# A beam trajectory correction tool for the FLASH undulators

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**Abstract**

In DESY's FLASH (Free electron LASer in Hamburg) accelerator, electron bunches are accelerated to energies of about 1GeV and put through undulators in order to produce laser pulses in the soft X-ray wavelength range. Accurate photon beam position at the output is required for high precision experiments carried out in the FEL halls. This report presents a tool that enables correction of a given deviation in the photon beam by manipulation of the electron beam, using the different elements along the accelerator beam-line (quadrupoles, corrector dipoles). It is to be used in the FLASH control room and will give operators a reccomendation of the required trajectory correction.

# Contents

# 1. Introduction

FLASH accelerator operating at DESY is a 315 meter long machine providing ultra-short femtosecond laser pulses in the soft X-ray wavelength range with unprecedented brilliance. Uses of Free Electron Laser (FEL) include (but are not limited to) studies of chemical reactions in very short time intervals, structural analysis of molecules and simulation of processes under extreme conditions (very high temperatures and pressures). Thus, it is clear that very high precision is required in all beam characteristics. One of such is the position of the beam as it exits the aperture of the first instrument in the experimental hall.
In FLASH, FEL is generated using undulators which are made up of a series of equal dipole magnets with alternating magnetic-field direction (figure 1). An electron beam moving through such a device will wiggle in a single plane, which causes it to emit light in a direction tangent to the trajectory. In the case of FLASH, the wiggling motion takes place in the horizontal plane. The exact mechanism of generating FEL is beyond the scope of this report [1].

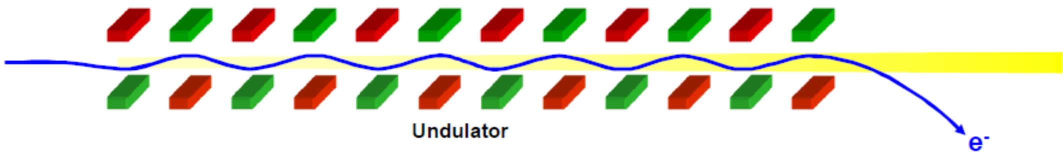



Figure 1: Schematic of an undulator, showing electron beam trajectory and emitted light.

In figure 2 a layout of FLASH is presented[2]. Note that the undulator section is located at the very end of the beam line, just before the experiment halls.

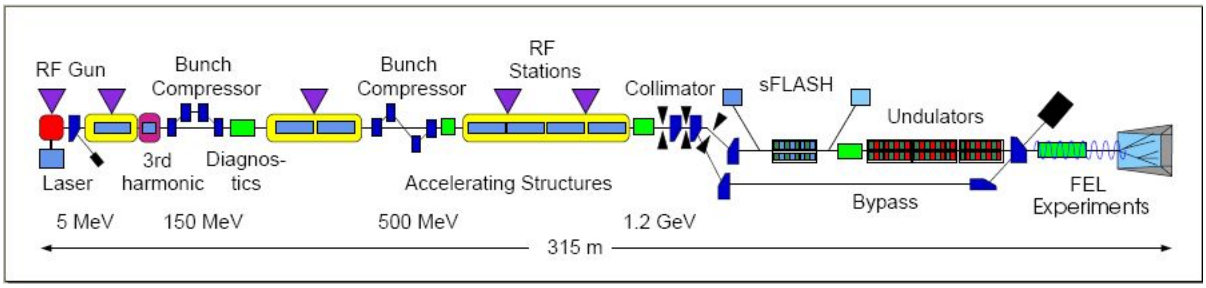



Figure 2: General layout of FLASH (not to scale)

In some cases the photon beam emerging from the accelerator deviates from the desired location in the transversal plane. The tool offers a solution for this to the operators in the control room by manipulation of the electron beam as it travels through the vacuum chamber. This is achieved using the existing elements along the undulator section, namely dipole bending magnets (also refered to as 'correctors' or 'steerers') and quadrupole magnets which are mounted on micromovers. For a dipole magnet, the bending angle rises proportionally to the supplied current. For a quadrupole magnet, deflection depends on the current supplied but also on the location of the particle with respect to the center of the quadrupole (a charged particle passing through the center will not be affected, whereas a particle passing far from the center will be deflected). Another important note is that all elements have a linear effect on the electron beam (in first order approximation).
The properties that can be changed for the purpose of correction are the currents supplied to the steerers and the transversal position of 14 quadrupoles which are located in the undulator section. The output of the program is a set of values (of currents or displacements) for each of these elements.

## 2. Linear Beam Dynamics

In beam dynamics, all calculations are made with respect to the curvilinear coordinate system which describes the path of an ideal beam (in our case this path is simply a straight line passing through the center of the vacuum chamber). The coodinates $(x, y)$ represent the deviations of the actual particle trajectory from that path. Throughout this discussion a point-like beam will be assumed (with no transversal dimensions), keeping in mind that since the system is linear any change inflicted on the center of the beam is proportional to that inflicted on the edge of the beam.
Considering a certain plane and applying Maxwell's equations for a beam of charged particles produces the equation of motion[3]:

$$x'' + K(s)x = 0 \tag{1}$$

where K(s) $\equiv k(s) + \frac{1}{\rho^2(s)}$. The factors $k(s)$ and $\rho(s)$ represent the focusing factor and the bending radius (respectively) of the magnets which are situated in the longitudinal location $s$ and are defined as following:

$$k = e\frac{g}{p} = \frac{e}{\beta E}g \tag{2}$$

$g \equiv \frac{ce}{E}\frac{\mathrm{d}B_\varphi}{\mathrm{d}r}$ is the field gradient (polar coordinates), $p$ is the particle momentum, $E$ the particle enregy, $\beta$ the particle speed (in our ultrarelativistic case can be assumed as 1) and $e$ is the charge (in the case of electrons).

$$\frac{1}{\rho_x} = \left|\tfrac{ce}{\beta E}B_y\right| \tag{3}$$

where $\rho_x$ is the bending radius for the $x$ axis and $B_y$ is the $y$ component of the magnetic field (the same applies for $\rho_y$ and $B_x$).

