Radiative corrections for electron-proton and positron-proton scattering

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1. Introduction.

Recent determinations of the proton electric to magnetic form factor ratio from polarization transfer measurements [1] indicate an unexpected and dramatic discrepancy with the form factor ratio obtained using the Rosenbluth separation technique in unpolarized cross section measurements[2]. This discrepancy has been explained as the effects of multiple photon exchange beyond the usual one-photon exchange approximation in the calculation of the elastic electron-proton scattering cross section. Since most of our understanding on the structure of the proton and atomic nuclei is based upon lepton scattering analyzed in terms of the single photon approximation; it is essential to definitively verify the contribution of multiple photon exchange.

In June 2007, OLYMPUS collaboration submitted to DESY a letter of intent to carry out an experiment to definitively determine the contribution of multiple photon exchange in elastic lepton-nucleon scattering. The most direct evidence for multiple photon exchange would be a deviation from unity in the ratio of positron-proton to electron-proton elastic scattering cross sections. The experiment would utilize intense beams of electrons and positrons in the DORIS ring incident on an internal hydrogen gas target at an incident energy of 2.01 GeV and precisely measure elastic scattering at polar angles between 20 and 80 with high statistical and systematic precision. For this experiment we proposed to use the existing Bates Large Acceptance Spectrometer Toroid (BLAST) from MIT and an unpolarized internal gas target similar to one used by the HERMES experiment at HERA.

The OLYMPUS (pOsitron-proton and eLectron-proton elastic scattering to test the hYpothesis ofMulti-Photon exchange Using doriS) collaboration comprises over fifty physicists from fifteen institutions in Germany, Italy, Russia, the United Kingdom, and the United States.

The experiment takes advantage of unique features of the BLAST detector combined with an internal hydrogen gas target and the DORIS storage ring operated with both electrons and positrons. The systematic uncertainties are controllable at the percent level, and with the superior luminosity that can be provided at DORIS, this experiment will not be limited in statistical precision [3].

The theme of this work is «Radiative corrections for electron-proton and positron-proton scattering». It is need to calculate radiative correction to the positron-proton and electron-proton elastic scattering cross sections neglecting excitation in proton intermediate state to get good accuracy of the experiment. It is important because this correction is different for electrons and positrons and could be a source of the systematic uncertainties.

2. Elastic scattering

2.1. Rosenbluth cross section

When an electron scatters elastically from a proton it exchanges a virtual photon with the proton as shown in Fig. 1. In electron scattering experiments the coupling constant ($\alpha \approx \frac{1}{137}$) is small so one can work only at the leading order of perturbation theory.

$$e(k) + P(p) \rightarrow e(k') + P(p') \quad (2.1)$$

where $k = (E, \vec{k})$ and $k' = (E', \vec{k}')$ are the four momenta of the initial and final electrons.

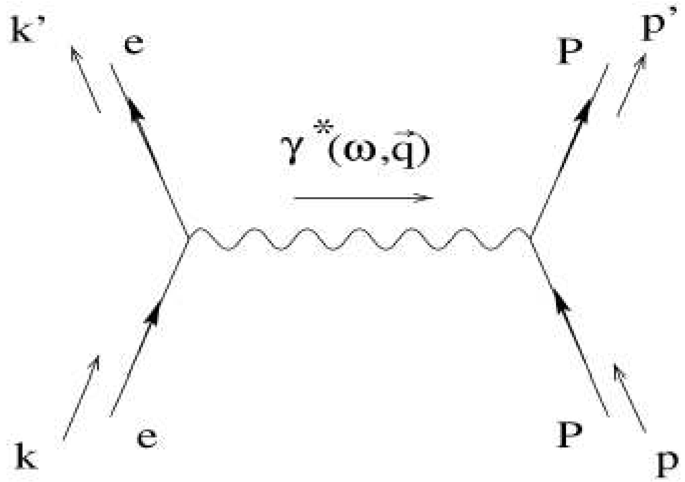


FIG. 1: Feynman diagram of elastic scattering of an electron off a proton in one photon exchange approximation (Born Approximation).

