Cherenkov Radiation in the Galileon Model

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1 Introduction

I have been working with the Galileon model, originally proposed by Nicolis et al in 2009 [1.]. The Galileon model is a possible explanation for the current acceleration of the expansion of the universe, in which gravity is modified on the very largest distance scales. The Galileon is a scalar field, π, coupled to the matter content of the standard model. The Galileon model differs from other scalar field models of dark energy as the galileon scalar field is non-linear. It thus avoids problems with fifth force experiments that are present in other models.

In the Nicolis et al paper [1.] it is claimed that the speed of the angular fluctuations of π propagate with a speed $v \sim 10^{-8}c$ for the sun-earth system. Thus any object with mass (e.g the earth) passing through π at speeds greater than this will emit Cherenkov radiation in the form of scalar field particles. If this effect is large enough, we can potentially detect the emission of scalar field particles by the substantial decelleration of the massive object passing through π.

I have been studying the equation given for the speed of the angular fluctuations in the Nicolis et al paper [1.], finding constraints for ${c_\Omega}^2$, and the model parameters d_i. In some cases, the possibility of Cherenkov radiation is not an issue. For the cases where Cherenkov radiation is possible, I have approximated the magnitude of the energy loss by Cherenkov radiation.

2 $\mathbf{c}_\Omega^2$

The expression for the speed of propagation of the angular fluctuations of π is given by

$$ {c_\Omega}^2 = \frac{K_\Omega}{K_t} \tag{1} $$

where K_i are the kinetic coefficients for the fluctuation. This is given in terms of the parameters of the model d_i, i=2,3,4,5. For $d_5 = 0$, this reduces to

$$ {c_\Omega}^2 = \frac{{d_2}^2 + 2d_2d_3y + 4{d_3}^2y^2 - 6d_2d_4y^2}{{d_2}^2 + 4d_2d_3y + 12{d_3}^2y^2 - 12d_2d_4y^2 + 24d_3d_4y^3 + 36{d_4}^2y^4} \tag{2} $$

In the Burrage and Seery paper [2.], we have an expression for y

$$y = \frac{\pi'}{r} = \frac{M}{4\pi r^3 d_2} g(r) \tag{3}$$

and for the two spatial regions, Region A and Region B, in which the solution can be approximated easily

1. Region A: $\alpha R_2 \ll r \ll R_1$, where

$$g \sim \left(\frac{r}{R_1}\right)^{\frac{3}{2}} \tag{4}$$

2. Region B: $0 < r \ll \alpha R_2$, where

$$g \sim \left(\frac{r}{R_2}\right)^2 \tag{5}$$

and R_1 and R_2 are lengthscales defined by

$${R_1}^3 = \frac{d_3 M}{2\pi {d_2}^2} \tag{6}$$

$${R_2}^6 = \frac{M^2 d_4}{8\pi^2 {d_2}^3} \tag{7}$$

We now consider the possible behaviours of ${c_\Omega}^2$ for the earth orbiting the sun.

3 Region A

The condition for being in Region A is

$$\left(\frac{R_1}{r}\right)^3 g^2 \gg \left(\frac{R_2}{r}\right)^6 g^3 \tag{8}$$

and substituting (4), (6) and (7) into this expression yields $d_4 \ll 10^{187.5} GeV^{-2}$. So we can disregard the possibilities where d_4 does not satisfy this completely. Further considering $12 d_2 d_4 y^2 \ll 24 d_3 d_4 y^3$ and $12 d_2 d_4 y^2 \ll 36 {d_4}^2 y^4$, gives negative speeds. These are unstable solutions and so we neglect them. Doing this places restrictions on our parameter space.

