Understanding the nature of Neutrinos: From Laboratory experiments to Leptogenesis

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Abstract

We study some aspects of the theory and modern experimental results of neutrino physics from oscillation experiments, cosmology, beta-decay and neutrinoless double beta decay both in the cases of Dirac and Majorana masses (generated by see-saw type I mechanism) and discuss bounds on the neutrino masses.

Contents

1 Introduction

Neutrino is the only electrically neutral fermion known so far. It was suggested by Pauli in 1930 to explain the continuous spectrum of beta-decay. Pauli thought that neutrinos will never be observed, but they were discovered 25 years later. Now neutrino telescopes are widely used in astrophysics, and also neutrino physics is one of low-energy frontiers of the physics beyond the Standard Model (SM). In SM neutrinos are massless, but from neutrino oscillation observations we know that neutrinos have masses. So, we should replace the massless neutrinos in the SM by the massive ones to describe the observations. It is very likely that new physics in large scales exists, described by so-called Grand Unified Theories (GUT), and our low-energy extensions of the SM should be a part of this extended theory. The next question is why neutrino masses (or couplings with the Higgs field) are so small. One may propose very small Dirac masses of the neutrinos, but it is likely that neutrinos are Majorana particles (proposed by E.Majorana in 1937, [1]).

In this review we will consider a model with heavy right-handed Majorana neutrinos (see-saw type I [2]). See-saw mechanism naturally appears in some of GUT theories (for example, with gauge group $SO(10)$[3]). Also, there are many other (more exotic) theories of the origin of neutrino masses, which we have not considered in this review. For example, see [3, 4].

In the see-saw model we can propose such phenomena as neutrinoless double beta decay and Leptogenesis in the Early universe. We will also consider cosmological conditions on neutrino masses (both in the cases of Dirac and Majorana neutrino masses), compare them with direct experimental measurements and put constraints on the neutrinoless double beta decay rate and Leptogenesis.

We start by recalling some basic properties of the Dirac and Weyl spinors and define Majorana spinors.

2 Dirac, Weyl and Majorana spinors

Let us consider Dirac equation for free particle:

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0, \tag{1}$$

where γ-matrices satisfy the conditions:

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}, \tag{2}$$

$$\gamma_0\gamma_\mu\gamma_0 = \gamma_\mu^\dagger. \tag{3}$$

There are many representations of γ-matrices, the widely used one is the chiral representation:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}. \tag{4}$$

Here $\sigma^\mu = (1, \vec{\sigma})$ and $\bar{\sigma}^\mu = (1, -\vec{\sigma})$, $\vec{\sigma}$ are the Pauli spin matrices. We can write massless Dirac equation in chiral representation:

$$\begin{pmatrix} 0 & i\sigma^\mu \partial_\mu \\ i\bar{\sigma}^\mu \partial_\mu & 0 \end{pmatrix} \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix} = 0. \tag{5}$$

Here ξ_L and ξ_R are two-component Weyl spinors.

Four-component spinors $\psi_L = \begin{pmatrix} \xi_L \\ 0 \end{pmatrix}$ and $\psi_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix}$ satisfy the massless Dirac equation. Any Dirac spinor can be considered as a sum of two Weyl spinors: $\psi_D = \psi_L + \psi_R$

Also, we have a general way to distinguish the right and left Weyl spinors. First, define $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ and projectors $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$. We can see (it is simple to check it in chiral representation) that:

$$\begin{aligned} P_L\psi_L = \psi_L, \qquad P_L\psi_R = 0, \\ P_R\psi_L = 0, \qquad P_R\psi_R = \psi_R. \end{aligned} \tag{6}$$

These equations can be considered as definition of the left and right Weyl fermions.

Dirac mass term $m\bar{\psi}\psi$ can be considered in terms of Weyl fermion as $m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$. The terms $\bar{\psi}_L\psi_L$ and $\bar{\psi}_R\psi_R$ are equal to zero.

2.1 Charge conjugation

Charge conjugation of the spinor field, by definition (see [6]), is a conversion, which inverses vector and axial-vector currents and does not change free Lagrangian. So, the terms

$$L_v = \bar{\psi}\gamma^\mu\psi, \quad L_{av} = \bar{\psi}\gamma^\mu\gamma_5\psi \tag{7}$$

should change sign by C-conjugation (for example, $\bar{\psi}\gamma^\mu\psi = -\bar{\psi}^C\gamma^\mu\psi^C$, here ψ^C is a C-conjugated spinor), and the terms

$$L_k = i\bar{\psi}\gamma^\mu\partial_\mu\psi, \quad L_s = \bar{\psi}\psi, \quad L_a = \bar{\psi}\gamma_5\psi \tag{8}$$

should not be changed. We will try to find ψ^C in the form

$$\psi^C = C\bar{\psi}^T, \tag{9}$$

and seek the conditions on the C-matrix. We can also notice that $\left(\psi^C\right)^C = \psi$, so

$$\bar{\psi}^C = \psi^T C^{-1\,T}. \tag{10}$$

The term $\bar{\psi}\psi$ converts as:

$$\bar{\psi}^C\psi^C = \psi^T C^{-1\,T} C\bar{\psi}^T = -\bar{\psi}C^T C^{-1}\psi = \bar{\psi}\psi. \tag{11}$$

Here we used anticommutation relations of spinor fields, and finally we obtain:

$$C = -C^T. \tag{12}$$

The mass term and the vector current convert as:

$$\bar{\psi}^C \psi^C = \psi^T \gamma_0^T C^\dagger \gamma_0 C \bar{\psi}^T = -\psi^T \bar{\psi}^T = \bar{\psi}\psi, \tag{13}$$

$$\bar{\psi}^C \gamma_\mu \psi^C = \psi^T \gamma_0^T C^\dagger \gamma_0 \gamma_\mu C \bar{\psi}^T = \psi^T \gamma_\mu^T \bar{\psi}^T = -\bar{\psi}\gamma_\mu \psi. \tag{14}$$