Using a "hard edge" model which approximates the field of a magnet as a step function rather then a curve (figure 3), it is possible to divide the beamline into segements with different elements in which $k$ and $\rho$ are constant.

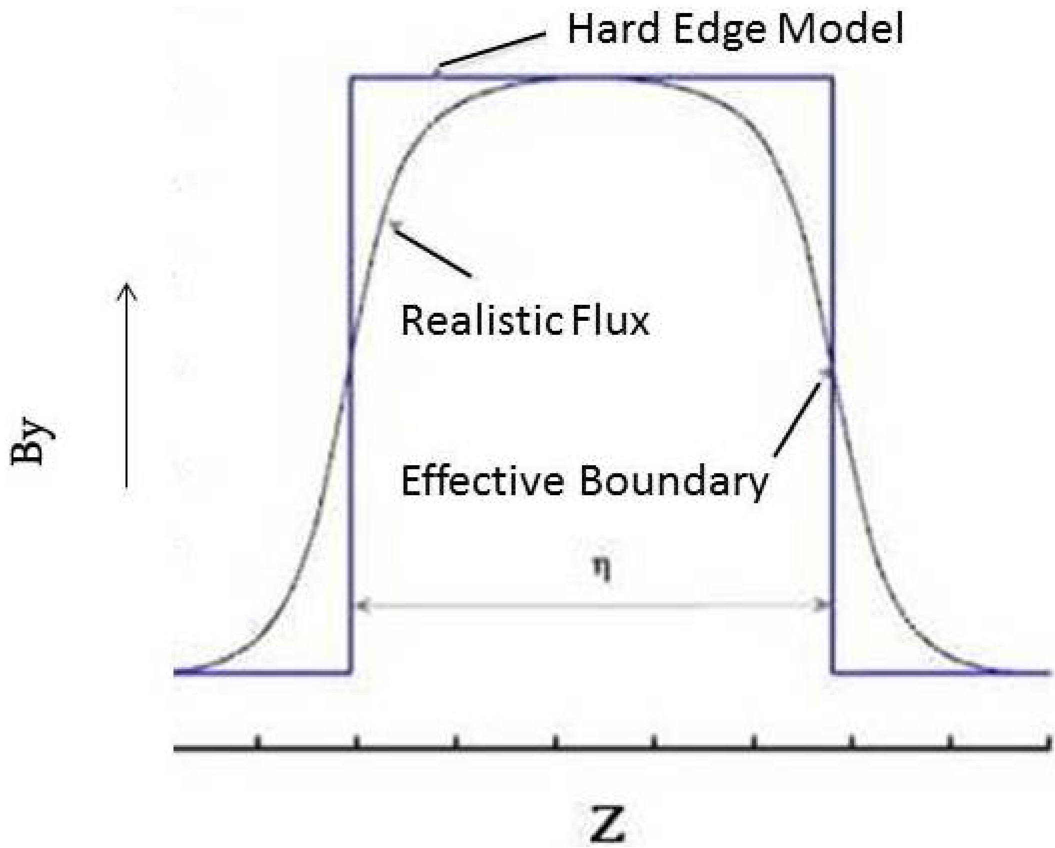



Figure 3: Hard Edge approximation for a magnetic field

If the magnets are sufficiently long and have a small enough aperture this model is a good approximation for any practical use. In other cases, perturbations may be inserted to correct the so called *fringe effects* of the magnets.
Inserting the assumption that these two parameters are constant into (1) generates a simple harmonic oscillator equation ($K = const$). The principal solutions are:
for $K > 0$

$$C(s) = \cos(\sqrt{K}s) \qquad \text{and} \qquad S(s) = \frac{1}{\sqrt{K}} \sin(\sqrt{K}s) \tag{4}$$

for $K < 0$

$$C(s) = \cosh(\sqrt{|K|}s) \qquad \text{and} \qquad S(s) = \frac{1}{\sqrt{|K|}} \sinh(\sqrt{|K|}s) \tag{5}$$

These can be used in matrix formalism to propagate a beam with the initial conditions $x_0$ and $x'_0$ (which is the beam angle $\frac{dx}{ds}$) past an individual element[3]:

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \tag{6}$$

These matrices are known as *transport matrices*, and are a powerful tool in linear beam dynamics.

## 2.1. Drift Space

A drift space is simply an area inside the vacuum chamber which does not feel the influence of any magnetic field. Thus, the transport matrix for the case of drift space is:

$$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \tag{7}$$

$L$ being the length of the space.

## 2.2. Corrector Dipoles

A corrector is a dipole magnet which under hard edge approximation has a constant field throughout its entire length. A charged particle entering this element with a speed perpendicular to the field will be bent in a circular arc. Equilibrium between the Lorentz force and the centrifugal force produces the following effect on the beam:

$$\begin{aligned} x &= x_0 + x'_0 L + \frac{L^2}{2\rho} \\ x' &= x'_0 + \frac{L}{\rho} \end{aligned} \tag{8}$$

$L$ is the length of the corrector, and $\rho$ is the bending radius as defined in (3).
An assumption has been made here that the bending angle is small.

