton distribution functions* (dPDFs), which are required to understand double parton scattering. This code takes input dPDFs at a certain $Q$ scale - $Q$ being the momentum transfer or momentum scale being considered - and uses the double parton version of the DGLAP evolution equations to calculate the corresponding dPDF at higher scales. I begin with a theoretical description of such functions and a description of the code used to generate them. I then describe the results of putting in a simple product of single parton distributions (sPDFs), and examine how the evolved dPDFs differ from simple products of sPDFs at higher energy scales. Such a product is chosen because it is the simplest approximation of dPDFs, which are not well understood. Another important aspect of calcutions involving dPDF is the significance of spin polarization correlations. Such spin correlations would have an important effect on the structure of dPDFs. I examine dPDFs for polarized partons, and particularly how the dPDFs for polarized partons are washed out by an evolution to higher energy scales. This has previously been done where the starting scale for evolution is $Q^2 = 1$ GeV$^2$, and where both partons are polarized in the same direction [5]. I compare this to the case where the starting scale is $Q^2 = 0.3$ GeV$^2$. Finally, I describe the results of changing the pre-existing code in order to deal with the case where one parton is polarized and one is unpolarized. I then compare these results to dPDFs with two polarized partons.

# 2 Theory

## 2.1 Parton distribution functions

A very important aim of QCD is the calculation of the cross-section of whatever interaction is being considered. To calculate the cross section for a proton-proton interaction, one often uses the factorization formula, which states that the cross section for such a reaction ($\sigma$) can be expressed as a convolution of the cross section for the reaction of constituent partons $i$ and $j$ ($\sigma_{ij}$), which can be calculated analytically, and the parton distribution functions (PDFs) of constituent partons $f_i(x_1)$ and $f_j(\overline{x}_1)$. Here the variable $x_1$ describes the fraction of the total proton longitudinal momentum possessed by the parton, and the parton distribution functions describe the probability of finding a parton with a momentum fraction $x$. Examples of PDFs are shown in Fig. 1. The factorization formula can be expressed thusly [5]:

$$\sigma = \sum_{ij} \int dx_1 \int d\overline{x}_1 f_i(x_1) f_j(\overline{x}_1) \sigma_{ij}. \tag{1}$$

The PDFs may be measured experimentally using deep inelastic scattering, but it is important to realize that these are not independent of the scale being considered. While this was once thought to be true (Bjorken scaling), it is now known that these functions vary logarithmically with $Q^2$. One reason for the violation of Bjorken scaling is the assumption made in the derivation of the parton model that the transverse momenta of partons is negligible in a large $Q^2$ limit, and that therefore only the longitudinal momentum fraction is significant. However, as described in [6], it is possible, for example, for a gluon to be emitted by quark in a transverse direction, thus causing the quark to obtain a non-trivial transverse momentum itself. As a result, while at low energies we may think of a proton as roughly composed of three (valence) quarks held together by gluons, at higher energy scales this becomes far more chaotic with gluons and (sea) quarks spontaneously forming, complicating enormously the picture of the proton. The evolution of the PDFs with $Q^2$ is described by the well-known Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation. In its simplest form this can be expressed as [5, 6]

$$\frac{\partial}{\partial \ln Q^2} f_{q_v}(x, Q) = \frac{\alpha_s(Q)}{2\pi} \int_x^1 \frac{dx'}{x'} P\left(\frac{x}{x'}\right) f_{q_v}(x', Q) \tag{2}$$

or more compactly

$$\frac{\partial}{\partial \ln Q^2} f_{q_v}(x, Q) = \frac{\alpha_s(Q)}{2\pi} P\left(\frac{x}{x'}\right) \otimes f_{q_v}(x', Q). \tag{3}$$

Here, $f_{q_v}(x, Q)$ is a valence quark distribution function, $\alpha_s(Q)$ is the running strong coupling constant, and $P$ is known as a splitting function. $P$ is properly expressed as an expansion in $\alpha_s$, but if we only take the leading order term in this expansion, we can interpret $P_{ab}(x)$ as the probability that a parton of type $b$ emits a parton of type $a$ with momentum fraction $x$ [6]. This equation, however, only applies to valence quark distributions - full quark distributions, which include the contribution from sea quarks, are coupled to gluon PDFs $f_g(x, Q)$ in the evolution equation. In order to simplify a calculation of the evolution of all thirteen PDFs (six quarks, six anti-quarks, and the gluon), an evolution basis is often used, whereby only one element of the basis - the so-called singlet distribution $\Sigma(x, Q)$ - is coupled to the gluon, and the other eleven elements of the basis can be calculated by using Eq. 2. These eleven elements are the valence distributions $V_i = f_{q_i} - f_{\overline{q}_i}$ and other flavour combination which are given on page 110 of [6]. $\Sigma(x, Q)$ is simply the sum of the all the quark and anti-quark distributions, and its evolution, along with that of the gluon, is described by

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} \Sigma(x, Q) \\ f_g(x, Q) \end{pmatrix} = \frac{\alpha_s(Q)}{2\pi} \int_x^1 \frac{dx'}{x'} \begin{pmatrix} P_{qq}\left(\frac{x}{x'}, \alpha_s(Q)\right) & 2n_f P_{qg}\left(\frac{x}{x'}, \alpha_s(Q)\right) \\ P_{gq}\left(\frac{x}{x'}, \alpha_s(Q)\right) & P_{gg}\left(\frac{x}{x'}, \alpha_s(Q)\right) \end{pmatrix} \times \begin{pmatrix} \Sigma(x', Q) \\ f_g(x', Q) \end{pmatrix}. \tag{4}$$

Here, $n_f$ is the number of quark flavours at a given energy scale. Given a PDF at a given input scale, then, we may calculate the corresponding PDF for other scales as well.

