Alignment with Millepede II using General Broken Lines for the Belle II Vertex Detector

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Abstract

This report describes the alignment procedure using Millepede II for the Belle II vertex detector. An interface between the track fitting with General Broken Lines and the GENFIT reconstruction toolkit within the Belle II software framework is developed and tested. The results of fitting and alignment are presented.

Contents

1 Belle and Belle II

1.1 Introduction

The progress of particle physics, initiated by the era of construction of large particle accelerators, provided an impressive insight into most fundamental aspects of nature. During past decades, the standard model (SM) of elementary particles and their interactions showed as one of the most successful theories ever developed. However conclusions from astronomical observations give hints, that this is not the full story. One of the most obvious facts is that our universe is composed almost entirely from matter, while antimatter is rather a rare substance. Before 1964 it was believed that matter and antimatter should obey mirror symmetry. But discovery of CP-violation in neutral kaons' systems completely changed this view. The possibility that CP-symmetry is broken at level of fundamental particle interactions offered a possible explanation of abundance of matter over antimatter created at early stage of universe's birth. Since then, the CP-violation was intensively studied in experiments and incorporated into physics theories.

One of the experiments that took part in investigation of CP-asymmetry was the Belle detector installed on the KEKB accelerator at the High Energy Accelerator Research Organization (KEK) in Japan. This accelerator belongs to so called B-factories, because it is designed to produce large amounts of B mesons and their antiparticles. The key idea behind B-factories is to use asymmetric (in energy) beams of electrons and positrons and collide them at the centre of mass energy tuned to the rest mass of the $\Upsilon(4S)$ resonance. This resonance instantaneously decays into $\mathrm{B}\overline{\mathrm{B}}$ pairs, whose successive decays are studied in the experiment.

1.2 The Belle Experiment

The Belle detector finished its operation after 10 years in 2010 and achieved unprecedented physical results. The machine, KEKB, holding a record in instantaneous and integrated luminosity, collected 1ab^{-1} of data. The analysis of the huge accumulated data sample continues till present. Belle was a forward/backward asymmetric detector with high vertex resolution, 1.5T solenoid magnetic field, high performance calorimeter and particle identification device. The main goal of Belle was a study of the CP symmetry violation in B meson decays. In 2001, the presence of CP violation in the B meson system was established by the Belle and BaBar collaboration, which observed time-dependent asymmetry in the decay process $B^0 \to J/\psi K_S^0$. Both, direct and indirect CP-violation was observed also in other decays. The time-dependent CP-violation measurements need a very good vertex spatial resolution, because the time measurement is translated to the distance measurement in that case. However, due to very short lifetimes of B mesons, the travelled distance (at almost speed of light) before decay is very small. This is also the reason why asymmetric beams are used. As the centre of mass after collision has a boost in the forward direction, the decay time is longer from the view of laboratory system at rest. In these experiments, SM with the Kobayashi-Maskawa mechanism of CP violation was confirmed and angles ϕ_2 and ϕ_3 of the CKM matrix and parameters of the unitarity triangle were measured. Although these examples belong to most known Belle achievements, important results were also obtained in the heavy quarkonium spectroscopy, tau lepton decays or two-photon physics [1].

1.3 The Belle II Experiment

After finishing KEKB/Belle operation, an upgrade to SuperKEKB/Belle II was approved leading to much better detector performance and machine luminosity. The luminosity is expected to be 40 times larger than in the previous machine. Upgrade of the detector will lead to better vertex resolution, higher performance of the calorimeter and the particle identification device. However, higher luminosity will lead to much higher background and will pose other challenges to individual sub detectors, namely the vertex, tracking and calorimeter parts. Upgrade of the detector will allow new possibilities in study of CP-violation, lepton flavour violation, physics beyond the SM as well as in many other fields of particle physics. With higher statistics, uncertainties not only in CKM matrix angles are to be lowered, but also some hints to discrepancies to the SM predictions are to be either confirmed or rejected. Thus, Belle II will join the efforts for searching new physics phenomena beyond the SM.

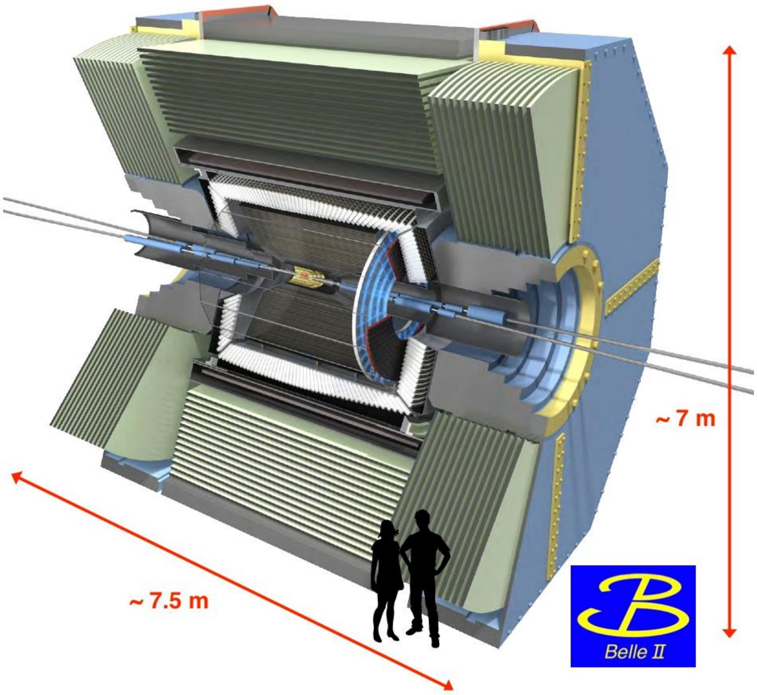

Figure 1: The Belle II detector. PXD and SVD sub-detectors are shown in red and yellow in the middle of the detector.