## 2.3. Quadrupoles

Quadrupole magnets are composed of four magnetic iron poles. For such an element, there is no bending affect ($\frac{1}{\rho} \to 0$), and so $K$ from (1) becomes just $k$ as defined in (2). The field distribution inside the quadrupole causes charged particles to be focused on one plane and defocused on the other meaning that a positive current given to the quadrupole will result in a positive $k$ on one plane and a negative $k$ on the other. The trasport matrix then assumes the form:

$$\begin{pmatrix} \cos(\sqrt{k}L) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}L) \\ -\sqrt{k}\sin(\sqrt{k}L) & \cos(\sqrt{k}L) \end{pmatrix} \tag{9}$$

for the focusing plane, and

$$\begin{pmatrix} \cosh(\sqrt{k}L) & \frac{1}{\sqrt{k}}\sinh(\sqrt{k}L) \\ \sqrt{k}\sinh(\sqrt{k}L) & \cosh(\sqrt{k}L) \end{pmatrix} \tag{10}$$

for the defocusing plane. $L$ is the length of the quadrupole.

## 2.4. Undulators

Undulators are made up of a series of equal dipole magnets with alternating magnetic field direction. For this reason, undulators cause a wiggling motion of the beam on the horizontal plane. Since the poles inside the undulator are relatively short, the fringe effects mentioned before have to be taken into account. A full analisys of these effects [4] shows that the undulator introduces a total focusing factor of

$$k = \frac{1}{2\rho_0^2} \tag{11}$$

in the vertical plane. $\rho_0$ is the bending radius in center point of the undulator pole in which the field reaches its maximum value.
In the horizontal plane, the undulator has no net effect on the beam and is therefore treated as a drift space.

# 3. Beam Correction

In some cases the photon beam emerging from the accelerator deviates from the desired location in the transversal plane. In order to compensate for that, corrections must be made to the trajectory in the undulator section. The required beam path is one in which light emitted tangently will reach the "screen" (a reference point just before the experimental hall) with an offset which is exactly equal to the given deviation, in the opposite direction. In the horizontal plane this is easily achieved by using two correctors (selected by the operator). In the vertical plane, the focusing effect introduced by the undulators makes the situation more complicated and in fact it is impossible to achieve exactly the desired path.

## 3.1. Correction in the Horizontal plane

A desired path can be achieved in two ways (discussed in the following subsections). Once a path has been chosen, it can be reached by using two steerers in order to bring the beam to a desired state $\begin{pmatrix} x \\ x' \end{pmatrix}$ in the begining of the undulator section. The quadrupoles in this region are movable, and can be set in such a way that the beam will pass through the center of each quadrupole and will not be affected by it. Once the beam has exited the undulator section it must be corrected back (using the exact same principle with two steerers) so that it is properly dumped and not lost in the vacuum chamber.

### 3.1.1. Introducing an offset

In order to get a photon beam that is $\Delta x$ to the right with respect to the intial situation, simply introducing an offset of $\Delta x$ to the whole trajectory before the undulator section will suffice. As shown in the figure below, this will require a shift of all of the quadrupoles in the undulator section of $\Delta x$ to the right, so that the beam will pass through their center.

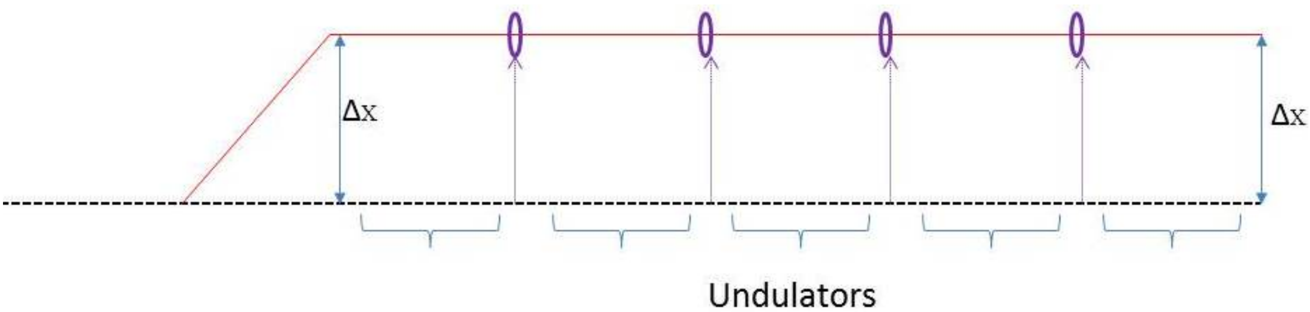



Figure 4: Intoducing an offset: The dashed black line represents the original trajectory and the red line - the corrected trajectory. Quadrupoles are represented by purple ellipses.

In figure 4 one can observe that the beam was manipulated in only two points. The beam is brought to the begining of the undulator section in a state $\begin{pmatrix}\Delta x\\0\end{pmatrix}$

#### 3.1.2. Introducing an angle

A different way to achieve the same affect, is introducing only an angle to the beam in the begining of the undulator section so that photons emmitted will reach the screen with a deviation of $\Delta x$. In this case, every quadrupole is shifted differently, in order for the beam to pass through its center.