From theese conditions we obtain:

$$\gamma_0^T C^\dagger \gamma_0 C = -1, \quad \gamma_0^T C^\dagger \gamma_0 \gamma_\mu C = \gamma_\mu^T. \tag{15}$$

Finally,

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T. \tag{16}$$

γ_5 is a product of even γ-matrices, so $C^{-1}\gamma_5 C = \gamma_5^T$. It means that the axial-vector current L_{av} should be inversed under C-conjugation, and the term L_a should not be inversed. Also, from the condition (16) we can prove that the kinetic term L_k should not be inversed under C-conjugation.

2.2 Majorana fermions

Firstly, let us check that C-conjugated Weyl spinor is a Weyl spinor with another chirality:

$$\frac{1+\gamma_5}{2}\psi_L = 0. \tag{17}$$

After conjugating this equation we obtain:

$$\gamma_0 C \frac{1+\gamma_5^*}{2}\psi_L^* = \frac{1-\gamma_5}{2}\psi_L^C = 0. \tag{18}$$

So, ψ_L^C is right-handed Weyl spinor.

Now we can define Majorana spinors - by definition, they are self-C-conjugare Dirac spinors. Dirac spinors are $\psi_D = \psi_L + \psi_R$, so Majorana spinors are

$$\psi_M = \frac{1}{\sqrt{2}}\left(\psi_{Weyl} + \psi_{Weyl}^C\right). \tag{19}$$

Factor $\frac{1}{\sqrt{2}}$ normalizes the kinetic term. In the literature Majorana spinors are often described in terms of left-handed Weyl spinor ψ_L and its C-conjugated partner ψ_L^C.

Majorana mass term can be written in terms of Weyl spinors as $\frac{m}{2}\bar{\psi}_L^C \psi_L + h.c.$ There is no reason why Majorana mass should be real. So, in the general case it is complex.

3 See-saw mechanism

Standard Model is a very good theory, describing our nature. Its dynamics is defined by the gauge symmetries $SU(3) \times SU(2)_L \times U(1)_Y$ of the Lagrangian. So, if we want to add some new terms to the SM Lagrangian, we should not break these symmetries. Left-handed leptons are $SU(2)_L$-doublet, and left-handed neutrinos have $U(1)_Y$-hypercharge $Y = 1$. They transforms under $U(1)_Y$ as $\nu_L \to \mathrm{e}^{i\alpha(x)}\nu_L$. The Lagrangian should not change under $U(1)_Y$ transformation.

So, let us try to transform Majorana mass term of left-handed neutrinos by $U(1)_Y$. We obtain:

$$m\bar{\nu}^C\nu + m^*\bar{\nu}\nu^C \to \mathrm{e}^{2i\alpha(x)}m\bar{\nu}^C\nu + \mathrm{e}^{-2i\alpha(x)}m^*\bar{\nu}\nu^C. \tag{20}$$

So, Majorana mass term is forbidden for left-handed neutrinos. Only neutral (singlet by any gauge group) fermions can have Majorana mass. Right-handed neutrinos are such neutral fermions, and we can add them to the SM.

Let us notice that Majorana mass term of right-handed neutrino breaks global (non-gauge) $U(1)_L$ symmetry $\nu_L \to \mathrm{e}^{iL}\nu_L$. But it does not change SM dynamics (also, $U(1)_L$ is broken in SM on quantum level too[5]). This effect means that it can exist some processes without lepton number conservation, such as neutrinoless double beta decay and leptogenesis.

It is also allowed by SM symmetries that right-handed neutrinos can interact with left-handed leptons and Higgs doublet $f\bar{l}_a\phi^a\nu_R + h.c.$ When Higgs field develops a VEV, we obtain Dirac mass term $m_D\bar{\nu}_L\nu_R + h.c.$

Thus, we obtain the following Lagrangian for the neutrinos:

$$L = i\bar{\nu}_L\gamma^\mu\partial_\mu\nu_L + i\bar{\nu}_R\gamma^\mu\partial_\mu\nu_R - m\bar{\nu}_R\nu_L - \frac{M}{2}\bar{\nu}_R^C\nu_R + h.c. \tag{21}$$

We can rewrite the mass term in (21) as:

$$L_m = -\frac{1}{2}\begin{pmatrix} \bar{\nu}_L^C & \bar{\nu}_R \end{pmatrix}\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} + h.c. \tag{22}$$

When we study neutrino propagation we should consider neutrino mass eigenstates. So, we should diagonalize the mass matrix $M^{D+M} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$. The matrix M^{D+M} is real and symmetric, so it can be presented in the form

$$M^{D+M} = Om_{diag}O^T, \tag{23}$$

where m_{diag} is a diagonal matrix, and O is an orthogonal matrix, parametrized as

$$O = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}. \tag{24}$$

So if we consider mixed neutrinos

$$\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = O\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}, \tag{25}$$

we will obtain a Lagrangian with diagonal mass matrix:

$$L_m = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_a^C & \bar{\nu}_b \end{pmatrix} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_b^C \end{pmatrix} + h.c., \tag{26}$$

where m_1 and m_2 are:

$$m_{1,2} = \frac{1}{2}\left(M \pm \sqrt{M^2 + 4m^2}\right). \tag{27}$$

If we should obtain a small Majorana mass of one of neutrinos, we should take a large value of M (and it agree GUT predictions) and usual (about lepton masses) value of mass m. In this case we can simplify this formula. So,

$$\begin{aligned} m_1 &\approx -\tfrac{m}{M^2}, \\ m_2 &\approx M. \end{aligned} \tag{28}$$

We can consider $(i\psi_a)$ as a fermion field, so the mass term of this model in the mass-basis neutrino is:

$$L_m = -\frac{1}{2}\left|m_1\right|(i\bar{\nu}_a)^C(i\nu_a) - \frac{1}{2}m_2\bar{\nu}_b^C\nu_b + h.c. \tag{29}$$

So, we obtain a small Majorana mass of our "quasi-left-handed" neutrinos.