The main concept of the detector remains the same as in the Belle experiment. The detector [2] shown in Figure 1 is a cylinder with a bit asymmetric internals, which cover larger angle in the forward direction (direction of electrons, which have higher energy than positrons). The key improvement of the detector is the replacement of the silicon strip detector near the interaction point by a pixel detector (PXD) equipped by the DEPFET technology. The strip detector (SVD) follows just after PXD and occupies larger volume than in Belle. For tracking, the central drift chamber (CDC) is used as in Belle. The particle identification device is completely new and offers almost the best achievable separation between pions and kaons. The calorimeter is going to be upgraded by pure CsI crystals in the end cap and much faster readout electronics. While the barrel part of the muon detector remains the same, end cap is going to be equipped with scintillators instead of resistive plate chambers. Outer part of the detector, outside the superconducting coil, serves for muon and K_L detection and its support structure works as a return circuit for magnetic field [1, 3].

1.4 The Belle II Vertex Detector

Momenta of produced particles at Belle II are at most at level of few GeVs, usually lower. Due to relatively low energy of particles and required high spatial resolution, the vertex detector of Belle II must be as lightweight as possible in order to not disturb particle trajectories by multiple scattering. For this reason, most of the vertex detector is actually an active (measuring) material, namely silicon, with self-supporting structure in the whole range of detector acceptance.

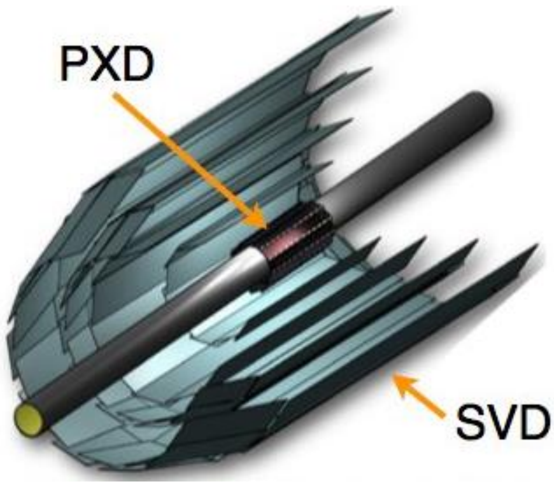

Figure 2: PXD and SVD detectors of Belle II.

PXD consists of 8 sensors in the first and 12 in the second layer arranged in a windmill structure. Thanks to the internal pre-amplification, the sensors can be made 75μm thin without reducing particle detection capabilities. SVD has in total 187 double-sided silicon strip sensors with thickness of 320μm with rectangular shape in barrel region and trapezoidal shape in the forward region, where sensors are slanted [2], see Figure 2. For both, PXD and SVD, the readout electronics and support structure is reduced to absolute minimum in the acceptance region of Belle II.

2 The Belle II Software Framework

2.1 Overview

In order to meet high requirements to the detector, a modern software framework, called basf2, is being developed for Belle II. It is written in C++ programming language and python steering scripts are used for user control over the framework. Its purpose will cover almost every aspect of related software issues, starting with Monte Carlo production, continuing to user physics analysis and last but not least real time applications in data acquisition and high level trigger functionality. It is designed to be composed from independent blocks, called modules, which serve for distinct tasks. Modules to be used, their order and parameters can be specified in the steering script [4].

In the basf2 framework, complete simulation of the Belle II detector can be done with detailed geometry, particle propagation (using GEANT4), simulation of the detector response (digitization) and other aspects of detector functionalities. Track finding modules, which search for track patterns in detector hits and construct track candidates to be later fitted and examined for their compatibility with the whole recorded event, are being developed. At the simulation stage, track candidates can be constructed from “true information”, so that hits in the detector are directly assigned to the particle that created them. In real reconstruction, this is obviously not possible and a lot of efforts is devoted to properly recognize footprints of individual particles in the detector, especially in much more challenging background conditions in comparison to the Belle experiment.

2.2 GENFIT Reconstruction Toolkit

For track reconstruction, it has been decided to use the GENFIT reconstruction toolkit [5]. Responses of various detectors producing spatial information on particle positions are combined into an input for track fitting. The aim is to estimate the momentum of a particle at any point along its trajectory. GENFIT is a generic framework for such a track reconstruction. The key idea behind GENFIT is to combine different kinds of detector measurements usually with different dimensionality in a generic way. The nature of individual measurements is hidden to actual fitting algorithm, while all possible correlations are taken into account. GENFIT also allows using different track representations and mechanism for particle propagation. The fitting algorithm can be, for example, a Kalman filter.

The Kalman filter is an iterative algorithm designed to estimate a state of a dynamic system in a series of measurements with known errors. It starts with initial value of state vector with a covariance matrix and propagates the state vector to the next measurement. Here, the state is updated to get closer to the measured position. The update depends on both, the covariance of the state vector (also propagated) and of the measurement, namely on their relative size. More precise measurement will enforce the algorithm to get closer to it. The Kalman filter uses different coordinate systems for measurements and the state vector and thus requires a linear transformation from state coordinates to the measurement system [5].

