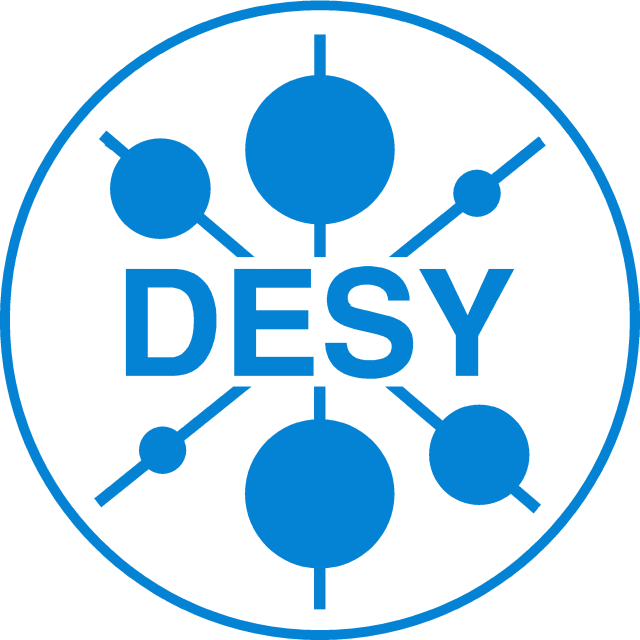
SUMMER STUDENT REPORT

DESY HAMBURG 2012

FLASH beam spatial coherence measured by non-redundant arrays of slits

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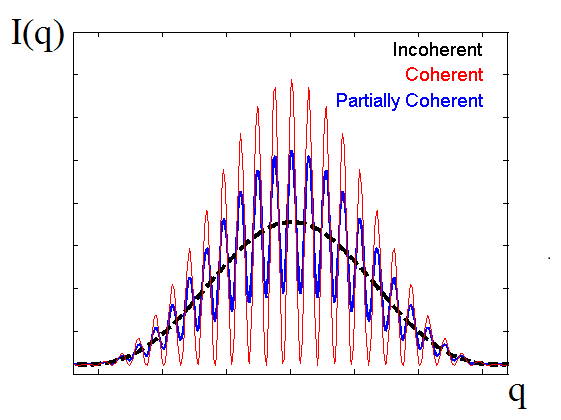
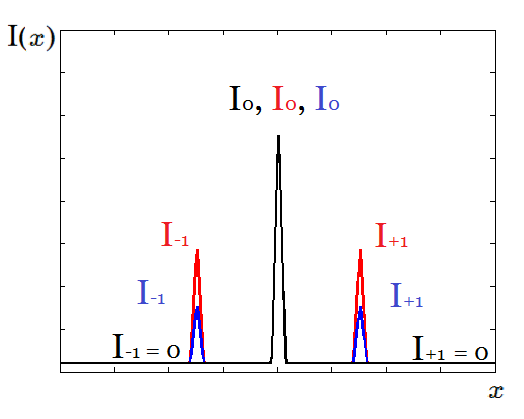
Free-electron lasers (FELs) based on the self-amplified spontaneous emission (SASE) principle produce extremely brilliant, highly coherent radiation in the extreme ultraviolet and hard x-ray range. Utilizing the high photon flux, the femtosecond pulse duration and the high degree of coherence, techniques like coherent x-ray diffraction imaging (CXDI) x-ray holography and, recently, nano-crystallography promise important new insights in biology, condensed matter physics and atomic physics. Some of these methods can be implemented only if the radiation is sufficiently coherent, both spatially and temporally. This means that the knowledge of the coherence is essential for the success of these experiments.

The classic Thomas Young’s experiment with double slits diffraction might be used to measure the spatial coherence properties of a light source, particularly FLASH beam. It is better to use an extended scheme that could obtain more coherence data for one single shot than to have the lesser ammounts as in classic setup due to FLASH instability.

The main problem of the experiment that was done on FLASH laser is that the intensity of the beam at the slits is unknown. So the spatial coherence properties cannot be found rigorously from a mathematical point of view. Yet, it seems to be possible to find the solution if one allows additional restrictions and assumptions for the problem. The work done at DESY Summerschool 2012 is an attempt to find these restrictions.

II. Theory

1. Young’s double slit diffraction

Classic Young’s double pinholes experiment setupmay be used to measure the spatial coherence properties of a light source, particularly, FLASH beam.

