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Analyses of the energy jitter in FLASH
(Free Electron Laser)

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CONTENTS

ABSTRACT

In this report, we compare two methods to obtain the electron beam energy from beam position measurement. The first method (called here the spectrometer method) uses the information from a single BPM (Beam Position Monitor) located downstream a dipole magnet. The second method applies the SVD factorization method to the beam monitor position measured with all the BPMs along FLASH for a large number of bunches. Both methods have been applied to the beam trajectory simulation data and experimental data and the results are shown. The accuracy of both methods is discussed in detail.

1. INTRODUCTION

Particle accelerators are based on the interaction of the electric charge of the accelerated particles with the static and dynamic electromagnetic fields produced by the accelerator itself. The technical realization of this interaction defines the different types of particle accelerators. Basically there exist two types of accelerators: linear and circular accelerators. In circular accelerators the particle track is bent into a circle whereas in the linear ones the track is straight.

The advantages of the circulars accelerator are that the ring topology allows for continuous acceleration, at the same energy reach they can have a smaller size than the linear accelerators and furthermore the continuous crossing of the beams increases the luminosity. For high energy accelerators the disadvantage is the loss of energy due to synchrotron radiation which limits the energy of the accelerator. This radiation however depends on the size of the accelerator (its radius) and the mass of the particles (the nature) being smaller for heavy particles (protons with respect to electrons, $\sim 1/m^4$, m is the mass of the particle). Due to the characteristic of this radiation this problem can turn into an advantage for fields like material science or biology allowing for detailed studies of the composition of materials and tissues, etc... Examples of circular high energy accelerators are LEP (e+e-) and LHC (proton-proton) at CERN (Geneva) with a circumference of about 30 km and there are many examples of accelerators using the synchrotron radiation light all over the world and especially at DESY with its future XFEL facility.

On the other hand in linear accelerators electrons do not loose much energy because of synchrotron radiation hence light particles like electrons (e-) or positrons (e+) can be accelerated to higher energies than in circular accelerators. The disadvantage in this case being the low luminosity acquired as the beams can cross only once. To overcome this feature it is then necessary to use very small beams (nano-beams) and to control their position with extremely high accuracy in order to optimize the beam colliding conditions.

FLASH is VUV (vacuum ultra violet) Free Electron Laser (FEL) driven by a 1 GeV (one billion Electron Volt) linear accelerator (linac) using a superconducting technology for accelerator cavities. It provides photons to experiments covering a wide range of wavelength possibilities from short ultraviolet radiation up to soft x rays. In FELs the electron beam is accelerated almost to the speed of the light, after which the relativistic beam is conducted through transverse a magnetic field which changes its direction periodically (an undulator magnet). The beam then emits very intense coherent synchrotron radiation as it is sketched in figure 1.

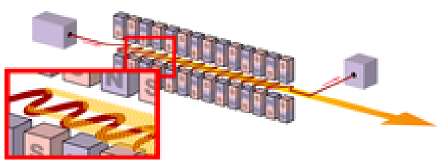

Figure 1: Schematic of the operation of the Free Electron Laser

An undulator is a periodic structure of dipole magnets along which the static magnetic field is alternating. With an undulator one can achieve very intense radiation, concentrated in narrow bands in the spectrum. The total length of the FLASH is 260 meters. The main parts of the FLASH are shown in figure 2 and are described following:

- the first component is a photo injector where the electrons are produced. An UV laser is used to produce ultra-short electron bunches from the photocathode,
- along the accelerator there are quadrupoles and sextupoles, magnetic lenses to focus and the beam,
- the next part consist of all the superconducting accelerator modules where the particles are accelerated. Each of the accelerator modules (so called cryomodules) contain a string of eight superconducting microwave cavities operating at a frequency of 1.3 GHz. The axis of these cavities and the quadrupoles need to be correctly aligned. Each of the cavities is encased in a titanium helium vessel. The operating temperature of the helium lines connected to the cavities and the magnets is 2 K and they withstand a pressure of 4 bar,
- the bunch compressor sections consist of bending magnets .Bunch compressors are used to reduce the bunch length from the electron gun to about 25 microns (required for the laser). In the bending magnet systems the compression is achieved using the path length difference for particles of different energies,
- the collimator section is used to remove large amplitude and off-energy electrons which otherwise would be lost in the undulator and damage it.