For the purposes of track reconstruction in Belle II, an upgraded version of the toolkit, GENFIT 2, is being developed [6].

3 Track Based Alignment

Requirements of precise vertex reconstruction for Belle II pose a challenge to the tracking part of the detector. High intrinsic resolution of the tracker itself could be devaluated by not precise alignment of individual parts of the sub detectors. Systematic errors of measurements might in that case exceed improved statistical uncertainties achieved by higher luminosity. Thus, precise alignment of the tracker is of high importance.

An initial alignment can be done by precise construction of the detector and additional, e.g., laser measurements. But to achieve the best possible accuracy, track based alignment is necessary. In track based alignment, the data from tracks coming from the interaction point or cosmic radiation serve as input data for precise alignment. Single measurements and tracks can contain unwanted correlations. To reduce the effect of the correlations, the data sample used for alignment must contain various track samples with different topologies: tracks from the interaction point, interactions of beams with the beampipe and cosmic rays.

Track based alignment can be described as a least squares (χ^2) minimization problem:

$$\chi^2(\boldsymbol{g},\boldsymbol{l}) = \sum_j^{\text{tracks}} \sum_i^{\text{hits}} \boldsymbol{r}_{ij}^T(\boldsymbol{g},\boldsymbol{l}_j)\, V_{ij}^{-1} \boldsymbol{r}_{ij}(\boldsymbol{g},\boldsymbol{l}_j), \tag{1}$$

where $\boldsymbol{r}_{ij}$ is a difference (residual) of predicted and measured position of a single hit i and track j, which depends on the alignment parameters $\boldsymbol{g}$ and parameters of the track $\boldsymbol{l}_j$. V_{ij} is an element of the covariance matrix, e.g., in the simplest case of uncorrelated measurements with equal errors, this is a diagonal matrix with elements equal to squared measurement errors (usually denoted as σ) [2, 7].

3.1 Millepede II

Millepede is a method developed for solution of specific least squares problems with a very large number of parameters, where the parameters can be divided in two groups:

- **Local parameters** are specific to a usually small subset of measurements. In our case, these parameters describe a single track and are obtained from a fit of the track. These parameters are not the subject of interest in the scope of the alignment procedure.
- **Global parameters** correspond to the previously discussed alignment parameters, which affect a large number of measurements. The estimation of these parameters is the goal of the alignment procedure.

Millepede is able to solve the minimization problem in one step by simultaneous fit of all local and global parameters while taking all correlations into account. This is in contrast to more popular and simpler methods of alignment where the mean values of residuals are used to estimate alignment parameters. Corrected alignment parameters are then used again in a new fit and the procedure is repeated until mean values of residuals are close to zero. Such method, however, can produce biased alignment parameters [9].

Millepede uses a global least squares fit with all local and global parameters with thousands or even millions of events by using special structure of the least squares matrix, where the dimension of the matrix for the problem to be solved is determined only by the number of global parameters. This method does not introduce any approximation to the problem. In the Millepede algorithm, the χ^2 expression to be minimized is linearized to first order [7 - 8]:

$$\chi^2(\boldsymbol{g},\boldsymbol{l}) = \sum_j^{\text{tracks}} \sum_i^{\text{hits}} \frac{1}{\sigma_{ij}^2}\left(\boldsymbol{m}_{ij} - \boldsymbol{p}_{ij}(\boldsymbol{g}_0,\boldsymbol{l}_0) - \frac{\partial \boldsymbol{p}_{ij}}{\partial \boldsymbol{g}}\Delta\boldsymbol{g} - \frac{\partial \boldsymbol{p}_{ij}}{\partial \boldsymbol{l}_j}\Delta\boldsymbol{l}_j\right)^2, \tag{2}$$

where $\boldsymbol{p}_{ij}$ is the prediction of the track model for the track j and hit i, $\boldsymbol{m}_{ij}$ is the measured value for this hit, $\boldsymbol{g}_0, \boldsymbol{l}_0$ are initial values for the global and local parameters and σ_{ij}^2 is the variance (uncorrelated measurements are supposed). Local parameters of a track are not related to the local parameters of other tracks, but only to (usually) small subset of all global parameters. This results in a special structure of the matrix (see Figure 3 [7]) of equations to be solved for corrections $\Delta\boldsymbol{g}$, $\Delta\boldsymbol{l}_j$.

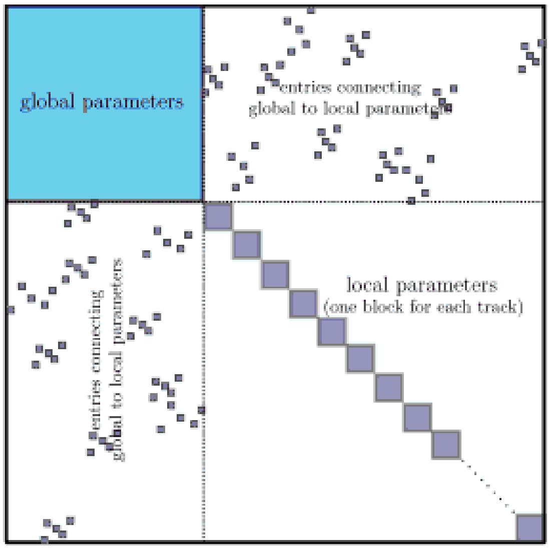

Figure 3: Structure of the matrix for the least squares problem with global and local parameters and their combinations.

