Implementation of the 2HDM in WHIZARD

David Gordo Gómez, University of Salamanca, Spain

Advisor: Jürgen Reuter

September 5, 2013

Abstract

We conduct a study of an extension of the Standard Model (SM) with two Higgs doublets, the so-called Two Higgs Doublet Model (2HDM). In general the 2HDM gives rise to flavour changing neutral currents (FCNC), that can be solved requiring the alignment of the Yukawa matrices that couple the leptons to the Higgs doublets, these are the Aligned 2HDM (A2HDM), that are a generalization of the previously analyzed 2HDM without FCNCs (type-I, type-II, type-X, type-Y). We calculate all the Feynman rules of the theory and implement them in the Monte-Carlo event-generator `WHIZARD`.

1 Introduction

The last year the ATLAS and CMS collaborations announced the discovery of a new neutral boson, and the measured data is compatible with the Standard Model (SM) Higgs boson. This boson appear to have all the properties expected for a Higgs like particle, associated with the spontaneous breaking of the electroweak symmetry: fermionic couplings proportional to the fermion mass and the expected coupling strength to the electroweak gauge bosons (W^+,W^-,Z,γ).

An important question to answer is if it is an unique Higgs boson, which is assumed in the SM, or if it is only a part of a richer scalar sector. The simplest extension to the minimal Higgs model consists in adding an extra scalar doublet, this is the so called *Two Higgs Doublet Model* (2HDM), where we have five physical scalar bosons: two charged fields $H^{\pm}$ and three neutral ones h, H and A. In principle, this gives us three possibilities for the recently discovered neutral boson.

In general, the multi Higgs doublet models give rise to *flavour changing neutral currents* (FCNC) which are absent in the minimal Higgs model and are very constrained by experiment. The tree-level FCNC can be eliminated requiring the alignment in flavour space of the Yukawa matrices that couple the leptons to the different Higgs bosons, these are the so called *Aligned Two Higgs Doublet Model* (A2HDM). The A2HDM constitutes a very general framework which includes, for particular values of its parameters, all the previously considered 2HDM without FCNCs and incorporates new sources of CP violation.

2 The Higgs potential

The 2HDM extends the SM with another scalar doublet with weak hypercharge $Y = \frac{1}{2}$, where $Q = T_3 + Y$ where Q is the electric charge in proton charge units and the T_i are the weak isospin operators. In the spontaneous symmetry breaking procedure the neutral component (the vacuum has to respect the electromagnetic gauge symmetry) of the scalar doublets acquire a, in general complex, *vacuum expectation value* (vev) given by $\left\langle 0 \right| \phi_j^T(x) \left| 0 \right\rangle = \frac{1}{\sqrt{2}}(0, v_j e^{i\theta_j})$. Using the $U(1)_Y$ symmetry we can enforce $\theta_1 = 0$, being only the relative phase $\theta = \theta_2 - \theta_1$ observable. Instead of working in the (ϕ_1, ϕ_2) basis we can perform a global $SU(2)$ transformation and work in the so-called Higgs basis (Φ_1, Φ_2), where only one of the doublets acquires a non-zero vev:

$$\left(\begin{array}{c} \Phi_1 \\ \Phi_2 \end{array} \right) = \left(\begin{array}{cc} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{array} \right) \left(\begin{array}{c} \phi_1 \\ e^{-i\theta}\phi_2 \end{array} \right)$$

with $\tan\beta = \frac{v_2}{v_1}$.

In that way Φ_1 plays the role of the SM Higgs doublet with a vev given by $v \equiv \sqrt{v_1^2+v_2^2} \sim \left(\sqrt{2}G_F\right)^{-1/2}$ and are parametrized by:

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v+S_1+iG^0\right) \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(S_2+iS_3\right) \end{pmatrix}$$

where $G^\pm$, G^0 are the Goldstone bosons and $H^\pm$, S_1, S_2, S_3 the charged and neutral Higgs bosons, all of them with zero vev. The proportionality constants have been chosen in order to have canonical kinetic terms in the Lagrangian and the mass eigenstates neutral Higgs bosons h, H, A are related to the S_i through an orthogonal transformation, that diagonalizes the mass matrix.

