



Implementation of a color sextet extension to the Color Flow Algorithm in the Matrix element generator O'Mega.

Robert Czechowski

Georg-August-Universität Göttingen, Germany

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Abstract

In my time as summer student in the DESY research center with help of my Supervisor Jürgen Reuter, I worked with the matrix element generator O'Mega, which generates the matrix elements for the event generator WHIZARD. I implement an extension to the color flow algorithm, that is used in O'Mega to calculate the color factor of matrix elements. This extensions allows to calculate the color factor for exotic particles which carry more than one color; the color sextets and the color decuplets. Mainly, I extended the vertex-merging function for the color flows to the cases that occur for these two kinds of particles. In this report, I will also describe the theoretical background and an outline of the color flow algorithm.

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1 Introduction

In the QCD there exists two kind of particles that carry the charge of the strong interaction: the color. There are the quarks which carry one color, respectively anti-quarks which carry the corresponding anti-color. And there are the gluons which carry one color *and* one anti-color¹. But of course it is possible to think of other particles which carry two or three colors. These particles are called color sextets or decuplets respectively. Since they have not been observed so far, one must expect them to be very heavy or to have a very weak coupling to standard model particles.

2 Theory

2.1 QCD

The quantum chromodynamics (QCD) is the theory of strong forces. It is obtained by introducing color charge and requiring gauge invariance for this kind of charge.

The QCD Lagrangian is [2]

$$\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} \quad (1)$$

where the covariant derivative D_μ is

$$D_\mu = \partial_\mu - igA_\mu^a t^a. \quad (2)$$

The field strength tensor in the non abelian gauge theory is defined as

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c. \quad (3)$$

For the generators of the SU(3) usually the 8 Gell-Mann matrices $t^a = \frac{1}{2}\lambda^a$ are chosen, where λ^a are the 8 λ -matrices:

$$\begin{aligned} \lambda^1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda^2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda^3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda^4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \lambda^5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} & \lambda^6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ \lambda^7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} & \lambda^8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{aligned} \quad (4)$$

¹This is not entirely correct, but holds true for either regarding U(N) instead of SU(N) or for the limit $N \leftarrow \infty$ [1]. The group SU(N) only has $N^2 - 1$ generators and hence $N^2 - 1$ gluons.

2.2 Color sextets

Color sextets are particles that does not only carry one color, but two. This would allow for 9 different color states, but since the state has to be symmetrical in its colors, we obtain the six states

$$\begin{aligned} &|rr\rangle, & |gg\rangle, & |bb\rangle, \\ &|rg\rangle + |gr\rangle, & |gb\rangle + |bg\rangle, & |br\rangle + |rb\rangle. \end{aligned} \quad (5)$$

These are related to the six σ -matrices, needed for calculate matrix elements in Feynman graphs:

$$\begin{aligned} \sigma^1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \sigma^2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \sigma^3 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \sigma^4 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & \sigma^5 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \sigma^6 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (6)$$

2.2.1 Decuplets

Color decuplets are in principle the same as color sextets, they are just carrying three colors instead of two. They have to be invariant under every permutation of its three colors, so we obtain the ten states

$$\begin{aligned} &|rrr\rangle, & |ggg\rangle, & |bbb\rangle, \\ &|rrg\rangle + |rgr\rangle + |grr\rangle, & |ggb\rangle + |gbg\rangle + |bgg\rangle, & |bbr\rangle + |brb\rangle + |rbb\rangle \\ &|rgg\rangle + |grg\rangle + |ggr\rangle, & |gbb\rangle + |gbg\rangle + |bbg\rangle, & |brr\rangle + |rbr\rangle + |rrb\rangle \\ &|rgb\rangle + |rbg\rangle + |grb\rangle + |gbr\rangle + |brg\rangle + |bgr\rangle. \end{aligned} \quad (7)$$

