A Comparison of the WHIZARD and ALPGEN Event Generators

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September 8, 2011

Abstract

We compare the (partonic) cross sections for a number of hard scattering processes at hadron colliders calculated with the WHIZARD 2 and the ALPGEN Monte Carlo event generators. In addition to the QCD calculations done by both codes we compare the results with the cross sections obtained in the full standard model using WHIZARD 2. We find no significant deviations between both codes. The effects of electroweak processes in addition to pure QCD amplitudes turn out to play a significant role for at least one processes.

Contents

1. Introduction

In hadron colliders composite particles – protons and protons or protons and antiprotons – are accelerated. These particles consists of quarks and gluons (called partons), strongly interacting fundamental particles. The fact that the colliding particles are non-elementary and that the resulting interactions between partons are dominantly QCD processes results in complicated event topologies and a large total cross section. To perform searches for interesting physics a huge background of known processes has to be understood and modeled exactly. The only practical method so far is to use Monte Carlo computer calculations in combination with analytical computations and empirical models.

At the most powerful hadron colliders – the Tevatron and the LHC – the energy is large enough to frequently result in final states with a large number of jets. On the parton level, each outgoing quark or gluon is (after showering and jet selection) responsible for exactly one jet. As the number of Feynman graphs at tree level grows roughly factorially with the number of outgoing particles, a naive treatment becomes highly problematic even for moderate jet numbers. During the last years more advanced algorithms have been developed to evaluate such processes numerically.

Another difficult point is the phase space integration itself. This integral has dimension $3n - 4$ for n outgoing particles. The only possibility to evaluate this for $n \gtrsim 3$ is to use Monte Carlo methods.

In this project report, we compare the total cross sections calculated by two Monte Carlo event generators – WHIZARD 2 [1] and ALPGEN [2] – for a set of QCD processes. WHIZARD 2 and ALPGEN use different approaches to reduce the complexity of the matrix element evaluation.

2. Theory

2.1. QCD and Strong Interactions

2.1.1. Beta function of QCD and Confinement

Renormalization is necessary to remove ultraviolet divergences if one computes observables using a perturbation series. This introduces an unphysical energy scale μ. To obtain μ-independent quantities, the coupling g has to become scale dependent. Its scale dependence is given by the equation

$$\mu \frac{\partial g}{\partial \mu} = \beta(g) = \frac{g}{4\pi} \sum_{n=1}^{\infty} \beta_n \left(\frac{g^2}{4\pi} \right)^n . \tag{1}$$

β denotes the beta function of the theory and can be calculated in perturbation theory as a power series in g, as indicated on the right-hand side.

For QCD the first coefficient is given by

$$\beta_1 = -\left(11 - \frac{2}{3}n_f\right), \tag{2}$$

where n_f denotes the number of flavours. As $n_f = 6$ (or $n_f = 5$ below the top threshold) in the standard model, the coupling decreases with energy. Above $\approx 2\,\mathrm{GeV}$, the coupling is small enough to allow perturbative calculations. The theory becomes asymptotically free at high scales. On the other hand, on low energy scales the coupling too strong for perturbative approaches to be applicable.

Another important phenomenon in QCD is confinement. Confinement ensures that no colored particle can be isolated and therefore no such particle can be observed. This leads to hadronization of jets, i.e. the recombination of quarks and gluons to color neutral objects.

2.1.2. QCD Radiation, Soft and Collinear Divergences

Radiating a gluon or a (light) quark can lead to divergences in the cross section. This is because this introduces another quark propagator proportional to

$$\frac{1}{q^2} = \frac{1}{(p_1 + p_2)^2} \approx \frac{1}{2E_1E_2(1 - \cos\theta_{12})}. \tag{3}$$

The cross section diverges whenever $E_j \to 0$ or $\theta_{12} \to 0$. These divergences are called infrared and collinear divergences respectively. All divergent parts usually cancel out if all higher order effects are taken into account. To get finite results at tree level one could consider *infrared/collinear safe* observables. The simplest would be to introduce an energy (or transverse momentum) and an angular cut.