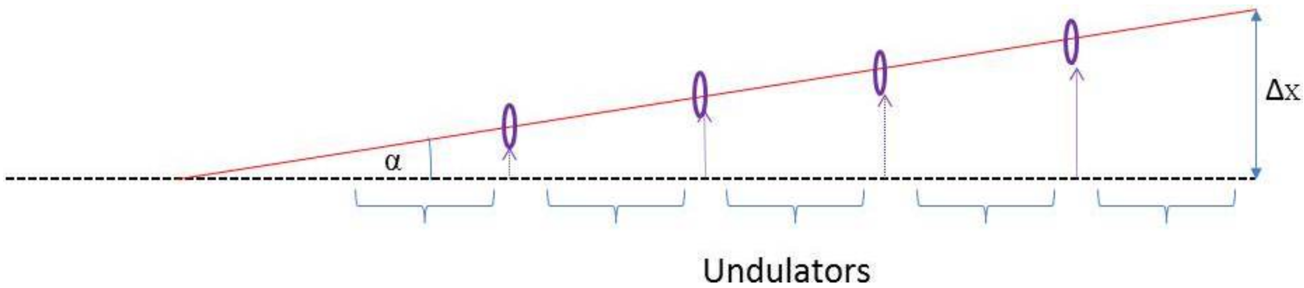



Figure 5: Introducing an angle: each quadrupole is shifted appropriately. The required angle is $\alpha \approx \tan\alpha = \frac{\Delta x}{\ell}$

In this case, the beam is brought to the begining of the undulator section in a state $\begin{pmatrix}0\\\alpha\end{pmatrix}$

#### 3.1.3. Combining the offset and angle

Each of the two solutions presented above can be used and since the system is linear, so can any linear combination of the two. In the program a fit was carried out to find the linear combination that minimizes current demand from the correctors or the square sum of all quadrupole shifts (at a user's choise).

### 3.2. Correction in the Vertical plane

In the vertical plane, applying the above solutions ("the gross correction") is not enough. Even if the beam is brought to the desired state in the begining of the undulator section and all the quadrupoles have been moved to the locations required by the previous solutions, it will not follow the ideal desired trajectory (which is a straight line) due to the focusing affect in this plane (figure 6).

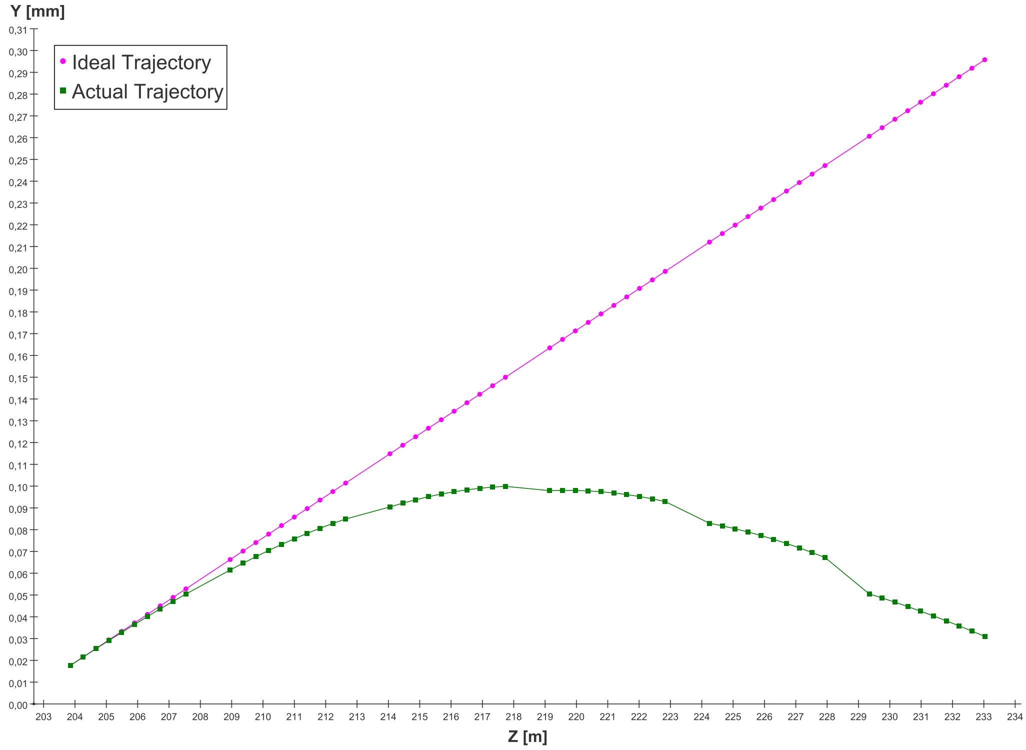



Figure 6: Ideal trajectory vs actual trajectory in undulator section. As can be seen, the beam is constantly being focused away from the desired path.

The only means that can be used to overcome this are applying small shifts in the quadrupoles which are located before the entrance of each undulator. Since correction is not possible inside the undulator, an ideal straight line will never be exactly achieved. The next best thing is to achieve such a trajectory, that the RMS (root-mean-square) of the differences from the ideal path is at a minimum. This RMS is measured in 10 equally spaced points inside the undulator. Two approaches were used to find the exact required shifts.

### 3.2.1. Analytical approach

Consider the following beam trajectory inside the undulator (derived from (9)):

$$y(\ell) = y_0 \cos(\sqrt{k}\ell) + y_0' \frac{\sin(\sqrt{k}\ell)}{\sqrt{k}} \tag{12}$$

in which $y_0$ and $y_0'$ are the position and angle at the entrance of the undulator, and $\ell$ is the longitudinal location inside undulator (with $\ell = 0$ as the undulator entry point). Whereas the *ideal* linear beam trajectory is:

$$\hat{y}(\ell) = a\ell + b \tag{13}$$

Where $a$ and $b$ depend on the gross correction chosen.
Our desire is to minimze:

$$RMS(y_0, y_0') = \sqrt{\frac{1}{L}\int_0^L (y - \hat{y})^2 \,\mathrm{d}\ell} \tag{14}$$

as a function of $y_0$ and $y_0'$. The result is the state $\begin{pmatrix} Y_0 \\ Y_0' \end{pmatrix}$ which will give the lowest RMS trajectory inside the undulator (for the detailed solution see appendix A). If two quadrupoles are switched on before each undulator this state can be reached with the same methods applied for the gross correction.