4 Mixing and oscillations

Many independent recent experiments showed that neutrinos can convert from one flavour state to another during propagation in Vacuum. These conversions were called neutrino oscillation. The explanation is in term of the neutrino flavor mixing. We have flavour basis $|\nu_f\rangle = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$ and mass basis $|\nu_m\rangle = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$, transition matrix U is defined by equation

$$|\nu_f\rangle = U \cdot |\nu_m\rangle. \tag{30}$$

In the case of Majorana neutrinos, Majorana mass term is $\bar{\nu}_i^C M^{ij}\nu_j$, where $i,j = 1..3$ means number of (flavour) families, M^{ij} is a hermitian non-diagonal mass matrix. We should diagonalize mass matrix and not change kinetic term. So, we have (D is a diagonal positive matrix):

$$U^\dagger U = 1, \quad U^\dagger M U = D. \tag{31}$$

The second conditions is checked automatically from the first by simple linear algebra. So, U is an unitary matrix, called PMNS (Pontecorvo[7], Maki,Nakagava, Sakata[8]), and after rotating phases of fermion fields it parametrizes as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{32}$$

Here $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$. δ is CP-violating phase. The last term is a phase of Majorana masses.

It is a similar situation in the mixing matrix of Yukawa couplings (or, Dirac masses). The difference is that left and the right neutrinos converse with different unitary matrices ($|\nu_{R,flavour}\rangle = A|\nu_{R,mass}\rangle$ and $|\nu_{L,flavour}\rangle = B|\nu_{L,mass}\rangle$), and the mass matrix M in unitary basis may be non-hermitian. We should make double unitary transformation:

$$A^{\dagger} M B = D. \tag{33}$$

Here D is also diagonal positive matrix, PMNS matrix has the same form as in thr Majorana case, without the last term, because pure Dirac masses should be real.

4.1 See-saw mechanism in the case of 3 generations

Previously we considered see-saw mechanism only for one generation for simplicity. Let us generalize it on the realistic case of three generations. The starting Lagrangian is:

$$L = i\bar{\nu}_L^a \gamma^\mu \partial_\mu \nu_L^a + i\bar{\nu}_R^a \gamma^\mu \partial_\mu \nu_R^a - \bar{\nu}_R^a m_D^{ab} \nu_L^b - \frac{1}{2}\bar{\nu}_R^{a,C} M^{ab} \nu_R^{b,C} + h.c. \tag{34}$$

Here a and b are flavour indices. The mass term of Lagrangian (34) can be rewritten as (analogy to (35), but now neutrinos have 3 generation components):

$$L_m = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L^C & \bar{\nu}_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^{\dagger} & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} + h.c. \tag{35}$$

After diaganalization of 6-component mass matrix by neutrino fields rotation we obtain:

$$M_{diag}^{see-saw} = \begin{pmatrix} -m_D M^{-1} m_D^{\dagger} & 0 \\ 0 & M \end{pmatrix}. \tag{36}$$

So, effective mass matrix of our (quasi-left-handed) neutrinos is $M_L = m_D M^{-1} m_D^{\dagger}$.

Let us notice, what if we want to work with chiral states, it is simpler to use diagonal mass matrix of right-handed neutrinos. Really, we rotate only right-handed neutrinos to diagonalize matrix M:

$$\begin{pmatrix} 0 & m_D \\ m_D^{\dagger} & M \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 \\ 0 & U^{\dagger} \end{pmatrix} \cdot \begin{pmatrix} 0 & m_D \\ m_D^{\dagger} & M \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & U \end{pmatrix} = \tag{37}$$
$$= \begin{pmatrix} 0 & m_D U \\ U^{\dagger} m_D^{\dagger} & u^{\dagger} M U \end{pmatrix} = \begin{pmatrix} 0 & \widetilde{m}_D \\ \widetilde{m}_D^{\dagger} & M_{diag} \end{pmatrix}.$$

Here we mean Dirac mass matrix as $m_D = y \cdot \frac{v}{\sqrt{2}}$, y is a matrix of Yukava couplings with Higgs field. We can use representation (37) also in the case of interactions with Higgs field. But if we want to study neutrino propagation, it is convenient to use the diagonal Majorana mass basis (36).

