



Report

Estimate of the dilepton invariant mass spectrum in $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ using data in $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ and heavy quark symmetry

Aleksey Rusov

Yaroslavl State University, Russia

Supervisor: **Ahmed Ali**

Theory Group, DESY, Germany

Abstract

This report is devoted to a short description of the experimental data and theoretical estimates concerning the decays $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ and $B^+ \rightarrow \pi^+ \ell^+ \ell^-$. Recent data from the BaBar and Belle collaborations on $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay are reviewed and the form factor shape $f_+(q^2)$ is extracted in terms of different parametrizations. In the second part of the report a short overview of the theoretical works devoted to the study of $b \rightarrow s(d) \ell^+ \ell^-$ processes is given. As a check, the numerical value of the branching fraction of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ is reproduced. We use the extracted $f_+(q^2)$ and heavy quark symmetry to derive the dilepton invariant mass spectrum in $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ in the interval $4m_\mu^2 \leq q^2 \leq 8 \text{ GeV}^2$

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1 Introduction

The physics of B -meson plays a fundamental role both in the precision tests of the Standard Model (SM) and in the search of New physics. In this connection it is important to determine different characteristics of B -meson decays, such as branching fractions (BFs), differential distribution, CP -asymmetry etc with high accuracy. The interest in B -physics is greatly stimulated by the experiments at the B -factories, BaBar and Belle, and the LHCb at the LHC, which provide a huge amount of new and accurate experimental data concerning B -meson and B -baryon decays.

Theoretical estimates provide a good description of the B -physics processes, but the results strongly depend on some uncertain phenomenological functions, such as the form-factor in B -meson decays, and several free parameters in the SM some of which are known with low accuracy (for example, the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements V_{ub}, V_{td}, V_{tb}) [1]. The analysis and measurements of more rare B -meson decays will be helpful.

In this work we give a short overview of the experimental data and theoretical estimates concerning the $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ and $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ decays, with the view of using the former to make precise prediction for the latter. In particular, we extract the form-factors in $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ using data and heavy quark symmetry to calculate the dilepton mass spectrum in the range $4m_\mu^2 \leq q^2 \leq 8 \text{ GeV}^2$.

2 The $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay: overview

Measurements of the $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay allow to extract the CKM matrix element V_{ub} and the form-factor $f_+(q^2)$ shape (see (4)). The CKM matrix is contained in the quark weak current entering the quark- W flavor-changing interaction, the Lagrangian of which takes the form:

$$L_W = -\frac{g}{2\sqrt{2}} W^\mu (\bar{u}, \bar{c}, \bar{t}) \gamma_\mu (1 - \gamma_5) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} + h.c. \quad (1)$$

This matrix defines the mixings in the quark sector, which arise from the Yukawa interactions of the quarks with the Higgs field [1]. This circumstance makes the flavor-changing process $\bar{b} \rightarrow \bar{u} \ell^+ \nu_\ell$ to be possible. Flavor-changing charged current $\bar{b} \rightarrow \bar{u} W^+$ induces the $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay, the rate of which takes the form [9], [5]:

$$\frac{d\Gamma}{dq^2}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2, \quad (2)$$

where G_F is the Fermi coupling, m_B is the B -meson mass, $\ell = e, \mu$ (masses of leptons are neglected), $q = P_\ell + P_\nu$ is the sum of the four-momenta of the final-state lepton and neutrino, and q^2 varies in the range $4m_\ell^2 \leq q^2 \leq (m_B - m_\pi)^2$, and

$$\lambda(q^2) = (q^2 - m_B^2 - m_\pi^2)^2 - 4m_B^2 m_\pi^2. \quad (3)$$

The corresponding Feynman diagram is shown in Fig.1. The function $f_+(q^2)$ defines

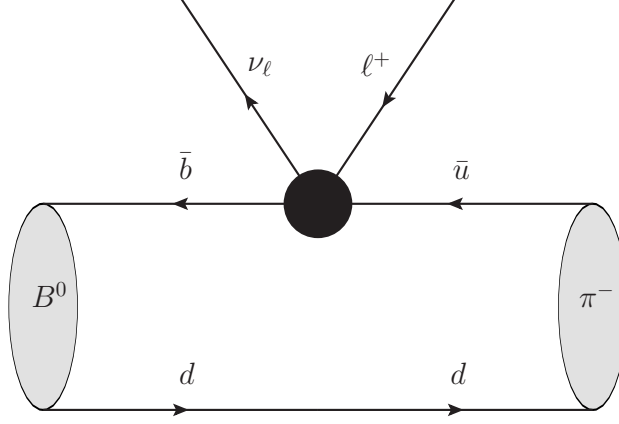


Figure 1: Feynman diagram for the decay $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$

the form-factor of the B^0 -meson decay, entering the $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ transition matrix element [7], [8]:

$$\langle \pi(P_\pi) | \bar{u} \gamma^\mu b | B(P_B) \rangle = f_+(q^2) \left(P_B + P_\pi - \frac{m_B^2 - m_\pi^2}{q^2} q \right)^\mu + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu. \quad (4)$$

Here P_B and P_π are the four-momenta of the B - and π -mesons, respectively, m_π is the pion mass. In practice, only $f_+(q^2)$ is measurable in $B \rightarrow \pi \ell \nu$ decays into the light leptons e or μ , since the contribution of the form-factor $f_0(q^2)$ to the decay rate is suppressed by a factor m_ℓ^2/m_B^2 [8] and the second term on the right-hand side of (4) can be neglected. The G_F , m_B , m_π values are known with high accuracy [1], while the derived value of V_{ub} strongly depends on the method of its extraction. Particle Data Group (PDG) [1] quotes the following values:

$$\begin{aligned} |V_{ub}| &= (4.11 \pm 0.15_{-0.19}^{+0.15}) \times 10^{-3} \quad [B \rightarrow X_u \ell \bar{\nu}], \\ |V_{ub}| &= (3.23 \pm 0.31) \times 10^{-3} \quad [B \rightarrow \pi \ell \bar{\nu}], \\ |V_{ub}| &= (5.10 \pm 0.47) \times 10^{-3} \quad [B \rightarrow \tau \bar{\nu}]. \end{aligned} \quad (5)$$

In brackets the processes from which the corresponding values of $|V_{ub}|$ are extracted are shown. One can see a significant discrepancy between central values of $|V_{ub}|$ derived from the analyses of the B -meson decays.