**c)**

**b)**

**a)**

**(q)**

**(x)**

**Detector plane**

**Double slits**

**Incoming beam**

**Figure 1**.**a**: Young’s double pinhole experiment setup

**1.b**: Intensity function I(q) of the detector plane (q) for two slits

**1.c**: Fourier transform of I(q)

Consider the experiment illustrated in Fig.1. An incoming light beam diffracts on double pinholes at the pinholes plain (coordinate x) and forms an interference image on the detector plain (coordinate q). Depending on the spatial coherence properties of the beam light in the slits, one might obtain different interference images. Assume we have uniform intensities within each slit and the shapes of slits are the same. If the light beam is completely incoherent, i.e. we assume a limit case of coherent properties, the intensity function on the detector plane could be represented as well known Airy disk for diffraction of light on single-slit in the case of far-field diffraction, shown on Fig.1.a with dashed black curve. If the incoming light beam is coherent, the intensity function spot would be modulated with oscillation, i.e. we would observe the interference phenomena. The value taken to measure the coherence of the light is the amplitude of this modulation. If the amplitude of modulation is so big that the intensity function periodically reaches zero value on the detector plane (Fig.1.a, red curve), we have a limit case of perfect spatial coherence of the light beam. In between these two limit cases lies the partially coherent diffraction case (Fig.1.a, blue curve). That is a very simple and short introduction to spatial coherence phenomena.

In Joseph’s W. Goodman “Statistical Optics” [1] it is shown that the intensity function on the detector, produced by light beam passing through two slits, is represented by following formula:

,

Where and are intensity functions on the detector plane produced by, respectively, first and second slits, d – distance between slits, – phase difference between slits and - modulus of complex degree of coherence for two slits. By definition, modulus of complex degree of coherence is

where and are electric fields at two points of the light beam, is the ensemble average and \* denotes the complex conjugate. If we assume that intensity is uniform inside each slit and the shapes of the slits are the same, we may define the intensity functions as following:

,

where (q) is a function, proportional to intensity function of a single slit, i.e. sinc2(q/D), where D is the slit diameter. Using these formulas we obtain

,

If we now take a Fourier transform of this intensity function, we would get

due to the convolution theorem, where denotes convolution, is a triangular function and denotes Dirac’s delta function. After convolution we finally obtain

That formula describes us exactly what we see on Fig.2c. Therefore, height of the central peak is the sum of intensities of the slits and heights of the two satellite peaks are the same and equal to . Value of modulus of complex degree of coherence is exactly the measure of the spatial coherence of the light beam. It is important to mention that this value is dependent on the slits distance:

|

We might measure this value on a set of pinholes at several distances to see how modulus of complex degree of coherence depends on the latter.

A.Singer et al.[2] previously obtained the results for spatial coherence properties of FLASH laser. Double pinhole diffraction patterns of single femtosecond pulses focused to a size of about 10x10 µm2 were measured. A transverse coherence length of 6.2 0.9 µm in the horizontal and 8.7 1.0 µm in the vertical direction were obtained.

The idea of the work that is described in this report is to measure the modulus of complex degree of coherence at several of distances for one shot, but not for a set of shots as it was done before. For that we may use an extended version of Young’s experiment.

1. Multiple slits diffraction

If we now assume that we have a set of slits at the slits plane, we would have a bit more complex picture, but very similar to one we have obtained before. Unlike double slit experiment, we would now obtain several satellite peaks in the Fourier space for diffraction image. If the slits array is not redundant, i.e. distance between any two slits is unique, each satellite peak would correspond to one pair of slits, as in the double slits experiment (fig.2).

**Figure 2.** Fourier transform of multiple slits diffraction image.

The formulas for multiple slits experiment are well described in A.I.Gonzales and Y.Mejia works [3, 4]:

,

Where *n* is thenumber of peaks, is the central peak height, – satellite peaks heights and – intensities of the slits as the were introduced in the double slit experiment. If we now solve this system of equations for all *i, j* we might obtain the complex degree of coherence at several distances at once.

(1)

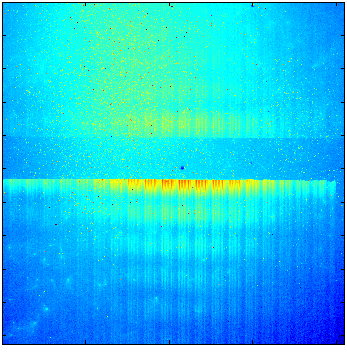
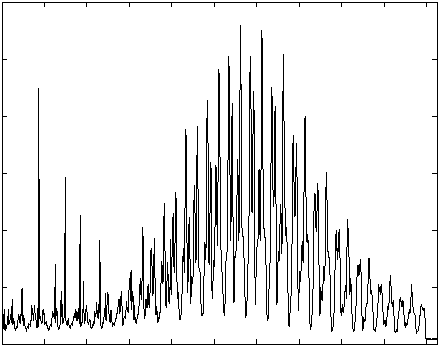
1. Image processing. “Spark” reduction

In November 2010 the experiment for measuring FLASH coherence properties was performed. Four sets of non-redundant arrays of slits in vertical and horizontal direction and of different counts of slits, precisely, five and six counts for one aperture were illuminated at a wavelength of 8.0 nm and the diffraction images on the detectors (20482048 pixels, 13.513.5 µm each) at a distance of 0.34 m from the slits plain were obtained.