2.3 Color factor

2.3.1 QCD

Lets consider the process of quark-quark-annihilation to a virtual gluon and its decay to two quarks again. For example $c\bar{c} \rightarrow u\bar{u}$. The corresponding Feynman diagram is drawn in fig. 1. From this we obtain by the Feynman rules of QCD the following amplitude, where the index s refers to any spin state for each particle

$$\mathcal{A}_{ijkl} = \bar{v}_i^s(p) \cdot ig\gamma^\mu t^a \cdot u_j^s(p) \cdot \frac{-ig_{\mu\nu}\delta^{ab}}{p^2 + i\epsilon} \cdot \bar{u}_k^s(p) \cdot ig\gamma^\nu t^b \cdot v_l^s(p) \quad (8)$$

$$= C_{ijkl} \cdot \bar{v}^s(p) \cdot ig\gamma^\mu \cdot u^s(p) \cdot \frac{-ig_{\mu\nu}}{p^2 + i\epsilon} \cdot \bar{u}^s(p) \cdot ig\gamma^\nu \cdot v^s(p), \quad (9)$$

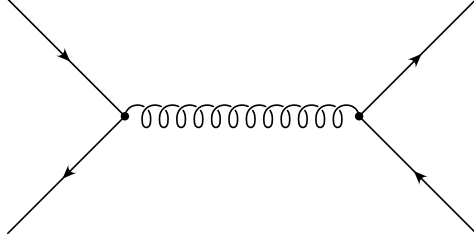


Figure 1: Feynman graph of the process $q\bar{q} \rightarrow g \rightarrow q'\bar{q}'$, two quarks annihilating to a gluons and the pair production of two other quarks in QCD.

where the color factor of the diagram C_{ijkl} was introduced. It is

$$C_{ijkl} = c_i t^a c_j \cdot \delta^{ab} \cdot c_k t^b c_l \quad (10)$$

$$= t_{ij}^a t_{kl}^a. \quad (11)$$

If one wants to calculate the absolute squared matrix element $|\mathcal{M}|^2$ in order to obtain a cross section or a decay width, the absolute square of the color factor is needed.

$$|C_{ijkl}|^2 = C_{ijkl} C_{ijkl}^* \quad (12)$$

$$= t_{ij}^a t_{kl}^a t_{ij}^{a'*} t_{kl}^{a'*} \quad (13)$$

$$= t_{ij}^a t_{ji}^{a'} t_{kl}^a t_{lk}^{a'}. \quad (14)$$

The last identity is due to the fact that the generators t^a are Hermitian matrices.

Summing over all possible configurations of incoming colors gives

$$\sum_{ij} t_{ij}^a t_{ji}^{a'} = \text{tr } t^a t^{a'}, \quad (15)$$

the same holds for outgoing colors. We finally obtain

$$\sum_{ijkl} |C_{ijkl}|^2 = \text{tr } t^a t^{a'} \cdot \text{tr } t^a t^{a'}. \quad (16)$$

This dependency only of the trace of products of generators of the SU(3) is an important perception of quantum chromodynamics. Since for the Gell-Mann matrices (4) holds

$$\text{tr } t^a t^{a'} = \frac{1}{2} \delta^{aa'}, \quad (17)$$

we can simply evaluate this to be 2.

When averaging over incoming color states, we have to add another factor of $\frac{1}{9}$ (there are 3 possible colors per incoming quark) to our result, $\frac{2}{9}$.

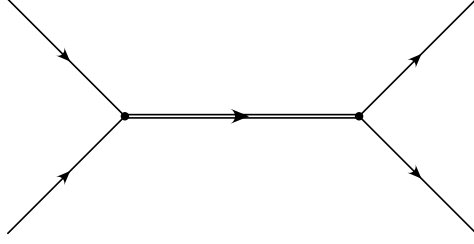


Figure 2: Feynman graph of the process $qq \rightarrow \chi \rightarrow q'q'$, two quarks annihilating to a color sextet particle χ and the pair production of two other quarks.

2.3.2 Sextets

For color sextets, we can consider the process $cc \rightarrow uu$ (note that there are no anti quarks involved here.) The corresponding Feynman diagram is drawn in fig. 2. By the Feynman rules we obtain the following amplitude

$$\mathcal{A}_{ijkl} = ig' \gamma^\mu \sigma^a \cdot u_i^s(p) \cdot u_j^s(p) \cdot \frac{-ig_{\mu\nu} \delta^{ab}}{p^2 + i\epsilon} \cdot \bar{u}_k^s(p) \cdot \bar{u}_l^s(p) \cdot ig' \gamma^\mu \sigma^b, \quad (18)$$

where we find a color dependent factor of

$$C_{ijkl} = \sigma^a c_i c_j \cdot \delta^{ab} \cdot c_k c_l \sigma^b \quad (19)$$

$$= \sigma_{ij}^a \sigma_{kl}^a. \quad (20)$$

We can now follow the steps (12) to (16) because the σ matrices are real and symmetrical. We obtain

$$\sum_{ijkl} |C_{ijkl}|^2 = \text{tr } \sigma^a \sigma^{a'} \cdot \text{tr } \sigma^a \sigma^{a'}. \quad (21)$$