2.2. Partonic Cross Sections and the Factorization Theorem

To describe hadronic interactions one can make the assumption that the processes which bound the initial state particles together, the parton-level scattering process and the formation of showers and hadronization occur on different energy/time scales. Then it is intuitively plausible that the whole process factorizes:

$$\mathrm{d}\sigma(pp \to X) = \sum_{i,j,k} \int \mathrm{d}x_i\,\mathrm{d}x_j f_i^p(x_i,\mu^2) f_j^p(x_j,\mu^2) \mathrm{d}\sigma(ij \to Y_k) P(Y_k \to X), \tag{4}$$

where f_i^p is the probability to find a parton i with momentum fraction x_i inside the proton, $\mathrm{d}\sigma(ij \to Y_k)$ is the parton level cross section and $P(Y_k \to X)$ is the hadronization probability. The scale dependence of f_i^p can be calculated within perturbative QCD (DGLAP equations). The initial condition can (universally) be determined by experiment. The parton level cross section can be calculated pertubatively. The hadronization can (in practice) not be calculated from first principles and has to be modeled.

3. Measurement

3.1. The WHIZARD Event Generator

WHIZARD is a multi-purpose Monte Carlo event generator written with the goal to avoid any hardcoding of processes into specialized libraries. It utilizes the O'Mega matrix element generator [3] and the Vamp integration library [4].

Processes in WHIZARD are controlled through a steering script, written in the scripting language Sindarin [1]. In this file the hard scattering (or decay) process can be set up and an analysis can be performed. WHIZARD parses the Sindarin script file and passes a system call to O'Mega which generates a Fortran code containing the matrix elements. This code gets compiled automatically and is then fed into the Vamp integration algorithm. The complexity of available processes is only limited by computation time.

WHIZARD can use different models for its calculations, for example the full standard mode, pure QCD or the MSSM. New models can easily be implemented by writing files for O'Mega which contain the necessary parameters and vertices for the desired model. It is even possible to use FeynRules [5] to generate the Feynman rules associated with a given Lagrangian and passing these directly to O'Mega.

3.2. The ALPGEN Event Generator

ALPGEN is a collection of codes for at the moment 16 different multi-parton processes. It uses the Alpha algorithm [6] to compute QCD matrix elements at leading order and can integrate the corresponding cross sections. ALPGEN can generate unweighted event samples which can in turn used as input for shower evolution MCs such as HERWIG [7] or PYTHIA [8].

All available process types in ALPGEN are hardcoded. The process specific part is separated from the common parts and can be modified to set up a specific analysis. A number of parameters specific for the chosen hard process (such as jet multiplicities) can be set using a simple text input file. The available choices for e.g. jet multiplicities are usually limited.

In contrast to WHIZARD, ALPGEN uses only pure QCD amplitudes in its calculations. For processes with massive vector bosons or a Higgs in the final state a single electroweak vertex gets attached to the process.

3.3. Some Technicalities of the Comparison

There a few subtle points to be considered when working with ALPGEN. First of all, the reference values given in the ALPGEN documentation [2] sometimes cannot be reproduced in a straightforward way because the necessary options are no longer available. This concerns for example the $b\bar{b} + n$ jets example, where the factorization scale can no longer be chosen in the way it has been for the reference values.

There is a similar problem with the PDF (see next section). Therefore, we will try to reproduce the reference calculations in the ALPGEN manual for only one process (see section 3.6) but then go on and use another PDF and another scale to only compare ALPGEN and WHIZARD in all other cases.

Another delicate point is the fact that ALPGEN uses at least one undocumented cut for the process $b\bar{b} + n$ jets, namely a cut on $\sqrt{\hat{s}}$ which is given by the sum of the b masses and all transverse momentum cuts. If there are n light jets and the minimal transverse momentum for each jet is 20 GeV, ALPGEN performs the cut

$$\sqrt{\hat{s}} < 2m_b + (n+2) \cdot 20\,\mathrm{GeV}. \tag{5}$$

We do not remove these cuts from ALPGEN but implement them in WHIZARD instead.

3.4. PDFs and α_s

The default PDF used in ALPGEN is CTEQ5L. Although CTEQ5L files for LHAPDF (which can be used by WHIZARD) are available and therefore the same PDF could be used in WHIZARD, ALPGEN uses a parametrized PDF instead of a tabulated one for this PDF which could lead to discrepancies at the 1% level for certain processes. We therefore decided to use CTEQ6L for every comparison we perform between ALPGEN and WHIZARD. Both programs use CTEQ6L in a tabulated form so there is no source of discrepancies. Details about this PDF can be found in [9].

ALPGEN automatically uses the same α_s parametrization which has been used in the fit of the PDF in use. This means that in the case of CTEQ6L a two-loop expression is used. The same can be achieved in WHIZARD by leaving the α_s evaluation to LHAPDF.