My work at DESY Summerschool is currently performed for one set of slits. Therefore, let us assume that further the talk would be only about the vertical non-redundant arrays of five slits. The slits coordinates are -7.5, -4.77, 2.05, 3.41 and 7.5 micrometers with respect to the aperture center. Their diameters in longitudal (i.e. along their) direction are 250 nm each.

1. Image processing. “Spark” reduction

A typical image of diffraction pattern for one shot of FLASH beam is shown on fig.3.a. The dark central area is a shadow of beamstop that prevents the detector damage. If we now take a vertical slice of 1 pixel width in the most intense vertical fringe of the diffraction pattern, we would obtain an intensity profile that is displayed on fig.3.b. In this profile we may see odd extra intense peaks that are not related to periodic picture we see at all the rest of the intensity function along the slice. Those peaks are pointed with arrows on fig.3.b. They would further be called “the sparks”. The “sparks” strongly affect the Fourier transform of the image. Even the neighbouring 1 pixel width slices might have different heights of the peaks in the Fourier space. These effects were verified in the first days of DESY Summerschool programme. “Sparks” contribution to diffraction pattern make any further data processing useless and therefore they were deleted in the following way.

**** ****

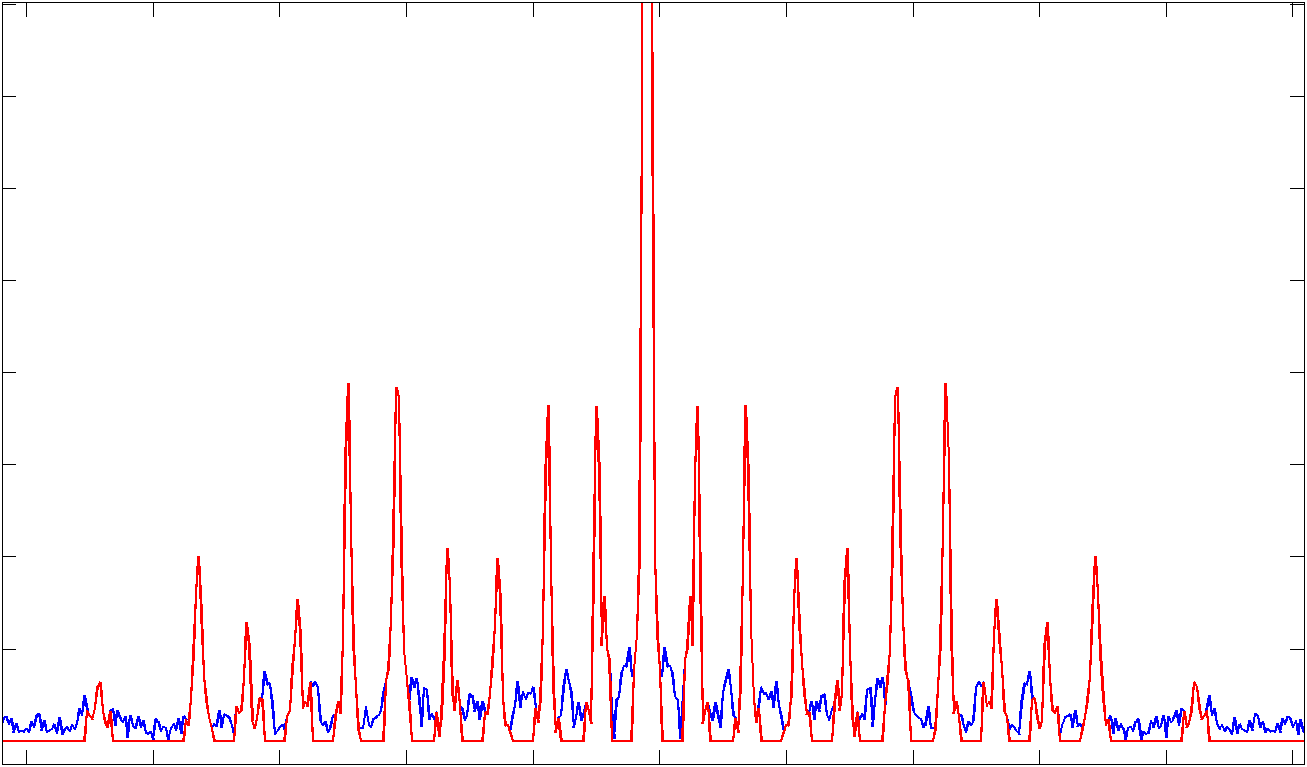
**a)**

**b)**

**Figure 3.a:** typical diffraction pattern for one shot.

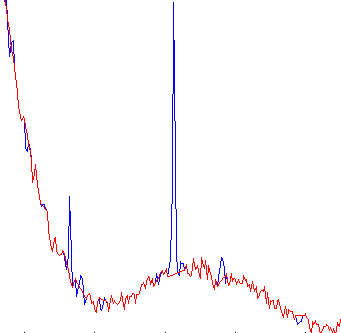
**3.b:** An intensity function of a “slice” in vertical direction from the maximal intensity fringe on fig.3.a (taken in region outlined with red color). The “sparks” are pointed out with red arrows.