Working in the Higgs basis, the most general potential which spontaneously breaks the $SU(2)_L \otimes U(1)_Y$ gauge group is:

$$V = \mu_1 \Phi_1^\dagger\Phi_1 + \mu_2 \Phi_2^\dagger\Phi_2 + \left(\mu_3 \Phi_1^\dagger\Phi_2 + \mu_3^* \Phi_2^\dagger\Phi_1\right) +$$

$$+\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) +$$
$$+\left[\lambda_5\left(\Phi_1^\dagger\Phi_2\right) + \lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) +$$
$$+\left[\lambda_5^*\left(\Phi_2^\dagger\Phi_1\right) + \lambda_6^*\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7^*\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_2^\dagger\Phi_1\right)$$

The hermiticity of the potential requires all the parameters to be real except μ_3, λ_5, λ_6 and λ_7 that can be in general complex, giving us a total of 14 real parameters. The minimization conditions impose the relations $\mu_1 = -\lambda_1 v^2$ and $\mu_3 = -\frac{1}{2}\lambda_6 v^2$.

Expanding the potential in terms of the fields we obtain the quadratic (mass), cubic and quartic interactions between the Higgs bosons and the Goldstone bosons. For example, the mass terms are (the Goldstone are massless in principle):

$$V_2 = M_{H^\pm}^2 H^+H^- + \frac{1}{2}(S1, S2, S3)\mathcal{M}\begin{pmatrix} S1 \\ S2 \\ S3 \end{pmatrix}$$

with

$$M_{H^\pm} = \mu_2 + \frac{1}{2}\lambda_3 v^2$$

and

$$\mathcal{M} = \begin{pmatrix} 2v^2\lambda_1 & v^2\lambda_6^R & -v^2\lambda_6^I \\ v^2\lambda_6^R & M_{H^\pm}^2 + \frac{1}{2}v^2(\lambda_4+2\lambda_5^R) & -v^2\lambda_5^I \\ -v^2\lambda_6^I & -v^2\lambda_5^I & M_{H^\pm}^2 + \frac{1}{2}v^2(\lambda_4-2\lambda_5^R) \end{pmatrix}$$

where λ_i^R and λ_i^I are the real and imaginary parts of λ_i. Diagonalizing $\mathcal{M}$ we obtain the masses of h, H and A, and the orthogonal matrix $\mathcal{R}$ which relates them with the S_i. It is worth noting that in the CP-conserving case $(\lambda_5, \lambda_6, \lambda_7) \in \mathbb{R}$ the S_3 field does not mix with the two others and the scalar spectrum contains a neutral CP-odd field $A = S_3$ and two neutral CP-even fields h and H, also the vertices involving an odd number of S_3 vanish.

To set the notation the S_i and the physical neutral Higgs bosons $h_i^0 = (h, H, A)$ are related by $h_i^0 = \mathcal{R}_{ij} S_j$.

3 The gauge bosons

We obtain the Higgs-gauge bosons interactions gauging the $SU(2)_L \otimes U(1)_Y$ group through the covariant derivatives of the Higgs doublets. The part of the Lagrangian involving the electroweak gauge bosons is:

$$\mathcal{L}_K + \sum_{j=1}^{2} (D_\mu \Phi_j)^\dagger D^\mu \Phi_j + \mathcal{L}_{GF}$$

where $\mathcal{L}_K$ is the usual kinetic term of gauge theories, $\mathcal{L}_{GF}$ is a gauge-fixing term and we can express the covariant derivative as[1] $D_\mu = \partial_\mu + ieQA_\mu + i\frac{g}{\sin\theta_W}(T_3 - Q\sin^2\theta_W)Z_\mu + ig\left(T^+ W_\mu^- + T^- W_\mu^+\right)$. The covariant derivative terms are responsible of giving mass to the gauge bosons satisfying $M_W = gv/2$ and $M_Z = gv/2\cos\theta_W$.