The four momentum transfer q carried by the virtual photon is constrained by momentum conservation $q = (k - k')$. The square of the four momentum transfer is a Lorentz invariant that can be expressed in terms of the incident energy E, final energy E' and the electron scattering angle θ as

$$Q^2 = -q^2 = -(\omega^2 - \vec{q}^{\,2}) = -(k - k')^2 = 4EE' \sin^2 \frac{\theta}{2} \quad (2.2)$$

where $\omega = \frac{Q^2}{2Mp}$ is the energy transferred by the virtual photon from the electron to the proton and $\vec{q}$ is the spatial component of the four momentum transfer. The mass of the electron is neglected because $E >> m_e$. Since a large Q^2 is associated with a very short wavelength, the virtual photon, $\gamma^*(\omega, \vec{q})$, can probe the internal structure of the proton.

The leptonic vertex, $e(k) \rightarrow e(k') + \gamma^*(\omega, \vec{q})$, where an electron emits a virtual photon, is fully described by QED (Quantum ElectroDynamics) and is well understood. However the hadronic vertex $\gamma^*(\omega,\vec{q})+P(p)\rightarrow P(p')$, where the virtual photon is absorbed by the proton, is not easy to calculate due to the structure of the proton.

To calculate the cross section of the reaction we need to calculate the amplitude of elastic scattering that depends on the leptonic and hadronic vertices. If the proton were point like, the cross section could be calculated within the framework of QED which gives

$$\sigma_{Mott}=\frac{d\sigma_{Mott}}{d\Omega}=\frac{E'}{E}\frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}} \quad (2.3)$$

where α is the fine structure constant. But the proton is not a point like particle. The spatial extent of the electromagnetic charge and current densities of the proton introduces the form factors in the cross section measurement. In this case we can express the cross section as

$$\frac{d\sigma}{d\Omega}=\sigma_{Mott}\left\{F_1(Q^2)+\frac{k_p^2Q^2}{4M_p^2}F_2(Q^2)+\frac{Q^2}{4M_p^2}\left[F_1(Q^2)+k_pF_2(Q^2)\right]\tan^2\left(\frac{\theta}{2}\right)\right\} \quad (2.4)$$

where kp=1.79 is the proton's anomalous magnetic moment. $F_1(Q^2)$ and $F_2(Q^2)$ are the Dirac and Pauli form factors respectively. These form factors depend only on Q^2 and contain the information about the internal structure of the proton.

We can simplify the above expression using the Sachs form factors $G_E(Q^2)$ and $G_M(Q^2)$. Sachs form factors can be expressed as a linear combination of the Dirac and Pauli form factors as

$$G_E(Q^2)=F_1(Q^2)-k_p\frac{Q^2}{4M_p^2}F_2(Q^2) \quad (2.5)$$

$$G_M(Q^2)=F_1(Q^2)+k_pF_2(Q^2) \quad (2.6)$$

At $Q^2=0$, $G_E(0)=1$ and $G_M(0)=\mu_p=1+k_p$ where μ_p is the proton magnetic moment. So we can rewrite the expression for the cross section as

$$\frac{d\sigma}{d\Omega}=\sigma_{Mott}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}+2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\right) \quad (2.7)$$

where $\tau=\frac{Q^2}{4M_p^2}$. This expression for the cross section is known as the Rosenbluth formula in the one photon exchange approximation (Born Approximation). Both $G_E(Q^2)$ and $G_M(Q^2)$ depend only on Q^2 [4].

2.2 Elastic cross section.

The Feynman diagrams contributing to the elastic scattering cross section to order α^3 are shown in Fig. 2.[5]

$$d\sigma_{elastic} = (2\pi)^2 \frac{E_1 E_2}{\left[(p_1 p_2)^2 - m^2 M^2\right]^{\frac{1}{2}}} \frac{1}{4} \int \delta(p_3 + p_4 - p_1 - p_2) d^3 p_3 d^3 p_4$$
$$\times \sum_{spin} \left[M_1^\dagger M_1 + \sum_{i=2}^{6} 2\,\mathrm{Re}(M_1^\dagger M_i) \right] \quad (2.8)$$