From each vertical slice with intensity was taken the Fourier transform and intensity F1(x) in the Fourier space was obtained. In that Fourier space (x-space) the intensity function at all points except the areas near the peaks positions were set to zero (see fig.4).



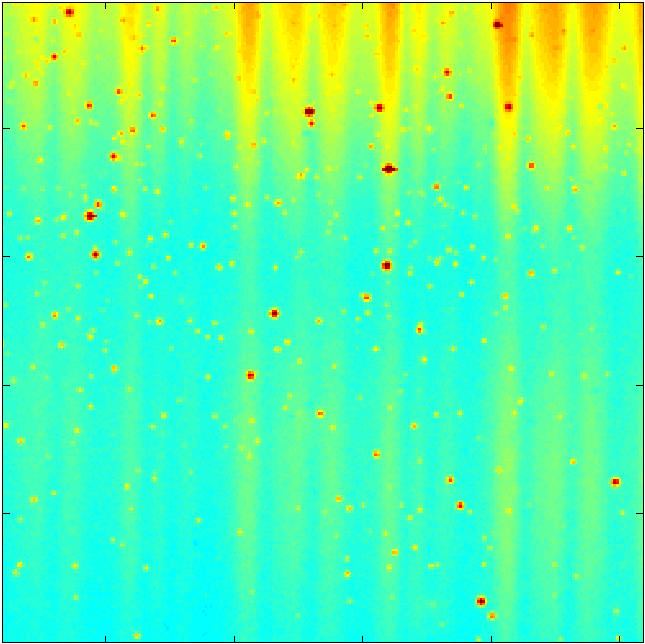
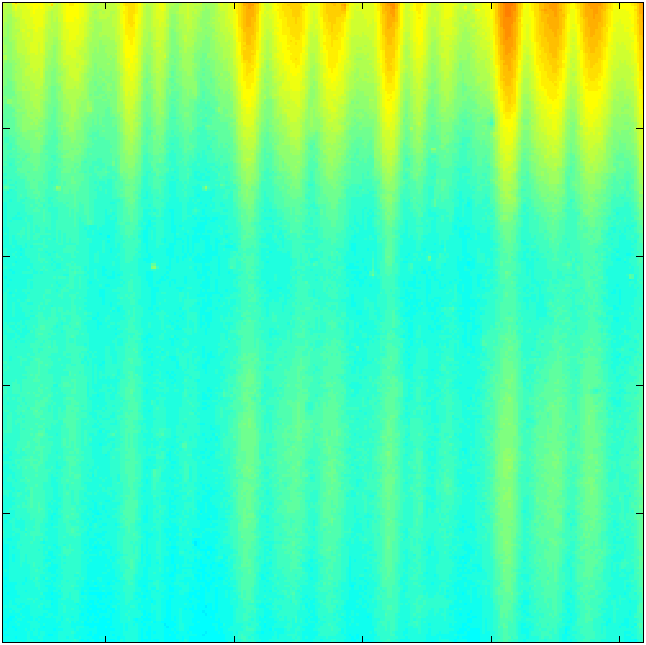
**Figure 4**.Fourier spectrum of noised slice F1 (blue curve), Cropped Fourier spectrum F2 (red curve). Red curve might overlap blue curve at the points where they are equal.

Then the inverse Fourier transform from that new intensity function F2(x) was taken, so that we would have a new intensity function in q-space. After that F2(x) was subtracted from F1(x) and the standard deviation of the result was calculated. If at certain point the value of |F2(x)-F1(x)| is bigger than 3 sigmas, this point was designated as a “spark” containing. In this way we found the “sparks” in each vertical slice, which means that we found sparks in the whole picture and their coordinates are known. The neighbourhood of the “sparks” coordinates was then substituted with first-order splines in every horizontal direction, i.e. each “left” and “right” ends of the intensity function at the ends of neighbourhood of the “spark” was connected with a line (see fig 5.a). The procedure is repeated two times.