Using (4), (6) and (3) we find the expression for y in Region A is

$$y = \frac{1}{2}\left(\frac{M}{2\pi r^3 d_3}\right)^{\frac{1}{2}} \tag{9}$$

Substituting this expression for y into (2), we can compare terms and find conditions for each term to dominate which gives us an allowed range for ${c_\Omega}^2$. For example, for ${d_2}^2 \ll 2d_2d_3y$ we have

$$\frac{d_2}{d_3y} \ll 2 \tag{10}$$

or, equivalently

$$\frac{d_2}{d_3^{\frac{1}{2}}} \ll \left(\frac{M}{2\pi r^3}\right)^{\frac{1}{2}} \tag{11}$$

which implies, using , $d_2 \sim {M_P}^2$ [2.], $d_3 \sim 10^{118}$ [2.] and the solar mass, that $r < 10^{33}GeV^{-1}$. So, considering just these two terms as the dominant terms in ${c_\Omega}^2$ and dividing by $4d_2d_3y$

$${c_\Omega}^2 = \left(\frac{1}{2} + \frac{d_2}{4d_3y}\right)\left(1 + \frac{d_2}{4d_3y}\right)^{-1} \tag{12}$$

and we can use the binomial expansion to give

$${c_\Omega}^2 = \frac{1}{2} + \frac{d_2}{8d_3y} + ... \tag{13}$$

which is valid for $\frac{d_2}{4d_3y} < 1$. This is equivalent to $r < 10^{33}GeV^{-1}$, consistent with the original condition. This is much larger than the solar radius ($\sim 10^{24}GeV^{-1}$) and so we are justified in using the binomial expansion.

Inserting (10) into (13) we have an upper and lower bound for ${c_\Omega}^2$,

$$\frac{1}{2} < {c_\Omega}^2 < \frac{3}{4} \tag{14}$$

In a similar fashion we can compare the other terms in the expression for ${c_\Omega}^2$ and find the possible, allowed behaviours:

1. ${d_2}^2 \ll 2d_2d_3y$ gives $r < 10^{33}GeV^{-1}$ and $\frac{1}{2} < {c_\Omega}^2 < \frac{3}{4}$
2. ${d_2}^2 \ll 4{d_3}^2y^2$ gives $r < 10^{33}GeV^{-1}$ and $\frac{1}{3} < {c_\Omega}^2 < \frac{5}{9}$
3. ${d_2}^2 \gg 6d_2d_4y^2$ gives $d_4 < 10^{177}GeV^{-2}$ and $1 < {c_\Omega}^2 < \frac{3}{2}$
4. ${d_2}^2 \ll 6d_2d_4y^2$ gives $10^{177}GeV^{-2} < d_4 \ll 10^{187.5}GeV^{-2}$ and $\frac{1}{4} < {c_\Omega}^2 < \frac{1}{2}$
5. $2d_2d_3y \gg 6d_2d_4y^2$ gives $d_4 < 10^{187.5}GeV^{-2}$ and $0 < {c_\Omega}^2 < \frac{1}{2}$
6. $4{d_3}^2y^2 \gg 6d_2d_4y^2$ gives $d_4 \ll 10^{187.5}GeV^{-2} < 10^{198}GeV^{-2}$ and $\frac{2}{9} < {c_\Omega}^2 < \frac{1}{3}$
7. ${d_2}^2 \gg 24d_3d_4y^3$ gives $d_4 \ll 10^{166.5}GeV^{-2}$ and $0 < {c_\Omega}^2 < 1$

8. ${d_2}^2 \ll 24 d_3 d_4 y^3$ gives $10^{166.5} GeV^{-2} \ll d_4 \ll 10^{187.5} GeV^{-2}$ and $0 < {c_\Omega}^2 < 1$
9. $4 d_2 d_3 y \gg 24 d_3 d_4 y^3$ gives $d_4 \ll 10^{177} GeV^{-2}$ and $0 < {c_\Omega}^2 < \frac{1}{2}$
10. $4 d_2 d_3 y \ll 24 d_3 d_4 y^3$ gives $10^{177} GeV^{-2} \ll d_4 \ll 10^{187.5} GeV^{-2}$, $0 < {c_\Omega}^2 < \frac{1}{2}$
11. $12 {d_3}^2 y^2 \gg 24 d_3 d_4 y^3$ gives $d_4 \ll 10^{187.5} GeV^{-2}$ and $0 < {c_\Omega}^2 < \frac{1}{3}$
12. ${d_2}^2 \gg 36 {d_4}^2 y^4$ gives $d_4 \ll 10^{177} GeV^{-2}$ and $0 < {c_\Omega}^2 < 1$
13. ${d_2}^2 \ll 36 {d_4}^2 y^4$ gives $10^{177} GeV^{-2} \ll d_4 \ll 10^{187.5} GeV^{-2}$ and $0 < {c_\Omega}^2 < 1$
14. $4 d_2 d_3 y \gg 36 {d_4}^2 y^4$ gives $d_4 \ll 10^{182.25} GeV^{-2}$ and $0 < {c_\Omega}^2 < \frac{1}{2}$
15. $4 d_2 d_3 y \ll 36 {d_4}^2 y^4$ gives $10^{182.5} GeV^{-2} \ll d_4 \ll 10^{187.5} GeV^{-2}$, $0 < {c_\Omega}^2 < \frac{1}{2}$
16. $12 {d_3}^2 y^2 \gg 36 {d_4}^2 y^4$ gives $d_4 \ll 10^{187.5} GeV^{-2}$ and $0 < {c_\Omega}^2 < \frac{1}{3}$
17. $12 d_2 d_4 y^2 \gg 36 {d_4}^2 y^4$ gives $d_4 \ll 10^{177} GeV^{-2}$ and $\frac{1}{2} < {c_\Omega}^2 < 1$