## 2.2 Double parton distribution functions

For the case of double parton scattering, an equation similar to Eq. 1 is used for the calculation of the cross-section, except that we are now dealing with double PDFs, $f_{ij}(x_1, x_2, \vec{y}, Q)$ which are a function of the longitudinal momentum fractions of two partons, the vector $\vec{y}$ which defines the relative position of the two partons, and of course $Q$. However, it is often assumed the $\vec{y}$-dependence can be factored out [5,7]. Then evolution $f_{ug}(x_1, x_2, Q)$, for example, then describes the probability of finding a u quark with momentum fraction $x_1$ and a gluon with momentum fraction $x_2$ at an energy scale Q. As we might expect, there is also a double DGLAP equation which describes the evolution of such dPDFs [8]:

$$\frac{\partial f_{ij}(x_1, x_2, \vec{y}, Q)}{\partial \ln Q^2} = \frac{\alpha_s(Q)}{2\pi}\left[\sum_{i'} \int_{x_1}^{1-x_2} \frac{dx_1'}{x_1'} f_{i'j}(x_1', x_2, \vec{y}, Q) P_{ii'} + \sum_{j'} \int_{x_2}^{1-x_1} \frac{dx_2'}{x_2'} f_{ij'}(x_1, x_2', \vec{y}, Q) P_{jj'}\right]. \quad (5)$$

As mentioned above, the $\vec{y}$ may be factored out and set to 1. This equation involves two separate integrals over $x_1$ and $x_2$. If we take the simplest assumption about the form of the dPDF, that it is a simple product of sPDFs [4,5], such that $f_{ij}(x_1, x_2, Q) = f_i(x_1, Q) f_i(x_2, Q)$, then we note that

$$\frac{\partial f_{ij}(x_1, x_2, Q)}{\partial \ln Q^2} = \frac{\partial f_i(x_1, Q)}{\partial \ln Q^2} f_j(x_2, Q) + f_i(x_1, Q) \frac{\partial f_j(x_2, Q)}{\partial \ln Q^2} \quad (6)$$

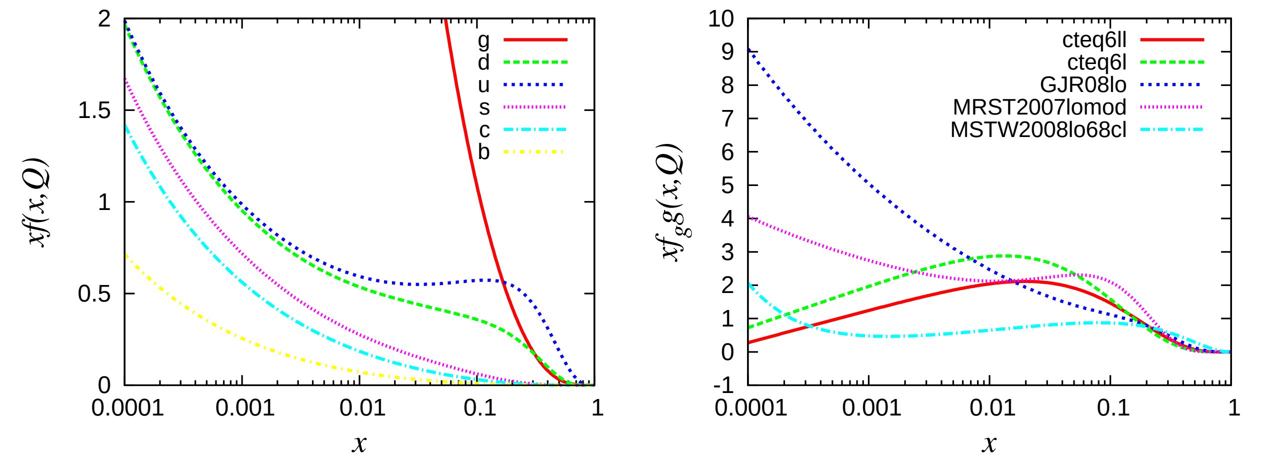
$$\approx \frac{\alpha_s}{2\pi}\left[\left(P_{ii'} \otimes f_i(x_1, Q)\right) f_j(x_2, Q) + f_i(x_1, Q)\left(P_{jj'} \otimes f_j(x_2, Q)\right)\right] \quad (7)$$

for non-singlet states, from Eq. 3. But Eq. 7 is just another way of writing Eq. 5, with the $\vec{y}$ dependence factored out. Hence the product of two sPDFs evolving separately is, ignoring one simplification, equivalent to the evolution of a dPDF. If we input a product of sPDFs and evolve them according to Eq. 5, then we should get a similar result to evolving both of those sPDFs separately and multiplying them, barring effects due to correlations between the partons in a single hadron.

To see how this result may be inaccurate, we notice that in deriving Eq. 7, we ignored the fact that the integration limits in the double DGLAP equation do not go up to 1 as they do in Eq. 2. This is due to the conservation of momentum - i.e. it expresses the obvious fact that the total momentum fraction of both partons cannot exceed 1. This introduces a simple correlation between the two partons which, as we will see, has an effect on the form of the dPDFs.

Correlations which may be significant for cross-section calculations are correlations due to the spin polarization of constituent partons, even if the protons themselves are unpolarized [7,9]. It is shown in [9] that the spin polarizations for two partons may be expressed as a spin density matrix, which may be used to establish upper limits for the possible polarization of partons within unpolarized hadrons. These spin density matrices includes elements with dPDFs $f_{ab}$, where $a$ and $b$ can be $q/g$ for unpolarized quarks and gluons, $\Delta q/\Delta g$ for quark and gluons polarized longitudinal to the beam direction, and $\delta q/\delta g$ for transversely polarized quarks and linearly polarized gluons. They may also be polarized or unpolarized anti-quarks $\overline{q}$. These polarized dPDFs are actually differences between the probabilities of finding the parton in certain spin configurations. It is interesting to observe how these polarized dPDFs evolve when put into equation 5, and we will examine such cases below. In order to evolve polarized distributions, it is necessary to use polarized splitting functions. All splitting functions are given in [9].

# 3 Evolution code




(a) Single PDFs at Q=10GeV for the MSTW2008 set for the gluon and quarks [10].

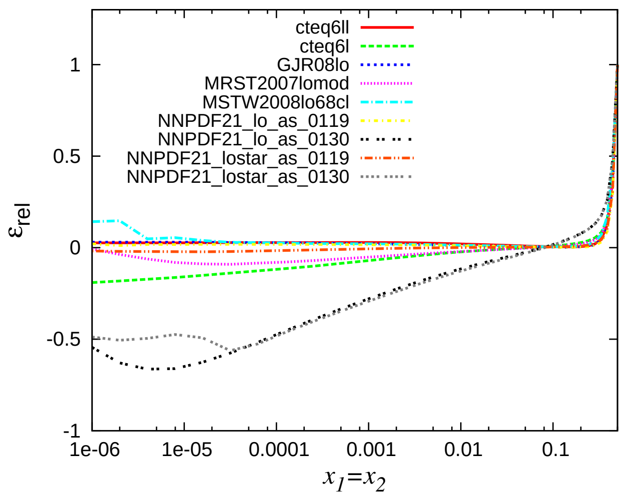
(b) Gluon PDFs at Q=1GeV, demonstrating the difference between various PDF sets.

Figure 1: Examples of PDFs available in the LHAPDF online repository.