3.5. $pp \to b\bar{b} +$ jets

First we analyzed the processes $pp \to b\bar{b} + n$ jets at $\sqrt{s} = 14\,\mathrm{TeV}$ (LHC) and $p\bar{p} \to b\bar{b} + n$ jets at $\sqrt{s} = 2\,\mathrm{TeV}$[1] (Tevatron). All additional jets (i.e. jets additional to $b\bar{b}$) are light, i.e. either gluon or quark jets with $m_q < m_b$. We used the following parameters

[1] As this is not a detailed physics study, we use 2.0 GeV, not 1.96 GeV.

and cuts:

$$
\begin{aligned}
& m_b = 4.7\,\text{GeV}, \quad m_t = 175\,\text{GeV} && (6)\\
& p_T^j > 20\,\text{GeV}, \quad |\eta_j| < 2.5, \quad \Delta R_{jj} > 0.7 && (7)\\
& p_T^b > 20\,\text{GeV}, \quad |\eta_b| < 2.5, \quad \Delta R_{b\bar{b}} > 0.7, \quad \Delta R_{bj} > 0.7 && (8)\\
& \sqrt{\hat{s}} = \sqrt{x_1 x_2 s} < 2m_b + (n+2) \cdot 20\,\text{GeV} && (9)\\
& Q_0^2 = \hat{s}, && (10)
\end{aligned}
$$

where Q_0 denotes the factorization and renormalization scale.

We consider gluons and all quarks up to (and including) charm quarks as initial state partons.

The results up to 3 additional jets are shown in tab. 1 for LHC and Tevatron. This table also shows the cross section which WHIZARD gives if one uses the full standard model (with unit CKM matrix) instead of only QCD processes. This is the most accurately studied process of this report and we find no significant deviations between ALPGEN and WHIZARD. Cross sections change at the percent level if we use full standard model amplitudes and this effect is clearly statistically significant, although it is much smaller than PDF uncertainties and NLO/NNLO corrections.

Table 1: Cross section of $pp \to b\bar{b} + n$ jets at LHC and Tevatron, computed with WHIZARD and ALPGEN.

n	experiment	σ[nb] (WHIZARD)	σ[nb] (ALPGEN)	rel. diff. $\frac{\lvert\sigma_{Wh}-\sigma_{Alp}\rvert}{\sqrt{\Delta\sigma_{Wh}^2+\Delta\sigma_{Alp}^2}}$	σ[nb] (WHIZARD SM)
0	LHC	1185.72(12)	1185.59(60)	0.22	1188.33(59)
	Tevatron	46.511(4)	46.48(2)	1.26	47.078(4)
1	LHC	202.6(1)	202.4(2)	0.5	203.3(3)
	Tevatron	4.244(5)	4.243(4)	0.28	4.342(5)
2	LHC	36.26(8)	36.28(6)	0.14	36.76(8)
	Tevatron	0.4598(9)	0.4609(3)	1.18	0.4740(10)
3	LHC	5.07(5)	5.080(8)	0.17	5.04(5)
	Tevatron	0.0352(4)	0.03559(4)	0.98	3.68(2)

3.6. $pp \to t\bar{t} +$ jets

We now compare the process $pp \to t\bar{t} + n$ jets. The summarized parameters and cuts are

$$m_b = 0\,\text{GeV}, \quad m_t = 175\,\text{GeV} \tag{11}$$
$$p_T^j > 20\,\text{GeV}, \quad |\eta_j| < 2.5, \quad \Delta R_{jj} > 0.7 \tag{12}$$
$$Q_0^2 = m_t^2. \tag{13}$$

In particular, there are no cuts on the top pair. We do not take top decays into account.

For this process we are able to redo the analysis in the ALPGEN manual. The results are shown in table 2. The results are compatible. It turned out that lot more statistics are needed to obtain agreement than the statistical error calculated by ALPGEN indicates, especially for larger jet numbers.

Table 2: Cross section of $pp \to t\bar{t}+n$ jets at LHC and Tevatron, computed with ALPGEN and compared to the values in the ALPGEN manual.

process	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
LHC [pb], reference	530.0(8)	462.6(6)	255(1)	111.5(5)	42.4(4)	14.07(16)	4.36(8)
LHC [pb], our run	528.1(7)	462.1(8)	251(2)	110.8(8)	39.5(8)	14.28(43)	4.26(6)
Tev. [fb], reference	6364(8)	1592(3)	282(1)	40.6(3)	4.83(4)	0.483(6)	0.0419(9)
Tev. [fb], our run	6380(10)	1597(5)	281(1)	39.2(5)	4.82(4)	0.483(4)	0.0446(24)

We now repeat the comparison from the last section for $t\bar{t} + n$ jets and $n \leq 3$. The results are shown in tab. 3. Again we see no significant deviations between WHIZARD (using the QCD model) and ALPGEN. The effects of additional electroweak interactions are significant but again below the PDF uncertainties and higher order corrections.