Using formula (2) one can see that the $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ differential decay rate $d\Gamma/dq^2 \sim |V_{ub} f_+(q^2)|^2$. The form-factor is a non-perturbative quantity and can be calculated for example in model-dependent Light Cone Sum Rules (LCSR) or Lattice QCD (LQCD). There also exist theoretical predictions concerning the form-factor shape which involve some phenomenological parameters. Using the experimental data on $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay, one can extract these parameters and the form-factor shape. This information is valuable as one can use the results obtained in the

calculation of other B -meson decays. An obvious application is the flavor-changing neutral current (FCNC) decay $B^+ \rightarrow \pi^+ \mu^+ \mu^-$, measured recently by the LHCb collaboration. We give below a short overview of the different types of the form-factor shape parametrization, testing them against data (χ^2 fits) in the next section.

3 Form-factor parametrizations

Several parametrizations of the pseudoscalar form factor $f_+(q^2)$ have been proposed in the literature. The following four parametrization discussed below are commonly used. All of them include at least one pole term at $q^2 = M_{B^*}^2$, with $m_{B^*} = 5.325 \text{ GeV} < m_B + m_\pi$ [5].

3.1 The Becirevic-Kaidalov (BK) parametrization

Here,

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha_{BK}q^2/m_{B^*}^2)}, \quad (6)$$

where the fitted parameters are $f_+(0)$ and α_{BK} . $f_+(0)$ sets the normalization of the form-factor, while the α_{BK} defines the shape [9]. This parametrization is one of the simplest ones.

3.2 The Ball-Zwicky (BZ) parametrization

Here,

$$f_+(q^2) = f_+(0) \left[\frac{1}{1 - q^2/m_{B^*}^2} + \frac{r_{BZ}q^2/m_{B^*}^2}{(1 - q^2/m_{B^*}^2)(1 - \alpha_{BZ}q^2/m_{B^*}^2)} \right], \quad (7)$$

where the fitted parameters are $f_+(0)$, α_{BZ} and r_{BZ} . $f_+(0)$ sets the normalization of the form-factor, while the α_{BZ} and r_{BZ} define the shape [10]. In particular, for $\alpha_{BZ} = r_{BZ}$ one reproduces the previous parametrization (6).

3.3 The Boyd-Grinstein-Lebed (BGL) parametrization

Here,

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, q_0^2)} \sum_{k=0}^{k_{max}} a_k(q_0^2) [z(q^2, q_0^2)]^k, \quad (8)$$

$$z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}}, \quad m_+ = m_B + m_\pi, \quad (9)$$

where the fitted parameters are a_k ($k = 0, 1, \dots, k_{max}$). q_0 is a free parameter that is often taken to be $q_0^2 = -m_-^2$, where $m_- = m_B - m_\pi$. In [5] it is proposed to choose the value $q_0^2 = -0.65m_-^2$; $P(q^2) = z(q^2, m_{B^*}^2)$ is the so-called Blaschke factor, which

accounts for the pole at $q^2 = m_{B^*}^2$, and $\phi(q^2, q_0^2)$ is an arbitrary analytic function, whose choice only affects the particular values of the series coefficients a_k , and is given by the expression [5]:

$$\begin{aligned} \phi(q^2, q_0^2) &= \sqrt{\frac{1}{32\pi\chi_J^{(0)}}} \left(\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2} \right) \\ &\times \left(\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - m_-^2} \right)^{3/2} \left(\sqrt{m_+^2 - q^2} + \sqrt{m_+^2} \right)^{-5} \frac{m_+^2 - q^2}{(m_+^2 - q_0^2)^{1/4}}, \end{aligned} \quad (10)$$

with the numerical factor $\chi_J^{(0)} = 6.889 \times 10^{-4}$. The shape of form-factor is given by the values of a_k , with truncation at $k_{max} = 2$ or 3 [11].

3.4 The Bourely-Caprini-Lellouch (BCL) parametrization

Here,

$$\begin{aligned} f_+(q^2) &= \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{k_{max}} b_k(q_0^2) \\ &\times \left([z(q^2, q_0^2)]^k - (-1)^{k-k_{max}-1} \frac{k}{k_{max} + 1} [z(q^2, q_0^2)]^{k_{max}+1} \right), \end{aligned} \quad (11)$$

where the variable z is defined as in (9). In this expansion, the shape of the form-factor is given by the values of b_k , with truncation at $k_{max} = 2$ or 3. The value of free parameter q_0^2 is proposed to be $q_0^2 = q_{opt}^2 \equiv (m_B + m_\pi) (\sqrt{m_B} - \sqrt{m_\pi})^2$ [12].

4 Experimental Data on $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$

The partial branching fraction (BF) of $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ has been measured by the CLEO, BaBar and Belle collaborations. As the BF is proportional to $|V_{ub}f_+(q^2)|^2$, experimental data allows to extract the form-factor shape together with V_{ub} . $f_+(q^2)$ is calculated in the unquenched lattice QCD (LQCD)[13], reliable only at large $q^2(\geq 16 \text{ GeV}^2)$, and in light-cone sum rules (LCSR) calculations ([14]), based on approximations only valid at small $q^2(\leq 16 \text{ GeV}^2)$.

In Fig. 2 we show the partial branching fraction of $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ measured by BaBar. The left plot corresponds to the data analysis of 2011 [2] and the right one to the data analysis of 2012 [4].

The experimental data from the Belle collaboration [3] is shown in Fig.3. In both figures, the data points are placed in the center of each bin and the error bars include total (statistical and systematic) uncertainties.

Below we give the total branching fraction of $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay [2], [4], [3], respectively, taking into account the recent data from the BaBar and Belle collabo-

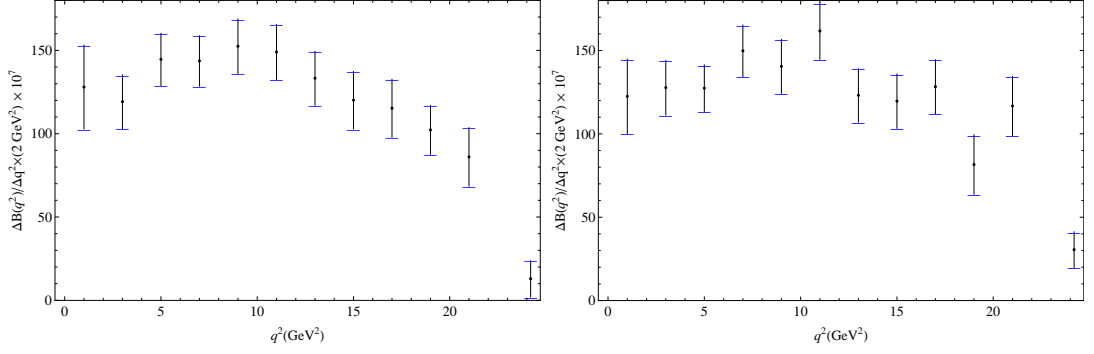


Figure 2: Partial $\Delta B(q^2)$ spectra in 12 bins of q^2 for $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay measured by the BaBar collaboration. The left plot corresponds to the data analysis of 2011 [2] and the right one to the data analysis of 2012 [4].