**Figure 5**: a horizontal “slice” in the maximal fringe region. Red curve is the intensity function after the “spark”-reduction procedure, blue curve – the intensity function before the procedure. The red curve overlaps blue one in regions where they are the equal.

ii. The “spark”-reduction procedure results

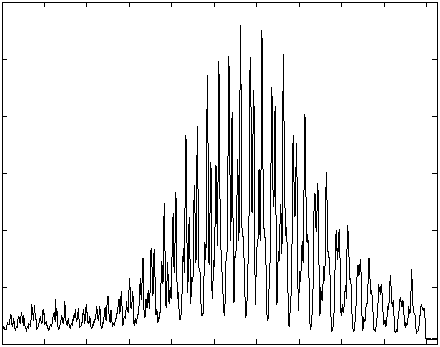
 

**b)**

**a)**

**Figure 6.a** and **6.b:** the intensity image on the detector plain before (b) and after (c) the “spark”-reducing procedure.

As a result we obtain filtered images. If we take a closer look to a region near the most intense fringe we would see that the new image is indeed smooth (fig.6.a) compared to original (fig.6.b). All further results are validated for each vertical slice inside the maximal intensity fringe for one shot and the described “spark”-reducing approach seems to work very well, i.e. the ratio between the satellite peaks are nearly exactly the same, but their ratio to the central peak is reduced in the direction from the center. The last problem to be resolved seems to correspond to the smooth hill-like raise of the intensity in the center of the detector image and in areas near the “sparks”. This raise might be seen in fig.5: the intensity function has to be sinc-squared-like, yet it does not reduce to zero in its local minima. Last problem is not yet really resolved, but it seems not to affect strongly at least some images.



**Figure 7.** The same slice as in figure 4 after the “spark”-reducing procedure.

iii. The model

Now, after we have obtained filtered images we may proceed with solving the problem. The main problem of the experiment is that it lacks the data for the beam intensity for all FLASH shots done. Therefore, the intensities of the beam at the slits are unknown (see system of eq.(1)) and the problem cannot be solved rigorously. The very essence of the work done emerges from the two following assumptions.

**First**, the FLASH beam is Gaussian, i.e. its cross-section intensity is:

Where is the beam center with respect to the multiple slits center, – the maxima intensity. For each set of (, ) we immediately have a unique solution of the mathematical problem, yet there is only one strict adequate solution of the physical problem. We need to find some kind of optima in space, constructed of all (, ) pairs. It is a complex problem to find out what kind of optima must be found.

Hence, **the second** assumption was made. The spatial coherence curve (i.e. absolute values of complex degree of coherence taken at set of distances) is monotonic. To formulate this mathematically, the monotony classes for spatial coherence curves were defined.

In this work the complex degree of coherence is expected to be found at ten distances and is expected to be equal to one at the central peak (by definition). Let us denote the distances as , where their values increase with the indices. Then, the spatial coherence curve, “fetched” on the corresponding module of complex degree of coherence values , belongs to N-th class of monotony if

I.e. first N -points decrease monotonically with allowing an error . All further work is done for . The perfectly monotonic coherence curve belongs to 10th class in the case of 5 slits diffraction. The latter restriction of the problem (monotony) is supposed to allow us find the near-optima, i.e. near-physical-solution points in (, ) space. Therefore, we might find the module of complex degree of coherence for each single FLASH shot. shall be further sometimes substituted with FWHM of the beam, which is the same in principle.