4.2 Neutrino oscillations

Let us look at neutrino propagation. Free neutrino with defined mass propagates to a distance L as

$$|\nu_i(t,L)\rangle = \mathrm{e}^{-i(E_i t - p_i L)}|\nu_i(0,0)\rangle. \tag{38}$$

Neutrino masses are very small, so all neutrinos we can observe are ultrarelativistic. So, $p_i = E_i - \frac{m_i^2}{2E}$, $t = L$, and neutrino propagates as:

$$|\nu_i(L)\rangle = \mathrm{e}^{-i\frac{m_i^2}{2E}}|\nu_i(0)\rangle. \tag{39}$$

When neutrino interacts with charged leptons, it is a neutrino in flavour basis. So, both emitted and adsorbed neutrinos are in flavour basis, but they propagate in the mass basis. Transition amplitude from one flavour neutrino $|\nu_\beta\rangle$ to another $|\nu_\alpha\rangle$ is

$$A = \sum_i \langle\nu_\beta|\nu_i(L)\rangle\langle\nu_i(0)|\nu_\alpha\rangle = \sum_i \mathrm{e}^{-i\frac{m_i^2}{2E}}\langle\nu_\beta|\nu_i\rangle\langle\nu_i|\nu_\alpha\rangle = \sum_i U_{\beta i}\mathrm{e}^{-i\frac{m_i^2}{2E}}U^*_{i\alpha}. \tag{40}$$

Transition probability is the square of the probability amplitude. For simplicity, in the case of two generations, probaility of transitions from electron neutrino to muon neutrino is:

$$P(\nu_e \mapsto \nu_\mu) = \sin^2\theta_{12}\sin^2\left(\frac{\Delta m_{12}^2}{4E}L\right). \tag{41}$$

So, if we know the distance to neutrino source and if we can measure its energy, we can determine the mixing angles θ_{ij} and mass squared differences Δm_{ij}^2.

There are some types of experiments to detect neutrinos of different generations to measure mixing angles and mass squared differences. First kind of experiments is to detect neutrinos from the Sun (solar neutrinos have the largest flux of observable neutrinos than neutrino from other sources). In thermonucleus reactions in the Sun a lot of electron neutrinos are emitted, but detectors measure only 1/3 of predicted flux. Experiments of this type are Kamiokande, Super-K (Japan), SNO (Canada), SAGE (Russia) etc. Electronic antineutrinos from nuclear reactors are measured in KamLAND experiment in Japan. Theis experiment yields the following values for the mixing angles and the mass difference squared [9]:

$$\Delta m_{12}^2 = (7.59 \pm 0.21)\cdot 10^{-5}\mathrm{eV}^2, \quad \tan^2\theta_{12} = 0.47 \pm 0.05. \tag{42}$$

In experiments with atmosheric neutrino(Super-Kamiokande, MINOS etc.), other mixing parameters are measured [10]:

$$\Delta m_{23}^2 = (2.43 \pm 0.13)\cdot 10^{-3}\mathrm{eV}^2, \quad \sin^2(2\theta_{23}) > 0.92. \tag{43}$$

For the last mixing angle θ_{13}, only an upper limit from the CHOOZ experiment[11] is obtained:

$$\sin^2(2\theta_{13}) < 5\cdot 10^{-2}, \tag{44}$$

and there is no significal constraint on the CP-violating phase δ.

Future experiments, such as T2K[12] and RENO[13] can improove $\sin^2(2\theta_{13})$ to sensitivity 0.006, $\sin^2(2\theta_{23})$ to sensitivity 0.01 and Δm^2_{23} to $10^{-4}\,\mathrm{eV}^2$[12].

From (43) we obtain widely used value $\sqrt{\Delta m^2_{23}} = (4.9 \pm 0.2) \cdot 10^{-2}\,\mathrm{eV}$, and for simplicity we will use the estimation 0.05 eV. We also notice that $|\Delta m^2_{12}| \ll |\Delta m^2_{23}|$,so $\Delta m^2_{13} \approx \Delta m^2_{23}$. Thus, from our data we can propose two variants of neutrino spectrum:

1. Normal spectrum: $m_1 < m_2 < m_3$,
2. Inverted spectrum: $m_3 < m_1 < m_2$,
3. Quasi-degenerated spectrum: $m_1 \approx m_2 \approx m_3$.

Let us notice that oscillation observations accurately measure the lower bound on the sum of neutrino masses ($\sum_i m_i > 0.05\,\mathrm{eV}$). Other processes we will considered make estimations on its upper bound.

5 Direct searches for neutrino mass

From oscillation observations we can observe only mass squared difference, but not absolute values of neutrino masses.

An obvious approach for a direct mass determination of the electron neutrino is to examine very precisely the end region of the electron spectrum in tritium beta-decay.

Current results, from Meinz[14] and Troitsk[15] groups, are:

$$m_{\nu_e} < 2.1\,\mathrm{eV}. \tag{45}$$

The next generation experiment is KATRIN (the KArlsrue TRItium Neutrino experiment) [16]. It will be able to achieve sensitivity

$$\delta m_{\nu_e} = 0.20\,\mathrm{eV}. \tag{46}$$

Let us define more exactly what means the value m_{ν_e}, measured in this type of experiments. Precisely, m_{ν_e} is not defined, and in theese experiments we measure m_1, m_2 and m_3. But even KATRIN sensitivity is significally more than difference in neutrino masses $\delta m > 0.05eV$, so if KATRIN seek a positive signal, it means that we have quasi-degenerate spectrum of neutrino masses, and $m_{\nu_e} \approx m_1 \approx m_2 \approx m_3 \approx \frac{\sum_i m_i}{3}$.

6 Cosmological constraints on neutrino mass

According to the big bang theory there exists a huge amount of neutrinos in the Universe left over from the big bang, the so-called cosmic background (CMB) neutrinos. If neutrinos are massless, their contribution to the energy density of the Universe is negligibly small (see

[17]), but contribution of massive neutrinos to energy density can be significant and can influence the expansion of the Universe. We know the energy density of the Universe very well because our Universe is flat, so we can calculate constraints on the neutrino masses.