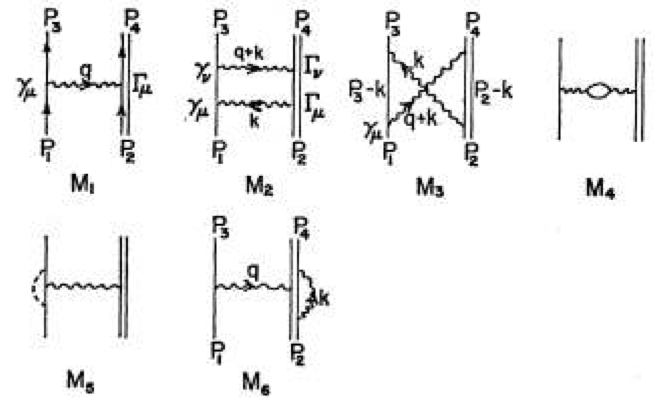


FIG. 2. Feynman diagrams for elastic scattering.

The expression for the elastic scattering cross section can be written as

where E_1 is the energy of input beam, m – mass of the electron, M – mass of the proton.

The first term in the square bracket of Eq.(2.8) represents the Rosenbluth cross section. The Rosenbluth cross section is given by Eq (2.7).

Neglecting the noninfrared terms in matrix elements we have

$$M_2 = -\frac{\alpha Z}{2\pi} M_1 \left[K(p_2, -p_1) + K(p_4, -p_3) \right] (2.9)$$

$$M_3 = \frac{\alpha Z}{2\pi} M_1 \left[K(p_2, -p_1) + K(p_4, -p_3) \right] (2.10)$$

$$M_6 = -\frac{\alpha Z^2}{2\pi} M_1 \left[K(p_2, p_4) - K(p_2, p_2) \right] (2.11)$$

For vacuum polarization (M_4) and electron vertex (M_5) diagrams, we have

$$M_4 = \frac{\alpha}{\pi} \left[-\frac{5}{9} + \frac{1}{3} \ln\left(\frac{Q^2}{m^2} \right) \right] M_1 (2.12)$$

$$M_5 = -\frac{\alpha}{2\pi} \left[K(p_1, p_3) - K(p_1, p_1) - \frac{3}{2} \ln\left(\frac{Q^2}{m^2} \right) + 2 \right] M_1 (2.13)$$

where $K(p_i,p_j)=(p_i,p_j)\int_0^1 \frac{dy}{p_y^2}\ln\frac{p_y^2}{\lambda^2}$, $p_y=p_i y+p_j(1-y)$.

Substituting expressions for matrix elements in Eq. (2.8), we obtain the elastic scattering cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic}=\left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth}\{1+\frac{\alpha}{\pi}[-K(p_1,p_3)+K(p_1,p_1)-ZK(p_2,p_1)$$
$$-ZK(p_4,p_3)+ZK(p_2,p_3)+ZK(p_4,p_1)-Z^2K(p_2,p_4) \quad (2.14)$$
$$+Z^2K(p_2,p_2)]+\frac{\alpha}{\pi}\left[-\frac{28}{9}+\frac{13}{6}\ln\frac{Q^2}{m^2}\right]\}$$

3. Bremsstrahlung

The Feynman diagrams for the matrix elements contributing to the inelastic cross section to order α^3 are shown in Fig. 3.[5]

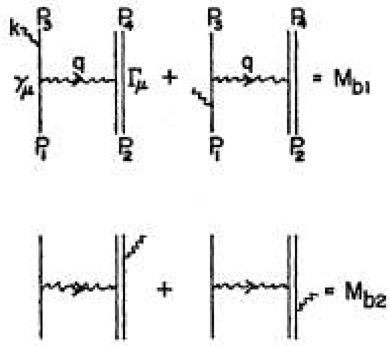


Fig. 3. Feynman diagrams for inelastic scattering.