### 3.2.2. Numerical approach

There are three general cases in which the analytical solution may not be valid:

1. Not all of the moveable quadrupoles are switched on.
2. The analitycal solution requires shifts that are beyond system constraints.
3. The condition to minimize (14) is not fullfilled (since this dependes only on the dimentions of the undulator and the beam energy - it is unlikely that this situation will occur so long as FLASH is not altered).

In these cases, a fit must be applied in order to find the lowest RMS possible. The fit parameters (degrees of freedom) are the shifts of the 14 moveable quadrupoles and the two corrector currents.

### 3.2.3. Combining the offset and angle

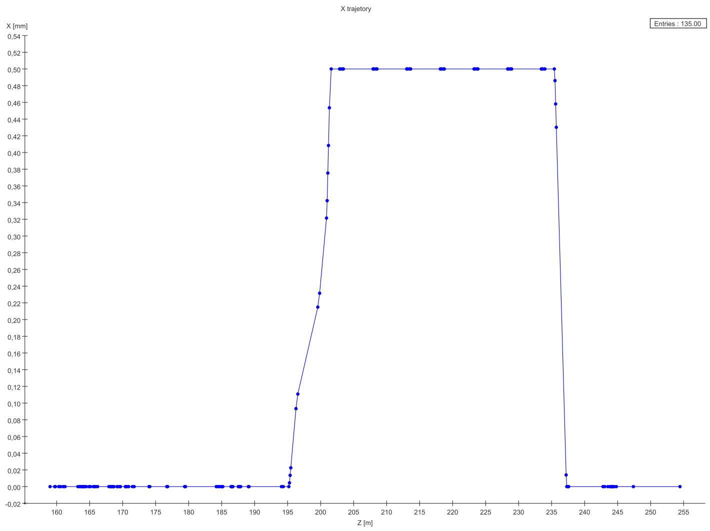
After applying one of the two methods above (analitycal solution or fit) to minimize the RMS for each undulator separately, a linear combination between the offset and the angle solutions mentioned in 3.1 must be chosen. Unlike the horizontal plane, here a fit minimizes either the RMS again or the total sum of squares of the quadrupole shifts (at a user's choise).

# 4. Results

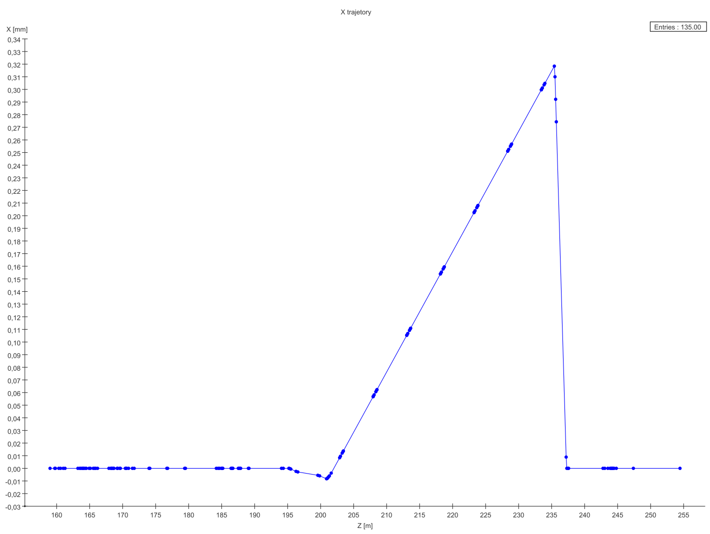
A trajectory simulation program was written in order to implement the solutions.

## 4.1. Horizontal Correction

Both solutions mentioned in 3.1 were carried out. The plots in figures 7a and 7b show the beam trajectory for both cases, with a given deviation of $0.5mm$.




(a) Offset solution




(b) Angle solution

Figure 7: Horizontal correction - the two possible solutions for a deviation of $0.5mm$. The undulators are located at $Z$ coordinates 202-235[m].

The path achieved fits the requirements inside the undulator section. Before the undulators the path is not a straight line due to the presence of active non-moveable quadrupoles along the way. Nevertheless, inside the undulators the beam follows the ideal trajectory perfectly. Note also that after the undulator section the beam is corrected back and reaches the dump in a state $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ as it began.

When combining the two solutions under the requirement to minimize current consumptions the angle solution governs. This is due to the fact that it requires smaller bending angles from the two steerers and thus reduces current demand.

## 4.2. Vertical Correction

As mentioned the gross correction is not sufficient to achieve the desired path in the vertical plane, as can be seen in figure 8.

