Constraints for MSSM parameters from (g-2) experment.

This paper describes constraints of the parameters of MSSM that one can get from (g-2) anomalous magnetic dipole-moment of muon and branching ratio MEG experiment.

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1. Introduction.

One can get constraints for the Minimal Supersymmetric Standard Model (MSSM) parameters from experiments that are enough sensitive to Supersymmetry. In this paper we study two of them: searching for muon decays into electron and gamma with lepton flavor violation and measuring of (g-2) anomalous magnetic dipole-moment of the muon.

One of the minimal extensions of the standard model with lepton flavor violation (LFV) is the model with non-vanishing neutrino masses. If the masses of the neutrinos are induced by the seesaw mechanism one has a new set of Yukawa couplings involving the right-handed neutrinos. Introduction of the new Yukawa couplings generally gives rise to the flavor violation in the lepton sector, similar to its quark sector counterparts
In non-supersymmetric standard models, however, the amplitudes of the LFV processes are proportional to inverse powers of the right-handed neutrino mass scale which is typically much higher than the electroweak scale, and as a consequence such rates are highly suppressed.

If there exists SUSY broken at the electroweak scale, we may expect that the rates of these LFV processes will be much larger than the non-supersymmetric case. The point is that the lepton-flavor conservation is not a consequence of the standard model gauge symmetry and renormalizability in the supersymmetric case, even in the absence of the right-handed neutrinos. Indeed, slepton mass terms can violate the lepton flavor conservation in a manner consistent with the gauge symmetry. Thus the scale of LFV can be identified with the electroweak scale, much lower than the right-handed neutrino scale. However, an order-of-unity violation of the lepton-flavor conservation at the electroweak scale would cause disastrously large rates for $\mu \rightarrow e\gamma$ processes.

All modern papers that includes calculations of the branching ratios and (g-2) of the muon in MSSM use as basic [1] Hisano's article, written in 1995.It includes details of the Feynman diagrams for a lot of processes and amplitudes, but the parameters of MSSM used in the paper are rather old (they used data known before 1995).

In section 2 modern MSSM frameworks are described (well, only some the parameters), which are used for calculations and results.
Section 2.1 describes mu to e-gmma process, amplitudes are written and reflected in Hisano's notations.

Section 2.2 describes how in SUSY (g-2) is calculated and some plots for the SUSY contribution to the anomalous magnetic dipole-moment of muon are presented as a function of the left-handed selectron mass are there.

2 MSSM Frameworks.

While a detailed scanning over the more-than-hundred-dimensional parameter space of the MSSM is clearly not practicable, even a sampling of the three- or four-dimensional parameter space of the SUSY breaking scenarios is beyond the present capabilities for phenomenological studies, in particular when it comes to simulating experimental signatures within the detectors. For this reason one often resorts to specific benchmark scenarios, i.e. one studies only specific parameter points or at best samples a one-dimensional parameter space, which exhibit specific characteristics of the MSSM parameter space. Benchmark scenarios of this kind are often used, for instance, for studying the performance of different experiments at the same collider. The Snowmass Points and Slopes (SPS) are based on an attempt to merge the features of the above proposals for different benchmark scenarios into a subset of commonly accepted benchmark scenarios. They consist of benchmark points and model lines (“slopes”).[4]

SPS	1a	1b	2	3	4	A	B
m_0	100	200	1450	90	400	500	500
A_0	−100	0	0	0	0	0	0
$m_{1/2}$	250	400	300	400	300	500	500
$\tan\beta$	10	30	10	10	50	40	10
$sgn(\mu)$	+1	+1	+1	+1	+1	+1	+1
$\mu(M_Z)$	352	507	422	516	388	614	629

- SPS1a and SPS1b are typical mSUGRA scenario. 1b is model with relativity high tan β.