From these possible allowed behaviours, we see that problematic results in terms of the emission of Cherenkov radiation are 5. and 7.-16..

Now, the average orbital speed of the earth is $29.78 km/s$ [3.]. Converting into a dimensionless speed gives $v_E = 9.9335 \times 10^{-5}$. For stable solutions we want ${c_\Omega}^2 > {v_E}^2$, otherwise, as the earth moves through the solar galileon field, it would emit Cherenkov radiation and so decellerate and lose energy. This would result in the earth spiralling into the sun. So we need ${c_\Omega}^2 > 10^{-8}$ to prevent this.

Now we can find further constraints on the parameters. Focusing on those that contain just 2 parameters we have 5. 11. and 16. with the same constraints on the parameters we have already found so all of these cases still pose a problem with the possibility of Cherenkov radiation. We shall see that some cases in Region B can be neglected after considering the orbital speed of the earth.

4 Region B

Conversely, to be in Region B we require $d_4 \gg 10^{187.5} GeV^{-2}$ and

$$y = \frac{1}{2} \left(\frac{M}{\pi r^3 d_4} \right)^{\frac{1}{3}} \tag{15}$$

and we can similarly find possible allowed behaviours for Region B

1. ${d_2}^2 \gg 2 d_2 d_3 y$ gives $d_4 > 10^{219} GeV^{-2}$ and $\frac{1}{2} < {c_\Omega}^2 < 1$
2. ${d_2}^2 \ll 2 d_2 d_3 y$ gives $10^{187.5} GeV^{-2} \ll d_4 < 10^{219} GeV^{-2}$ and $\frac{1}{2} < {c_\Omega}^2 < \frac{3}{4}$
3. ${d_2}^2 \gg 4 {d_3}^2 y^2$ gives $d_4 > 10^{219} GeV^{-2}$ and $\frac{1}{3} < {c_\Omega}^2 < 1$