In the process of the expansion of the Universe, CMB neutrino decoupling occurs when the average time between neutrino interactions (and a time of departure from thermal bath) is approximately the Hubble time, or the life-time of the Universe. After that these neutrinos did not interact with the rest. Neutrinos at the decoupling time were ultrarelativistic (we will check it later) and were just in thermal bath, so their average time between interactions is

$$\tau \approx \frac{1}{G_F^2 T^5} \approx \frac{M_{PL}^*}{T^2} = H^{-1}. \tag{47}$$

Here G_F is the Fermi constant, H is the Hubble constant, and $M_{PL}^* = \frac{M_{PL}}{1.66\sqrt{g^*}}$. g^* is the number of degrees of freedom for the SM fields, which were in thermal bath at the moment of the neutrino decoupling. So, neutrino decoupling temperature is about $2\,\mathrm{MeV}$, which means that neutrinos were really ultrarelativistic at that moment. But by the expantion of the Universe neutrinos cooled, and their effective (because they are not in the thermal bath now) temperature is $2 \cdot 10^{-4} eV$. So, at least two of the neutrino types should be non-relativistic.

Thus, neutrino energy density is $\rho_\nu = \sum_i m_i \cdot n_\nu$. Here n_ν is a density of neutrinos, $n_\nu \approx 112 cm^{-3}$. Critical energy density is $\rho_c = h^2 \cdot 10^{-5} \frac{GeV}{cm^3}$. Neutrinos can make contribution to dark matter, so

$$\Omega_\nu = \frac{\rho_\nu}{\rho_c} \approx \left(\frac{\sum_i m_i}{\mathrm{eV}}\right) \cdot 10^{-2} h^{-2} < 0.2. \tag{48}$$

Using the value $h = 0.7$, we obtain:

$$\sum_i m_i < 10\,\mathrm{eV}. \tag{49}$$

But now we can significally improve this criterium. Firstly, neutrinos cannot make up the dominant fraction of the dark matter. Most part of dark matter should be non-relativistic at the decoupling time (cold dark matter) to form cluster structure of the Universe. Secondly, theory of primordial Nucleosynthesis predicts accurate estimation of the rate of Universe expansion at that moment. Also, anisotropy in CMB (measured by WMAP and now being measured by Planck) and redshift galaxy observations can make a contribution to theese data. So, modern path to determine sum of neutrino masses includes statistical analysis of amounts of experimental data [18].

Modern estimation of the upper bound on the sum of the neutrino masses is[18]:

$$\sum_i m_i < 0.5\,\mathrm{eV}. \tag{50}$$

Analysis, made in [19] yield:

$$\sum_i m_i < 0.28\,\mathrm{eV}. \tag{51}$$

But it should be checked by another measurements.

Future astrophysical measurements can decrease the upper limit on neutrino masses. Planck will measure the CMB anisotropy more accurately than WMAP, galaxy and cluster observations can yield more precise data about the rate of expansion of the Universe. So, one anticipates to reach a sensitivity 0.05 eV within the next decade.

7 Neutrinoless double beta decay

Previously described processes can occur both in cases of Dirac and Majorana neutrino masses. The following process, neutrinoless double beta decay, can occur only if neutrinos are Majorana particles.

Beta-decay (single beta-decay) is a process, in which neutron in unstable nuclei decays to proton, electron and electron antineutrino:

$$(A, Z) \to (A, Z+1) + e^- + \bar{\nu}^e. \tag{52}$$

In some nuclei ($^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe$ etc.) single beta-decay is energetically forbidden, but concerning beta-decay of virtual $(A, Z+1)$-nuclei, we obtain double beta decay:

$$(A, Z) \to (A, Z+2) + 2e^- + 2\bar{\nu}^e. \tag{53}$$

Double beta decay ($2\nu\beta\beta$) is a very rare process, suppressed by G_F^2. But if neutrino is a Majorana particle, we should also consider another process, where two electrons are emitted from a nucleon without any neutrinos. This process is called neutrinoless double beta decay ($0\nu\beta\beta$, see Fig.1):

$$(A, Z) \to (A, Z+2) + e^- + e^-. \tag{54}$$

The mass of the nucleon is many times more than the electron mass, so the electron energy spectrum is very close to a delta-function. So, it can be detected on continuous double beta decay spectrum background. Let us discuss the matrix element of the neutrinoless double beta decay. This process is low-energy ($\Delta E \approx 2MeV \ll M_W$), so we can use effective Fermi theory to describe it. The Lagrangian is:

$$L_{int} = 2\frac{G_F}{\sqrt{2}} \cdot \bar{p}\Gamma^\mu n \cdot \bar{e}_L \gamma_\mu U_{ei} \nu_{i,L} + m\bar{\nu}^C_{i\,L}\nu_{i\,L} + h.c. \tag{55}$$

Here U_{ei} is the first row of the PMNS matrix, associated with electron neutrino, $\nu_{i\,L}$ are mass-basis neutrinos, Γ^μ is a nuclear formfactor.

Further we will call $\bar{p}\Gamma^\mu n$ as hadronic current J^μ. So, tree-level S-matrix element is

$$M = \frac{4}{2!}\frac{G_F}{\sqrt{2}}^2 \langle N_f \bar{e}\bar{e}| \int d^4x d^4y J^\mu(x) \bar{e}_L(x)\gamma_\mu U_{ei}\nu_{i\,L}(x) J^\lambda(y) \left(\bar{e}_L(y)\gamma_\lambda U_{ei}\nu_{i,L}(y)\right)^T |N_i\rangle. \tag{56}$$

We use the Wick theorem and obtain (in k-space):

$$M = \frac{4}{2!}\frac{G_F}{\sqrt{2}}^2 \left(\langle N_f| T\left(J^\mu J^\lambda\right)|N_i\rangle \bar{u}(p_1) P_R U_{ei}\gamma_\mu \langle 0| T\left(\nu_{i\,L}(k)\nu_{i\,L}(k)^T\right)|0\rangle \gamma_\lambda^T U_{ei}^T P_R^T \bar{u}_L^T(p_2) - (p_1 \leftrightarrow p_2)\right). \tag{57}$$