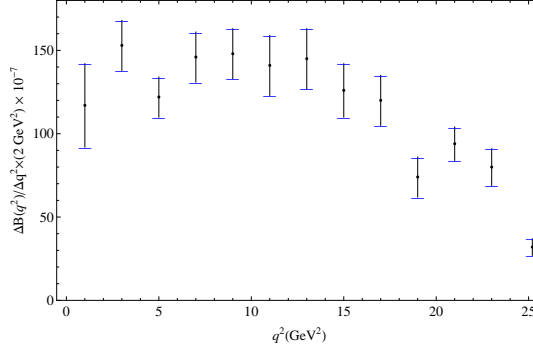


Figure 3: Partial $\Delta B(q^2)$ spectra in 13 bins of q^2 for $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay measured by the Belle collaboration [3].

rations:

$$\text{Br}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = \begin{cases} (1.42 \pm 0.05_{\text{stat}} \pm 0.07_{\text{syst}}) \times 10^{-4} & [\text{BaBar, 2011}] \\ (1.45 \pm 0.04_{\text{stat}} \pm 0.06_{\text{syst}}) \times 10^{-4} & [\text{BaBar, 2012}] \\ (1.49 \pm 0.04_{\text{stat}} \pm 0.07_{\text{syst}}) \times 10^{-4} & [\text{Belle, 2011}]. \end{cases} \quad (12)$$

All these measurements are in excellent agreement with each other.

5 Extraction of the $f_+(q^2)$ form-factor shape

5.1 Method

This section is devoted to extraction of the $f_+(q^2)$ form-factor in $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decay by using the experimental data, given in section 4, and the different parametrisations, which were shortly described in section 3. We follow the standard procedure of deriving best-fit values applying the χ^2 distribution function. The fitted form-factor is a function of q^2 , including some set of k parameters $\alpha_1, \dots, \alpha_k$:

$$f_+(q^2) = f(q^2; \alpha_1, \dots, \alpha_k). \quad (13)$$

We introduce χ^2 distribution function, which takes the form [1]:

$$\chi^2 = \sum_{i=1}^N \left(\frac{y_i - F(x_i; \alpha_1, \dots, \alpha_k)}{\sigma_i^2} \right)^2, \quad (14)$$

where N is the number of the experimental points, y_i imply the experimental values of partial branching fractions $\Delta B(q^2)$ in bins of q^2 , σ_i are the corresponding uncertainties, $F(x_i; \alpha_1, \dots, \alpha_k)$ imply the theoretical values of the partial branching fractions $\Delta B(q^2)$ for the given parametrization:

$$F(x_i; \alpha_1, \dots, \alpha_k) = \frac{1}{\Gamma_B} \int_{x_i - a_i/2}^{x_i + a_i/2} \frac{d\Gamma}{dq^2} dq^2, \quad (15)$$

where a_i is the range of the corresponding bin, and Γ_B is the decay width of the B -meson. One follows the standard procedure of minimizing of the χ^2 -function. The extracted minimum of this function χ_{min}^2 corresponds to certain values $\alpha_{1,min}, \dots, \alpha_{k,min}$, which are considered to be the best-fit values of these parameters.

The results obtained by using the four FF parametrization for different set of experimental data (BaBar and Belle collaboration) are represented in Fig. 4 - 7 and the numerical values are given in Tables 1 and 2.

	BK [9]	BZ [10]	BGL [11]	BCL [12]
BaBar 2011 [2]	9.93/10	4.80/9	4.12/9	3.75/9
BaBar 2012 [4]	11.11/10	9.28/9	9.30/9	9.35/9
Belle 2011 [3]	15.86/11	14.55/10	12.97/10	14.44/10
BaBar 2012 & Belle 2011	27.06/23	27.07/22	24.31/22	26.40/22

Table 1: Summary of χ^2 / ndf values for different set of experimental data (rows) and different FF parametrizations (columns), ndf - number of degree of freedom.

	BK [9]	BZ [10]	BGL [11]	BCL [12]
BaBar 2011 [2]	45%	85%	90%	93%
BaBar 2012 [4]	35%	41%	41%	41%
Belle 2011 [3]	15%	15%	23%	15%
BaBar 2012 & Belle 2011	25%	21%	33%	24%

Table 2: Summary of the Probability (χ^2) values corresponding to the χ^2 / ndf values in Table 1 for different set of experimental data (rows) and different FF parametrizations (columns).