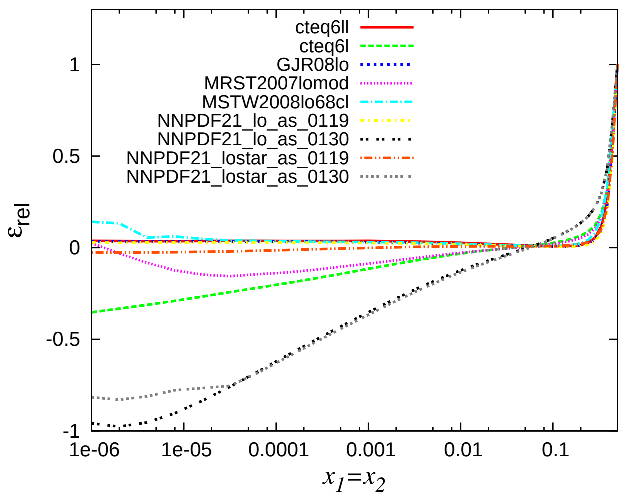
In order to evolve various dPDFs, we use a code developed by Gaunt and Stirling [4]. This code uses Newton-Cotes rules to solve the integrals on the RHS of Eq. 5 and then uses a fourth-order Runge-Kutta method to solve the resultant ordinary differential equation. The grid spacings are logarithmic - they are spaced evenly in $u = \ln \frac{x_i}{1-x_i}$. For all of our studies, we have $x_{min} = 10^{-6}$, to reflect the minimum $x$ of many of the publicly available LHAPDF single PDF sets [11]. Examples of the PDFs in such sets, and how they may differ, are shown in Fig. 1. As described in section 2.1, evolution may take place with leading order (LO) or higher order splitting functions. This code is designed to deal only with LO splitting functions, and only LO sPDF sets are used for comparison. Evolution is carried out in the evolution basis described in section 2.1, where the input values at the starting energy scale are simple products of the evolution basis single parton distributions at that scale. The code uses a variable flavour number (VFN) scheme in the evolution. This means that contributions from charm and bottom quarks are introduced once the energy scale rises above their respective masses. Their masses are therefore taken as inputs to the code, as well as the value of $\alpha_s(Q)$ at the starting scale. The evolution of $\alpha_s$ in the code is also leading order. It is also possible, however, to modify the code such that the value of $\alpha_s$ at every scale is taken directly from LHAPDF.

# 4 Comparison of dPDFs with products of sPDFs at different energy scales

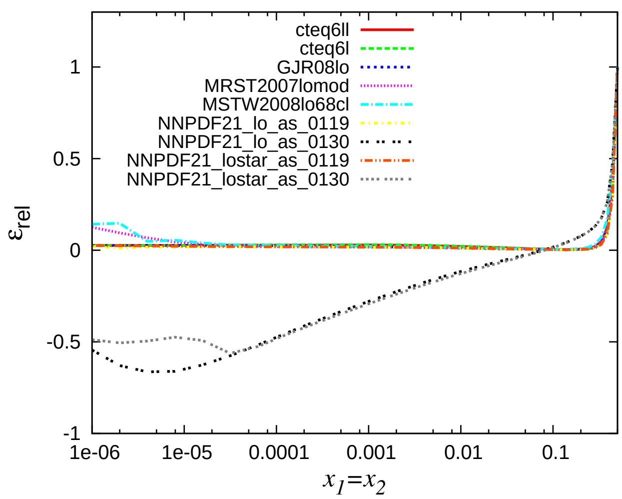
We begin by putting in products of sPDFs into the evolution code in order to examine how far they diverge from simple products of sPDFs at energies higher than the starting scale. There are many different PDF sets given in the LHAPDF for LO, reflecting different assumptions and methodologies for generating the PDF sets. Fig.1b, for example, shows how drastically



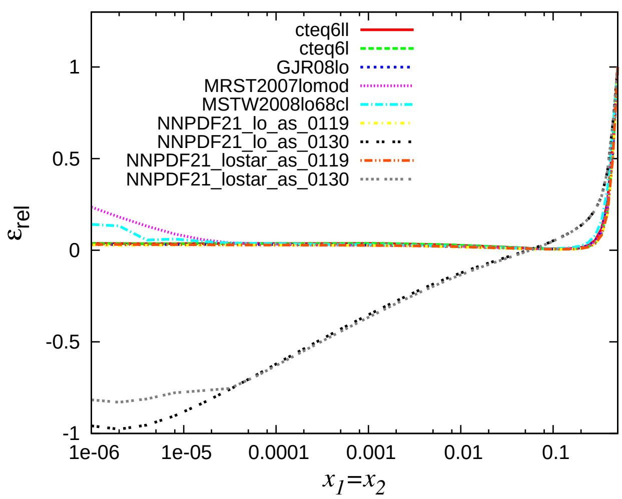

(a) The case where $\alpha_s$ is calculated by the code (Q=10GeV).




(b) The case where $\alpha_s$ is calculated by the code (Q=100GeV).




(c) The case where $\alpha_s$ is taken from LHAPDF (Q=10GeV).




(d) The case where $\alpha_s$ is taken from LHAPDF (Q=100GeV).

Figure 2: The fractional difference, as defined in Eq. 8, between the dPDF $f_{uu}$ evolved by our evolution code and a product of sPDFs taken from LHAPDF. The starting scale for the evolution is $Q = 1.414$GeV.

the gluon PDFs for different sets disagree at $Q = 1$GeV. Every PDF set we examine uses a VFN scheme, and every set has a LO calculation of $\alpha_s$, with the exception of CTEQ6l which uses a next-to-leading order (NLO) calculation for $\alpha_s$ [12]. Using the CTEQ6l set gives us the opportunity to observe the importance of how $\alpha_s$ is calculated. We use a starting scale for the evolution of $Q^2 = 2\text{GeV}^2$. The reason for this is that the starting value of $Q^2$ for the NNPDF sets is $2\text{GeV}^2$, and the extrapolation subroutine contained in LHAPDF seems to be unable to output PDF values for lower scales [13]. It is also questionable how reliable this extrapolation method may be, and it is therefore safer to use a starting scale equal to or higher than the lowest non-extrapolated value of the PDF sets in LHAPDF. We compare the outputted dPDFs from our evolution code to a product of sPDFs at scales higher than the starting scale for each of nine PDF sets, where the sPDFs at higher values are taken directly from LHAPDF. We obtain the fractional difference, defined as

$$\epsilon_{rel} = 1 - \frac{f_{ij}(x_1, x_2, Q)}{f_i(x_1, Q) f_j(x_2, Q)}. \tag{8}$$