Following [1] we adopt the R_ξ gauge-fixing term with $\xi = 1$:

$$\mathcal{L}_{GF} = -\frac{1}{2}(\partial_\mu A^\mu)^2 - \frac{1}{2}\left(\partial_\mu Z^\mu + M_Z G^0\right)^2 - \left(\partial^\mu W_\mu^+ + iM_W G^+\right)\left(\partial^\nu W_\nu^- - iM_W G^-\right)$$

which has the advantage of canceling the quadratic terms involving gauge and Goldstone bosons interactions and provides the Goldstone bosons with the masses $M_G^\pm = M_W$ and $M_G^0 = M_Z$.

Expanding the three terms we obtain the kinetic terms of the Higgs bosons, the kinetic and mass terms of the Goldstone bosons, the usual Lagrangian of the gauge bosons (including their self-interactions and the masses of $W^\pm$ and Z) and interaction terms of the form ϕVV, $\phi\phi V$ and $\phi\phi VV$ (where ϕ is a Higgs or Goldstone boson and V a gauge boson).

[1] θ_W is the weak mixing angle which relates the A_μ and Z_μ with the original gauge bosons of the $SU(2)_L \otimes U(1)_Y$ group. It satisfies $g\sin\theta_W = e$.

4 The fermions

Like in the SM, to give mass to the fermions we have to couple them to the Higgs doublets via Yukawa couplings. In the 2HDM, the couplings to Φ_1 are fixed, because the interactions have to be diagonal in flavour space in the mass eigenstates basis; but the coupling to Φ_2 can be non-diagonal and unrelated to the fermion masses. Then, we have to different Yukawa matrices coupled to the right-handed fermion fields that cannot be, in general, diagonalized simultaneously; giving rise to FCNCs.

The most general Yukawa Lagrangian is [2]:

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v}\{\bar{Q}'_L\,(M'_d\Phi_1 + Y'_d\Phi_2)\,d'_R + \bar{Q}'_L(M'_u\widetilde{\Phi}_1 + Y'_u\widetilde{\Phi}_2)u'_R +$$

$$+\bar{L}'_L\,(M'_l\Phi_1 + Y'_l\Phi_2)\,e'_R + h.c.\}$$

where Q'_L and L'_L are the left-quark and left-leptonic $SU(2)_L$ doublets, $\widetilde{\Phi}_j = i\tau_2\Phi_j^*$ are the charge-conjugate fields, $f_{R,L} = \frac{1\pm\gamma_5}{2}f$ the chirality projections, all the fields are written as vectors in the flavour space[2] $d' = (d', s', b') = Vd$, $u' = (u', c', t') = u$, $e' = (e', \mu', \tau') = e$ and $\nu' = (\nu'_e, \nu'_\mu, \nu'_\tau) = \nu$ and the M'_f and Y'_f $(f = d, u, l)$ are 3×3 matrices in the flavour space.

The Yukawa matrices in the mass eigenstates basis are related to the primed ones by $M'_d = VM_dV^\dagger$, $Y'_d = VY_dV^\dagger$, $M'_{u,l} = M_{u,l}$ and $Y'_{u,l} = Y_{u,l}$. Expanding the Lagrangian, the part corresponding to the vev of Φ_1 is:

$$\mathcal{L}_{FM} = -\bar{d}M_dd - \bar{u}M_uu - \bar{e}M_le$$

that corresponds to the mass term of the Dirac Lagrangian.

4.1 The A2HDM

Like we have seen the Y_f matrices can be in general non-diagonal, generating FCNCs. The standard way of solving this problem is, working in the (ϕ_1, ϕ_2) basis, imposing that every right-handed fermion field couples to only one of the Higgs douplets. In that way there is only one matrix that can be diagonalized. There are different choices that result in the so-called type-I, II, X, Y models.

Model	d_R	u_R	e_R
Type-I	ϕ_2	ϕ_2	ϕ_2
Type-II	ϕ_1	ϕ_2	ϕ_1
Type-X	ϕ_2	ϕ_2	ϕ_1
Type-Y	ϕ_1	ϕ_2	ϕ_2

Model	ζ_d	ζ_u	ζ_l
Type-I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type-II	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
Type-X	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type-Y	$-\tan\beta$	$\cot\beta$	$\cot\beta$
Inert	0	0	0

[2] V is the CKM or quark mixing matrix.