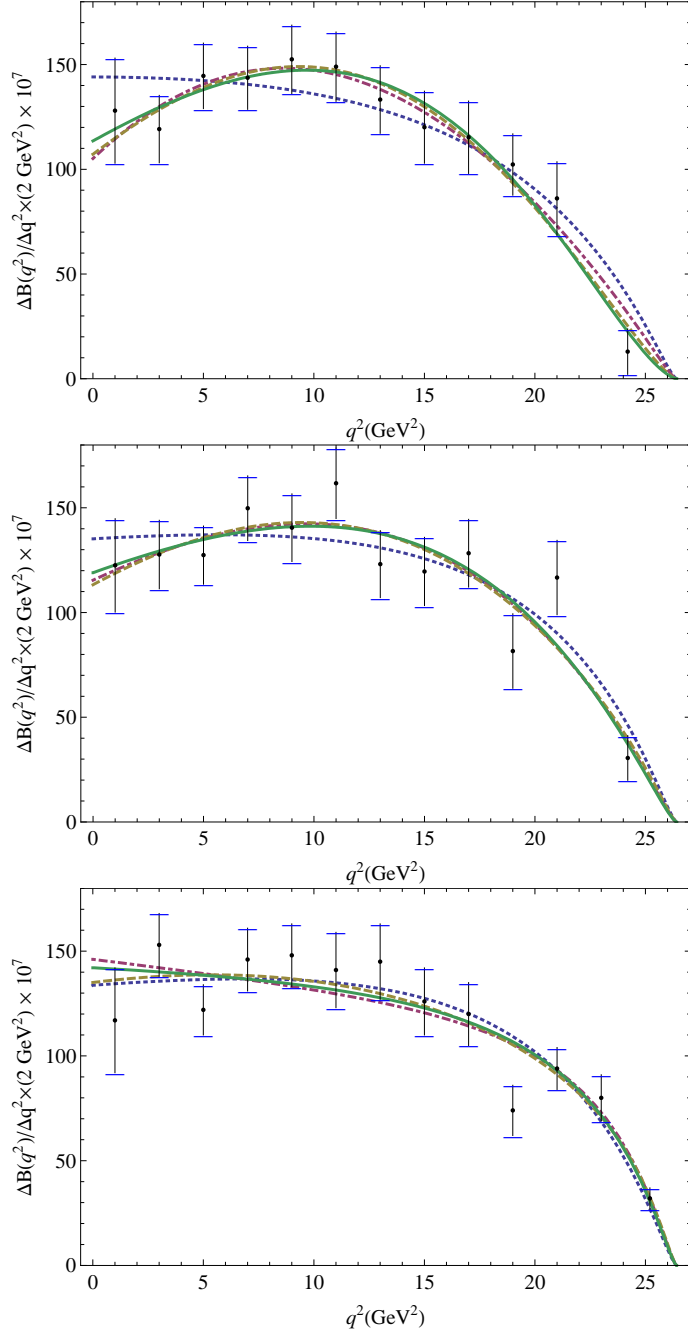


Figure 4: (Color online) Partial $\Delta\text{Br}(q^2)$ spectrum for $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decays. The data point (black dots) are placed in the middle of each bin. The error bars (blue) include the total experimental uncertainties. The curves show the results of the fit to the data: BK (thick dotted blue line) (see (6)), BZ (thick dashed purple line) (see (7)), BGL (thick dot-dashed yellow line) (see (8); $k_{\max} = 2$), and BCL (thick solid green line) (see (11); $k_{\max} = 2$) parametrization. Top plot corresponds to the BaBar 2011 data analysis [2], the plot in the middle is for the BaBar 2012 data analysis [4], and the bottom plot is for the Belle 2011 data analysis [3].

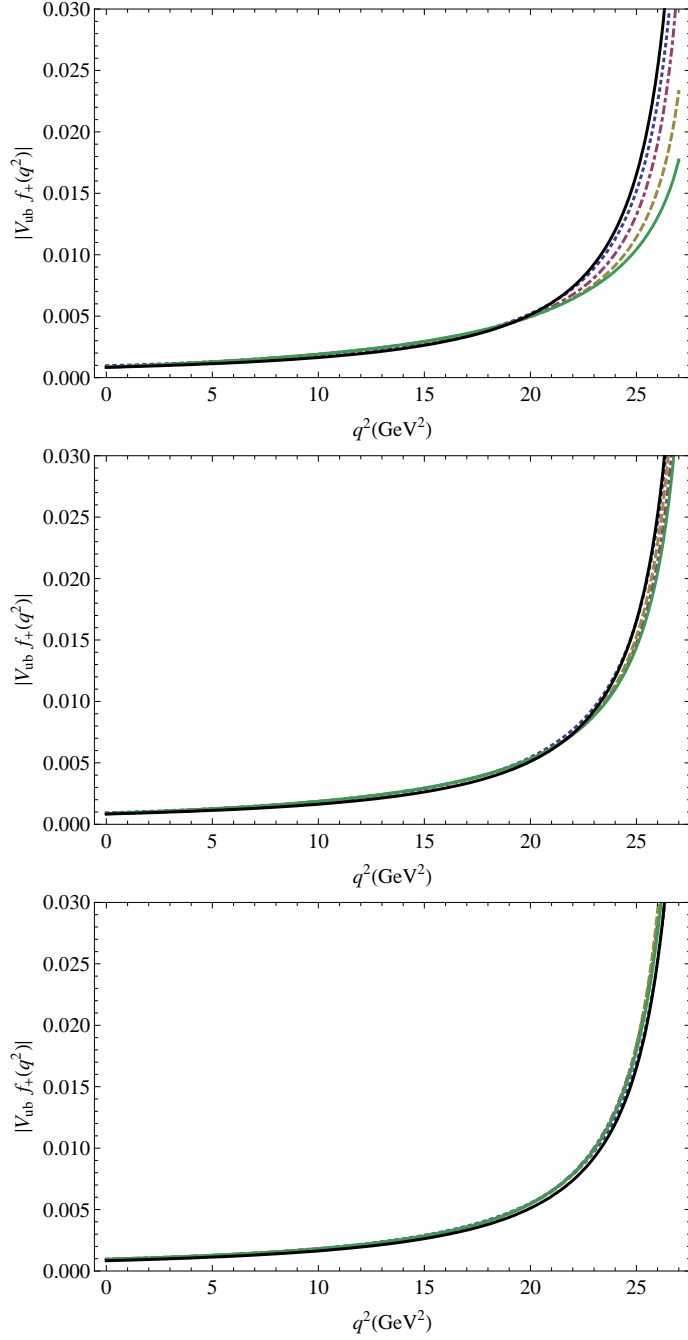


Figure 5: (Color online) The $|f_+(q^2)|$ form-factor shapes multiplied by $|V_{ub}|$. The curves show the results of the fit to the data: BK (thick dotted blue line) (see (6)), BZ (thick dashed purple line) (see (7)), BGL (thick dot-dashed yellow line) (see (8); $k_{max} = 2$), and BCL (thick solid green line) (see (11); $k_{max} = 2$) parametrization. The thick solid black curve corresponds to the form-factor shape extracted from the LCSR [10] (for this we used the value $|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$ [1]).

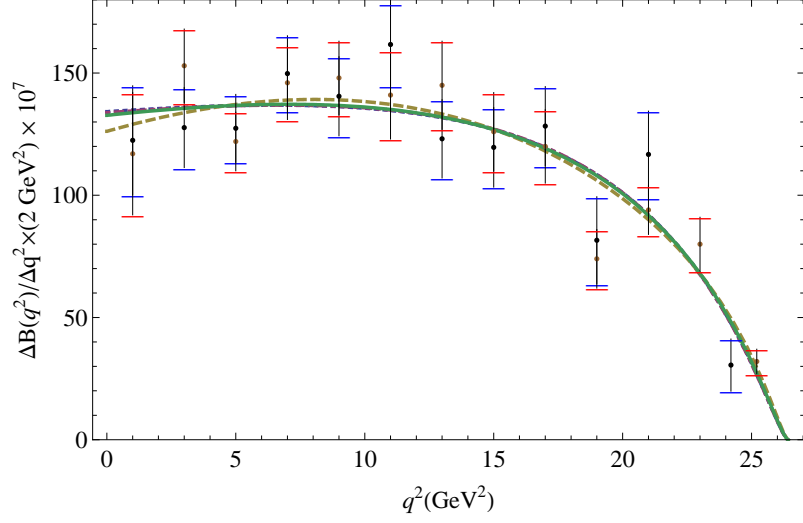


Figure 6: (Color online) Partial $\Delta\text{Br}(q^2)$ spectrum for $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ decays. The data point are from BaBar 2012 [4] (black dots) and Belle [3] (brown dots). The curves show the results of the fit to the data: BK (thick dotted blue line) (see (6)), BZ (thick dashed purple line) (see (7)), BGL (thick dot-dashed yellow line) (see (8); $k_{max} = 2$), and BCL (thick solid green line) (see (11); $k_{max} = 2$) parametrization.