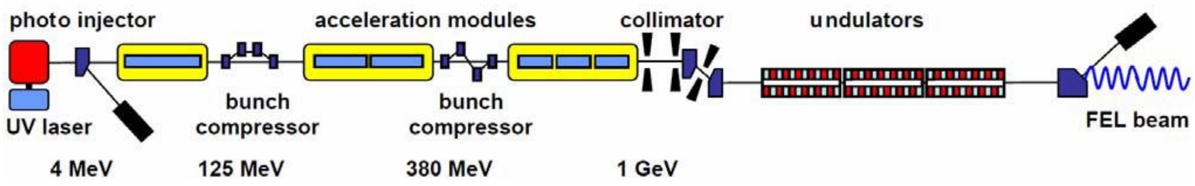

Figure 2: Schematic view of FLASH

In this work thus the measurement of the bema energy of the beam in FLASH is examined. The methodology is described in the section 2 and the results are presented in section 3. Finally the main conclusions are summarized in section 4.

2. SETUP AND METHODOLOGY

Currently over 15 TBytes of data were collected during a period of two weeks experimental run at FLASH using the Data Acquisition System (DAQ). This work however considers a small part of this data set corresponding to only 213 pulses. A pulse is a “train of bunches” at frequency of 200 kHz each bunch and being 800 microsecond long. The trains are pulsed at 5 Hz.

We use data from a Toroid which is a device to measure the charge of the particles in the beam. In fact the first Toroid, the Toroid/3GUN, serves to know if the signal corresponds to a real bunch. Beam Position Monitor (BPM) measure the position of the beam.

The energy of the beam is measured after the accelerator by a dipole with a BPM (the BPM 6 BYP) used together as a spectrometer.

A BPM is a device to detect and record the transverse location of the beam bunches moving through a particular segment of the accelerator as it is shown in figure 3. The BPM shown in the figure 3 consist of four metal strips on the inside of the accelerator structure, connected to wires that extend outside the structure and are grounded. If the beam does not pass through the centre of the monitor, there is a potential difference resulting, and these signals allow us to determine the offset of the beam. The BPMs are very useful to monitor the trajectory of the beams. Since the energy of the beam is correlated with its trajectory deviations of the beam trajectory will also correspond to deviations in the energy of the beam. In this way it is possible to detect or understand causes of the beam deviation or detect a malfunctioning of the BPMs themselves.

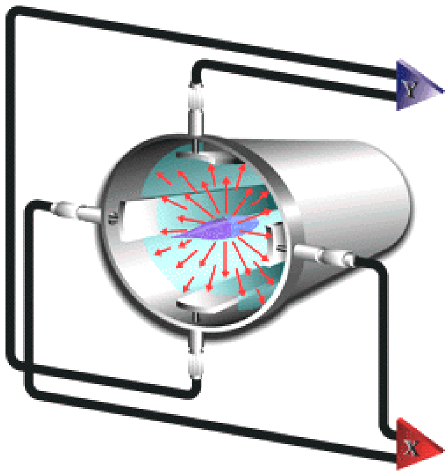

Figure 3. Scheme of a BMP.

The energy of the beam was measured by two methods. The first method used both a dipole and a BPM as a spectrometer (BPM 6BYP). The second method was the Singular Value Decomposition (SVD) method which is defined below.

The principles of the first method are described in the following.When a particle moves into a magnetic field the particle suffers the Lorentz force which is given by the following equation:

$$\vec{F} = q(\vec{v} \times \vec{B}) \ (1)$$

Where q is charge of the particle, v is its velocity and B intensity of the magnetic field.

If the magnetic field is perpendicular to the trajectory, the trajectory of the particle describes an arc. The radius of the orbit of the particle can be determinate applying the dynamic equation of uniform circular motion: force equals mass times normal acceleration:

$$|F| = qvB = m\frac{v^2}{r} \ \rightarrow \ \frac{1}{r} = e\frac{B}{mv} \quad (2)$$

In the above equation m is the relativistic mass of the particle, r the radius of the orbit described by the particle and v the velocity of the particle.

When the particle crosses the dipole its trajectory is diverted by an angle α. This angle is defined as:

$$\alpha = \int \frac{ds}{r} = \frac{e}{p}\int B\, ds \ \ (3)$$

Figure (4) shows the trajectory of a charge particle after it crosses a dipole providing a constant magnetic field and all the parameters that are going to be discussed next.

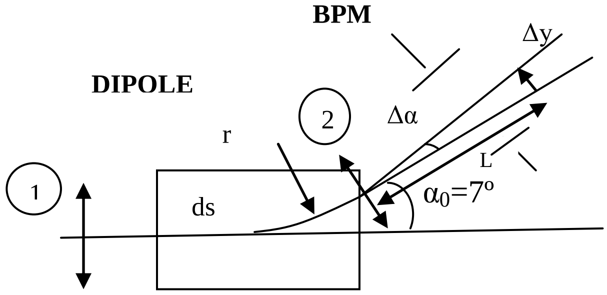

Figure (4). Scheme of the trajectory of a charge particle crossing a dipole.