A more general way of getting rid of the tree-level FCNCs is imposing the alignment in flavour space of the Yukawa couplings to the two Higgs doublets, this guarantees that the Y_f and M_f are proportional and can be diagonalized simultaneously. Setting

$$Y_{d,l} = \zeta_{d,l} M_{d,l}, \quad Y_u = \zeta_u^* M_u$$

we have three complex parameters that generates models with fermionic coupligs to the scalar fields proportional to the fermion's masses, V as the only source of flavour changing phenomena and possibility of having new sources of CP-violation through the complex nature of the ζ_f.

5 WHIZARD

`WHIZARD`[3] is a Monte-Carlo event generator, designed for the efficient calculation of multiparticle scattering cross sections and simulated event samples. It generates complete tree-level matrix elements by calling the matrix-element generator `O'Mega`[4], integrates them over phase space and evaluates distribution of observables. `WHIZARD` can describe the physics of the Standard Model and other well-known extensions of it, like the minimal supersymmetric Standard Model (MSSM). The purpose of this project has been to implement the 2HDM and also a simplified version of it, the A2HDM with trivial CKM matrix.

5.1 *model*.mdl

A model is collection of particles (with their properties), numerical parameters (coupling constants, masses, widths, ...) and the interactions (Feynman rules). In order to implement all that in `WHIZARD` the first step is to create a file *model*.mdl, located in the `share/models` subdirectory of the `WHIZARD` installation, which contains all the basic parameters of the model with their values, the particle content with their quantum numbers (electric charge, spin, isospin, $SU(3)_C$ representation, ...) and a list of all the interaction vertices. This is used only for the generation of the phase space, the Feynman rules are stored in a different file within the `O'Mega` directory.

5.2 parameters.*model*.f90

We also need a fortran file parameters.*model*.f90 located in the `src/models` subdirectory. This file contains all the functional relations among the coupling constants and the basic parameters that we have define in the previous file. In the 2HDM, due to the great number of vertices and the existence of several mixing matrices (neutral Higgs bosons mixing matrix, CKM matrix) this process is error-prone and has been done with the help of the symbolic computation software $Wolfram\ Mathematica^{®}$.

```
parameter V_33     = 1.000    # CKM Matrix
# Dependent parameters
derived v      = 1 / sqrt (sqrt (2.) * GF)     # v (Higgs vev)
derived cw     = mW / mZ                        # cos(theta-W)
derived sw     = sqrt (1-cw**2)                 # sin(theta-W)
derived ee     = 2 * sw * mW / v                # em-coupling (GF scheme)
derived alpha_em_i = 4 * pi / ee**2             # inverse fine structure const

########################################################################
# Particle content

# The quarks
particle D_QUARK 1  parton
  spin 1/2  charge -1/3   isospin -1/2  color 3
  name d down
  anti dbar D "d~"
  tex_anti "\bar{d}"
```

Figure 1: Example of the file *model*.mdl.

```
    gHpHm23 = lamb3*R21*R31 + lamb7R*R22*R31 - lamb7I*R23*R31 + &
         lamb7R*R21*R32 + 2*lamb2*R22*R32 - lamb7I*R21*R33 + 2*lamb2*R23*R33
    gHpHm33 = (lamb3*R31**2)/2. + lamb7R*R31*R32 + lamb2*R32**2 - &
         lamb7I*R31*R33 + lamb2*R33**2
    !!! Quartic neutral higgs
    gh1111 = (lamb1*R11**4 + R11**3*(2*lamb6R*R12 - 2*lamb6I*R13) + &
         2*R11*(lamb7R*R12 - lamb7I*R13)*(R12**2 + R13**2) + lamb2*(R12**2 + &
         R13**2)**2 + R11**2*((lamb3 + lamb4 + 2*lamb5R)*R12**2 - &
         4*lamb5I*R12*R13 + (lamb3 + lamb4 - 2*lamb5R)*R13**2))/4.0
    gh1112 = (R11**3*(2*lamb1*R21 + lamb6R*R22 - lamb6I*R23) + &
         (R12**2 + R13**2)*(lamb7R*R12*R21 - lamb7I*R13*R21 + &
         2*lamb2*R12*R22 + 2*lamb2*R13*R23) + R11**2*(R12*(3*lamb6R*R21 + &
         (lamb3 + lamb4 + 2*lamb5R)*R22 - 2*lamb5I*R23) + &
         R13*(-3*lamb6I*R21 - 2*lamb5I*R22 + (lamb3 + lamb4 - &
         2*lamb5R)*R23)) + R11*(R13**2*((lamb3 + lamb4 - &
         2*lamb5R)*R21 + lamb7R*R22 - 3*lamb7I*R23) + R12**2*((lamb3 + &
         lamb4 + 2*lamb5R)*R21 + 3*lamb7R*R22 - lamb7I*R23) - &
         2*R12*R13*(2*lamb5I*R21 + lamb7I*R22 - lamb7R*R23)))/2.0
```