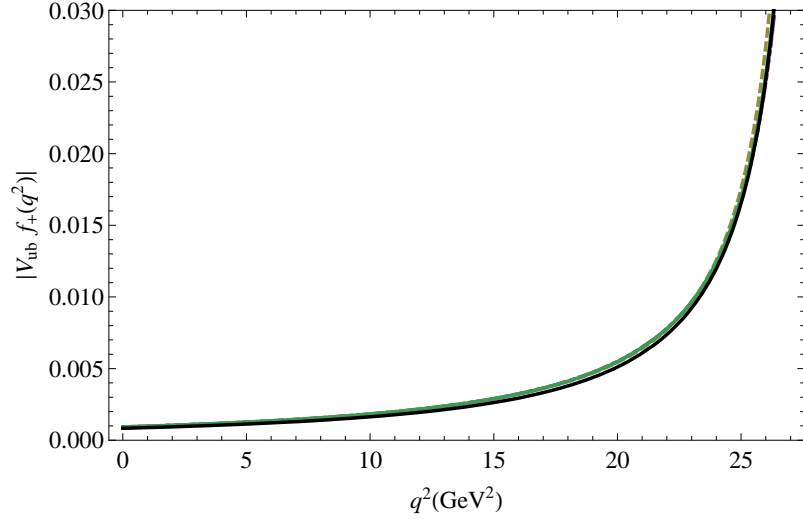


Figure 7: (Color online) The $|f_+(q^2)|$ form-factor shapes multiplied by $|V_{ub}|$. Results are obtained by combining the experimental data of BaBar 2012 [4] and Belle 2011 [3]. The curves show the results of the fit to the data: BK (thick dotted blue line) (see (6)), BZ (thick dashed purple line) (see (7)), BGL (thick dot-dashed yellow line) (see (8); $k_{max} = 2$), BCL (thick solid green line) (see (11); $k_{max} = 2$) parametrization. The thick solid black curve corresponds to the form-factor shape extracted from LCSR [10] (for this we used the value $|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$ [1]).

6 The $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay: overview

Rare B meson decays, induced by flavor-changing neutral current (FCNC) $b \rightarrow s(d)$ transitions, provide potentially stringent test of the standard model in flavor physics. FCNC are forbidden at the tree level and they proceed at a low rate via penguin and box diagram in the SM. In addition, these transitions may also be parametrically suppressed by the CKM matrix elements [17].

In the context of the SM, decay $b \rightarrow s \ell^+ \ell^-$ has been studied in a number of papers [16] — [22], and the partial results are known to NNLO. The rare B decay induced by $b \rightarrow d \ell^+ \ell^-$ transition is similar to the $b \rightarrow s \ell^+ \ell^-$ decay with some obvious modifications.

Quite recently, the LHCb collaboration has measured the decay $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ induced by the $b \rightarrow d$ transition. In this section we aim to reproduce the SM expression for the decay rate of $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ and obtain theoretical estimate for $\text{Br}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$.

Let us introduce the effective weak Hamiltonian encompassing the transitions $b \rightarrow d \ell^+ \ell^-$ [17], [18]:

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{td} \sum_{i=1}^{10} [C_i(\mu) O_i(\mu)], \quad (16)$$

where V_{tb} , V_{td} are the CKM matrix elements, C_i ($i = 1, \dots, 10$) are the Wilson coefficients, depending on the renormalization scale μ , O_i ($i = 1, \dots, 10$) are dimension-six operators at the scale μ and take the form [18]:

$$O_1 = (\bar{d}_L \gamma_\mu T^A c_L) (\bar{c}_L \gamma^\mu T^A b_L), \quad (17)$$

$$O_2 = (\bar{d}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L), \quad (18)$$

$$O_3 = (\bar{d}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q), \quad (19)$$

$$O_4 = (\bar{d}_L \gamma_\mu T^A b_L) \sum_q (\bar{q} \gamma^\mu T^A q), \quad (20)$$

$$O_5 = (\bar{d}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q), \quad (21)$$

$$O_6 = (\bar{d}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^A b_L) \sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^A q), \quad (22)$$

$$O_7 = \frac{e m_b}{g_{\text{st}}^2} (\bar{d}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \quad (23)$$

$$O_8 = \frac{m_b}{g_{\text{st}}} (\bar{d}_L \sigma^{\mu\nu} T^A b_R) G_{\mu\nu}^A, \quad (24)$$

$$O_9 = \frac{e^2}{g_{\text{st}}^2} (\bar{d}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell} \gamma_\mu \ell), \quad (25)$$

$$O_{10} = \frac{e^2}{g_{\text{st}}^2} (\bar{d}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell} \gamma_\mu \gamma_5 \ell), \quad (26)$$

where the subscripts L and R refer to the left- and right- handed components of the fermion fields, $\psi_{L,R} = 1/2(1 \mp \gamma_5) \psi$. Sums over q and l denote sums over all the light quarks and the leptons, respectively.

Exclusive decay $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ is described in terms of the matrix elements of the quark operators between meson states, which are expressed in terms of several independent form factors. For the process $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ the non-vanishing matrix elements are [15], [17]:

$$\langle \pi(p_\pi) | \bar{b} \gamma^\mu d | B(p_B) \rangle = f_+(s) \left[p_B + p_\pi - \frac{m_B^2 - m_\pi^2}{s} q \right]^\mu + f_0(s) \frac{m_B^2 - m_\pi^2}{s} q^\mu, \quad (27)$$

$$\langle \pi(p_\pi) | \bar{b} \sigma^{\mu\nu} q_\nu d | B(p_B) \rangle = i [(p_B + p_\pi)^\mu s - q^\mu (m_B^2 - m_\pi^2)] \frac{f_T(s)}{m_B + m_\pi}, \quad (28)$$

where p_B and p_π are the four-momenta of B and π mesons, respectively, $q = p_+ + p_- = p_B - p_\pi$, where p_+ and p_- are the four-momenta of ℓ^+ and ℓ^- leptons, respectively, $s \equiv q^2$. $f_+(s)$, $f_0(s)$ and $f_T(s)$ are the form-factors in $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay.