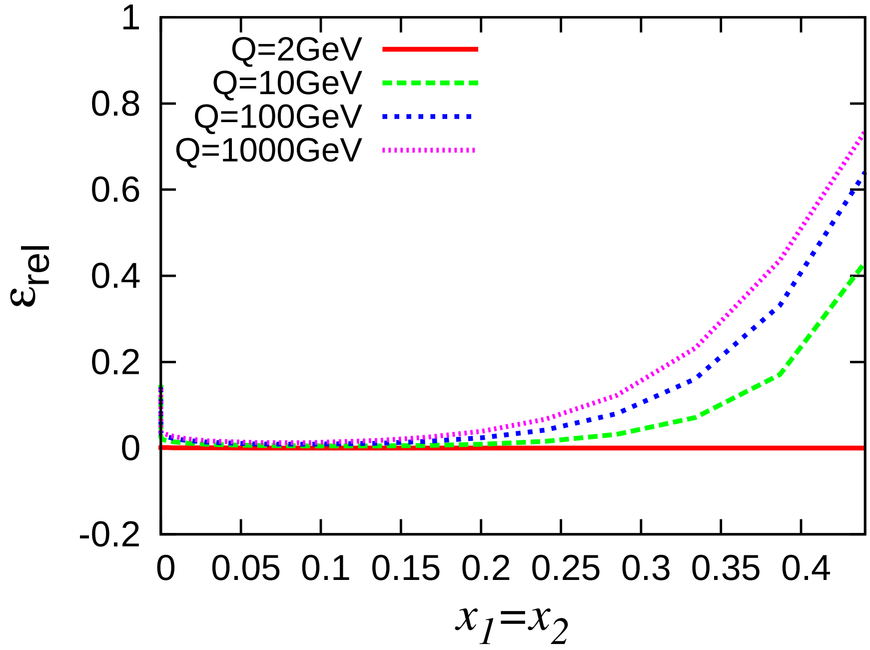


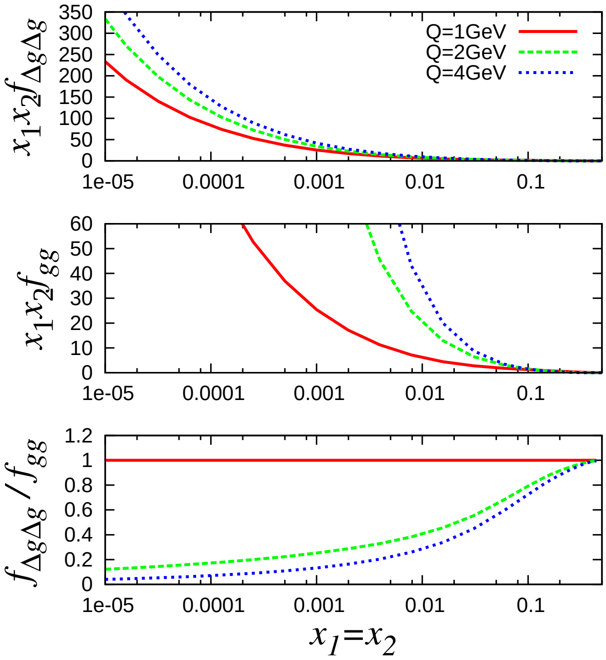

Figure 3: Fractional difference, as defined in Eq. 8, between the evolved dPDF $f_{uu}$ and a product of sPDFs at different energy scales for the MSTW2008lo set. The starting scale is $Q = 1.414$ GeV.

The results of this comparison are presented in Fig. 2 for $Q = 10$GeV and $Q = 100$GeV for the case $x_1 = x_2$ and where $\alpha_s$ is calculated in the code. (We will use the line $x_1 = x_2$ in all our analyses of dPDFs.) They are also presented for the case where $\alpha_s$ is taken directly from LHAPDF, rather than explicitly calculated in the code. We notice several features in these graphs. Firstly, the products of NNPDF sets where the value of $\alpha_s(M_Z)$ is taken to be 0.130 do not seem to agree very closely with our dPDFs, especially at lower $x$ values. We have not been able to explain this difference. However, for other cases there is good agreement at lower $x$ values. The CTEQ6l distribution, as we might expect, does not present very good agreement when $\alpha_s$ is calculated in the code - for the simple reason that the code calculates it to LO while CTEQ6l uses a NLO calculation. However, when the LHAPDF values of $\alpha_s$ are used, it presents much better agreement. At lower $x$ values, MRST2007 [14] also tends to disagree somewhat, but since these sPDFs only have a starting $x$ value of $10^{-5}$, they rely upon the LHAPDF extrapolation subroutine below this value, which may account for the discrepancy. For MSTW2008lo, there is a also a noticeable divergence at lower $x$ values. This is present to a roughly equal extent at all energy scales, and may be due to some numerical instability at very low $x$ values. In Fig. 3, we present the fractional difference for just the MSTW2008lo PDF set at different values of $Q$. This highlights the main point in which the dPDFs and the products of sPDFs disagree. At higher $x$ values, the fractional difference grows towards 1 as we get closer to $x_1 = x_2 = 0.5$. For higher $Q$ values, this begins earlier and earlier. This is of course due to the fact that the dPDFs must be zero for values of $x_1 + x_2 > 1$ due to momentum conservation, whereas products of sPDFs are not constrained to be non-zero. We see, however, that this effect is localized to higher $x$ values, and that for lower $x$ values this effect becomes relatively insignificant.

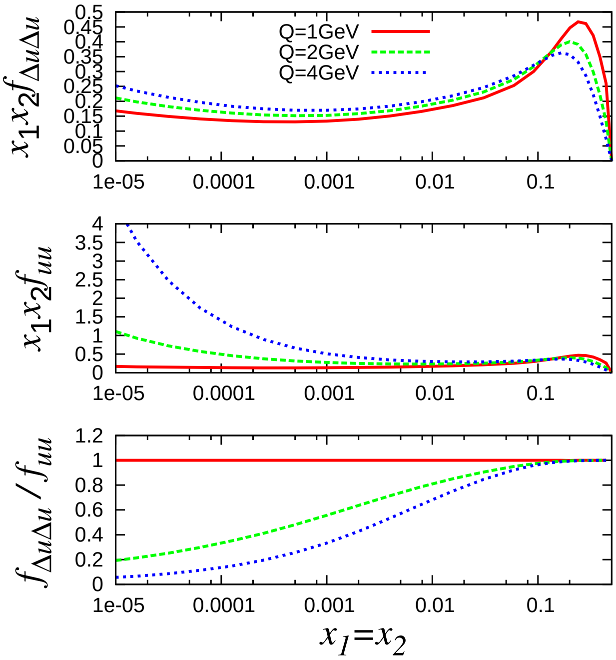
# 5 Polarized PDFs at low starting scales

In [5], the evolution of the dPDFs for the case where both partons are polarized in the same direction is examined. It is there noted that if we assume the polarized distribution is equal to the unpolarized at the starting scale, then the polarized distribution is quickly suppressed relative to the unpolarized because of the differing structure of the polarized and unpolarized splitting functions. The assumption that the polarized distribution is equal to the unpolarized comes from allowing it to be maximally polarized within the bounds established in [9] and mentioned above. For the purposes of comparison, we reproduce in Fig. 4 the results for both gluons and $u$ quarks with both longitudinal and transverse/linear polarization, for the GJR08lo PDF set [15]. We notice that the polarized dPDFs are suppressed quite quickly under evolution at low $x$ values. The GJR08lo set is particularly interesting because it provides single PDFs down to a value of $Q = 0.5477\text{GeV}$, which allows us to compare the evolution of the polarized distributions at this very low starting scale with those at the starting scale of $Q = 1\text{GeV}$ presented in Fig. 4. In Fig. 5 we present the case where the starting scale of the evolution is $Q = 0.5477\text{GeV}$.