The overall matrix is split into sub-matrices, from which many elements are zero, some contain only global or local parameters and some combine both of them. By using block matrix theorems, the matrix can be rearranged while correcting the parts with only global parameters with information from local parameters. Finally, only a problem with matrix with the size equal to the number of global parameters remains to be solved. Thus, possibly millions of rows (global parameters + local parameters × number of tracks) are reduced to the number of alignment parameters without any approximation.

Millepede allows also application of additional constraints, which can restrict the solution. Constraints are added to the problem as Lagrange multipliers. Also, an internal iteration for rejection of outliers in the data, which can disturb the result, can be performed by Millepede.

Millepede II is the second version of the Millepede algorithm optimized for solution of very large problems with more than 100 000 global parameters. Millepede II ships as a standalone program, which is divided to two parts:

- **Mille** is a subroutine for data accumulation and writing of binary files for further processing,
- **Pede** takes the binary file as an input and obtains the solution as explained above.

For running of Millepede, the following data has to be provided by the user for each measurement (hit i of track j) in each dimension [7, 9]:

- the number of local parameters and derivatives of the track model w.r.t. (relevant) local parameters $\frac{\partial p_{ij}}{\partial \boldsymbol{l}_j}$,
- the number of (relevant) global alignment parameters and corresponding global derivatives $\frac{\partial p_{ij}}{\partial \boldsymbol{g}}$ with an integer label for each of them,
- the residual $m_{ij} - p_{ij}(\boldsymbol{g}, \boldsymbol{l}_j)$ and
- the standard deviation of the measurement σ_{ij}.

3.2 General Broken Lines

General Broken Lines (GBL) [11] is a track model with proper description of multiple scattering. The matrix for linear equations for fit of a track produced by GBL has a special structure, which allows one to obtain the fit with complexity depending linearly on the number of measurements. Also, the full covariance matrix along the particle trajectory allows using Millepede's global approach for the alignment. Output of the GBL track model can be used instead of the Mille routine to directly produce binary files for Pede assuming that the user provides additional information about the global parameters. Required local derivatives are provided by the GBL fit automatically.

GBL can account for multiple scattering in material between the measurement planes by equivalent thin scatterers. For the Belle II vertex detector, only scattering from the detector planes is considered.

In construction of the GBL trajectory, two orthonormal coordinate systems are introduced. At the measurement (or scattering) plane it is the local system $(u,\ v,\ w)$ with w in the direction perpendicular to the plane and u, v in the measurement directions (or for scatterer with no measurement, one of these coordinates must be perpendicular to the track direction). The track can be described in a curvilinear frame $(x_\perp,\ y_\perp,\ z_\perp)$ with $z_\perp$ pointing in the track direction and $x_\perp$ lying in the global $(x,\ y)$-plane. For a particle moving in the magnetic field, the curvature of the trajectory is described by inverse momentum q/p. At each thin scatterer a two-dimensional offset $\boldsymbol{u} = (u, v)$ in the local frame is defined as a fit parameter. In the local frame, the track can be described by the offset $\boldsymbol{u}$ and slopes $\boldsymbol{u}' = \frac{\partial \boldsymbol{u}}{\partial w}$. In case of small corrections to the track parameters $(\Delta q/p, \Delta\boldsymbol{u}', \Delta\boldsymbol{u})$, they propagate as described in [11]:

$$\Delta\boldsymbol{u}_{i+1} = \frac{\partial \boldsymbol{u}_{i+1}}{\partial \boldsymbol{u}_i}\Delta\boldsymbol{u}_i + \frac{\partial \boldsymbol{u}_{i+1}}{\partial \boldsymbol{u}'_i}\Delta\boldsymbol{u}'_i + \frac{\partial \boldsymbol{u}_{i+1}}{\partial q/p}\Delta q/p, \tag{3}$$

where the derivatives can be obtained from the Jacobian $\frac{\partial(q/p,\ \boldsymbol{u}',\ \boldsymbol{u})_{i+1}}{\partial(q/p,\ \boldsymbol{u}',\ \boldsymbol{u})_i}$ for the track parameter propagation between the adjacent (scattering) planes. With three such planes, this can be solved for corrections for the slopes and curvature. The change of track directions (difference between slopes of the track before and after a scattering plane), called *kinks* can be solved as well. Kinks are results of multiple scattering, mainly due to Coulomb interactions of the particle with electrons of atoms in the scattering material (Figure 4). The variance for the kinks can be obtained from formulae describing mean angle deflections (see below in the implementation). If at least one coordinate of the local system is perpendicular to the track direction, this variance is a diagonal matrix (because along the track trajectory, the angular deflection due to multiple scattering makes a symmetric "cone"), which is required by GBL.

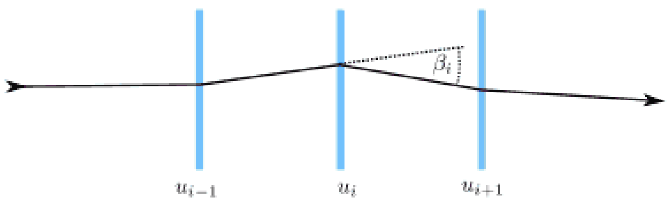

Figure 4: Simplified sketch of a particle trajectory undergoing multiple scattering deflections in scatterers.