Since we are interested only in the soft foton emission, the vertex function connecting the real photon k and the proton current may be approximated by γ_ν. Thus the matrix elements M_{b1} and M_{b2} may be written as

$$M_{b1}=\frac{e^3}{(2\pi)^{7/2}}\frac{mMZ}{(2\omega E_1E_2E_3E_4)^{1/2}}\bar{u}(p_3)[e\frac{p_3+k+m}{2p_3k}\gamma_\mu-\gamma_\mu\frac{p_1-k+m}{2p_1k}e]$$
$$\times u(p_1)\bar{u}(p_4)\Gamma_\mu u(p_2)\frac{1}{(p_1-p_3-k)^2} \quad (3.1)$$

$$M_{b2}=-\frac{e^3}{(2\pi)^{7/2}}\frac{mMZ^2}{(2\omega E_1E_2E_3E_4)^{1/2}}\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)$$
$$\times\left[e\frac{p_4+k+M}{2p_4k}\Gamma_\mu-\Gamma_\mu\frac{p_2-k+M}{2p_2k}e\right]\frac{u(P_2)}{(p_1-p_3)^2} \quad (3.2)$$

with the approximation $\Delta E(1+2\frac{E_1}{M}) << E_3$ (where ΔE is the maximum energy loss of the electron or the maximum energy of a photon which can be emitted, E_3 is the energy of scattering electron) we may write

$$M_{b1}+M_{b2} \approx \frac{i}{\pi^2}\left(\frac{\alpha}{2}\right)^{1/2} M_1 \frac{1}{(2\omega)^{1/2}}\left[\frac{p_3 e}{p_3 k}-\frac{p_1 e}{p_1 k}+\frac{Z p_4 e}{p_4 k}+\frac{Z p_2 e}{p_2 k}\right]$$

The inelastic cross section can be calculated by using the formula

$$\begin{aligned} d\sigma_b = (2\pi)^2 \frac{E_1 E_2}{\left[(p_1 p_2)^2 - m^2 M^2\right]^{1/2}} \frac{1}{4} \int d^3 p_3 d^3 p_4 d^3 k \delta(p_3+p_4+k-p_1-p_2) \\ \times \sum_{spin} (M_{b1}^{\dagger}+M_{b2}^{\dagger})(M_{b1}+M_{b2}) \end{aligned} \quad (3.3)$$

This expression can be transformed to

$$\left(\frac{d\sigma}{d\Omega}\right)_b = -\left(\frac{d\sigma}{d\Omega}\right)_{Rosenluth} \frac{\alpha}{8\pi^2} \int_{2\lambda M}^{2M\eta\Delta E} \frac{x dx}{2(x+m^2)} \int d\tilde{\Omega}_k \chi^2 \quad (3.4)$$

where $\chi^2 = \left[\frac{p_3}{p_3 k}-\frac{p_1}{p_1 k}-\frac{Z p_4}{p_4 k}+\frac{Z p_2}{p_2 k}\right]^2$

Finally inelastic cross section can be written as

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{elastic} + \left(\frac{d\sigma}{d\Omega}\right)_b \equiv \left(\frac{d\sigma}{d\Omega}\right)_{Rosenluth} (1+\delta) \quad (3.5)$$

where

$$\delta = -\frac{\alpha}{\pi}\{\frac{28}{9} - \frac{13}{6}\ln\left(\frac{Q^2}{m^2}\right) + \left(\ln\left(\frac{Q^2}{m^2}\right) - 1 + 2Z\ln\eta\right)\left(2\ln\frac{E_1}{\Delta E} - 3\ln\eta\right) - \Phi\left(\frac{E_3 - E_1}{E_3}\right) - Z^2\ln\frac{E_4}{M}$$

$$+ Z^2\ln\frac{M}{\eta\Delta E}\left(\frac{1}{\beta_4}\ln\frac{1+\beta_4}{1-\beta_4} - 2\right) + \frac{Z^2}{\beta_4}\left[\frac{1}{2}\ln\frac{1+\beta_4}{1-\beta_4}\ln\frac{E_4+M}{2M} - \Phi\left(-\left(\frac{E_4-M}{E_4+M}\right)^{\frac{1}{2}}\left(\frac{1+\beta_4}{1-\beta_4}\right)^{\frac{1}{2}}\right)\right]$$