The radius of the curved trajectory of the particle inside the dipole is *r*, ds is the differential arc, α_0 is a fix angle that defines the kick of the beam for the design energy, α is the angle that determines the deflection due to the energy uncertainty of the particle, L is the distance to which the position of the bunch is measured, and Δy is the position of the bunch with respect to the monitor centre. It is important to say that the coordinate system changes when the particles cross the dipole and they are kicked. In the figure 4 this is indicated by a circle with a (1) as the first system of coordinates and an arrow determining the y direction and analogously with a circle (2) as the second coordinate system.

In the equation 3 the integral goes along the trajectory. Using the two previous equations the momentum of the particle can be obtained as:

$$p = \frac{e}{\alpha}\int B\ ds \quad (4)$$

And finally considering that the particles accelerated in accelerators are ultrarelativistics and taking into account that in this case the energy is given by E=pc the following relation is obtained:

$$E = \frac{e}{\alpha}\int B\ ds \ \rightarrow\ E = \frac{C}{\alpha} \ (5)$$

where C is a constant.

From equation (5) thus one can conclude that the angle is inversely proportional to the energy.

Another quantity which is important to consider is the so called dispersion, *D*, which relates the variation of the energy with the variation of the beam position. The definition of the dispersion is:

$$D = \frac{y}{\Delta E/E} \ (6)$$

The dispersion obviously can change along the machine and depends on the system but if the value of the dispersion is known the energy can be obtained as:

$$\frac{\Delta E}{E} = \frac{y}{D} \ (7)$$

For illustration, the expression of the dispersion in the simplest case is going to be derived below. This case corresponds to the situation when a beam goes through a dipole and between the dipole and the BPM there are no additional magnets. Unfortunately this is not completely the real situation because in FLASH there are optics components between the dipole and the BPM that we use to measure the energy (there is a sextupole and the 6BYP BPM is situated in the middle of a quadrupole). It is however interesting to show how to get the expression for the dispersion in the simplest case (the real case is too complicated to treat here). To follow the demonstration it is necessary to refer again to the figure 4.In the there the following relation is satisfied:

$$y = L * \Delta\alpha = L * (\alpha - \alpha_0) \ (8)$$

Differentiating equation (8) we obtain:

$$dy = L * d\alpha \ \ (9)$$

Substituting in equation (6):

$$D = \frac{L * d\alpha}{dE/E} \ \ (10)$$

Then solving α from equation (5), differentiating and introducing the result again in equation (9):

$$\alpha = \frac{C}{E} \ \rightarrow \ d\alpha = \frac{-C * dE}{E^2} \ \ \rightarrow \ D = \frac{-L * C * dE/(E^2)}{dE/E} = -L * \alpha \ \ (11)$$

And so the dispersion for the simple case is obtained. In the real case at FLASH the procedure must include the effect of the optics components which here has been neglected. In the following figure we can see the dispersion function for FLASH.

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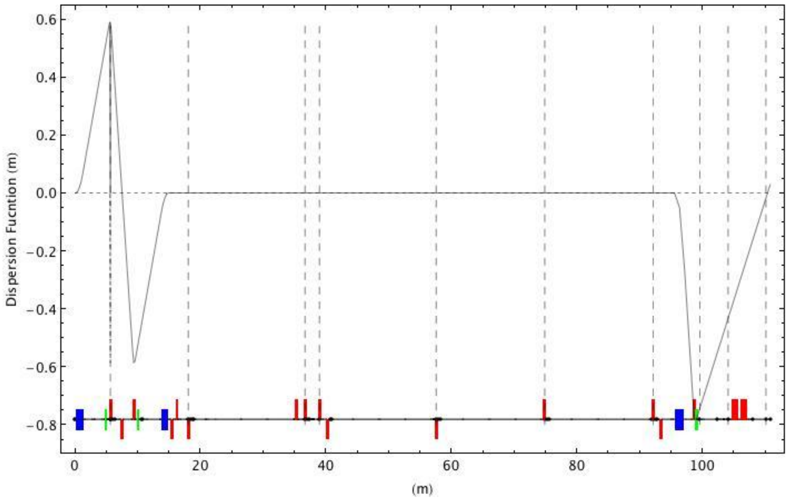

Figure (5).Dispersion for the FLASH bypass. The BPM 6BYP correspond with the first dashed lines around 6 m from the 0. The colour coding for the magnet key is blue for dipoles, red for quadrupoles (up focusing, down defocusing) and green for sextupoles.