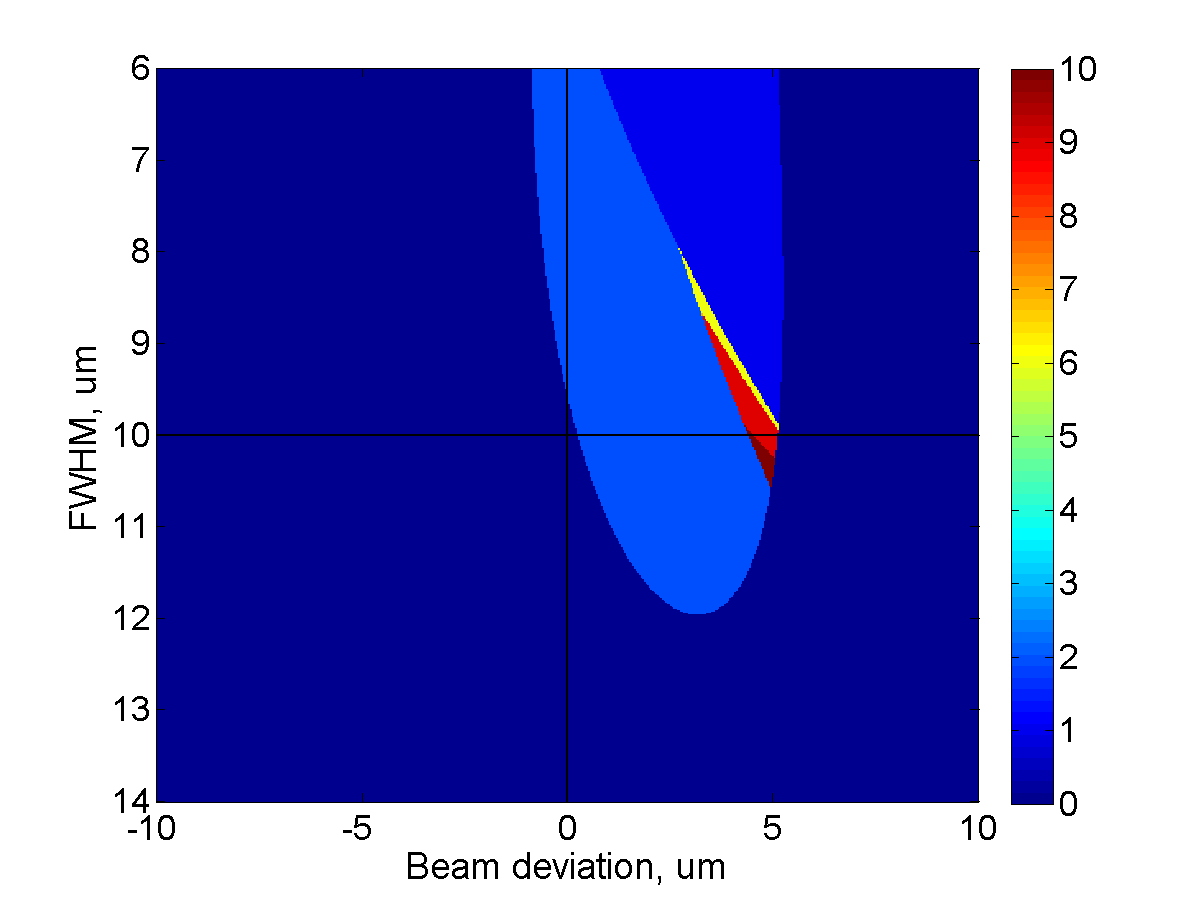
The algorithm

The code for solving the problem was developed in Matlab. According to the proposed model, a single shot image (e.g., as in fig.3.a) was first twice processed with “spark”-reduction procedure. After that the “right” and the “left” end (in horizontal direction) of the maximal intensity fringe was found. Further, a vertical slice is taken near to the “right” end (direction where the beamstop shadow appears) and is Fourier transformed. Then the peaks in the Fourier space are detected and their heights are found. After that an array of 501501 of (, ) pairs in the intervals of FWHM = (8 µm, 16 µm) and = (-10 µm, 10 µm) is created. For each single (, ) pair the intensities at the slits are known and, therefore, the coherence curve is found and its monotony class is determined.

1. The model results

As a result for one single FLASH shot we obtain (, ) spaces with each point corresponding to a certain coherence curve monotony class. These spaces are showed as diagrams on fig. 7:

**2nd class**



**6th class**

**functions area**

**9th class**

**functions area**

**Most monotonic**

**functions area**

**(10th class)**

**1st class**

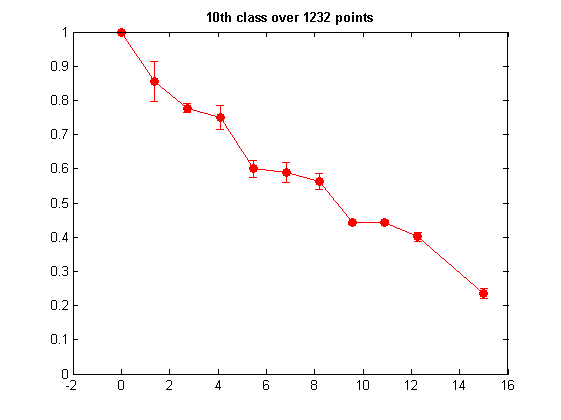
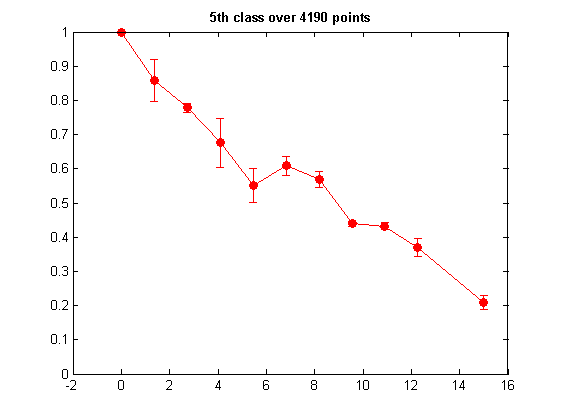
**Figure 8.** (, ) space diagram for one shot. Beam deviation denotes , the colored areas are certain coordinates that correspond with certain coherence curve. The class-to-color correspondence is indicated on the color bar.

It is important to notice that for most of the shots the maximal monotony class (it is not always 10th) the FWHM lies near value of 10µm. That corresponds with the results obtained by A.Singer et al. [2]. The precise statistics is not done yet.