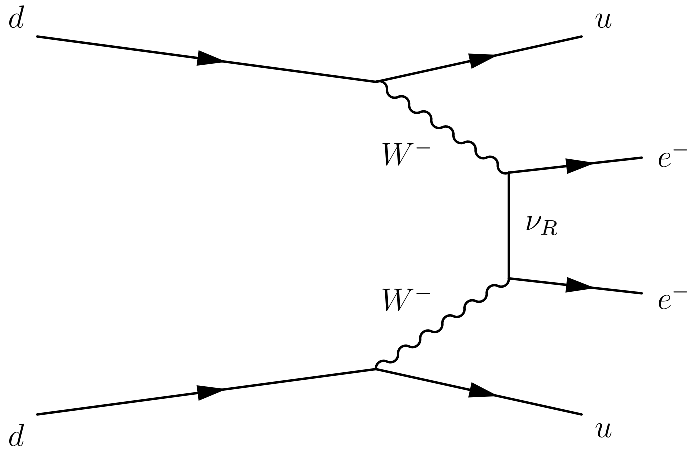

Figure 1: Quark-level diagram of $0\nu\beta\beta$-decay

Here p_1 and p_2 are 4-momenta of the final electrons, k is a 4-momentum of the virtual neutrino. Let us consider the neutrino propagator:

$$\langle 0|T\left(\nu_{i,L}(k)\nu^T_{i,L}(k)\right)|0\rangle = \frac{1-\gamma_5}{2}\langle 0|T\left(\nu_i(k)\nu_i^T(k)\right)|0\rangle \frac{1-\gamma_5^T}{2}, \tag{58}$$

where $\nu_i(k)$ is a 4-component Majorana spinor. From conditions (10) and (12) we obtain: $\nu_i^T = -\bar{\nu}_i(k)C$. We also notice, that $\langle 0|T\left(\nu(k)\bar{\nu}(k)\right)|0\rangle = i\frac{\hat{k}+m}{k^2-m^2}$ is the Dirac fermion propagator. Finally, the Majorana propagator (58) can be presented as:

$$\langle 0|T\left(\nu_{i,L}(k)\nu^T_{i,L}(k)\right)|0\rangle = i\frac{m_i}{k^2-m_i^2}\frac{1-\gamma_5}{2}C. \tag{59}$$

The average momentum of virtual neutrino can be approximated by the uncertainty relation: $|k| \approx \frac{1}{r}$, where r is the average distance between two neutrons in nucleus. Thus, $r \approx 1\text{fm}$, and we have $|k| \approx 200\text{MeV} \gg m_\nu$. So, we can neglect m_i^2 in the denominator of the Majorana propagator. So, transition matrix element M depends on neutrino mass parameters as

$$M \propto \sum_i U_{ei}^2 m_i \equiv m_{\beta\beta}. \tag{60}$$

Its absolute value $|m_{\beta\beta}|$ is called effective Majorana mass. Also S-matrix element depends on nuclear matrix elements, but computing them is very complicated, see [20].

Thus, the half-life (associated with $0\nu\beta\beta$-decay) of some nuclei is proportional to the inverse square of the effective Majorana mass:

$$T^{0\nu}_{1/2} \propto \frac{1}{|m_{\beta\beta}|^2}. \tag{61}$$

So, if we measure some events of $0\nu\beta\beta$-decay in a $2\nu\beta\beta$-decaying nucleus (for example, ^{76}Ge), we count the half-life, and calculate the effective Majorana mass $|m_{\beta\beta}|$.

Taking into account PMNS-matrix parametrization (32) and a negligible small value of θ_{13}, we obtain:

$$|m_{\beta\beta}| = \left| m_1 \cos^2\theta_{12} e^{2i\alpha_1} + m_2 \sin^2\theta_{12} e^{2i\alpha_2} + m_3 \sin^2\theta_{13} \right|. \tag{62}$$

We do not have any other sources to measure the Majorana phases α_i, so they are free parameters. Mass of (for example) the first neutrino m_1 is a free parameter too. Previously we have considered three types of neutrino mass spectrum. Let us consider effective Majorana mass in theese three cases.

1. Normal spectrum, $m_1 \ll m_2 < m_3$. So, $m_2 \approx \sqrt{\Delta m^2_{12}}$ and $m_3 \approx \sqrt{\Delta m^2_{13}}$. After putting theese conditions and numerical values of PMNS matrix parameters in (62), we obtain:
$$0.3 \cdot 10^{-3}\,\mathrm{eV} < |m_{\beta\beta}| < 5.3 \cdot 10^{-3}\,\mathrm{eV}. \tag{63}$$
2. Inverted spectrum, $m_3 \ll m_1 < m_2$. Here we have $m_1 \approx m_2 \approx \sqrt{\Delta m^2_{13}}$. Concerning numerical values, we have:
$$0.02\,\mathrm{eV} < |m_{\beta\beta}| < 0.05\,\mathrm{eV}. \tag{64}$$
3. Quasi-degenerate spectrum, $m_1 \approx m_2 \approx m_3 >> \sqrt{\Delta m^2_{ij}}$. So, we have:
$$0 < |m_{\beta\beta}| < \frac{\sum_i m_i}{3}. \tag{65}$$

If we take into account cosmological estimates (50), we will have in all cases:

$$m_{\beta\beta} < 0.16\,\mathrm{eV}. \tag{66}$$

This is the current upper bound on the effective mass of $0\nu\beta\beta$-decay.