For further analysis it is convenient to introduce the following kinematic variables normalized in the terms of the B -meson mass m_B :

$$\hat{s} = (\hat{p}_+ + \hat{p}_-)^2, \quad \hat{u} = (\hat{p}_B - \hat{p}_-)^2 - (\hat{p}_B - \hat{p}_+)^2, \quad (29)$$

where $\hat{p}_{B,-,+} = p_{B,-,+}/m_B$. These quantities are bounded as:

$$(2\hat{m}_\ell)^2 \leq \hat{s} \leq (1 - \hat{m}_\pi)^2, \quad (30)$$

$$-\hat{u}(\hat{s}) \leq \hat{u} \leq \hat{u}(\hat{s}), \quad (31)$$

where $\hat{m}_{\ell,\pi} = m_{\ell,\pi}/m_B$ and the variable \hat{u} can be written as follows [17]:

$$\hat{u}(\hat{s}) = \sqrt{\lambda \left(1 - 4 \frac{\hat{m}_\ell^2}{\hat{s}} \right)}, \quad (32)$$

$$\lambda \equiv 1 + \hat{m}_\pi^4 + \hat{s}^2 - 2\hat{s} - 2\hat{m}_\pi^2(1 + \hat{s}). \quad (33)$$

Finally, the differential decay branching fraction for $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ takes the form [17]:

$$\frac{d\text{Br}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)}{d\hat{s}} = \frac{G_F^2 \alpha_{em}^2 m_B^5 \tau_B}{2^{10} \pi^5} |V_{tb}^* V_{td}|^2 \hat{u}(\hat{s}) F_\pi(\hat{s}), \quad (34)$$

$$F_\pi(\hat{s}) = (|A|^2 + |C|^2) \left(\lambda - \frac{\hat{u}(\hat{s})^2}{3} \right) + |C|^2 4\hat{m}_\ell^2 (2 + 2\hat{m}_\pi^2 - \hat{s}) + \text{Re}(CD^*) 8\hat{m}_\ell^2 (1 - \hat{m}_\pi^2) + |D|^2 4\hat{m}_\ell^2 \hat{s}, \quad (35)$$

where the functions $A(\hat{s})$, $C(\hat{s})$, $D(\hat{s})$ are defined as:

$$A(\hat{s}) = \tilde{C}_9^{eff}(\hat{s}) f_+(\hat{s}) + \frac{2\hat{m}_b}{1 + \hat{m}_\pi} \tilde{C}_7^{eff}(\hat{s}) f_T(\hat{s}), \quad (36)$$

$$C(\hat{s}) = \tilde{C}_{10}^{eff}(\hat{s}) f_+(\hat{s}), \quad (37)$$

$$D(\hat{s}) = \tilde{C}_{10}^{eff}(\hat{s}) f_-(\hat{s}), \quad (38)$$

with

$$f_{-}(\hat{s}) = \frac{1 - \hat{m}_{\pi}^2}{\hat{s}} (f_0(\hat{s}) - f_{+}(\hat{s})). \quad (39)$$

The effective Wilson coefficients are written as [18]:

$$\begin{aligned} \tilde{C}_7^{eff}(\hat{s}) &= \left(1 + \frac{\alpha_s(\mu)}{\pi} \omega_7(\hat{s})\right) A_7 \\ &\quad - \frac{\alpha_s(\mu)}{4\pi} \left(C_1^{(0)} F_1^{(7)}(\hat{s}) + C_2^{(0)} F_2^{(7)}(\hat{s}) + A_8^{(0)} F_8^{(7)}(\hat{s})\right), \end{aligned} \quad (40)$$

$$\begin{aligned} \tilde{C}_9^{eff} &= \left(1 + \frac{\alpha_s(\mu)}{\pi} \omega_9(\hat{s})\right) (A_9 + T_9 h(\hat{m}_c^2, \hat{s}) + U_9 h(1, \hat{s}) + W_9 h(0, \hat{s})) \\ &\quad - \frac{\alpha_s(\mu)}{4\pi} \left(C_1^{(0)} F_1^{(9)}(\hat{s}) + C_2^{(0)} F_2^{(9)}(\hat{s}) + A_8^{(0)} F_8^{(9)}(\hat{s})\right), \end{aligned} \quad (41)$$

$$\tilde{C}_{10}^{eff} = \left(1 + \frac{\alpha_s(\mu)}{\pi} \omega_9(\hat{s})\right) A_{10}. \quad (42)$$

The functions $h(\hat{m}_c^2, \hat{s})$ and $\omega_9(\hat{s})$ are given in [20]:

$$h(z, x) = -\frac{4}{9} \ln(z) + \frac{8}{27} + \frac{4}{9} x \quad (43)$$

$$-\frac{2}{9}(2+x)\sqrt{|1-x|} \begin{cases} \ln \left| \frac{\sqrt{1-x}+1}{\sqrt{1-x}-1} \right| - i\pi, & \text{for } x \equiv 4z/\hat{s} < 1, \\ 2 \arctan(1/\sqrt{1-x}), & \text{for } x \equiv 4z/\hat{s} > 1, \end{cases} \quad (44)$$

$$\begin{aligned} \omega(\hat{s}) &= -\frac{4}{3} Li_2(\hat{s}) - \frac{2}{3} \ln(1-\hat{s}) \ln \hat{s} - \frac{2}{9} \pi^2 - \frac{5+4\hat{s}}{3(1+2\hat{s})} \ln(1-\hat{s}) \\ &\quad - \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})} \ln \hat{s} + \frac{5+9\hat{s}-6\hat{s}^2}{6(1-\hat{s})(1+2\hat{s})}, \end{aligned} \quad (45)$$

and the functions $\omega_7(\hat{s})$ and $F_{1,2,8}^{(7,9)}(\hat{s})$ are defined in [21] and reproduced below:

$$\begin{aligned} \omega_7(\hat{s}) &= -83 \ln \left(\frac{\mu}{m_b} \right) - \frac{4}{3} Li_2(\hat{s}) - \frac{2}{9} \pi^2 - \frac{2}{3} \ln(\hat{s}) \ln(1-\hat{s}) \\ &\quad - \frac{1}{3} \frac{8+\hat{s}}{2+\hat{s}} \ln(1-\hat{s}) - \frac{2}{3} \frac{\hat{s}(2-2\hat{s}-\hat{s}^2)}{(1-\hat{s})^2(2+\hat{s})} \ln(\hat{s}) - \frac{1}{18} \frac{16-11\hat{s}-17\hat{s}^2}{(2+\hat{s})(1-\hat{s})}, \end{aligned} \quad (46)$$

$$F_1^{(7)}(\hat{s}) = -\frac{208}{243} \ln \left(\frac{\mu}{m_b} \right) + f_1^{(7)}, \quad (47)$$

$$F_2^{(7)}(\hat{s}) = \frac{416}{81} \ln \left(\frac{\mu}{m_b} \right) + f_2^{(7)}, \quad (48)$$

$$\begin{aligned} F_8^{(7)}(\hat{s}) &= -\frac{32}{9} \ln \left(\frac{\mu}{m_b} \right) + \frac{8}{27} \pi^2 - \frac{44}{9} - \frac{8}{9} i\pi + \left(\frac{4}{3} \pi^2 - \frac{40}{3} \right) \hat{s} \\ &\quad + \left(\frac{32}{9} \pi^2 - \frac{316}{9} \right) \hat{s}^2 + \left(\frac{200}{27} \pi^2 - \frac{658}{9} \right) \hat{s}^3 - \frac{8}{9} \ln(\hat{s}) (\hat{s} + \hat{s}^2 + \hat{s}^3), \end{aligned} \quad (49)$$

$$F_8^{(9)}(\hat{s}) = \frac{104}{9} - \frac{32}{27}\pi^2 + \left(\frac{1184}{27} - \frac{40}{9}\pi^2\right)\hat{s} + \left(\frac{14212}{135} - \frac{32}{3}\pi^2\right)\hat{s}^2 \\ + \left(\frac{193444}{945} - \frac{560}{27}\pi^2\right)\hat{s}^3 + \frac{16}{9}\ln(\hat{s})(1 + \hat{s} + \hat{s}^2 + \hat{s}^3), \quad (50)$$

$$F_1^{(9)} = \left(-\frac{1424}{729} + \frac{16}{243}i\pi + \frac{64}{27}\ln\left(\frac{m_c}{m_b}\right)\right)\ln\left(\frac{\mu}{m_b}\right) - \frac{16}{243}\ln\left(\frac{\mu}{m_b}\right)\ln(\hat{s}) \\ + \left(\frac{16}{1215} - \frac{32}{135}\hat{m}_c^{-2}\right)\ln\left(\frac{\mu}{m_b}\right)\hat{s} + \left(\frac{4}{2835} - \frac{8}{315}\hat{m}_c^{-4}\right)\ln\left(\frac{\mu}{m_b}\right)\hat{s}^2 \quad (51)$$

$$+ \left(\frac{16}{76545} - \frac{32}{8505}\hat{m}_c^{-6}\right)\ln\left(\frac{\mu}{m_b}\right)\hat{s}^3 - \frac{256}{243}\ln^2\left(\frac{\mu}{m_b}\right) + f_1^{(9)}, \quad (52)$$

$$F_2^{(9)} = \left(\frac{256}{243} - \frac{32}{81}i\pi - \frac{128}{9}\ln\left(\frac{m_c}{m_b}\right)\right)\ln\left(\frac{\mu}{m_b}\right) + \frac{32}{81}\ln\left(\frac{\mu}{m_b}\right)\ln(\hat{s}) \\ + \left(-\frac{32}{405} + \frac{64}{45}\hat{m}_c^{-2}\right)\ln\left(\frac{\mu}{m_b}\right)\hat{s} + \left(-\frac{8}{945} + \frac{16}{105}\hat{m}_c^{-4}\right)\ln\left(\frac{\mu}{m_b}\right)\hat{s}^2 \quad (53)$$

$$+ \left(-\frac{32}{25515} + \frac{64}{2835}\hat{m}_c^{-6}\right)\ln\left(\frac{\mu}{m_b}\right)\hat{s}^3 + \frac{512}{81}\ln^2\left(\frac{\mu}{m_b}\right) + f_2^{(9)}, \quad (54)$$

where [21]

$$f_a^{(b)} = \sum_{i,j} k_a^{(b)}(i,j)\hat{s}^i \ln^j(\hat{s}), \quad (a = 1, 2; b = 7, 9; i = 0, \dots, 3; j = 0, 1). \quad (55)$$

The numerical values for the quantities $k_a^{(b)}(i,j)$ are given in Tables 1 and 2 of [21].

The auxiliary quantities $A_7, A_8, A_9, A_{10}, T_9, U_9, W_9$ are expressed as linear combinations of the Wilson coefficients $C_i(\mu)$. The numerical values for these coefficients are obtained after solving the renormalization group equations for the Wilson coefficients $C_i(\mu)$, using the matching conditions from [20] and the anomalous dimension matrices from [20]. The numerical values for $A_7, A_8, A_9, A_{10}, T_9, U_9, W_9$ up to NLO are introduced in Table 1 of [18].

As for the strong coupling constant $\alpha_s(\mu)$ we introduce its two-loop expression [1]:

$$\alpha_s(\mu^2) \simeq \frac{1}{\beta_0 t} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\ln t}{t}\right), \quad (56)$$

with:

$$t \equiv \ln \frac{\mu^2}{\Lambda^2}, \quad \beta_0 = \frac{33 - 2n_f}{12\pi}, \quad \beta_1 = \frac{153 - 19n_f}{24\pi^2}, \quad (57)$$

where n_f is the number of quark flavors.

In [17] the authors used the LCSR results for the form-factors involving the $B \rightarrow \pi$ transitions [10]. Using these results for form factors and the values of different parameters contained in (34) (using 2008 data), the authors have obtained

the following value for the branching fraction of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay (in the framework of the SM):

$$\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) = (1.96 \pm 0.21) \times 10^{-8}. \quad (58)$$

This result is in agreement with the experimental one measured recently by the LHCb collaboration (see (59)), and we have verified the numerical result, presented in (58).

7 The $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay: first observation

Quite recently the LHCb collaboration reported the observation of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay with 5.2σ significance, using 1.0 fb^{-1} integrated luminosity in pp collisions at $\sqrt{s} = 7 \text{ TeV}$ collected during 2011 [6]. This decay is observed for the first time, as unlike the $b \rightarrow s \ell^+ \ell^-$, no $b \rightarrow d \ell^+ \ell^-$ transition has previously been observed. Hence the phenomenological analysis of this process will provide us a lot of new information concerning B physics and ultimately allow to check the SM. It is also a powerful tool in the search of new physics.