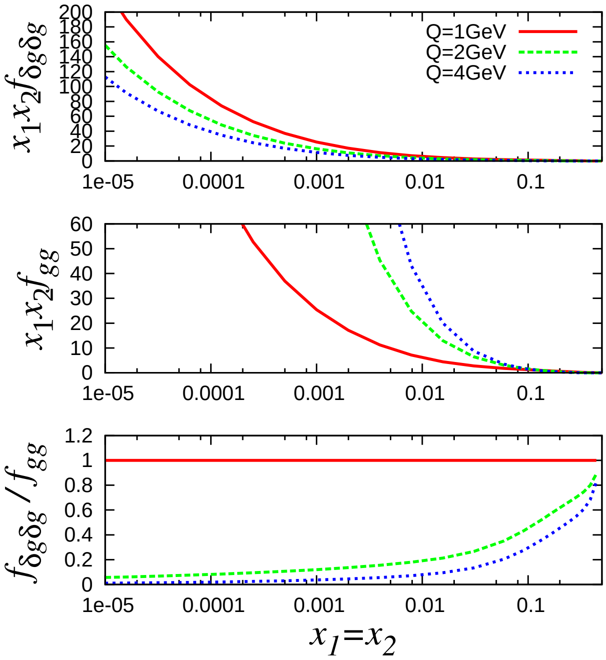
One interesting feature of these graphs is the differing evolution of the longitudinally and linearly polarized gluons, as in the case of the evolution with the higher starting scale of Fig. 4. This is evidently due to the differing structure of the respective splitting functions. In both cases, the polarized distribution is quickly suppressed relative to the unpolarized distribution, but in the longitudinal case, the ratio remains relatively constant after the initial suppression due to the growth in the polarized distribution at higher $x$ values compensating for the growth of the unpolarized distribution. In the linearly polarized case, the distribution continues to be further suppressed as $Q$ increases. Another interesting feature is that, particularly at lower $x$ values, the suppression happens much more quickly for the gluon distribution in the case of the low starting scale. This is due to the differing structure of the polarized distribution at the lower scale, where it is roughly centred around 0.3, and the different evolution which follows - it doesn't blow up at lower $x$ values as it does in the case of the higher starting scale, which has much higher values at lower $x$ at its starting scale. For the $u$ quark distributions, on the other hand, it is interesting to note that the suppression occurs more slowly than for the higher energy case for lower $x$ values. For example, the difference between the ratios of polarized to unpolarized distributions at twice the starting scale for both scales is presented in Fig. 6. This demonstrates that the $u$ quark is suppressed at a slower rate for lower $x$ values, but that the gluon is suppressed more quickly.



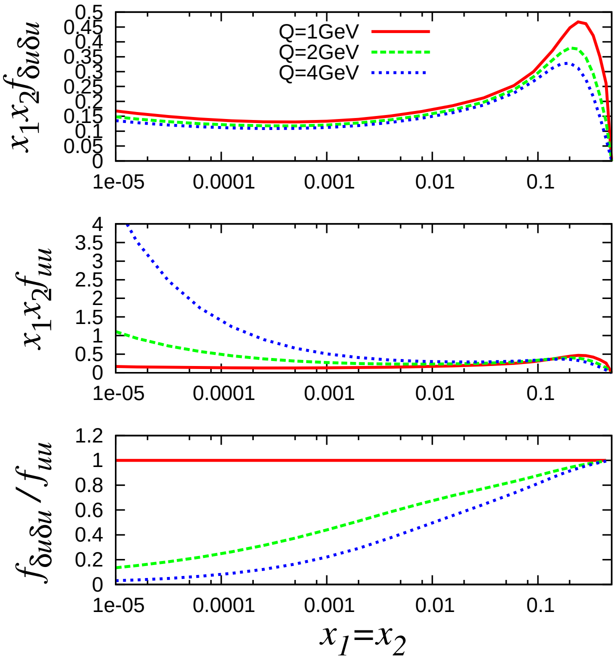

(a) Longitudinal polarization of gluons




(b) Longitudinal polarization of u quarks.

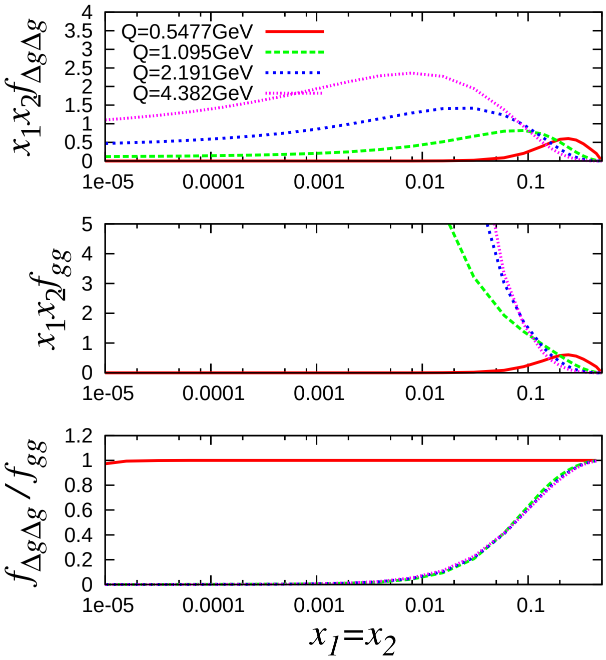



(c) Linear polarization of gluons.