$$+ Z\left[\Phi\left(-\frac{M-E_3}{E_1}\right) - \Phi\left(\frac{M(M-E_3)}{2E_3E_4 - ME_1}\right) + \Phi\left(\frac{2E_3(M-E_3)}{2E_3E_4 - ME_1}\right) + \ln\left|\frac{2E_3E_4 - ME_1}{E_1(M-2E_3)}\right|\ln\left(\frac{M}{2E_3}\right)\right] \quad (3.6)$$

$$- Z\left[\Phi\left(-\frac{E_4-E_3}{E_3}\right) - \Phi\left(\frac{M(E_4-E_3)}{2E_1E_4 - ME_3}\right) + \Phi\left(\frac{2E_1(E_4-E_3)}{2E_1E_4 - ME_3}\right) + \ln\left|\frac{2E_1E_4 - ME_3}{E_3(M-2E_1)}\right|\ln\left(\frac{M}{2E_1}\right)\right]$$

$$- Z\left[\Phi\left(-\frac{M-E_1}{E_1}\right) - \Phi\left(\frac{M-E_1}{E_1}\right) + \Phi\left(\frac{2(M-E_1)}{M}\right) + \ln\left|\frac{M}{2E_1 - M}\right|\ln\left(\frac{M}{2E_1}\right)\right]$$

$$+ Z\left[\Phi\left(-\frac{M-E_3}{E_3}\right) - \Phi\left(\frac{M-E_3}{E_3}\right) + \Phi\left(\frac{2(M-E_3)}{M}\right) + \ln\left|\frac{M}{2E_3 - M}\right|\ln\left(\frac{M}{2E_3}\right)\right]\}$$

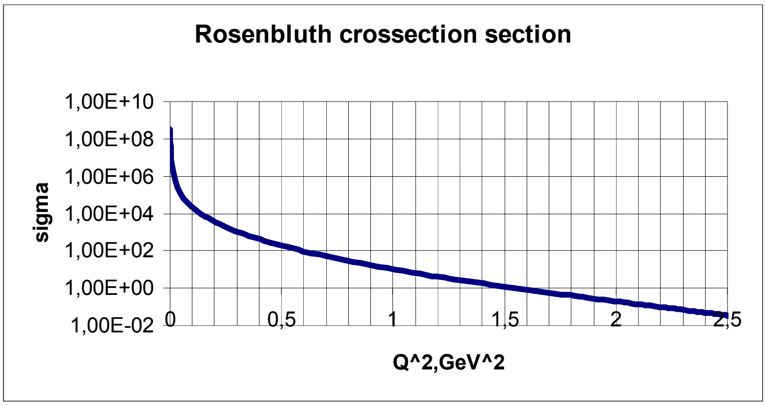
Where $\Phi(x)$ is the Spence function

$$\Phi(x) = \int_0^x \frac{-\ln|1-y|\,dy}{y}, \; E_4 = E_1 + M - E_3, \; \beta_4 = (E_4^2 - M^2)^{1/2} / E_4$$

We have kept Z in our formula for convenience of discussion. Z is equal to +1 for $e^- + p$ scattering and is equal -1 for $e^+ + p$ scattering.

4. Calculations.

The goal of this work was to calculate radiative correction for the case of one photon bremsstrahlung. First of all I calculate elastic cross section in Born approximation (Rosenbluth cross section) from kinematic variables. Here are plots of Rosenbluth cross section from the angle of scattering electrons (positrons) and from four momentum Q^2.



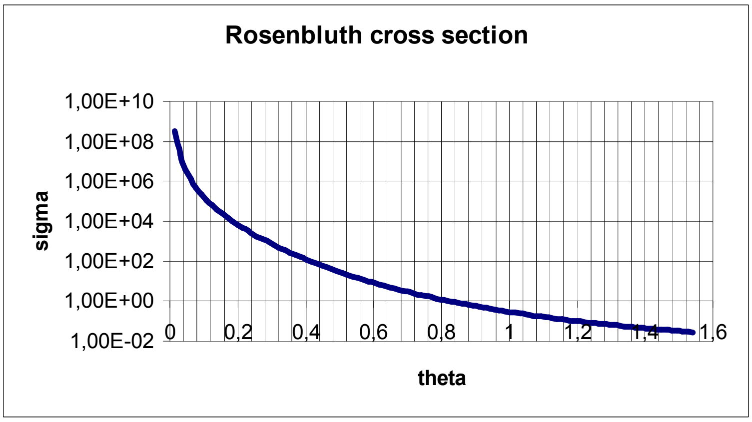


Fig 4. Rosenbluth cross section (in nb^-1) from kinematic variables.