The task of GBL is to fit parameters $\boldsymbol{x} = (\Delta q/p, \boldsymbol{u}_1, \dots, \boldsymbol{u}_{\#\,\text{of scatterers}})$ and thus obtain the kinks and their errors at scattering planes by minimization of the following expression (for the case of zero initial kinks and no external seed, see [11] for details):

$$\chi^2(\boldsymbol{x}) = \sum_{i=1}^{n_{\#\,\text{meas}}} (\boldsymbol{H}_{m,i}\boldsymbol{x} - \boldsymbol{m}_i)^T \boldsymbol{V}_{m,i}^{-1} (\boldsymbol{H}_{m,i}\boldsymbol{x} - \boldsymbol{m}_i) + \sum_{i=2}^{n_{\#\,\text{scat}}} (\boldsymbol{H}_{k,i}\boldsymbol{x})^T \boldsymbol{V}_{k,i}^{-1} (\boldsymbol{H}_{k,i}\boldsymbol{x}), \quad (4)$$

where $\boldsymbol{V}_{m,i}, \boldsymbol{V}_{k,i}$ are the variances of the measurements and kinks and $\boldsymbol{H}_{m,i}, \boldsymbol{H}_{k,i}$ are the matrices of the derivatives $\frac{\partial \boldsymbol{m}}{\partial \boldsymbol{x}}, \frac{\partial \boldsymbol{k}}{\partial \boldsymbol{x}}$ of the measurements, respective kinks w.r.t. to the fit parameters. Due to the special structure of this problem, computational complexity of the solution of the minimization problem can be significantly reduced.

The approach of GBL is proven to be mathematically equivalent to the Kalman filter, however it is different computationally. The complexity of the Kalman filter for n measurements is of order $O(n) \times 5^3$ while for GBL it is $O(n_{par}) \times (5+1)^2$ with $n_{par} = 5 \dots (4n+1)$ depending on the number of scatterers [10, 11].

4 GBL Interface Implementation

The main part of the project is the implementation of an interface between GENFIT 2, basf2 GENFIT2Module and GBL C++ implementation such that all required information for GBL fitting is collected from the framework and transformed into proper form which could be provided to the GBL. However, GENFIT 2 is not in a final shape and thus some workarounds have to be implemented.

Currently, the Kalman filter fitting with a reference track has to be run before GBL to attach information on the reference track and construct detector planes in the genfit::Track class. Track candidates are obtained from "true" information provided by basf2 MCTrackFinder module, which assigns hits in the detector to the corresponding particle. Hits are provided by TrueHit classes, thus no digitization and clustering is done. If using TrueHits, both PXD and SVD detectors provide 2D spatial information. If clusters have to be used, SVD sensors produce two 1D spatial measurements. Different hit dimensionality would require changes in the interface implementation and information about which kind of detector produced the hit (however, this information is not correctly provided by the current basf2 GENFIT 2 implementation, but this will be corrected in the future). True positions of hits are smeared according to a simple estimation of detector resolutions based on the pitch size (smearing is a Gaussian with $\sigma = \text{PITCH}/\sqrt{12}$ for each direction) before being passed to the track fitting.

Track description and extrapolation is provided by the Runge-Kutta track representation. Track parameters are stored in a track state, from which, after the extrapolation, initial predictions of the track momentum, position, charge or particle type can be recovered. Individual detector measurements are stored as track points, from which information about measured position are retrieved. In the future, the track should contain also track points without measurement describing inactive material passed by the track, which can be used for easier GBL interface implementation.

In GENFIT 2 the track state is represented at each plane by local plane coordinates $(\Delta q/p,\ \boldsymbol{u}', \boldsymbol{u})$ and thus the track positions predicted by the representation in the detector plane are simply the last two elements of the state vector. The difference of the measured and predicted track position is computed to get the residual. The covariance matrix of the measurement can be retrieved as well. From extrapolation between planes, the transportation Jacobian can be retrieved. Description of the track provided by GENFIT is unfortunately not compatible with GBL with scatterer planes identical to the detector planes. The measurement coordinates are not perpendicular to the track coordinates and thus the covariance matrix of multiple scattering becomes non-diagonal, which is currently not supported by GBL. Therefore, an additional Jacobian transformation must be introduced. (In the future, however, it might be possible to get a curvilinear propagation Jacobian instead of the local Jacobian provided by GENFIT from the Runge-Kutta track representation). This approach with an additional transformation of the Jacobian [12] has been implemented and tested, but satisfying results could not be reached and further investigation of this problem is required. Instead, a simplified Jacobian for the track propagation is computed. The transportation Jacobian for track parameters propagation for the constant magnetic field $\boldsymbol{B} = (0, 0, B_z)$ in the limit of small curvature $q/p \rightarrow 0$ is a function of arc-length difference Δs between adjacent planes [11]:

$$\frac{\partial(q/p,\ x'_\perp,\ y'_\perp,\ x_\perp,\ y_\perp)_{i+1}}{\partial(q/p,\ x'_\perp,\ y'_\perp,\ x_\perp,\ y_\perp)_i} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -B_z \Delta s \cos\lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -B_z \frac{1}{2} \Delta s^2 \cos\lambda & \Delta s & 0 & 1 & 0 \\ 0 & 0 & \Delta s & 0 & 1 \end{pmatrix}, \tag{5}$$

where $\cos\lambda = \sqrt{(p_x^2 + p_y^2)/|\boldsymbol{p}|}$ with λ being the dip angle (helix inclination in global *z*-direction, e.g., $\lambda = 0$ for track moving only in the global (x, y)-plane). The projection matrix from the track coordinates to the measurement coordinate system is computed as [12]:

$$\boldsymbol{H} = \frac{\partial(u,\ v)}{\partial(x_\perp,\ y_\perp)} = \frac{1}{\boldsymbol{T} \cdot \boldsymbol{I}} \begin{pmatrix} \boldsymbol{V} \cdot \boldsymbol{K} & -\boldsymbol{U} \cdot \boldsymbol{K} \\ -\boldsymbol{V} \cdot \boldsymbol{J} & \boldsymbol{U} \cdot \boldsymbol{J} \end{pmatrix}, \tag{6}$$

where $\boldsymbol{I}, \boldsymbol{J}, \boldsymbol{K}$ are unit vectors pointing in the direction of measurement coordinates ($\boldsymbol{I}$ is in the direction perpendicular to the plane) and $\boldsymbol{T}, \boldsymbol{U}, \boldsymbol{V}$ are unit vectors of the track coordinate system axes ($\boldsymbol{T}$ is the track direction).

A GBL track point is constructed from the Jacobian. Then a measurement is added to this point for which the projection matrix, the residual and measurement variance are provided.

The (diagonal) covariance matrix of multiple scattering corresponding to the propagation Jacobian (5) is:

$$\boldsymbol{V}_k = \begin{pmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_\theta^2 \end{pmatrix}. \tag{7}$$

The variance of the multiple scattering deflection angles (for particle with unit charge, velocity $v \sim c$ and momentum magnitude p in units of GeV) is computed using [13]:

$$\sigma_\theta = \frac{0.0136}{p} \sqrt{x/X_0} [1 + 0.038 \ln(x/X_0)], \tag{8}$$

where x is the track length passed inside the sensor material with radiation length X_0. Sensor's material radiation length (and its basf2 sensor ID not provided by GENFIT) is retrieved from the geometry using TGeoManager. The length passed inside the sensor is computed by extrapolation from sensor centre to its boundary and multiplied by a factor of two (assuming the change of track curvature inside a thin sensor to be negligible). A scatterer is then added to the GBL point with computed multiple scattering variance and zero initial kinks. An external additional GBL iteration has also been implemented. In that case, the GBL trajectory is constructed again from the results of the previous fit, which contain estimations of the kinks.

Finally, the global derivatives of the sensor measurement in its local system versus local alignment parameters $u, v, w, \alpha, \beta, \gamma$ are computed and labelled using previously retrieved sensor ID. The labels and the values for global derivatives are then provided to the point for the use in alignment with Millepede.

After all points are added to the GBL trajectory, the trajectory is fitted by GBL and a record is written to a binary file for Millepede. Results of the GBL fit are at this development stage outputted to a separate file together with the results obtained from the Kalman filter for comparison.

5 Results

The implementation of the GBL interface and alignment with Millepede II are tested directly using the basf2 framework with full Monte Carlo simulation and geometry restricted to the vertex detector. Particles for presented results are generated using the particle gun generator (basf2 module), which allows to set particle types and distribution of momenta, angles and vertex positions. Only tracks produced by primary particles from the particle gun were passed for fitting. Also a small modification of GENFIT2Module was done, to restrict the maximum number of hits to 12 (in each layer of sensors, 2 hits are generally possible because of the sensor overlaps) in whole vertex detector to discard possible curling tracks (though some of them still might be present). Except the alignment/misalignment test, all plots are generated with the conditions specified in Table 1.

Parameter	Value
Generated particles	μ^+, μ^- or e^-
Momentum	1.6 GeV < p < 2.4 GeV (uniform)
Azimuthal angle	$70° < \theta < 110°$ (uniform)
Polar angle	$0° < \varphi < 360°$ (uniform)
Vertex position	[0, 0, 0] (fixed)

Table 1: Particle gun setting for particle generation.

Included figures contain two kinds of histograms for investigation of the fit performance:

- Histograms with (normalized) distribution of χ^2 of fitted tracks (sum of residuals from all points of the track) divided by the number of degrees of freedom (ndf), which is in typical situation of 6 (2D) hits and 5 fitted track parameters equal to 7 ($6 \times 2 - 5$). In ideal case, the mean value of such distribution should be equal to 1.
- A distribution of p-values, also called the fit probability. The fit probability is defined as $p = \int_{\chi^2_{\text{observed}}}^{\infty} \chi^2(x, \text{ndf}) \mathrm{d}x$. In other words, this is the probability to obtain a larger value of χ^2 (residual) than observed. If the track model is a valid hypothesis to describe the track, the *p*-values should be distributed uniformly from 0 to 1. In these histograms, the red line shows the result of the fit to the distribution with a constant function. The values of χ^2/ndf of this fit are displayed in the statistics legend.

Figure 4 shows results from the Kalman filter and GBL fit for comparison in case of muons, while Figure 5 for electron. As electrons undergo Bremsstrahlung, there are a lot of outliers visible as a narrow spike at the lowest *p*-values. In Figure 6, the result of an additional external GBL iteration is shown for electrons with visible improvement of the fit. For muons, no difference after an additional iteration is visible (not shown here).