4. $d_2{}^2 \ll 4d_3{}^2y^2$ gives $10^{187.5}GeV^{-2} \ll d_4 < 10^{219}GeV^{-2}$ and $\frac{1}{3} < c_\Omega{}^2 < \frac{5}{9}$
5. $d_2{}^2 \ll 6d_2d_4y^2$ gives $d_4 \gg 10^{187.5}GeV^{-2} > 10^{156}GeV^{-2}$ and $\frac{1}{4} < c_\Omega{}^2 < \frac{1}{2}$
6. $2d_2d_3y \ll 6d_2d_4y^2$ gives $d_4 > 10^{187.5}GeV^{-2}$ and $0 < c_\Omega{}^2 < \frac{1}{2}$
7. $4d_3{}^2y^2 \gg 6d_2d_4y^2$ gives $10^{187.5}GeV^{-2} \ll d_4 < 10^{198}GeV^{-2}$ and $\frac{2}{9} < c_\Omega{}^2 < \frac{1}{3}$
8. $4d_3{}^2y^2 \ll 6d_2d_4y^2$ gives $d_4 > 10^{198}GeV^{-2}$ and $\frac{1}{2} < c_\Omega{}^2 < \frac{2}{3}$
9. $d_2{}^2 \ll 24d_3d_4y^3$ gives $r \ll 10^{33}GeV^{-1}$ and $0 < c_\Omega{}^2 < 1$
10. $4d_2d_3y \ll 24d_3d_4y^3$ gives $d_4 \gg 10^{187.5}GeV^{-2} > 10^{156}GeV^{-2}$, $0 < c_\Omega{}^2 < \frac{1}{2}$
11. $12d_3{}^2y^2 \ll 24d_3d_4y^3$ gives $d_4 \gg 10^{187.5}GeV^{-2}$ and $0 < c_\Omega{}^2 < \frac{1}{3}$
12. $12d_2d_4y^2 \gg 24d_3d_4y^3$ gives $d_4 \gg 10^{219}GeV^{-2}$ and $\frac{1}{2} < c_\Omega{}^2 < 1$
13. $d_2{}^2 \ll 36d_4{}^2y^4$ gives $d_4 \gg 10^{187.5}GeV^{-2} > 10^{156}GeV^{-2}$ and $\frac{1}{2} < c_\Omega{}^2 < 1$
14. $4d_2d_3y \ll 36d_4{}^2y^4$ gives $d_4 \gg 10^{187.5}GeV^{-2} > 10^{177}GeV^{-2}$ and $0 < c_\Omega{}^2 < \frac{1}{2}$
15. $12d_3{}^2y^2 \ll 36d_4{}^2y^4$ gives $d_4 \gg 10^{187.5}GeV^{-2}$ and $0 < c_\Omega{}^2 < \frac{1}{3}$

We see that problematic results that may indicate Cherenkov radiation are 6., 9.-11. and 13.-15..

Like for Region A we now use the orbital speed of the earth to calculate further constraints on the parameters for 9., 11., 13. and 15. (6. gives the same constraints that we already know and 10. 13. and 14. all have 3 parameters present).

For example, looking at 9., we have $10^{-8} < c_\Omega{}^2 < 1$ and inserting the unique expression for $c_\Omega{}^2$ for this case gives

$$10^{-8} < \frac{d_2{}^2}{24d_3d_4y^3} < 1 \tag{16}$$

where we have neglected the square term from the expression for $c_\Omega{}^2$ as 9. implies $\frac{d_2{}^2}{24d_3d_4y^3} \ll 1$. This implies

$$\frac{3Md_3}{\pi d_2{}^2} \times 10^{-8} < r^3 < \frac{3Md_3}{\pi d_2{}^2} \tag{17}$$

which gives the additional constraint for 9. as $10^{30.33}GeV^{-1} < r \ll 10^{33}GeV^{-1}$ and $10^{-8} < c_\Omega{}^2 < \frac{1}{2}$.

Similarly, for 11. we have $10^{187.5}GeV^{-2} \ll d_4 < 10^{199.5}GeV^{-2}$ and $10^{-8} < c_\Omega{}^2 < \frac{1}{3}$. For 13., we have $10^{156}GeV^{-2} \ll d_4 < 10^{168}GeV^{-2}$ so we can neglect this result as the condition for being in Region B is not satisfied. And for 15. we have $10^{187.5}GeV^{-2} \ll d_4 < 10^{193.5}GeV^{-2}$ and $10^{-8} < c_\Omega{}^2 < \frac{1}{3}$. So, the parameter space is restricted further ensuring there is no Cherenkov radiation for cases 9., 11. and 15..

We can now consider Cherenkov radiation in more detail for the cases which suggest the possibility of Cherenkov radiation, those being 5.and 7.-16. in Region A and 6., 10. and 14. in Region B.

5 Cherenkov Radiation

To calculate the energy lost by the earth as a result of emission of Cherenkov radiation as the earth moves through the solar galileon field at speeds greater than the angular fluctuations of the galileon, we use the π-action

$$S_\pi = \int d^4x \left(\mathcal{L}_\pi + \pi T^\mu_\mu\right) \tag{18}$$

Now the solar galileon field and the galileon field of the earth have non-linearities so we are unable to add the fields for the sun and earth together to do this integral, as we could for linear fields. So instead, if we consider a very small mass, such as a dust grain, we can use the linear solution for π

$$\pi(r) = \pi_0 - \frac{M}{4\pi d_2 r} \tag{19}$$

Considering only the kinetic term of the lagrangian, for simplicity, we can obtain an order of magnitude estimate for the energy loss. As we are considering a spherically symmetric galileon $\pi = \pi(r)$ we have

$$S = \int d^4r \left(\frac{-d_2}{2}\left(\frac{\partial\pi}{\partial r}\right)^2\right) \tag{20}$$