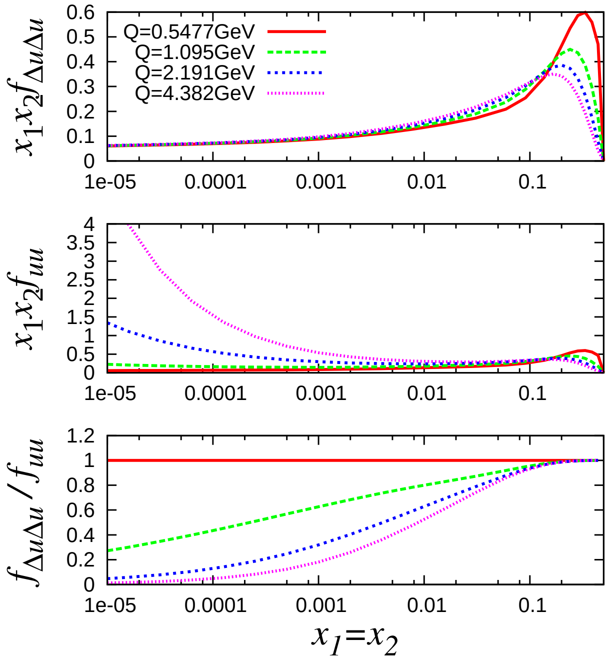



(d) Transverse polarization of u quarks.

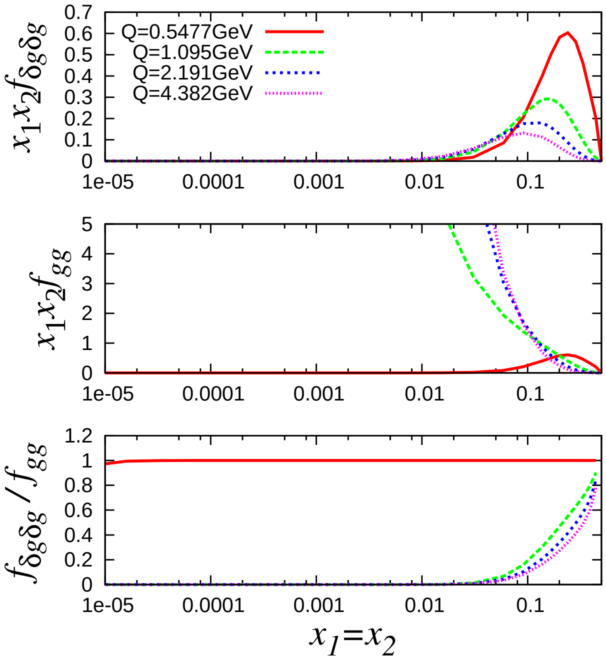
Figure 4: Comparison between polarized and unpolarized dPDFs under evolution, with a starting scale of $Q = 1\text{GeV}$ (GJR08lo) [15].



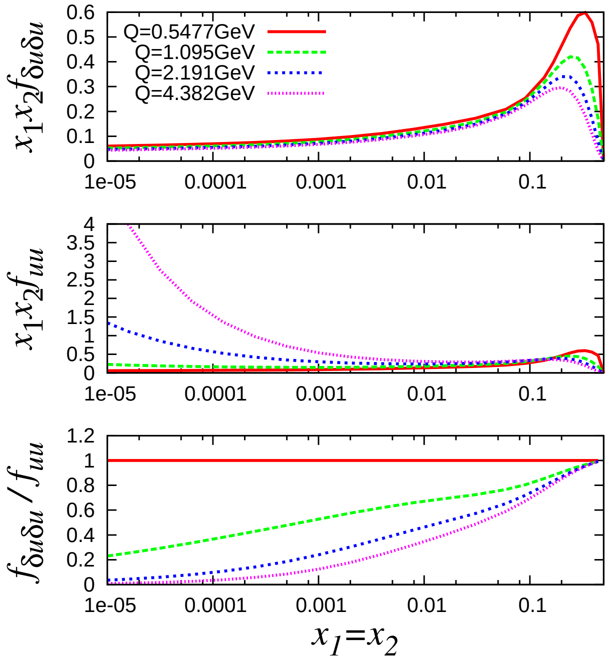

(a) Longitudinal polarization of gluons




(b) Longitudinal polarization of u quarks.

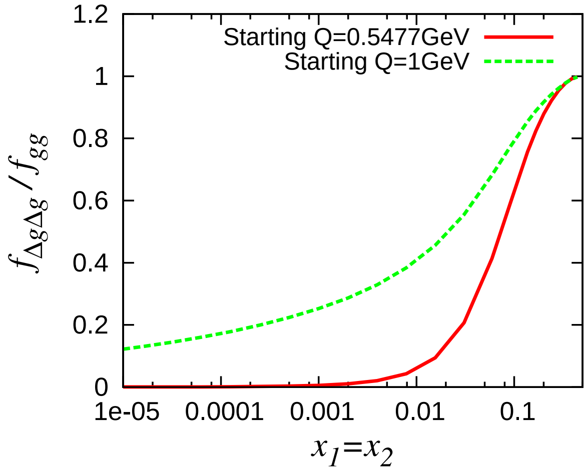



(c) Linear polarization of gluons.

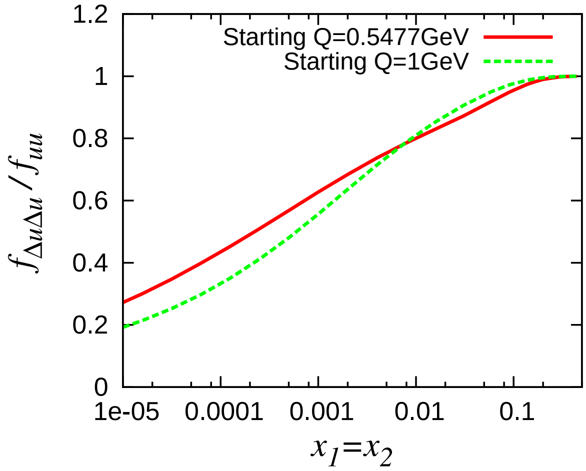



(d) Transverse polarization of u quarks.

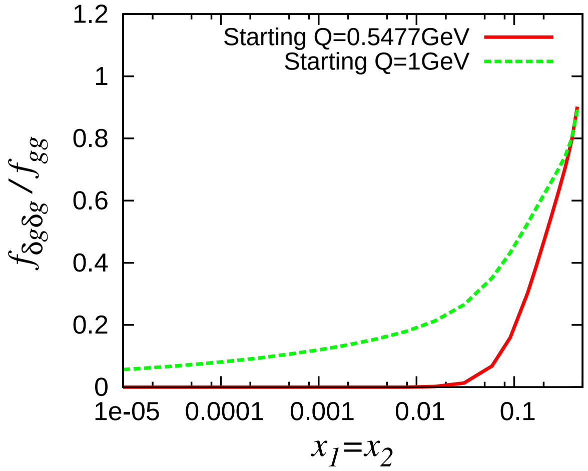
Figure 5: Comparison between polarized and unpolarized dPDFs under evolution, with a starting scale of $Q = 0.5477\text{GeV}$ (GJR08lo).