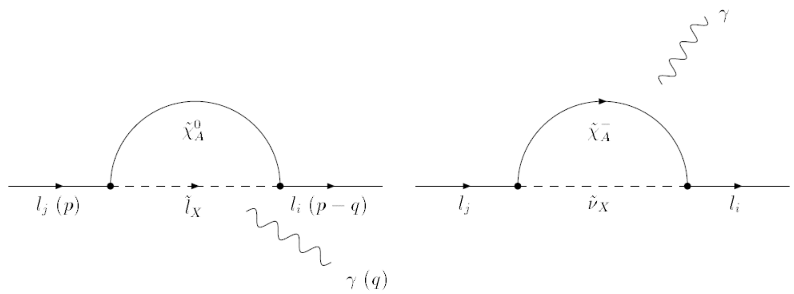
tan β is a parameter of the MSSM and it is defined as the ratio of the two vacuum expectation values of the two Higgses, tan β = V_2/V_1

- SPS2 – This scenario features relatively heavy squarks and sleptons, while the charginos and the neutralinos are fairly light and the gluino is lighter than the squarks.
- SPS3 – The model line of this scenario is directed into the “coannihilation region”, where a sufficiently low relic abundance can arise from a rapid coannihilation between the Lightest Supersymmetric Particle (LSP) and the (almost mass degenerate) NSLP, which is usually the lighter tau-slepton. Accordingly, an important feature in the collider phenomenology of this scenario is the very small slepton–neutralino mass difference.
- SPS4 – The large value of tan β in this scenario has an important impact on the phenomenology in the Higgs sector. The couplings of A,H to b¯b and τ+τ− as well as the H±t¯b couplings are significantly enhanced in this scenario, resulting in particular in large associated production cross sections for the heavy Higgs bosons.
- Scenarios A and B are more exotic.

In this paper we will use these parameters for testing.

2.1 Muon to electron and gamma experiment.

In [1], the processes of decaying of $\mu \to e\gamma$ is given by thge floowing Faynman diagrams.



Amplitude for $l_j^- \to l_i^- \gamma^*$

generally written as

$$T = e\epsilon^{\alpha *}\bar{u}_i(p-q)\left[q^2\gamma_\alpha(A_1^L P_L + A_1^R P_R) + m_{l_j} i\sigma_{\alpha\beta}q^\beta(A_2^L P_L + A_2^R P_R)\right]u_j(p)$$

in the limit of q → 0 with q being the photon momentum. Here, e is the electric charge, ϱ the photon polarization vector, u_i (and v_i in the expressions below) the wave function for (anti-) lepton, and p the momentum of the particle l_j .

$$A_a^{L,R} \;=\; A_a^{(n)L,R} + A_a^{(c)L,R} \qquad (a = 1, 2)$$

Amplitudes from the neutralino loops are:

$$\begin{aligned}
A_1^{(n)L} &= \frac{1}{576\pi^2}N_{iAX}^{R(l)}N_{jAX}^{R(l)*}\frac{1}{m_{\tilde{l}_X}^2}\frac{1}{(1-x_{AX})^4}\times(2-9x_{AX}+18x_{AX}^2-11x_{AX}^3+6x_{AX}^3\ln x_{AX})\\
A_2^{(n)L} &= \frac{1}{32\pi^2}\frac{1}{m_{\tilde{l}_X}^2}\left[N_{iAX}^{L(l)}N_{jAX}^{L(l)*}\frac{1}{6(1-x_{AX})^4}\right.\\
&\quad\times(1-6x_{AX}+3x_{AX}^2+2x_{AX}^3-6x_{AX}^2\ln x_{AX})\\
&\quad\left.+N_{iAX}^{L(l)}N_{jAX}^{R(l)*}\frac{M_{\tilde{\chi}_A^0}}{m_{l_j}}\frac{1}{(1-x_{AX})^3}(1-x_{AX}^2+2x_{AX}\ln x_{AX})\right]\\
A_a^{(n)R} &= A_a^{(n)L}|_{L\leftrightarrow R} \qquad (a=1,2),
\end{aligned}$$