Using the identity

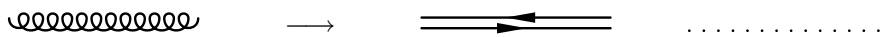
$$\text{tr } \sigma^a \sigma^{a'} = \delta^{aa'}, \quad (22)$$

we find the color factor to be 6 in this case.

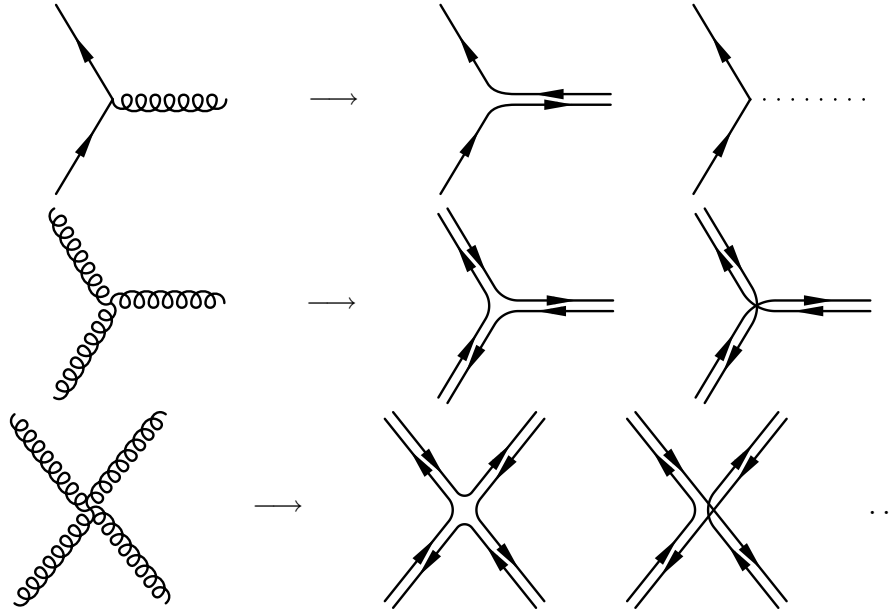
2.4 Color-Flow Algorithm

The Color-Flow-Algorithm [3] gives another possibility to calculate the color factor of the absolute squared matrix element $|\mathcal{M}|^2$ for an arbitrary color algebra $\text{SU}(N)$ with N colors.

Every quark is replaced by a color line. Every gluon by two color lines in opposing directions. Because that would give us one gluon too much, there is also the possibility to replace a gluon by a ghost particle, which cancels exactly one gluon.

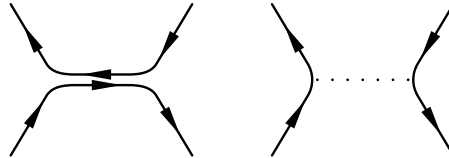


The QCD vertices become

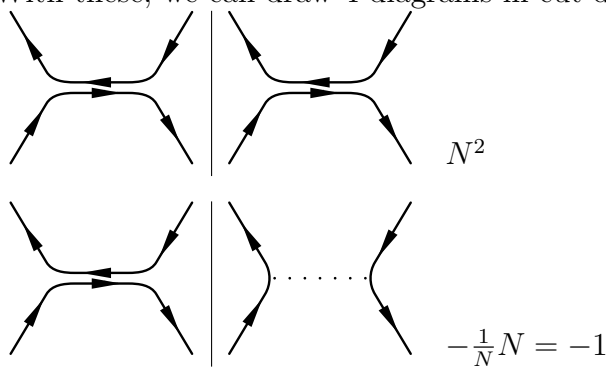


The color factor can now be obtained by writing down the absolute squared matrix element in cut notation, and counting the number of color flows. Each distinct line of color gives a factor of N , each ghost propagator gives a factor of $-\frac{1}{N}$ for canceling the unnecessary gluon. Each vertex is additionally weighed with $\frac{1}{\sqrt{2}}$, which gives a constant factor for every color flow representation of a diagram.