Figure 2: Example of the file parameters.*model*.f90.

5.3 omega_*model*.ml

Finally, all the Feynman rules are encoded in the O'Mega file omega_*model*.ml, located in the src/omega/src subdirectory. The file is written in O'Caml, which is a object-oriented programming language. Each vertex is described by the particles involved and the type of interaction; all the kind of interactions allowed and the syntax structure are described in the O'Mega manual.

```
    let gauge_higgs =
      [ (G Ga, O Hp, O Hm), Vector_Scalar_Scalar 1, G_AHpHm;
        (G Z, O Hp, O Hm), Vector_Scalar_Scalar 1, G_ZHpHm;
        (G Z, O Hh, O HH), Vector_Scalar_Scalar 1, G_Zh1h2;
        (G Z, O Hh, O HA), Vector_Scalar_Scalar 1, G_Zh1h3;
        (G Z, O HH, O HA), Vector_Scalar_Scalar 1, G_Zh2h3;
        (G Wp, O Hm, O Hh), Vector_Scalar_Scalar 1, G_WpHmh1;
        (G Wp, O Hm, O HH), Vector_Scalar_Scalar 1, G_WpHmh2;
        (G Wp, O Hm, O HA), Vector_Scalar_Scalar 1, G_WpHmh3;
        (G Wm, O Hp, O Hh), Vector_Scalar_Scalar 1, G_WmHph1;
        (G Wm, O Hp, O HH), Vector_Scalar_Scalar 1, G_WmHph2;
        (G Wm, O Hp, O HA), Vector_Scalar_Scalar 1, G_WmHph3;
        (O Hh, G Z, G Z), Scalar_Vector_Vector 1, G_h1ZZ;
        (O HH, G Z, G Z), Scalar_Vector_Vector 1, G_h2ZZ;
        (O HA, G Z, G Z), Scalar_Vector_Vector 1, G_h3ZZ;
        (O Hh, G Wp, G Wm), Scalar_Vector_Vector 1, G_h1WpWm;
        (O HH, G Wp, G Wm), Scalar_Vector_Vector 1, G_h2WpWm;
        (O HA, G Wp, G Wm), Scalar_Vector_Vector 1, G_h3WpWm ]
```

Figure 3: Example of the file omega_*model*.ml.

References

[1] Alejandro Celis, Victor Ilisie, and Antonio Pich. LHC constraints on two-Higgs doublet models. *JHEP*, 1307:053, 2013.

[2] Antonio Pich and Paula Tuzon. Yukawa Alignment in the Two-Higgs-Doublet Model. *Phys.Rev.*, D80:091702, 2009.

[3] Wolfgang Kilian, Thorsten Ohl, and Jurgen Reuter. WHIZARD: Simulating Multi-Particle Processes at LHC and ILC. *Eur.Phys.J.*, C71:1742, 2011.

[4] Mauro Moretti, Thorsten Ohl, and Jurgen Reuter. O'Mega: An Optimizing matrix element generator. 2001.

[5] John F. Gunion, Howard E. Haber, Gordon L. Kane, and Sally Dawson. THE HIGGS HUNTER'S GUIDE. *Front.Phys.*, 80:1–448, 2000.

[6] Michael E. Peskin and Daniel V. Schroeder. An Introduction to quantum field theory. 1995.

[7] F. Halzen and Alan D. Martin. QUARKS AND LEPTONS: AN INTRODUCTORY COURSE IN MODERN PARTICLE PHYSICS. 1984.

[8] M. Bohm, Ansgar Denner, and H. Joos. Gauge theories of the strong and electroweak interaction. 2001.