7.1 Experiments searching $0\nu\beta\beta$-decay

There are many experiments searching for $0\nu\beta\beta$-decay using different nuclei, but there is no strict evidence for it yet. These experiments yield only lower bound on the half-life of these nuclei, and hence yield an upper bound on the neutrino masses.

The lowest upper bound on effective Majorana mass was achieved in the Heidelberg-Moscow experiment. This experiment searched for the transition $^{76}Ge \to ^{76}Se + e^- + e^-$. From its data the upper bound on the effective Majorana mass was obtained:

$$|m_{\beta\beta}| < 0.35\,\mathrm{eV}. \tag{67}$$

This bound does not contradict the cosmological predictions (66). But the future experiments can have upgraded sensitivity than the bound (66), and possibly can detect $0\nu\beta\beta$-decay. New technique [21] will be used, and in the CUORE [22] experiment sensitivity $|m_{\beta\beta}| \approx 3 \cdot 10^{-2}\,\mathrm{eV}$ should be achieved, and EXO [23] expepiment will be able to measure an effectve Majorana mass $|m_{\beta\beta}| > 1.5 \cdot 10^{-2}\,\mathrm{eV}$.

8 Leptogenesis

The origin of the matter-antimatter asymmetry in the Universe is one of the most important questions in cosmology. This asymmetry can be quantatively described by the value

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} = (6.1 \pm 0.3) \cdot 10^{-10}. \tag{68}$$

It is unkilely to be primordial baryon asymmetry after inflation, it seems it was creating during the hot Big Bang phase. The process creating baryon-antibaryon asymmetry is called Baryogenesis. Sakharov [24] formulated three conditions for Baryogenesis.

1. Firstly, Baryon (or Lepton) number should be violated.

2. Secondly, one must have also C and CP - violation.

3. Thirdly, these processes should be out of thermal equilibrium.

So, if theese processes are in thermal equilibrium, no baryon asymmetry will be generated.

It is impossible to explain baryogenesis only in pure SM. There is $U_{B+L}(1)$-violation in SM on quantum level, but the probability of such processes at low energies is very small, suppressed by

$$\mathrm{e}^{-4\pi\alpha_W} \approx 10^{-170}. \tag{69}$$

In thermal bath it changes dramatically: the rate of transitions with $(B+L)$-violation is suppressed only by the Boltzmann factor $\mathrm{e}^{-\frac{E_{sph}(T)}{T}}$ [26]. At values $T > E_{sph}$ there are classical $(B+L)$-violating transitions. Here $E_{sph}(T)$ is the energy of the sphaleron in the electroweaak theory with Higgs doublet. Detailed calculation and comparison with the expansion rate of the Universe shows [27] that this process happens at temperatures $T > 100\,\mathrm{GeV}$. So, if we have $(B-L)$-asymmetry, created by non-SM processes, it follwos in thermal bath to [28]:

$$B = C \cdot (B-L), \qquad L = C \cdot (B-L). \tag{70}$$

Here $C \approx 0.4$.

This (B-L)-asymmetry may be created by the Leptogenesis (proposed by Fukugita and Yanagida [25]). Let us consider the see-saw Lagrangian (34), considering Dirac mass term as Yukawa couplings with Higgs doublet. Yukawa coupling matrix should have nonzero CP-violating phase. Let us consider ν_{R1} to be the lightest of right-handed neutrinos, so asymmetry of the heavier neutrino decays will be washed out on thermal bath, and we should count the decay rate of these processes: $\nu_{R,1} \to H + l_j$ and $\nu_{R,1} \to H + \bar{l}_j$. Taking into account tree-level contribution and one-loop corrections, we will obtain:

$$\Gamma(\nu_{R1} \to lH) \propto \sum_\alpha \left| y_{1\alpha} + \sum_{\beta,\gamma} D\left(\frac{M_1}{M_\gamma}\right) \cdot y^*_{1\beta} y_{\gamma\alpha} y_{\gamma\beta} \right|^2. \tag{71}$$

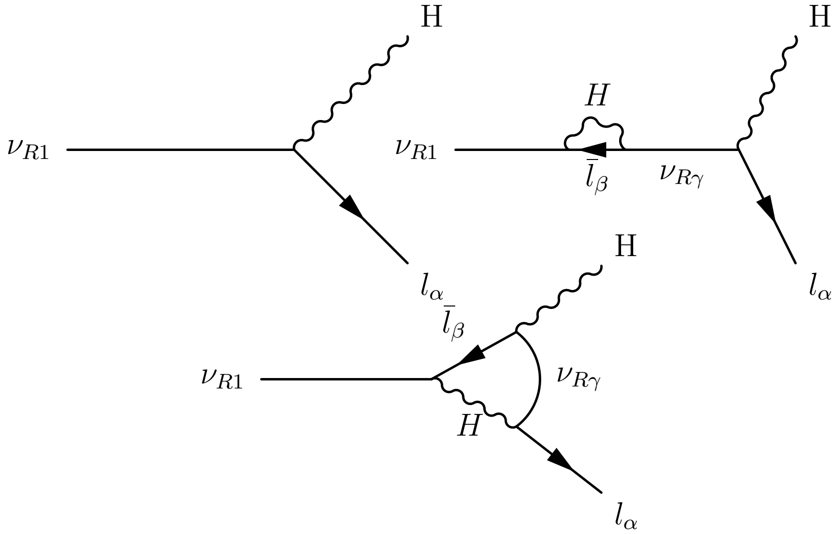

Figure 2: Tree-level and one-loop contribution to Leptogenesis

Here $D(x)$ means one-loop contribution, $f(x) = 8\pi \cdot \mathrm{Im}\, D(x)$.