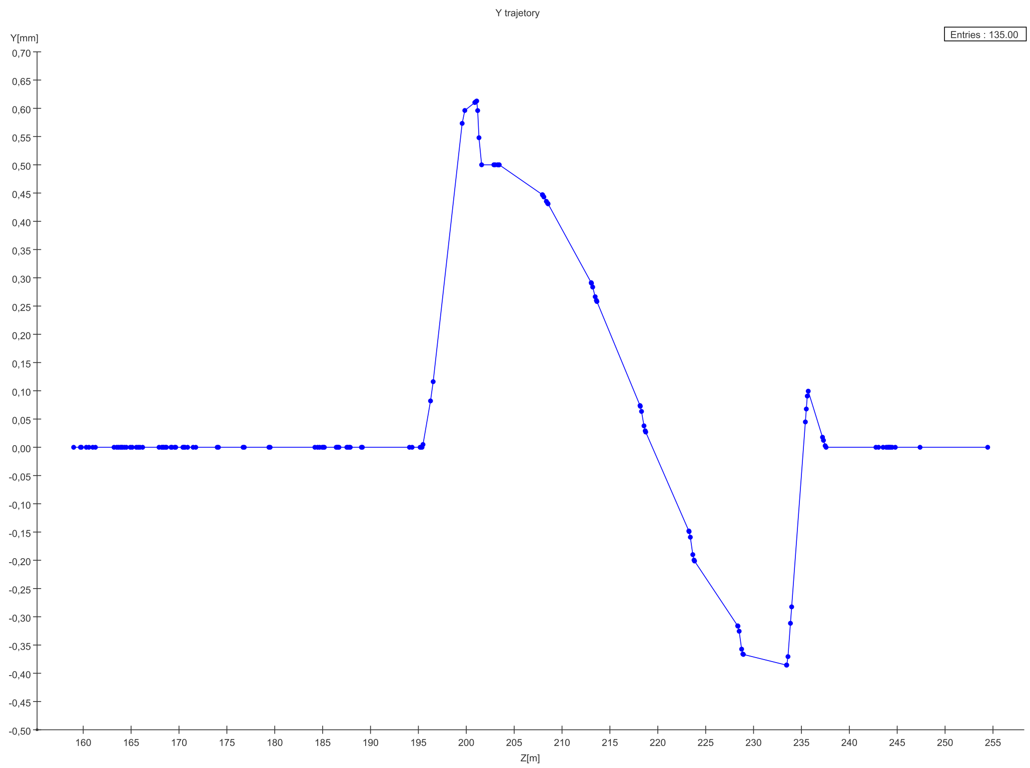
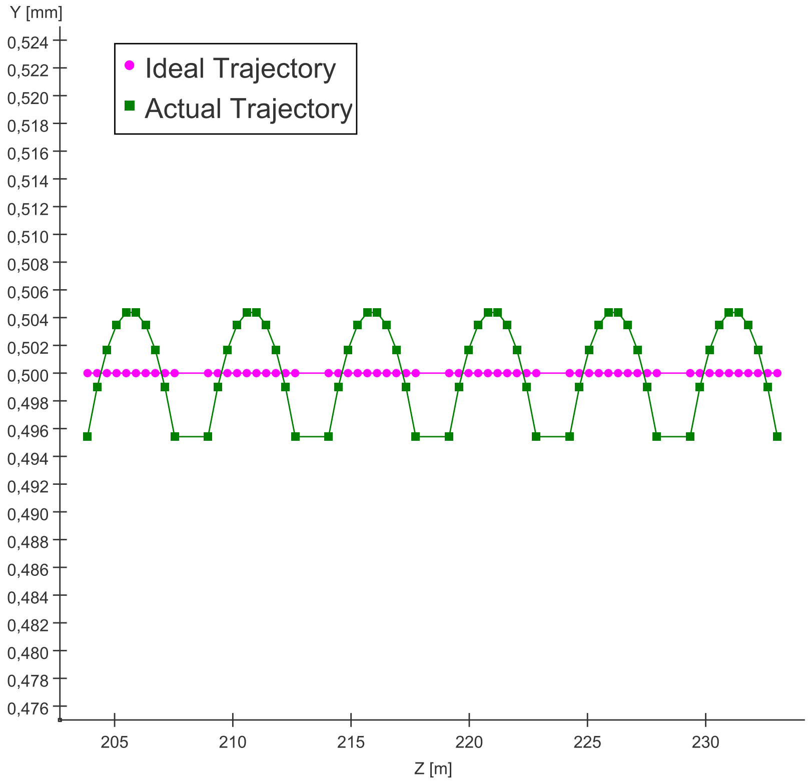
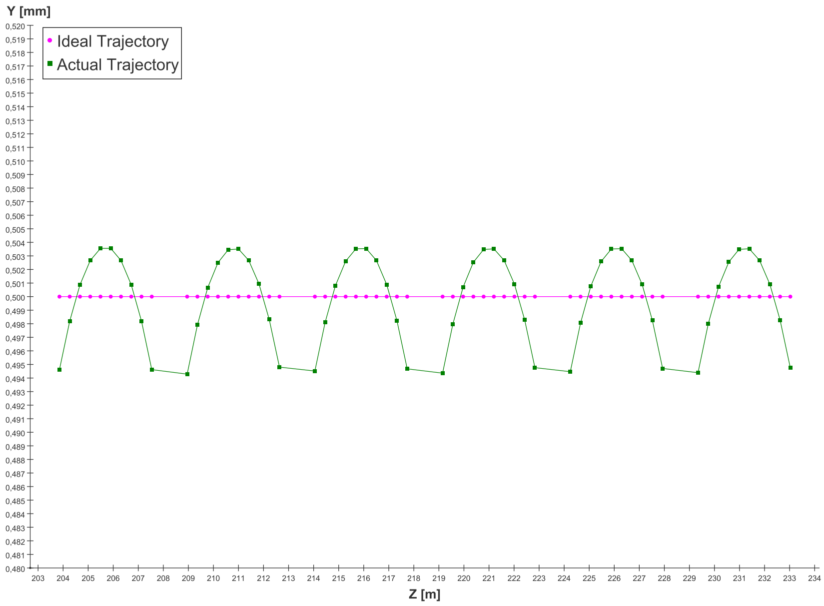



Figure 8: Beam trajectory after applying only the gross correction (offset solution) in the vertical plane for a deviation of $0.5mm$

Both approaches mentioned in 3.2 were implemented. Naturally, the analytical solution produced a (slightly) better RMS and a more symmetric trajectory with respect to the ideal path.