To find cross section was used equation (2.7).

Then the calculations were made for radiative correction δ (Eq. (3.6)) which includes matrix elements (2.9)-(2.13), (3.1),(3.2). The Spence function in equation was calculated by using numerical integration (trapezoidal method with using an adaptive algorithm) with ROOT data analysis Framework [6]. The results of calculations are presented on Figs. 5,6

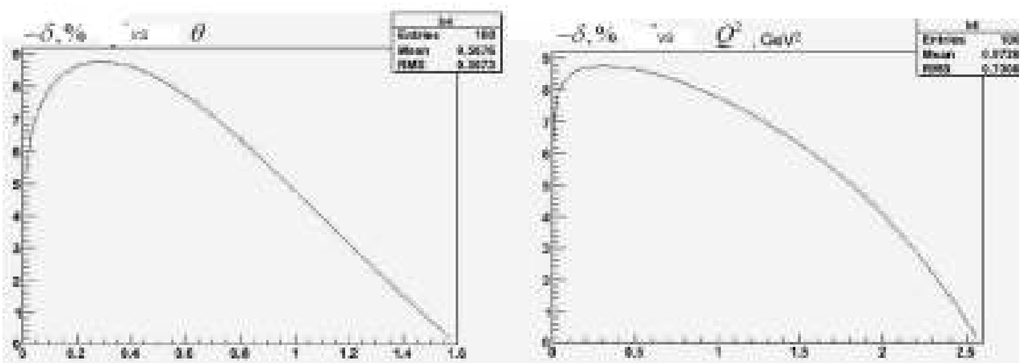


Fig 5. Radiative correction for electron-proton scattering.

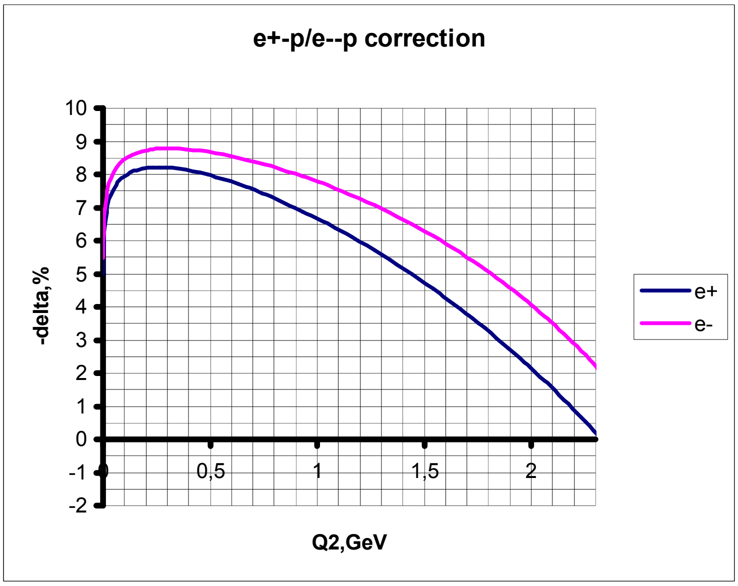


Fig. 6. Radiative corrections for electron-proton and positron-proton scattering.

5. Conclusion.

In this work, there were calculated the radiatiative corrections δ_+,δ_- for electron-proton and positron-proton scattering at the beam energy of 2.01 GeV, respectively. Following to [4], the calculations have been done under the assumption that the proton excitations in the intermediate state are neglected (amplitudes M2, M3 in Fig.2). The obtained results are to be used in the analysis of OLYMPUS experiment for a simple estimation of the two photon exchange effect on the measured e+p and e-p differential cross sections. It has been shown that for OLYMPUS kinematics the size of possible beam charge asymmetry due to the radiative correction is on the level of 1.5%. Further, it is assumed that the obtained result will be used in the OLYMPUS Monte Carlo where the detector acceptance will be taken into account.

6. References

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