We define microscopic lepton asymmetry as

$$\delta = \frac{\Gamma(\nu_{R1} \to lH) - \Gamma(\nu_{R1} \to \bar{l}H)}{\Gamma_{total}} = \tag{72}$$

$$= \frac{1}{8\pi} \sum_{\gamma=2,3} f\left(\frac{M_1}{M_\gamma}\right) \cdot \frac{\mathrm{Im}\left(\sum_{\alpha\beta} y_{1\alpha} y_{1\beta} y^*_{\gamma\alpha} y^*_{\gamma\beta}\right)}{\sum_\alpha |y_{1\alpha}|^2}.$$

We note that lepton asymmetry does not directly depend on CP-violating phase δ in PMNS matrix as we do not use flavour basis in Leptogenesis. In the case $M_1 \ll M_{2,3}$ we can simplify equation (72):

$$\delta = -\frac{M_1}{6\pi\, v^2} \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta} \mathrm{Im}\left(y_{1\alpha} m^*_{\alpha\beta} y_{1\beta}\right), \tag{73}$$

where $m_{\alpha\beta} = \sum_{\gamma=2,3} y_{\gamma\beta} \frac{v^2}{2M_\gamma} y_{\gamma\alpha}$, and v is VEV of the Higgs field.

Let us propose that the most part of lepton asymmetry was created near the decoupling moment:

$$\Gamma_{tot} \approx H(T = M_1). \tag{74}$$

Using values of the Hubble constant and the total decay rate, we obtain:

$$\widetilde{m}_1 \approx \frac{4\pi}{M^*_{PL}} \cdot v^2 \approx 10^{-3} eV, \tag{75}$$

where

$$\widetilde{m}_1 \approx \sum_\alpha \frac{|y_{1\alpha}|^2}{2M_1} \cdot v^2 \geq \sum_\alpha \frac{|y_{1\alpha}|^2}{2M_\alpha} \cdot v^2 = m_1. \tag{76}$$

So, in this case the mass of the lightest neutrino should be smaller than 10^{-3}eV. The microscopic lepton asymmetry is

$$\delta < \frac{M_1}{6\pi v^2} m_{atm}. \tag{77}$$

Lepton asymmetry Δ_L can be calculated as $\Delta_L \approx \frac{\delta}{g_*}$. So, from the condition (68) we obtain $M_1 \approx 10^8$GeV. In this case sum of the neutrino masses is about $m_{atm} = 0.05$eV.

We can also consider the case where the proposal (74) does not hold, and we should consider the processes of creating and washing out $(B - L)$-asymmetry in the thermal bath. Detailed calculations [17] shows that the mass of lightest right-handed neutrino should be $10^8 - 10^{12}$GeV, and yields the upper bound on sum of neutrino masses $\sum_i m_{\nu_i} < 0.3$eV. These data are in good agreement with the cosmology data from the Universe expansion!

9 Conclusion

In this review we defined Majorana neutrinos, considered see-saw mechanism, and reviewed recent data from neutrino oscillations, beta-decay and neutrinoless double beta decay, and cosmology. We also reviewed the bounds on the neutrino masses from them.

Both in the cases of Dirac and Majorana neutrinos we have a lower bound on the sum of the neutrino masses $m_{atm} \approx 0.05$ eV from the neutrino oscillation measurements.

In the case of Dirac neutrinos we can obtain an upper bound for the neutrino masses from direct measurements (tritium beta decay) and cosmology. The first one is more direct, the cosmology constraint can have large systemstic error. Not considering this probable error, this method is the most accurate and made an upper bound for neutrino masses 0.5 eV. So, it is unlikely that the future experiments (KATRIN) for direct searching neutrino masses will succeed. But if it succeded, it will be a signal for something new in cosmology.

In the case of Majorana neutrino masses we reviewed also neutrinoless double decay experiments. The past observations yielded no evidence for $0\nu\beta\beta$-decay in agreement with cosmological data. The measured value, $|m_{\beta\beta}|$, consists Majorana phase, and $0\nu\beta\beta$-decay is an exclusive experiment to measure it. In future experiments we can observe $0\nu\beta\beta$-decay if we are lucky and Majorana phases act coherently, and will not be able to observe it if theese phases lead to distinguish significally. If neutrinoless beta decay is established, it will be a strong confirmation of the Majorana nature of the neutrinos. But the absence of $0\nu\beta\beta$-decay in future experiments is not able to reject theories with Majorana masses.

If neutrinos are really Majorana particles and they recieve their masses by see-saw mechanism, it seems natural that baryon-antibaryon asymmetry was created by Leptogenesis. We can obtain an upper bound on the sum of neutrino masses $\sum_i m_i < 0.3$ eV. But it is not a reliable method for make accurate estimation of the neutrino masses.

These measurements can succeed in near future only in the case of quasi-degenerate neutrino masses. If we have strict hiearchy of neutrino masses, the present upper bounds are far from the realistic sum of neutrino masses.

The present bounds on the sum of neutrino masses are $0.05\text{eV} < \sum_i m_i < 0.5\text{eV}$. It is not a large window. Future experiments can significantly improve these bounds. Firstly, we wait for the progress in cosmology measurements. By optimistic proposals, one can decrease the upper bound on the neutrino masses to 0.05 eV. It means we will really find this value. If we are not so optimistic, we can decrease this bound too. Secondly, we are able to find neutrinoless double beta decay. And may be we will suddenly find the evidence for the electron neutrino mass in the KATRIN experiment.

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