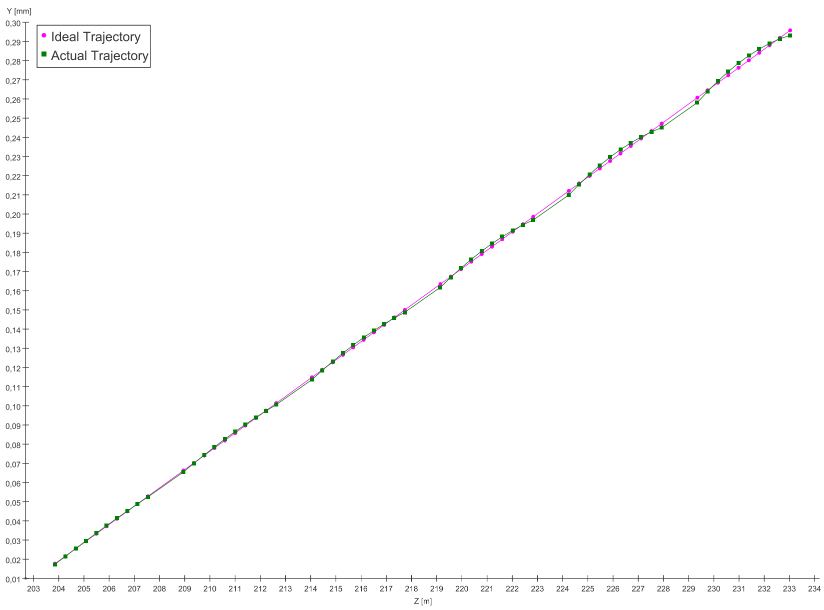



(a) Trajectory in the undulator section - analytical approach

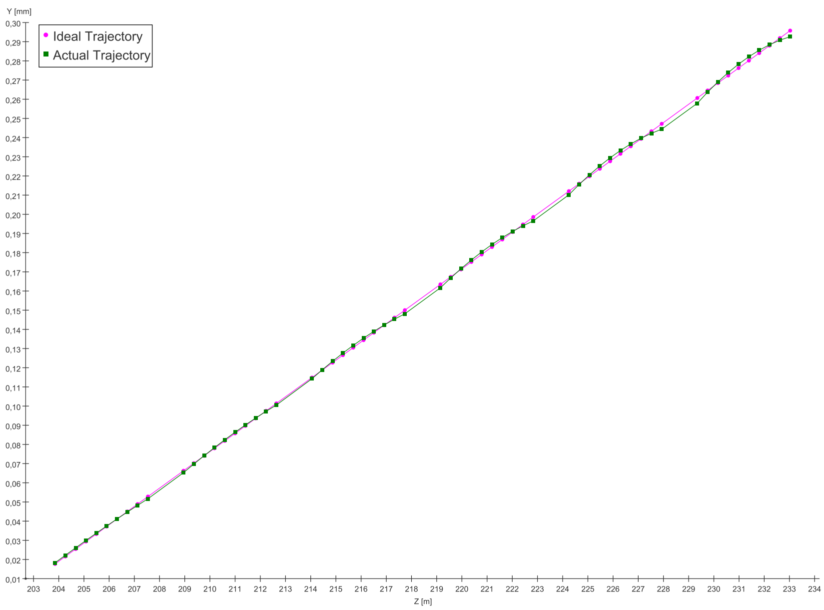



(b) Trajectory in the undulator section - fit approach

Figure 9: Vertical correction - the two possible approaches for a deviation of $0.5mm$ (offset solution)




(a) Trajectory in the undulator section - analytical approach




(b) Trajectory in the undulator section - fit approach

Figure 10: Vertical correction - the two possible approaches for a deviation of $0.5mm$ (angle solution)

For a sample deviation of $0.5mm$ in the offset solution RMS about $3.3\mu m$, while in the angle solution RMS about $1.1\mu m$. For this reason it is easier to distinguish between the two approaches in the offset solution.

Combination the two solutions in the vertical plane is under the requirement to minimize once again the RMS or alternatively the total sum of squares of the quadrupole shifts. In both cases, unlike in the horizontal plane, the outcome received resembles the one shown in figure 11.

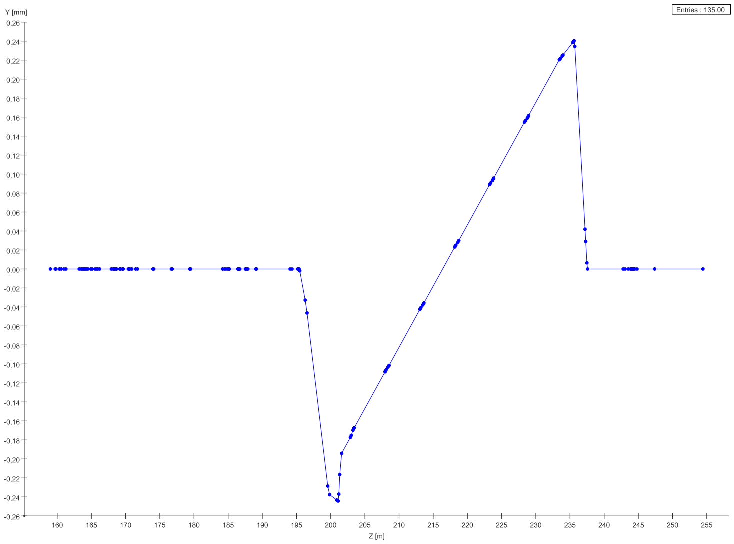



Figure 11: The selected angle+offset linear combination that minimizes the RMS - $0.5mm$ deviation

This solution represents a combination of negative offset and an angle which is larger than the one from the angle only solution. This can be attributed to the fact that the closer the beam is to the center of the undulator, the weaker the focusing affect becomes. This solution keeps the beam as close to the center as possible while achieving the desired final correction of $0.5mm$. The resulting RMS in this example is about $0.7\mu m$.