The value of total branching fraction of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ measured by the LHCb is [6]:

$$\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) = (2.4 \pm 0.6(\text{stat}) \pm 0.2(\text{syst})) \times 10^{-8}. \quad (59)$$

In the SM, $b \rightarrow d \ell^+ \ell^-$ is suppressed by a factor $|V_{td}/V_{ts}|$ relative to $b \rightarrow s \ell^+ \ell^-$. This suppression does not apply to new physics beyond the SM. Even with constraints from $b \rightarrow s \ell^+ \ell^-$ processes, the $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ branching fraction can be enhanced by new physics model.

8 $B^+ \rightarrow \pi^+ \ell^+ \ell^-$: HQS limit

The B^+ -meson is a bound state of the heavy \bar{b} and light u quarks, hence one can apply the so-called heavy-quark symmetry (HQS), which is valid in the large recoil limit (small values of q^2). The theory describing such processes is a heavy quark symmetry (HQS), which is considered in a number of papers (for example, [7], [15]). Using the HQS allows one to significantly simplify the description of $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay at small $q^2 \leq 7 \text{ GeV}^2$, namely, applying heavy-quark symmetry results in reducing the number of independent form-factors of the $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ process from three to one. As it is shown in [15], in the HQS limit (without taking into account symmetry-breaking corrections) the relations between the three form-factors $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$ are the following:

$$f_0(q^2) = \left(\frac{m_B^2 + m_\pi^2 - q^2}{m_B^2} \right) f_+(q^2), \quad (60)$$

$$f_T(q^2) = \left(\frac{m_B + m_\pi}{m_B} \right) f_+(q^2). \quad (61)$$

As one can see, in this case there is only one independent form-factor $f_+(q^2)$, the shape of which can be extracted from the $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ process. The decay rate of $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ in the HQS limit is simplified and takes the form:

$$\frac{d\Gamma}{dq^2} (B^+ \rightarrow \pi^+ \ell^+ \ell^-) = \frac{G_F^2 \alpha_{em}^2}{1024 m_B^3 \pi^5} |V_{tb}|^2 |V_{td}|^2 |f_+(q^2)|^2 F(q^2), \quad (62)$$

where

$$F(q^2) = \sqrt{1 - \frac{4m_\ell^2}{q^2}} \sqrt{\lambda} \left[\frac{2}{3} \lambda \left(1 + \frac{2m_\ell^2}{q^2} \right) \left(|C_9|^2 + 4 \frac{m_b}{m_B} \text{Re}(C_9^* C_7) + 4 \frac{m_b^2}{m_B^2} |C_7|^2 \right) + \frac{2}{3} \lambda |C_{10}|^2 + \frac{4m_\ell^2}{q^2} |C_{10}|^2 \left(4m_B^2 m_\pi^2 - \frac{1}{2} \lambda \right) \right], \quad (63)$$

$\lambda(q^2)$ is defined by the (3). Using the $f_+(q^2)$ form-factor shape extracted in the terms of the BGL parametrization and the combined BaBar and Belle data and the numerical values of the different quantities entering (62) given in [1], we have obtained the following branching fraction distribution (for $q^2 \leq 8 \text{ GeV}^2$) shown in Fig. 8. The upper bound in q^2 is imposed to avoid the background from the process $B^+ \rightarrow \pi^+ J/\psi \rightarrow \pi^+ \mu^+ \mu^-$. The numerical value of the partial branching ratio of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ in the range $4m_\mu^2 \leq q^2 \leq 8 \text{ GeV}^2$ is given below:

$$\text{Br}(B^+ \rightarrow \pi^+ \mu^+ \mu^-; 4m_\mu^2 \leq q^2 \leq 8 \text{ GeV}^2) \simeq 0.81 \times 10^{-8} \quad (64)$$

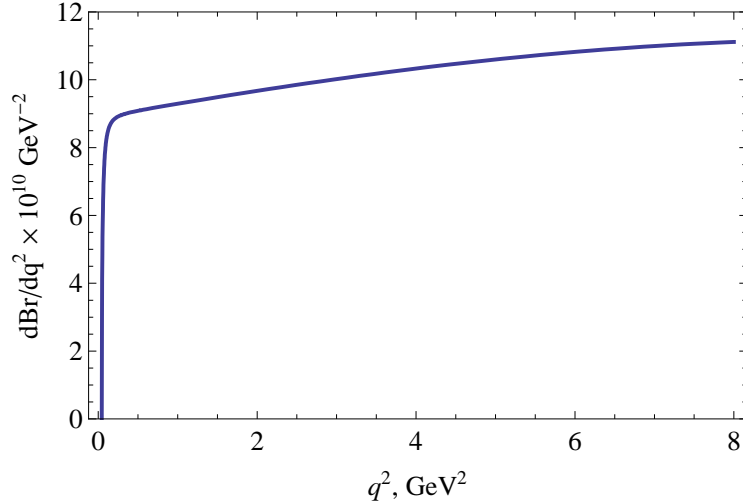


Figure 8: The branching fraction distribution for the decay $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ in the HQS limit for q^2 in the range $0 \leq q^2 \leq 8 \text{ GeV}^2$.

9 Summary and outlook

We have given an overview concerning the experimental data and theoretical calculations of the decays $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ and $B^+ \rightarrow \pi^+ \ell^+ \ell^-$. From the data of the BaBar and Belle collaborations in terms of the different form-factors parametrizations we have extracted the $f_+(q^2)$ form-factor shape. It was found that the best fit values are achieved with the BGL parametrization [11].

Also we made a short discussion concerning the $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay. The numerical value of the branching fraction for the decay $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ given in the paper [17] was reproduced and compared with the experimental one measured recently by the LHCb collaboration. To reduce the number of unknown independent form-factors of the $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ process, heavy quark symmetry was applied and the expression for the decay rate was obtained in HQS limit which is valid only for small values q^2 but involves only one independent form-factor $f_+(q^2)$. Using the results of the FF extraction and the HQS limit expression we have obtained the prediction for the branching partial fraction distribution (in the framework of the SM) for $q^2 \leq 8 \text{ GeV}^2$, which can be measured at the LHCb collaboration. Predicting the total branching ratio in the decay $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ involves some approximations due to modeling the form-factors $f_0(q^2)$ and $f_T(q^2)$ in the large q^2 range. This will be studied later and the results will be published as a paper soon.

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