In the figure (6) we can see the design vertical disperseon for the FLASH bypass. On the other hand we can see that the horizontal design dispersion which is zero. The dashed lines indicate the location of the BPMs in the bypass. The plot shows the dispersion after the first dipole at FLASH.

The error associated to the position variation given by the BPM situated after the dipole is:

$$\frac{\Delta E}{E} = \frac{y_{noise} + D\left(\frac{\Delta E}{E}\right) + y_{trajectories\ oscilations}}{D} \quad (12)$$

Where $y_{\text{trajectory oscillation}}$ is the uncertainty associated to the different initial conditions of the particles entering the dipole (the angle and position). Other sources of uncertainties in this expression are the noise contribution and a systematic error from the BPM. In this work there is no discuss on the systematic errors associated to the BPM and only the electronic noise contribution is considered.

The second method to obtain the energy is using the Singular Value Decomposition factorization. The basic principle consists in identifying correlated beam motion at the different BPMs by a singular value decomposition of a BPM matrix that contains consecutively measured trajectory vectors. The trajectory vector contains the information of the trajectory of each bunch for one pulse given by all the BPMs. The number of significant singular values equals the number of independently jittering variables in the system, eg.energy, offset and angle. The remaining values are measured of uncorrelated noise, for example, caused by the BPM electronics.

In the present case this is the decomposition of the orbit matrix:

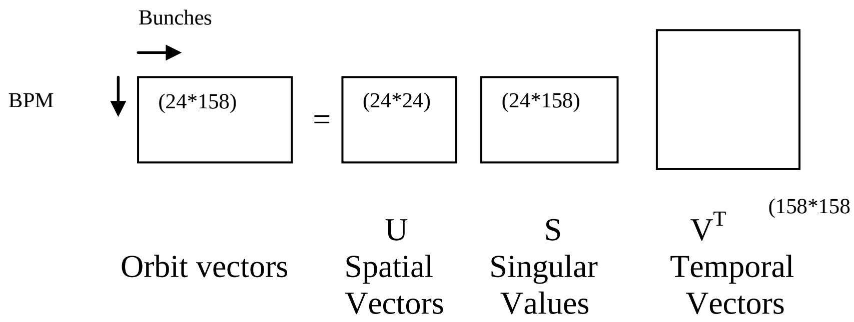

Figure (6). Scheme of the matrix decomposition given by SVD

The U and V matrices are orthonormal and the S is a diagonal matrix and contains the singular values. It turns out that the row vectors of the matrix U point into the direction of linear independent modes of orbit jitter. They are called spatial vectors and are orthogonal to each other. Typical patterns are trajectory oscillation or patterns proportional to the dispersion function, caused by energy jitter.

The column vectors of V describe the time development of the corresponding orbit patterns in U and are called temporal vectors. The vectors with the largest corresponding singular values contain the prominent jitter modes.

In the present work the SVD method is used to obtain the energy from simulated beam measurement and also to obtain the energy from real beam measurements.

3. RESULTS

3.1 STATISTICAL ANALYSIS

We started doing a statistical analysis of the data given by the BPMs. The objective of this part is to compare the contribution of the variation in the energy of each bunch in a determinate pulse and the noise in the measure of the position given by the BPMs. We start doing a plot for each BPM, position as a function of energy, to see the correlations between the energy and the position. For the BPM 6BYP we expected to get linear correlation because the energy is calculated from the signal taken from this BPM, we can see this in the following plot:

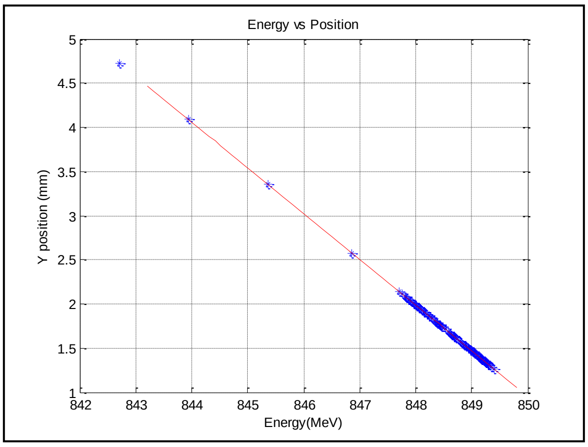

Figure (6). Position versus energy for the BPM 6BY

From the other BPM we don't expect to get so good correlation. This is the plot for one of the BPMs.