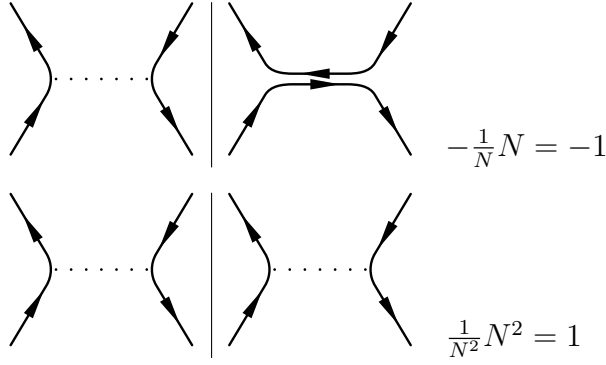
Example Considering the Process of sec. 2.3 again, the following color flow diagrams are obtained:



With these, we can draw 4 diagrams in cut diagram notation².



²This is somewhat better explained in the papers on color flow. A detailed description can be found on page 161 of the CTEQ handbook [4].



The color flow algorithm gives $N^2 - 1 - 1 + 1 = 8$ for $N = 3$ colors. Adding the factor of $\frac{1}{4}$ for 4 vertices we obtain 2, the same as in direct computation of the color factor.

The Color-Flow-Algorithm can easily be extended to color sextets and decuplets, as shown in [5]

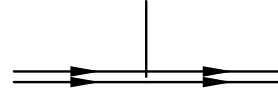
3 Implementation

My task was the implementation of the color sextets and decuplets in the color flow algorithm of O'Mega [6] which is the matrix element generator for WHIZARD [7], the event generator.

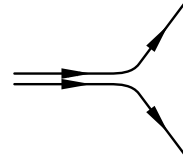
Mainly I altered the functions `colorize_fusion2` and `colorize_fusion3` in the `colorize` module of O'Mega. These are responsible for merging the colors of two, respectively three incoming propagators to the colors of the outgoing propagator of a vertex.

The sextet vertices implemented are:

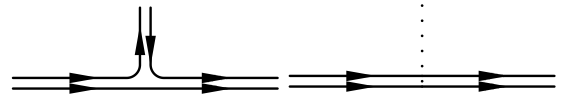
- sextet – sextet – singlet
(3 vertex & 3 orientations)



- sextet – triplet – triplet
(4 vertices & 2 orientations)



- sextet – sextet – octet (gluon)
(5 vertices & 3 orientations)



- sextet – sextet – octet – octet
(7 vertices & 3 orientations + 4 vertices & 4 orientations).



The decouplett vertices implemented are, similar to those above:

- decuplet – decuplet – singlet
(6 vertices & 3 orientations)
- decuplet – sextet – triplet
(6 vertices & 3 orientations)
- decuplet – triplet – triplet – Triplett
(12 vertex & 3 orientations)
- decuplet – decuplet – octet
(7 vertices & 3 orientations)
- decuplet – decuplet – octet – octet
(13 vertices & 3 orientations + 6 vertices & 4 orientations).

3.1 Results

Unfortunately, the right color factors were not calculated by O’Mega at the end of the summer students program. This is due to the symmetrization not fully being implemented in O’Mega so far. Another point where the calculations fails so far is the averaging over incoming color states.

For the example, the process of a scalar heavy color sextet particle into a scalar light color sextet and a light scalar color singlet (colorless particle) was considered. The calculated decay width obtained by WHIZARD is 9 times larger than the theoretical prediction.

The missing symmetrization is the reason for the other diagram, that gives a contribution of 3 instead of 9 is omitted. Otherwise, the average factor, the predicted value of 6, would be obtained. By averaging over incoming color states, the factor of 6 should be divided out again, which gives a final color factor of 1. A hand waving argument for this factor is, that the incoming quark has to transfer its color state exactly to the outgoing color sextet. So there is no difference to a decay of a color singlet in two others with the same masses and coupling.

References

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