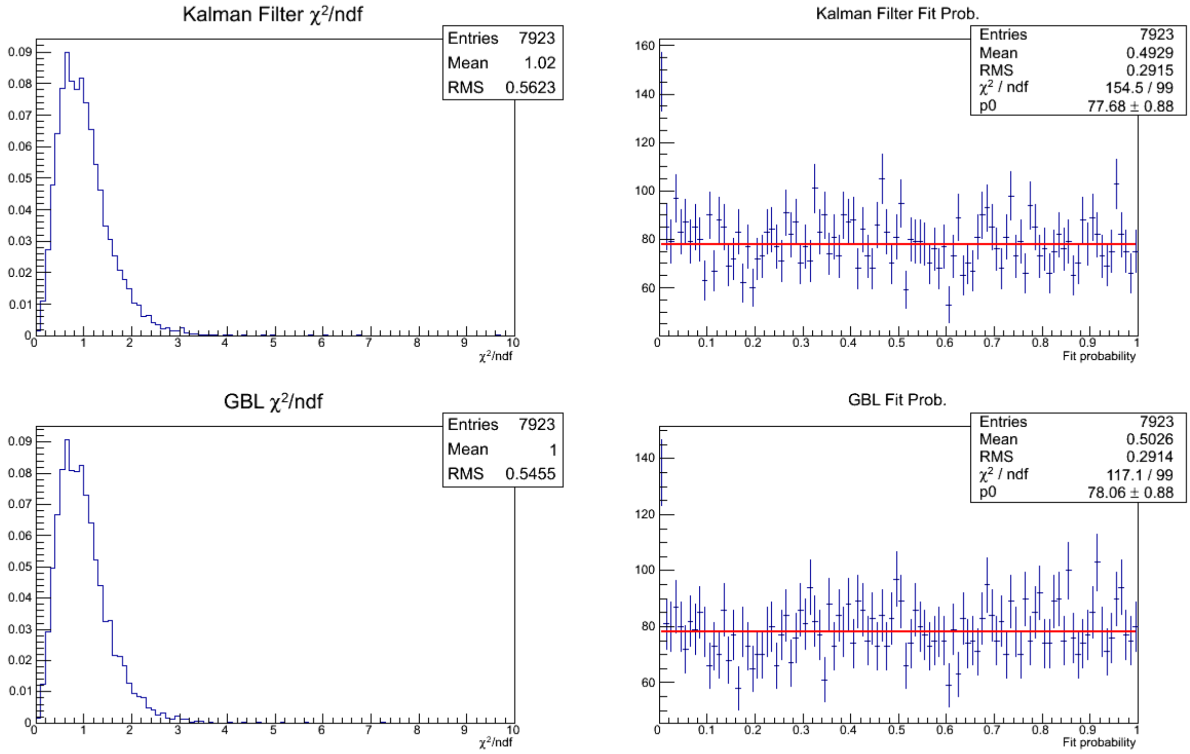

Figure 4: Results of fit for the Kalman filter (top row) and GBL (bottom row) for muons/antimuons.

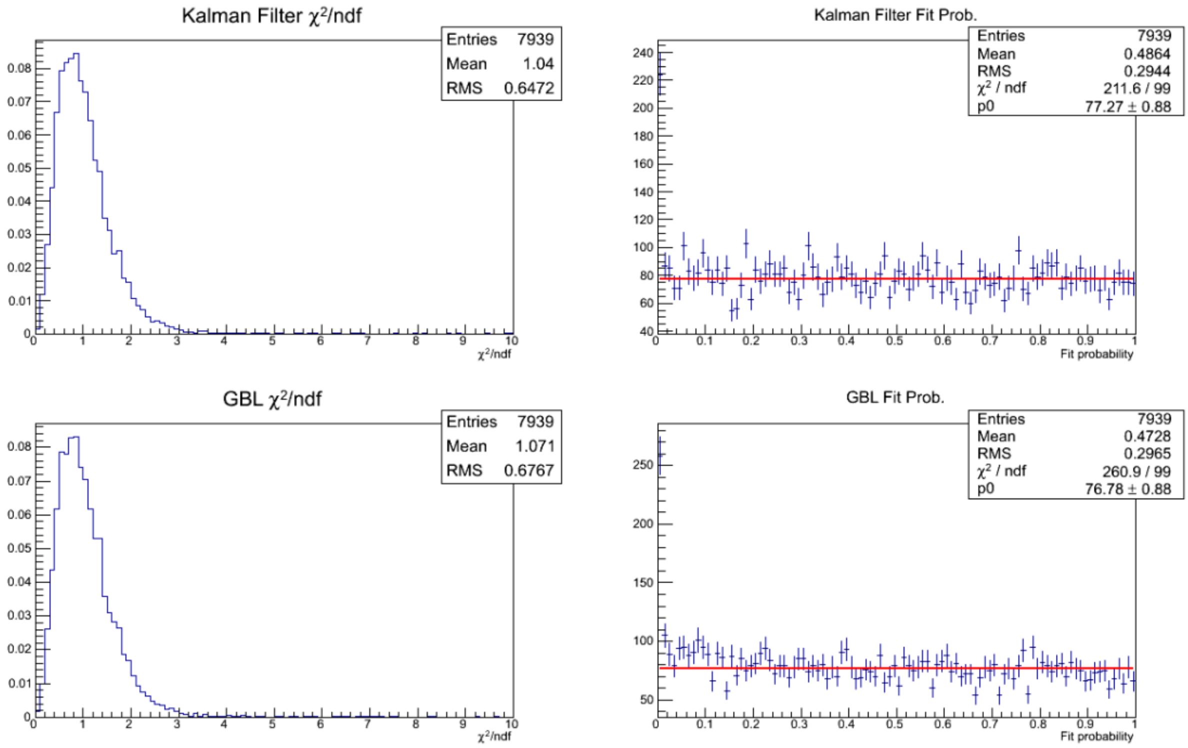

Figure 5: Results of fit for the Kalman filter (top row) and GBL (bottom row) for electrons.