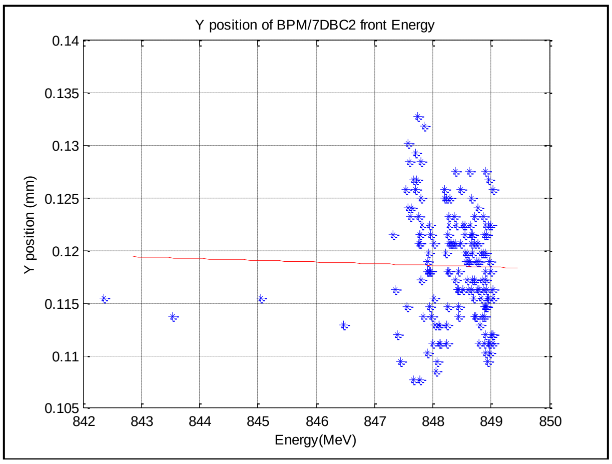

Figure (7). Position versus energy for the BPM 10DUMP

First we wanted to compare the energy and noise contributions to the uncertainty of the beam position (X and Y) taking data that corresponded to one pulse (including all the positions of the bunches along the accelerator). For this we calculated first the standard deviation the beam position (X and Y) in data for all the bunches corresponding for one pulse (blue line in the following plot). Then we calculated the standard deviation about the same data but subtracting the energy contribution. For this we proceeded as it is indicated in figures (6) and (7), then we calculated the fit for each BPM and we extracted at each point the corresponding fit (red line in figure 8). Later we calculated the standard deviation corresponding to the signals without charge thus obtaining the standard deviation of the noise (green line in the following plot).

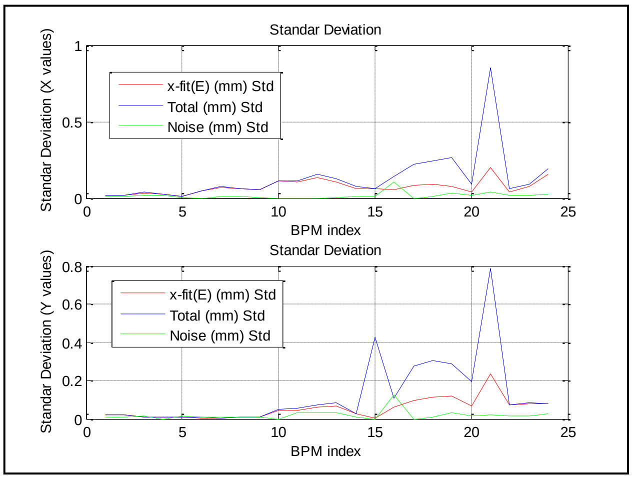

Figure (8). Position versus energy for the BPM 10DUMP

With this exercise we concluded that when the ''energy jitter'' is large, there is also a large energy correlation (dispersion). This is why the jitter is reduced when this correlation (linear fit) is removed. To be noticed that up to BPM 14 removing the fit does not really reduce the noise, indicating that this jitter is not correlated with the energy (low dispersion).

3.2 SIMULATION RESULTS

We made simulated BPM noise which is a matrix M^{noise} with a size (24*158) with Gaussian random numbers with a σ =0.01mm.The number of the rows correspond to the number of BPMs and the number of columns correspond to the number of bunches.

The dispersion for each BPM is equal:

$$D_i = \begin{cases} 0 & (i < 15) \quad (13) \\ \sin\left((i-14) \times {}^{\pi}/_{3}\right) & (i \geq 15) \end{cases}$$

where i is the index of BPM, and π is a constant. D is in units of meters, r_j is a random number from interval $[-1,1]$ which represents the $\frac{\Delta E}{E}$ jitter in the units of 10^{-3} ,and the index j is the number of the bunches.

Then we made another matrix M which is define below:

$$M_{ij}^{E} = D_i \times r_j = D_i \times \frac{\Delta E_j}{E} \quad (14)$$

$$M^{sim} = M^{E} + M^{noise} \quad (15)$$

where M^{sim} is a matrix of simulated beam measurement shows in figure 9.