The averaged over class coherence curves from different classes of two shots are displayed on fig. 9:

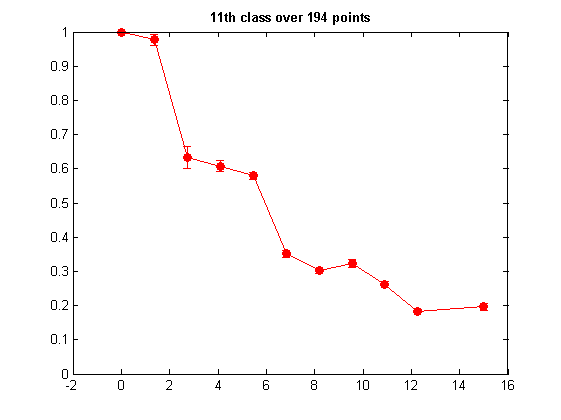
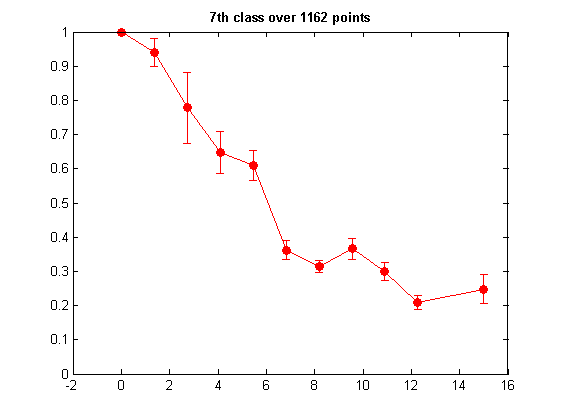
Shot #1 (4th class)

Shot #1 (10th class)



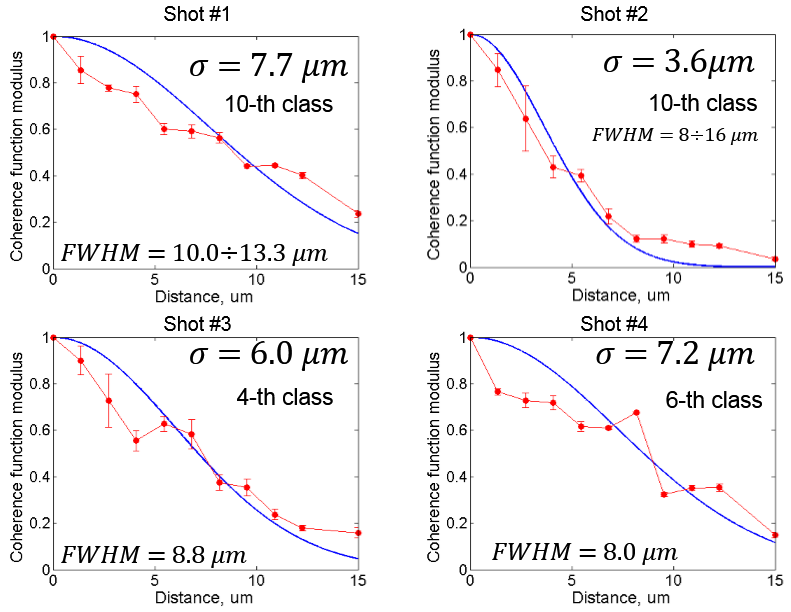
Shot #2 (10th class)

Shot #2 (6th class)



**Figure 9.** Averaged over corresponding class area coherence curves for two shots.

Fitting of coherence curves with Gaussian function was also obtained so it would be possible to estimate the transverse coherence length. Those are displayed on figure 10:

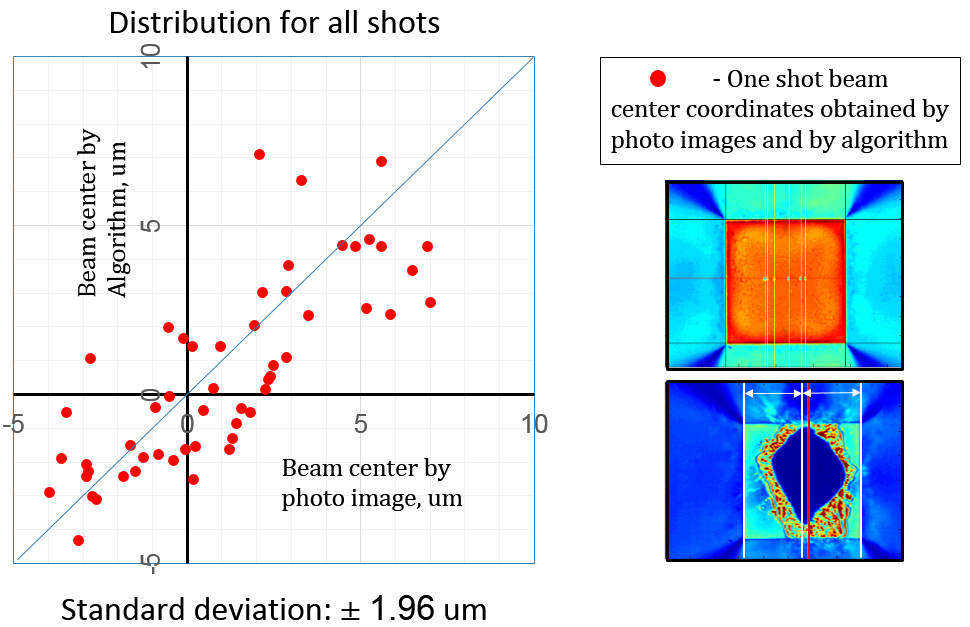


**Figure 10.** Coherence curves from different shots fitted with Gaussian functions. σ denotes transverse coherence length. It is found from Gaussian fit function sigma. FWHM denotes the FWHM interval or value (i.e. the interval is very short) with which the particular coherence curve corresponds.

The expected transverse coherence length is expected to be 8.7 1.0 µm [2]. In some cases we have a correspondence with previously done double pinhole experiments, but it seems to be that in most we do not. That might be because the hill-like intensity raise is not solved yet (see end of “spark-reduction procedure results”) and this effect, as well as image noise that is still not resolved, contributes to the central peak intensity, so that whole coherence curve, except the 0 at r0 = 0 µm is shifted down. Such shifting of the coherence function, while saving its 1, 2, … 10 ratios (i.e. curve shape), was observed when validating the “spark”-reduction procedure. The coherence curve was shifting down when moving outwards the center of the image.

Also, beam center position was validated with additional experimental data. The photo images of shot apertures were taken on the microscope\* and the beam center coordinate was obtained as a center of mass of hole left by beam position shift with respect to center of slits. Figure 11 describes this part of the work.

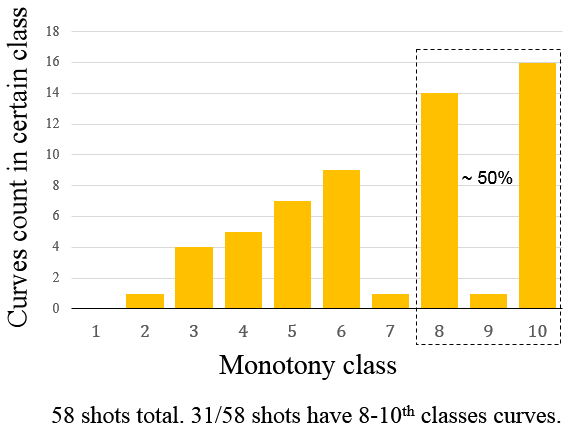
\* Thanks to XFEL optics group for letting use the microscope!



**Figure 11.** On the left: beam center obtained by algorithm-obtained by photo image distribution. On the right: processed photo images of slits array (not shot) and an after-shot array.

It is really hard to tell about strict coincidence, yet it is possible to say that the corellation to one (e.g., x < 0) direction or another is strong. Non-strictness of the corellation might be due to error in beam center estimation algorithm or to photo camera resolution error (visible light diffraction limit). It might more likely be due to beam instability. The center of the beam obtained by algorithm is taken as center obtained by photo image nearest neighbour point from FWHM interval on which the most monotonic coherence curves are defined.

Also, the statistics of maximal classes of monotony was calculated, that is shown on figure 12:

**Figure 12.** Maximal classes counts for all shots. 

V. Conclusions

As it was mentioned before, the problem cannot be solved strictly, because of the intensity data lacking. Therefore, additional assumptions might help to find the right solution. The proposed model is an attempt to estimate the first approximation of spatial coherence properties of FLASH beam. Summing up the results, we may conclude that the model does not perfectly fit the expected results for all shots, yet it does really good results in some cases. The mismatch might be due to several problems:

1. The beam cross-section intensity is not Gaussian-like. And that is the most likely reason.
2. The module of complex degree of coherence is not surely monotonic in all cases.
3. A big part of mismatch is surely due to hill-like intensity raise on the detector plane (see end of “spark-reduction procedure results” and text after fig.10).

According to that, a new attempt is recently developed. It is related to restriction on the coherence curve, but not on the intensity of the beam as it is done in this report. Some first results obtained show good correspondence with expected parameters of the FLASH beam as well. Perhaps, a combination of several approaches would resolve the problem.

The project is complex and not finished yet. It would surely be continued. As for me, I surely wish to participate!

References

1. J.W. Goodman, Statistical optics

2. A. Singer et al. *Optics Express*, **20**, 17480 (2012)

3. Y. Mejia and A. I. Gonzalez, *Opt. Commun.*, **273**, 428 (2007)

4. A. I. Gonzalez and Y. Mejia, *JOSA*, **28**, 1107 (2011)