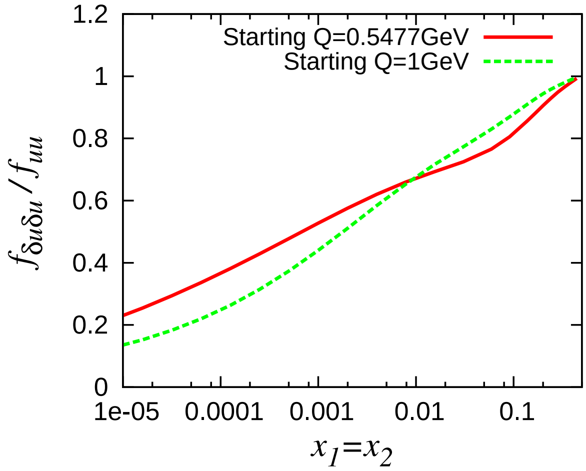

(a) Longitudinally polarized gluons.




(b) Longitudinally polarized u quarks.




(c) Linearly polarized gluons.

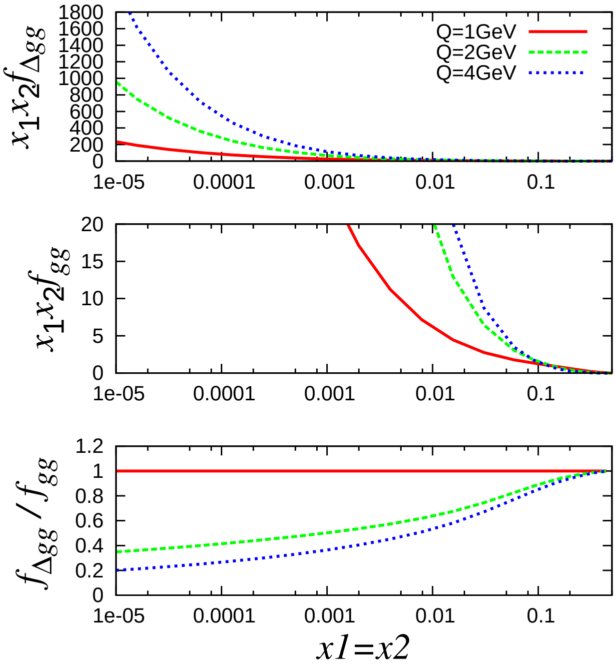



(d) Transversely polarized u quarks.

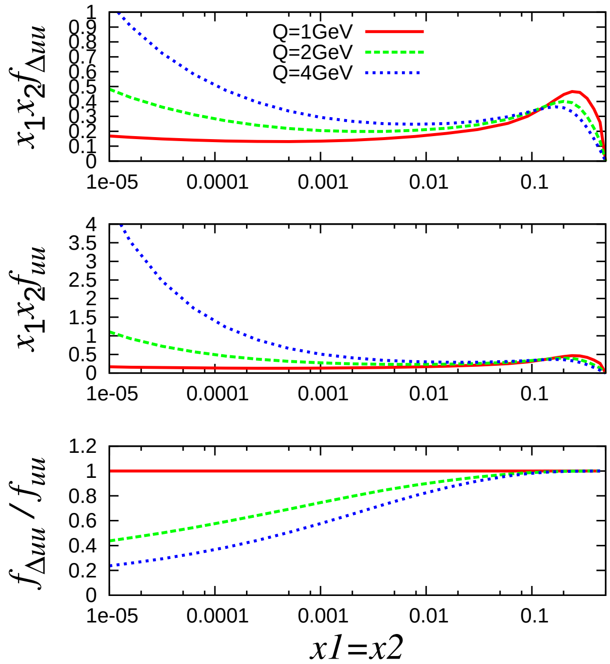
Figure 6: Ratio of polarized to unpolarized distributions for twice the starting scale at different evolution starting scales.

# 6 dPDFs for one polarized and one unpolarized parton

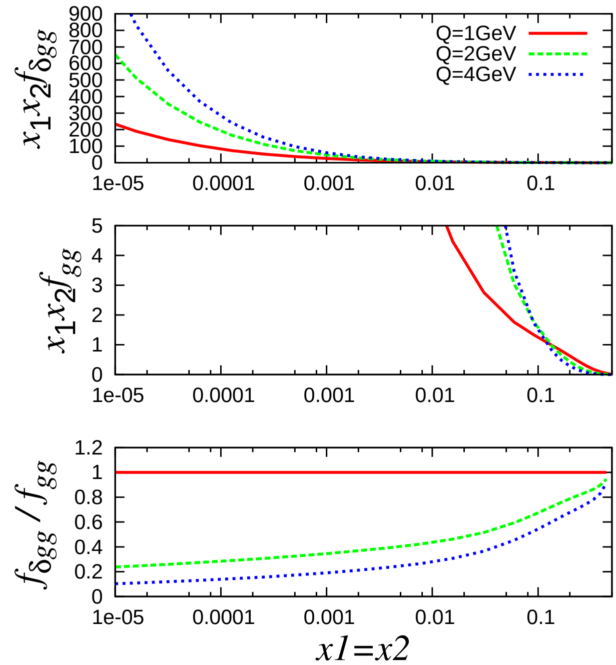
Finally we look at the case where one parton is polarized and one is unpolarized. Calculating dPDFs for such cases required the evolution code described in section 3 to be modified, as it was previously only capable of dealing with cases where both partons were polarized in the same direction. We looked at a case where the starting scale was $Q = 1\text{GeV}$ and examined how such dPDFs were affected by evolution. In Fig. 7 we present a comparison between the dPDFs for one polarized parton and for two unpolarized partons, all for the GJR08lo PDF set. Comparing them with the doubly polarized dPDFs in Fig. 4, we notice a number of differences. For example, the dPDF of longitudinally polarized gluons increases at smaller $x$ values here, whereas it decreases for the doubly polarized case. We can understand this by remembering



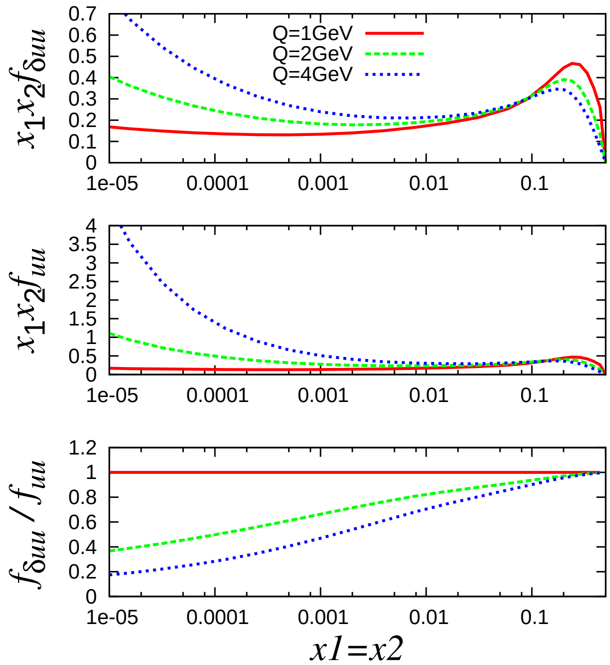

(a) Longitudinally polarized gluon and unpolarized gluon.