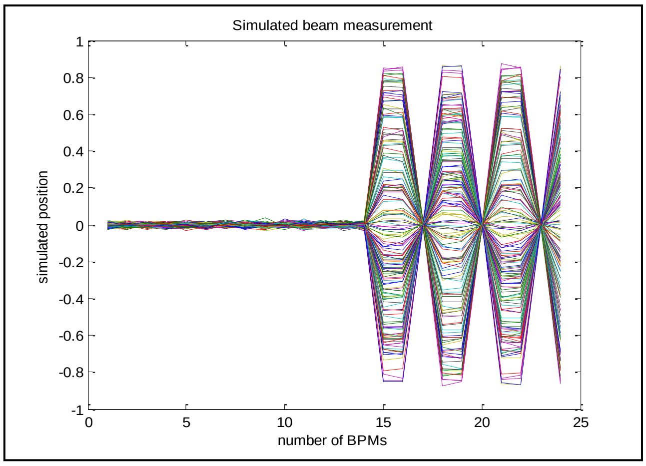

Figure (9). Simulated beam measurement

We apply SVD to M^{sim} .In the figure (10) we can see the singular values. As we can see the first singular value is the biggest and it shows that there is a correlation between several BPMs and it corresponds to the motion that we put as an input in matrix M^E .

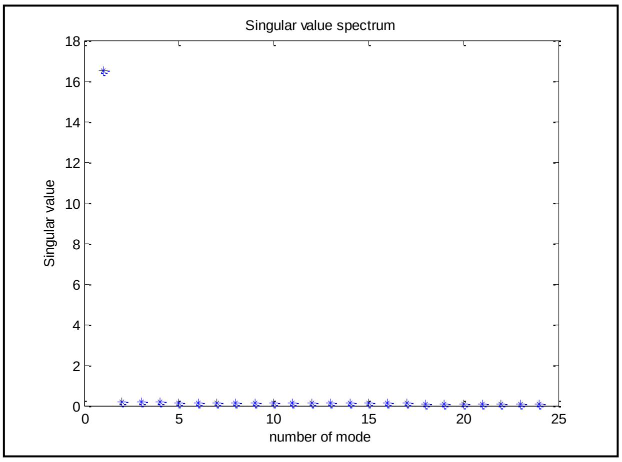

Figure (10). Singular value spectrum

Now we want to calculate energy extracted from SVD.

The spatial vector corresponding to the singular value S_{11} is U_{i1} , and corresponding temporal vector is V_{j1}.

We use the same BPM as in the experiment data from the FLASH. That is BPM6BYP.

Here is the expression of dispersion for the BPM6BYP:

$$D_{15} = \sin\frac{\pi}{3} \quad (16)$$

First we calculate y position data from this BPM for each bunch:

$$y_{15,j} = U_{15,1} \times S_{1,1} \times V_{j,1} \quad (17)$$

So the relative energy extracted from SVD is equal:

$$\frac{\Delta E^{SVD}}{E}_{j} = \frac{y_{15,j}}{D_{15}} \quad (18)$$

Then we made the plot of $\frac{\Delta E}{E}^{SVD}{}_{j}$ from r_j .This plot gives as an idea about the error of measurement from SVD method. As you can see for the chosen σ it is almost linear.

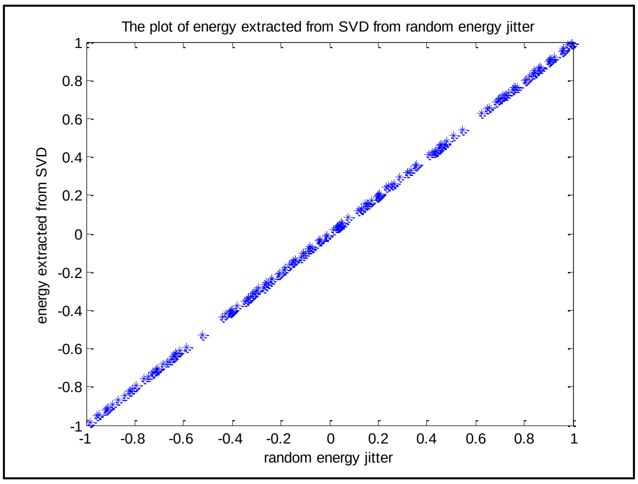

Figure (11). Energy extracted from SVD from random energy jitter.

Then we calculate the standard deviation of $\left(\frac{\Delta E}{E}^{SVD}{}_{j} - r_j\right)$ which is the error of the simulated energy measurement from SVD method. The sigma is approximately 0.004.

The energy from spectrometer is the following:

$$\left(\frac{\Delta E}{E}\right)_j^{spect} = \frac{M_{15,j}^{sim}}{D_{15}} \quad (19)$$

The standard deviation is 0.0123 values. The standard deviation of $(\frac{\Delta E}{E}^{spect}{}_{j}$-$r_j)$ is error of the simulated energy measurement from spectrometer.

Then we made a plot of error of the simulated energy measurement from spectrometer and error of the simulated energy measurement from SVD method for different BPM noise (see figure 11).