3.7. $pp \to t\bar{t}b\bar{b} +$ jets

We now consider the slightly more complicated case of $pp \to t\bar{t}b\bar{b}+$jets. The parameters and cuts are

$$m_b = 4.7\,\text{GeV}, \quad m_t = 175\,\text{GeV} \tag{14}$$
$$p_T^j > 20\,\text{GeV}, \quad |\eta_j| < 2.5, \quad \Delta R_{jj} > 0.7 \tag{15}$$
$$p_T^b > 20\,\text{GeV}, \quad |\eta_b| < 2.5, \quad \Delta R_{b\bar{b}} > 0.7, \quad \Delta R_{bj} > 0.7 \tag{16}$$
$$Q_0^2 = m_t^2. \tag{17}$$

The results are shown in tab. 4.

Table 3: Cross section of $pp \to t\bar{t} + n$ jets at LHC and Tevatron, computed with WHIZARD and ALPGEN.

n	experiment	σ[nb] (WHIZARD)	σ[nb] (ALPGEN)	rel. diff. $\frac{\lvert\sigma_{Wh}-\sigma_{Alp}\rvert}{\sqrt{\Delta\sigma_{Wh}^2+\Delta\sigma_{Alp}^2}}$	σ[nb] (WHIZARD SM)
0	LHC	489.19(6)	489.1(4)	0.25	489.724(6)
	Tevatron	5.5459(8)	5.547(4)	0.30	5.5798(8)
1	LHC	392.9(4)	393.4(6)	0.69	395.0(4)
	Tevatron	1.282(2)	1.2822(5)	$2.7 \cdot 10^{-4}$	1.286(1)
2	LHC	0.1987(2)	0.1986(2)	0.41	0.1997(2)
	Tevatron	$0.21003(9) \cdot 10^{-3}$	$0.2098(2) \cdot 10^{-3}$	1.20	$0.2224(2) \cdot 10^{-3}$
3	LHC	n/a	0.0798(1)	n/a	n/a
	Tevatron	n/a	$0.02787(3) \cdot 10^{-3}$	n/a	n/a

Table 4: Cross section of $pp \to t\bar{t}b\bar{b} + n$ jets at LHC and Tevatron, computed with WHIZARD and ALPGEN.

n	experiment	σ[fb] (WHIZARD)	σ[fb] (ALPGEN)	rel. diff. $\frac{\lvert\sigma_{Wh}-\sigma_{Alp}\rvert}{\sqrt{\Delta\sigma_{Wh}^2+\Delta\sigma_{Alp}^2}}$	σ[fb] (WHIZARD SM)
0	LHC	1055(2)	1057(1)	0.83	1170(2)
	Tevatron	2.618(4)	2.616(3)	0.24	3.352(5)
1	LHC	1041(3)	1042(2)	0.44	1111(5)
	Tevatron	0.6699(16)	0.66790(98)	1.11	0.837(2)
2	LHC	n/a	612(7)	n/a	n/a
	Tevatron	0.107(9)	0.1006(2)	0.78	0.197(27)

In this case we find a large difference between pure QCD and the full standard model, especially for 2 additional light jets. As the available statistics are not very good, this may very well be an artifact. We did not investigate this excess further.

3.8. $pp \to b\bar{b}b\bar{b} +$ jets

The last process we consider here is $pp \to b\bar{b}b\bar{b} + n$ jets. Parameters and cuts are given by

$$m_b = 4.7\,\text{GeV}, \quad m_t = 175\,\text{GeV} \tag{18}$$

$$p_T^j > 20\,\text{GeV}, \quad |\eta_j| < 2.5, \quad \Delta R_{jj} > 0.7 \tag{19}$$

$$p_T^b > 20\,\text{GeV}, \quad |\eta_b| < 2.5, \quad \Delta R_{b\bar{b}} > 0.7, \quad \Delta R_{bb} > 0.7, \quad \Delta R_{bj} > 0.7 \tag{20}$$

$$Q_0^2 = \hat{s}. \tag{21}$$

The results are shown in tab. 5. There is one highly signficant deviation for $n = 2$ at the LHC. This may well be an artifact due to too low statistics and badly adapted grids in WHIZARD although the cross section seems to be convergent. Recently we learned that ALPGEN considers b-jets as light jets if there are already 4 other b-jets in the process because the maximum number of b-tags at the LHC is 4. This means that we have to consider the process $pp \to b\bar{b}b\bar{b}b\bar{b}$ as well. Preliminary studies indicate that this contribution is well below the deviation found (it seems to be of the order ~ 0.1pb).