(b) Longitudinally polarized u quark and unpolarized u quark.



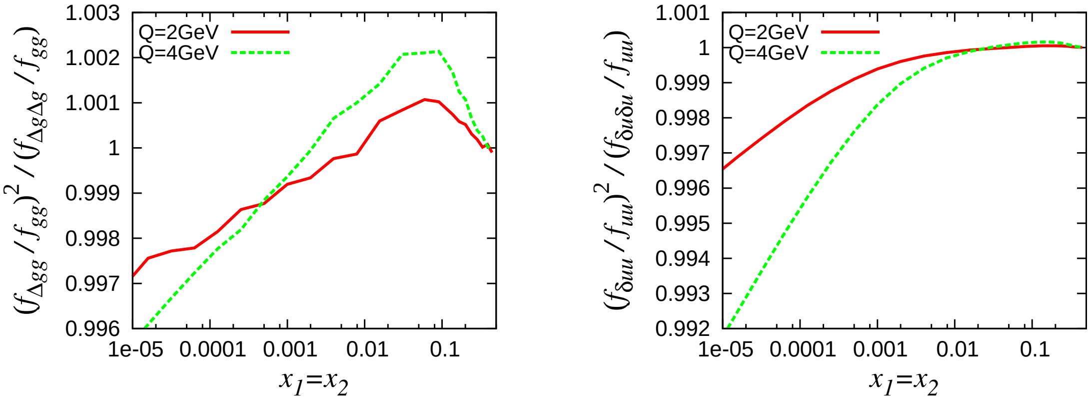

(c) Linearly polarized gluon and unpolarized gluon.




(d) Transversely polarized u quark and unpolarized u quark.

Figure 7: Comparison between dPDFs with one polarized and one unpolarized parton, and dPDFs with two unpolarized partons. The evolution has a starting scale of $Q = 1\text{GeV}$ (GJR08lo).

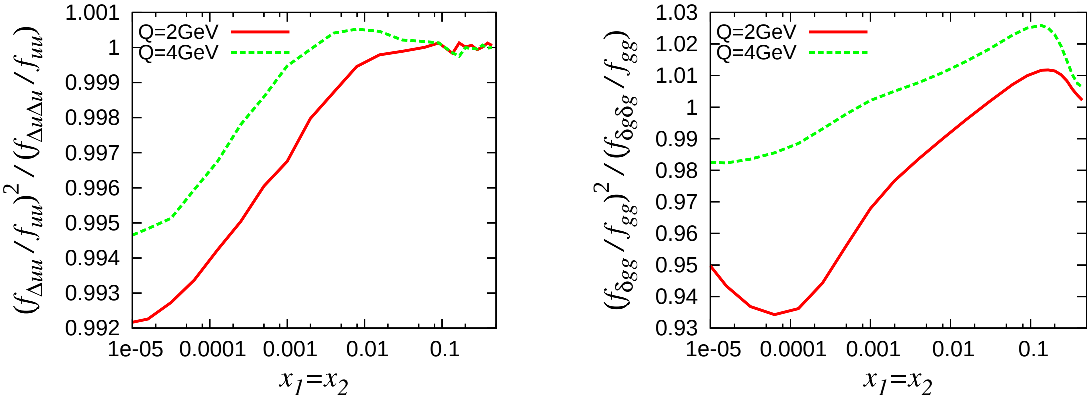
section 4, where we demonstrated that dPDFs are roughly equal to a product of single PDFs, barring effects due to conservation momentum. We can think of the dPDF as being composed of a polarized PDF (the square root of the doubly polarized PDF) and an unpolarized PDF. The growth in the unpolarized gluon PDF is so great that it outweighs the contribution due to a single polarized parton. Taking the lessons from section 4 further, if we assume that dPDFs are a product of sPDFs, then the ratio of singly polarized to unpolarized dPDFs given in Fig. 7 should be the square root of the corresponding ratios given in Fig. 4. To test this we plotted the square of the ratios in Fig. 7 diveded by the ratios in Fig. 4. Some typical results for GJR08lo are shown in Fig. 8. As we can see, they are very close to the expected value of 1. However, we did the same thing for the MSTW2008lo PDF set, and as we can see in Fig. 9, for the gluon case there is some not insignificant divergence from the expected value of 1 at lower $x$ values.




(a) Longitudinally polarized gluons.

(b) Transversely polarized u quarks.

Figure 8: The square of the ratios in Fig. 7 divided by the ratios in Fig. 2 (GJR08lo).




(a) Longitudinally polarized u quarks.

(b) Linearly polarized gluons.

Figure 9: The same as Fig. 8 but for the MSTW2008lo PDF set.

# 7 Conclusion

In this paper, we examined how the evolution code described in [4] can be used and modified in order to gain a fuller understanding of how spin correlations between the partons in double parton scattering can affect the calculation of QCD cross-sections. Hopefully, once these things are fully understood, greater light will be thrown upon the phenomenology of Higgs coupling to other particles, among other things. We first examined how closely simple products of the various publically available leading order PDF sets correspond to the double PDF generated by this code at higher energies. We concluded that, for lower $x$ values, they corresponded very closely to each other. We then showed how the speed with which polarized dPDFs are washed out can be affected by the starting scale of the evolution. Finally, we looked at the case of dPDFs where one parton is polarized and one is unpolarized. We observed how the ratio of such a dPDF to the corresponding unpolarized dPDF is close to the exact square root of the corresponding doubly polarized ratio. There are many ways in which the work described here could be extended. The dPDFs described in section could be examined for many different PDF sets, to observe how differences in the single PDF sets contribute to differences in the resultant dPDFs. Another interesting project would be to expand the code to include next-to-leading order splitting functions. Also, the analysis of the polarized dPDFs could be conducted with the *single feed* term added - i.e. an ill-understood term in the double DGLAP which includes a splitting function interpreted as the probability of a single parton splitting into two partons [4, 8]. In any event, there is much work to be done before double parton scattering is fully understood.

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