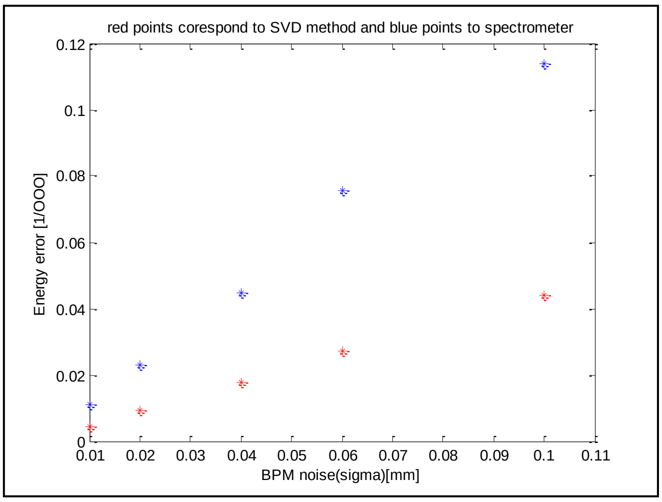

Figure (12). Error of simulated energy measurement from BPM noise

As we can see the error of the measurement from spectrometer is larger than the error of measurement in SVD method.

3.3 EXPERIMENTAL RESULTS

The trajectory measured by 24 BPM over 158 bunches is shown in figure 13.

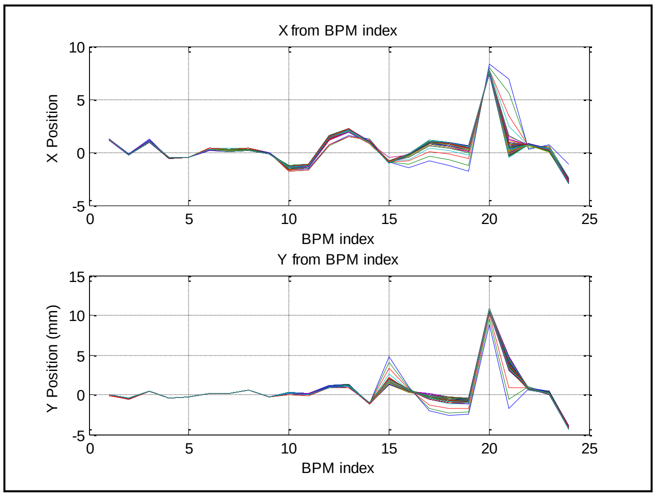

Figure (13). Trajectory of all the bunches in x any plane.

Using the SVD method we factorized the trajectory matrix with 24 BPMs and 158 bunches. The trajectory matrix contain real information about the trajectory of all the bunches that compose a single pulse. The values that compose the trajectory matrix are given by the following:

$$\hat{y}_{ij} = y_{ij} - \overline{y_{iJ}} \quad (20)$$

where *i* is the number of the BPM and *j* the number of the bunch.

The procedure is the same of that in the simulated matrix used in the previous part. SVD decomposition can be applied to the trajectory matrix. And the plot corresponding to the singular values is the following:

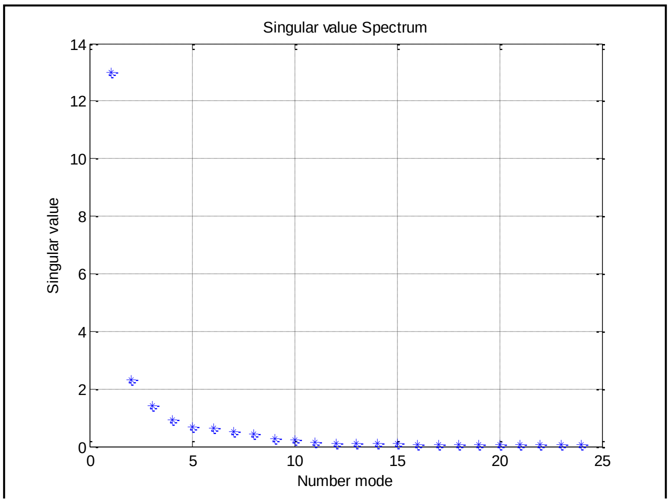

Figure (14). Singular value Spectrum.