# 5. Conclusion

It has been shown that beam correction can be achieved perfectly in the horizontal plane and to a very high degree of accuracy in the vertical plane, with a total RMS of differences between the ideal path and the actual trajectory in the order of a few microns (for an initial deviation which is in the order of $0.5mm$). This number is small compared to the transversal size of the beam itself (which is in the order of $50\mu m$).
In the horizontal plane the solution is purely analytical and is optimized by minimizing the corrector current demands, by selecting the best linear combination of the two basic solutions: introducing an offset to the beam and introducing an angle to the beam.
In the vertical plane an analytical solution exists and for the cases in which it is not valid a numerical one is also available. Both minimize the RMS of the achieved trajectory. Optimization of the angle and offset combination reduces furthermore the RMS, or alternatively the shifts required from the moveable quadrupoles.
An additional feature that has not been included in the program yet is the consideration of constraints, such as maximum steerer currents or quadrupole displacements. The constraints will then be used to rule out an analytical solution or set boundaries for parameters in a fit solution.

# References

[1] Peter Schmser, Martin Dohlus, Jrg Rossbach, *Ulatraviolet and Soft X-Ray Free-Electron Lasers* (Springer, 2008)

[2] http://flash.desy.de/

[3] Helmut Wiedemann, *Particle Accelelrator Physics*, 1st edition. (Springer-Verlag, Berlin Heidlelberg 1993)

[4] Helmut Wiedemann, *Particle Accelelrator Physics II*, 2nd edition. (Springer-Verlag, Berlin Heidlelberg 1999)

# A. The analytical solution for RMS minimization

Minimization of:

$$\sqrt{\frac{1}{L}\int_0^L (y-\hat{y})^2\,\mathrm{d}\ell} = \sqrt{\frac{1}{L}\int_0^L (y_0\cos(\sqrt{k}\ell) + y_0'\frac{\sin(\sqrt{k}\ell)}{\sqrt{k}} - a\ell - b)^2\,\mathrm{d}\ell} \tag{15}$$

as a function of $(y_0, y_0')$ is equivalent to the minimization of:

$$\int_0^L (y_0\cos(\sqrt{k}\ell) + y_0'\frac{\sin(\sqrt{k}\ell)}{\sqrt{k}} - a\ell - b)^2\,\mathrm{d}\ell \tag{16}$$

Since $(y_0, y_0')$ are independent of $\ell$ this is simply a second degree bivariate polynomial function. For convenience it will be denoted as following:

$$f(y_0, y_0') = Ay_0^2 + By_0'^2 + C + 2Dy_0y_0' + 2Ey_0 + 2Fy_0' \tag{17}$$

where $A$-$F$ are constant coeffcients.
Finding an extremum point for such a function is done by comparing each of the first derivatives to zero:

$$\frac{\partial f}{\partial y_0} = 2Ay_0 + 2Dy_0' + 2E = 0 \tag{18}$$

$$\frac{\partial f}{\partial y_0'} = 2By_0' + 2Dy_0 + 2F = 0 \tag{19}$$

Solving this linear system of equation produces the desired result:

$$y_0 = \frac{FD - BE}{AB - D^2} \qquad \text{and} \qquad y_0' = \frac{DE - AF}{AB - D^2} \tag{20}$$

In order for this solution to be a minimum, this function has to have a Hessian matrix with eigenvalues that are all positive. For this case, the matrix is:

$$\begin{pmatrix} \frac{\partial^2 f}{\partial^2 y_0} & \frac{\partial^2 f}{\partial y_0 \partial y_0'} \\ \frac{\partial^2 f}{\partial y_0' \partial y_0'} & \frac{\partial^2 f}{\partial^2 y_0'} \end{pmatrix} = \begin{pmatrix} 2A & 2D \\ 2D & 2B \end{pmatrix} \tag{21}$$

And so the condition for a minimum becomes:

$$\lambda_{1,2} = A + B \pm \sqrt{A^2 + B^2 - 2AB - 4D^2} > 0 \tag{22}$$

Performing the integration gives us the coeffcients $A$-$F$ in terms of the problem's constants:

$$\begin{aligned}
A &= \int_0^L \cos^2(\sqrt{k}\ell)\,\mathrm{d}\ell = \frac{L}{2} + \frac{1}{4\sqrt{k}}\sin(2\sqrt{k}L) \\
B &= \int_0^L \frac{1}{k}\sin^2(\sqrt{k}\ell)\,\mathrm{d}\ell = \frac{L}{2k} - \frac{1}{4\sqrt{k^3}}\sin(2\sqrt{k}L) \\
C &= \int_0^L (-a\ell - b)^2\,\mathrm{d}\ell = \frac{a^2}{3}L^2 + b^2 L \\
D &= \int_0^L \frac{1}{\sqrt{k}}\cos(\sqrt{k}\ell)\sin(\sqrt{k}\ell)\,\mathrm{d}\ell = \frac{1}{4k}(1 - \cos(2\sqrt{k}L)) \\
E &= \int_0^L \cos(\sqrt{k}\ell)(-a\ell - b)\,\mathrm{d}\ell = \frac{a}{k}(1 - \cos(\sqrt{k}\ell)) - \sin(\sqrt{k}\ell)\frac{aL + b}{\sqrt{k}} \\
F &= \int_0^L \frac{1}{\sqrt{k}}\sin(\sqrt{k}\ell)(-a\ell - b)\,\mathrm{d}\ell = \cos(\sqrt{k}\ell)\frac{aL + b}{k} - \frac{a}{\sqrt{k^3}} - \frac{b}{k}
\end{aligned} \tag{23}$$