Table 5: Cross section of $pp \to b\bar{b}b\bar{b} + n$ jets at LHC and Tevatron, computed with WHIZARD and ALPGEN.

n	experiment	σ[pb] (WHIZARD)	σ[pb] (ALPGEN)	rel. diff. $\frac{\lvert\sigma_{Wh}-\sigma_{Alp}\rvert}{\sqrt{\Delta\sigma^2_{Wh}+\Delta\sigma^2_{Alp}}}$	σ[pb] (WHIZARD SM)
0	LHC	138.18(5)	138.07(9)	0.77	145.29(7)
	Tevatron	1.2229(4)	1.2235(8)	0.64	1.3574(4)
1	LHC	42.02(6)	41.95(4)	1.13	42.99(9)
	Tevatron	0.221(1)	0.2213(2)	0.72	0.234(3)
2	LHC	6.1(3)	8.876(7)	11.54	6.9(4)
	Tevatron	0.24(1)	0.02486(3)	0.94	0.021(4)

4. Conclusions and Outlook

After clarification of conventions and adjustment of input parameters we found a good agreement between ALPGEN and WHIZARD for almost every considered process. The only significant deviation found, in $pp \to b\bar{b}b\bar{b}+2$ jets, is likely to diminish or even vanish completely once more statistics is available.

While comparing QCD calculations with full SM calculations we found some significant deviations in the $pp \to t\bar{t}b\bar{b} + n$ jets case which have not been investigated more thoroughly so far. In all other cases the effect of electroweak contributions seems to be negligible.

An important next step would be to compare more complicated observables, in particular differential cross sections. In addition, more complicated processes should be compared, especially processes with massive gauge bosons or a Higgs in the final state. In this case different models (i.e. full standard model in contrast to QCD amplitudes plus a weak vertex for the final state gauge boson) could result in significant differences.

Even higher jet multiplicities than presented here are difficult to obtain because the Fortran code generated by WHIZARD becomes too large ($\sim$ 100MB) to be compiled by the recent GCC compiler. This problem is currently under investigation. In the meantime, it can be circumvented by splitting the process into subprocesses and calculate these separately. Due to time constraints this has only been partially done, but it seems to work fine.

As LHC data interpretation rely on Monte Carlo predictions for background and signal modeling validation and comparison of such programs is an important task. The results obtained here are therefore quite reassuring. Another noteworthy point is that WHIZARD, which started as an BSM signal Monte Carlo and has then be extended to standard model background processes in hadron colliders has successfully been compared to ALPGEN, a program dedicated to such tasks.

A. All results

Table 6: All results of the comparison. Deviations smaller than 2 standard deviations are shown in green, larger deviations in red.

n	experiment	σ[nb] (WHIZARD)	σ[nb] (ALPGEN)	rel. diff.	σ[nb] (WHIZARD SM)
			$pp \to b\bar{b} + n$ jets		
0	LHC	1185.72(12)	1185.59(60)	0.22	1188.33(59)
	Tevatron	46.511(4)	46.48(2)	1.26	47.078(4)
1	LHC	202.6(1)	202.4(2)	0.5	203.3(3)
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			$pp \to t\bar{t} + n$ jets		
0	LHC	489.19(6)	489.1(4)	0.25	489.724(6)
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3	LHC	n/a	0.0798(1)	n/a	n/a
	Tevatron	n/a	$0.02787(3) \cdot 10^{-3}$	n/a	n/a
			$pp \to t\bar{t}b\bar{b} + n$ jets, cross sections $\times 10^{-6}$		
0	LHC	1055(2)	1057(1)	0.83	1170(2)
	Tevatron	2.618(4)	2.616(3)	0.24	3.352(5)
1	LHC	1041(3)	1042(2)	0.44	1111(5)
	Tevatron	0.6699(16)	0.66790(98)	1.11	0.837(2)
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	Tevatron	0.107(9)	0.1006(2)	0.78	0.197(27)
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2	LHC	6.1(3)	8.876(7)	11.54	6.9(4)
	Tevatron	0.24(1)	0.02486(3)	0.94	0.021(4)

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