Figure 8 shows that the larger correlation corresponds with the energy. Therefore we extract the correlation motion modes corresponding with the energy from the SVD decomposition for each bunch if we considered the first singular value of the matrix S of the SVD decomposition and the corresponding spatial value (for the BPM 6BYP) and the temporal vector from the U and V matrix. We can do this because according with the results corresponding Taking in account the figure (4) and the decomposition of the orbit matrix=U*S*V^T we do the following calculation:

$$Y_{j1} = U_{15,1} * S_{1,1} * V_{j,1} + \overline{M_{15,J}} \quad (21)$$

Where *j* is the bunch number, 15 is the index of the BPM that it's used to measure the enrgy,and then we add the mean of the orbit matrix from the BPM 6BYP because we want to obtain at the end he energy (if we don't add this we calculate the variation of energy).

We take the equation corresponding with the fit into the plot position versus energy corresponding with the BPM 6BYP because we want to calculate the energy from the position measure, see the figure (6). But in this case Y is given by the SVD decomposition equation 12:

$$Y_j = a_1 E_{svd,j} + a_0 \quad (22)$$

$$E_{svd,j} = \frac{Y_j - a_0}{a_1} \quad (23)$$

Where a_1, a_0 are the parameters of the fit corresponding to the figure (6),a_1=*-0.5175 (mm/MeV)and* a_0=*440.8542(mm)* , *j* determinate the number of bunch.

This is the way to extract the energy using the SVD method, now we plot this energy from the experimental data energy to see if the results are consistent (figure 15 upper sides). In the following figure we have also a plot with the dispersion of each BPM with data given by the SVD decomposition. In the figure 15 we have also the dispersion of each BPM. We have calculated the dispersion with this expression:

$$D_i = U_{i,1} \times \frac{a_1}{U_{15,1}} \quad (24)$$

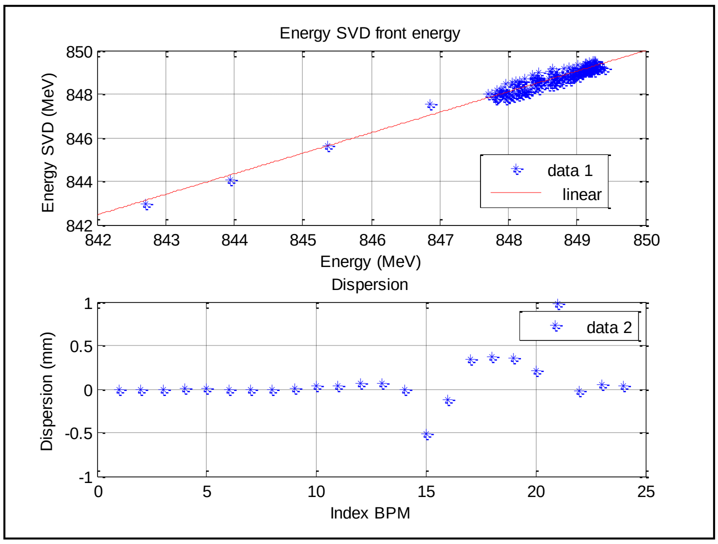

Figure (15). Energy from SVD method front experimental energy data from the spectrometer method. And the second plot shows the dispersion of each BPM.

Now we can calculate the standard deviation of the difference between the energy from the SVD method and the energy measured by the spectrometer to estimate the difference between these two measures. We consider that the values from the SVD method are better that the measures from the spectrometer for two reasons. The first reason is that with the SVD method we get the energy values from data of all the BPMs, we have more measurements and therefore a minor error. The second reason is that with the SVD method we separate the correlation, we take only energy mode (the stronger signal value), and we filter out betatron oscillation, for example. The standard deviation of the differences ($E_{SVD} - E_{spect}$):

$$\sigma\left(E_{SVD} - E_{spect}\right) = \sqrt{\sigma_{SVD}^2 + \sigma_{spect}^2} \quad (25)$$

If we considers $\sigma_{SVD}^2 \ll \sigma_{spect}^2$:

$$\sigma\left(E_{SVD} - E_{spect}\right) = \sigma_{spect} \quad (26)$$

If we apply this equation to the data from the figure 14 we obtain the following result

$$\sigma_{spect} \cong \sigma\left(E_{SVD} - E_{spect}\right) \cong 0.2\ MeV \quad (27)$$

4.CONCLUSIONS

High-precision knowledge of the beam can enable higher precision control of the beam. Control of the beam energy is critical for the stability of both the wavelength and the beam position. We apply the SVD method to a large number of trajectories to extract the correlated position changes observed at several BPMs due to beam energy changes. We have shown that the energy measurement obtained using the spectrometer method. The statistical error is smaller because the information from different BPMs is used. Morover, the error introduced by beam trajectory instabilities (originated upstream the spectrometer dipole magnet) is filtered out with the SVD method.

Aknowledgements.

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