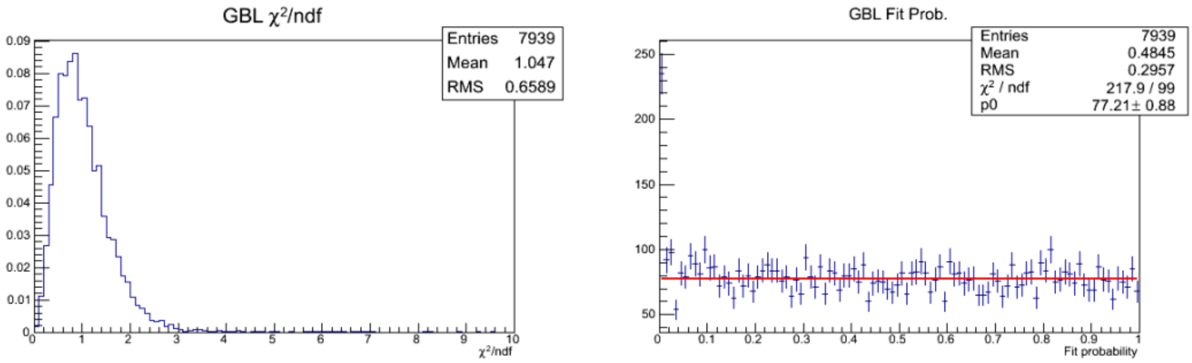

Figure 6: Results of the GBL fit for electrons with an additional external iteration. Note the improvement in comparison to Figure 5 (bottom row).

For test of alignment with Millepede II, only one PXD sensor has been selected for alignment. Particle gun settings are adjusted to achieve high efficiency of hitting the sensor for alignment and to collect enough data. The simulation has then been repeated with the same settings, but a misalignment of 50μm in the *u*-direction is artificially introduced in the GBL interface implementation. Figure 7 clearly shows dramatic change in the distribution of the *p*-values in the case of misalignment. Table 2 shows the results of Millepede II alignment for both scenarios together with the errors of estimated parameters. The misalignment is successfully recovered and the errors of parameter estimation are below 1μm for the displacements (except the perpendicular *w*-coordinate) and 1mrad for the angles (error for β angle is about 2mrad).

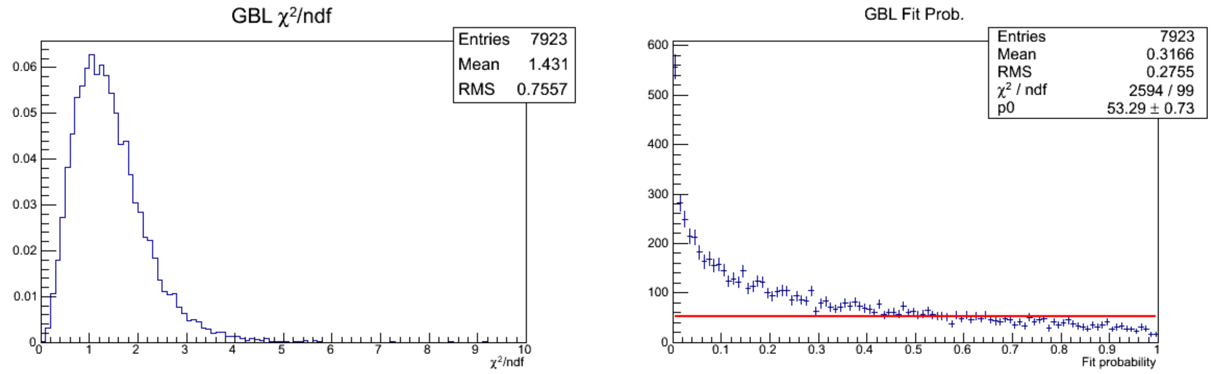

Figure 7: Results of the GBL fit with one sensor misaligned by 50μm in the *u*-direction (all tracks included in the fit go through this sensor).

Perfect alignment				Misalignment: *u* + 50μm			
Parameter	Unit	Correction	Error	Parameter	Unit	Correction	Error
u	μm	0,14	0,43	*u*	μm	50,14	0,43
v	μm	0,07	0,39	*v*	μm	0,07	0,39
w	μm	-1,12	2,20	*w*	μm	-1,04	2,21
α	mrad	0,08	0,34	*α*	mrad	0,08	0,34
β	mrad	-1,62	1,08	*β*	mrad	-1,62	1,08
γ	mrad	0,03	0,07	*γ*	mrad	0,03	0,07

Table 2: Results of Millepede II alignment for the single sensor in the perfect alignment scenario (left) and for misalignment (right) with estimated errors for parameters in the local sensor plane.

6 Conclusion

First integration of the General Broken Lines track model and fitting in the GENFIT 2 reconstruction toolkit running in the basf2 framework with the full Monte Carlo simulation has been successfully developed and tested. Millepede II alignment has been tested in a simple scenario for the Belle II vertex detector for the first time directly in the basf2 framework. Despite some approximations, obtained results are very good and promising. Further GENFIT 2 development will allow more straightforward GBL integration, which is going to become an independent fitter next to the Kalman filter.

The experience from the implementation issues will also lead to an upgrade of GBL, which will allow the reduction of necessary steps for the proper implementation in the future.

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