Differentiating $\pi(r)$ and substituting into S we find

$$\frac{\partial S}{\partial t} = \int r^2 \sin\theta dr d\theta d\phi \left(\frac{-d_2}{2}\frac{M^2}{16\pi^2 {d_2}^2 r^4}\right) \tag{21}$$

and performing the integral over θ and ϕ gives

$$\frac{\partial S}{\partial t} = \frac{M^2}{8\pi d_2}\int_{R_\odot}^{\infty} \frac{1}{r^2} dr \tag{22}$$

where the lower limit of integration is the radius of the sun because at shorter distances, inside the sun, other effects need to be taken into account and the galileon does not dominate or accurately describe the behaviour here.

Then the magnitude of the energy loss by scalar Cherenkov radiation is

$$\frac{\partial S}{\partial t}\left(1 - \frac{{c_\Omega}^2}{{v_g}^2}\right) \tag{23}$$

where v_g is the velocity of dust grains.

Performing the above integration and substituting we have the energy loss by Cherenkov radiation as

$$\frac{M^2}{8\pi d_2 R_\odot}\left(1-\frac{{c_\Omega}^2}{{v_g}^2}\right) \tag{24}$$

Now interstellar grains move at 26 km/s relative to the sun [4.]. Thus ${v_g}^2 \sim 10^{-8}$ which is of the same order of the speed of the earth. We know the cases for which Cherenkov radiation is possible for the earth and these will be the same for the dust grain. Further, the range for ${c_\Omega}^2$ will be the same as for the earth (and indeed for any object) as a result of using the binomial expansion. So, the maximum instantaneous energy loss for a dust particle is

$$\frac{{m_g}^2}{8\pi d_2 R_\odot} \sim 10^{-26} GeV \tag{25}$$

where $m_g \sim 10^{18} GeV$, calculated from $\rho_g = 2gcm^{-3}$ [5.] and the size of a dust grain, 0.1mm [5.], implying $V_g = 10^{-6}cm^3$.

We can compare this to the instantaneous kinetic energy of a dust grain as it orbits the sun. If the energies are comparable then the energy loss due to Cherenkov radiation is significant. This poses a problem for the model as the dust grain spirals in to the sun as a result of its energy loss, an effect we do not observe. The kinetic energy of the grain is $K_g = \frac{1}{2}m_g {v_g}^2$ and so $K_g \sim 10^{10} GeV$. So we see the energy loss due to Cherenkov radiation is a tiny fraction of the kinetic energy of the dust grain meaning the dust grain does not emit Cherenkov radiation, and so does not decellerate, significantly.

Additionally, looking at the behaviour of $y = \frac{\pi'}{r}$ which is proportional to $g(r)$, the Burrage and Seery paper [2.] showed that the earth has to be within the lengthscales R_1 and R_2 to avoid the appearance of fifth forces in the galileon model. Here, $g(r) \to 0$ and so there is no interaction between the solar galileon field and the earth, implying no possibility for Cherenkov radiation.

6 Conclusion

I have found that Cherenkov radiation by the earth as it moves through the solar galileon field is only possible under certain conditions on d_4. If bounds for d_4 can be found, like those for d_2 and d_3, then more of the above possible cases for Cherenkov radiation can be eliminated.

Unfortunately, it has not been possible to estimate the magnitude of energy loss by Cherenkov radiation for the earth due to the non-linearities of π in the model. However, the energy loss by a small mass, like a dust grain, has been approximated as we are able to use the linear solution for π for a small mass. I found the maximum energy loss is $\sim 10^{-26} GeV$ which is insignificant compared to the kinetic energy of the dust grain, implying there is no Cherenkov radiation and thus no problem in the galileon model.

Briefly mentioned was the possibility that the solar galileon field and the galileon field of the earth don't couple due to $g(r) \to 0$ and thus there would not be a problem with the galileon model in terms of Cherenkov radiation.

References

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