where $x_{AX} = M_{\tilde{\chi}_A^0}^2/m_{\tilde{l}_X}^2$

and for the chargino loops one has

$$\begin{aligned}
A_1^{(c)L} &= -\frac{1}{576\pi^2}C_{iAX}^{R(l)}C_{jAX}^{R(l)*}\frac{1}{m_{\tilde{\nu}_X}^2}\frac{1}{(1-x_{AX})^4}\\
&\quad\times\left\{16-45x_{AX}+36x_{AX}^2-7x_{AX}^3+6(2-3x_{AX})\ln x_{AX}\right\},\\
A_2^{(c)L} &= -\frac{1}{32\pi^2}\frac{1}{m_{\tilde{\nu}_X}^2}\left[C_{iAX}^{L(l)}C_{jAX}^{L(l)*}\frac{1}{6(1-x_{AX})^4}\right.\\
&\quad\times(2+3x_{AX}-6x_{AX}^2+x_{AX}^3+6x_{AX}\ln x_{AX})\\
&\quad\left.+C_{iAX}^{L(l)}C_{jAX}^{R(l)*}\frac{M_{\tilde{\chi}_A^-}}{m_{l_j}}\frac{1}{(1-x_{AX})^3}(-3+4x_{AX}-x_{AX}^2-2\ln x_{AX})\right]\\
A_a^{(c)R} &= A_a^{(c)L}|_{L\leftrightarrow R} \qquad (a=1,2).
\end{aligned}$$

where $x_{AX} = M_{\tilde{\chi}_A^-}^2/m_{\tilde{\nu}_X}^2$

and C and N are coefficients in the fremion-sfermion-chargino and the fremion-sfermion-neutralino* parts of the Lagrangian

$$\begin{aligned}
\mathcal{L}_{\rm int} &= \bar{l}_i(C_{iAX}^{R(l)}P_R + C_{iAX}^{L(l)}P_L)\tilde{\chi}_A^-\tilde{\nu}_X\\
&\quad+\bar{\nu}_i(C_{iAX}^{R(\nu)}P_R + C_{iAX}^{L(\nu)}P_L)\tilde{\chi}_A^+\tilde{l}_X\\
&\quad+\bar{d}_i(C_{iAX}^{R(d)}P_R + C_{iAX}^{L(d)}P_L)\tilde{\chi}_A^-\tilde{u}_X\\
&\quad+\bar{u}_i(C_{iAX}^{R(u)}P_R + C_{iAX}^{L(u)}P_L)\tilde{\chi}_A^+\tilde{d}_X + h.c.,
\end{aligned}$$

$$\mathcal{L}_{\rm int} = \bar{f}_i(N_{iAX}^{R(f)}P_R + N_{iAX}^{L(f)}P_L)\tilde{\chi}_A^0\tilde{f}_X$$

(*) for $\mu \to e\gamma$ we do not need a $N_{iAX}^{R(d)}$, etc.

$$
\begin{aligned}
N_{iAX}^{R(l)} &= -\frac{g_2}{\sqrt{2}}\{[-(O_N)_{A2}-(O_N)_{A1}\tan\theta_W]U_{X,i}^l+\frac{m_{l_i}}{m_W\cos\beta}(O_N)_{A3}U_{X,i+3}^l\} \\
N_{iAX}^{L(l)} &= -\frac{g_2}{\sqrt{2}}\{\frac{m_{l_i}}{m_W\cos\beta}(O_N)_{A3}U_{x,i}^l+2(O_N)_{A1}\tan\theta_W U_{X,i+3}^l\}, \\
N_{iAX}^{R(\nu)} &= -\frac{g_2}{\sqrt{2}}[(O_N)_{A2}-(O_N)_{A1}\tan\theta_W]U_{X,i}^{\nu}, \\
N_{iAX}^{L(\nu)} &= 0, \\
C_{iAX}^{R(l)} &= -g_2(O_R)_{A1}U_{X,i}^{\nu}, \\
C_{iAX}^{L(l)} &= g_2\frac{m_{l_i}}{\sqrt{2}m_W\cos\beta}(O_L)_{A2}U_{X,i}^{\nu}, \\
C_{iAX}^{R(\nu)} &= -g_2(O_L)_{A1}U_{X,i}^{l}, \\
C_{iAX}^{L(\nu)} &= g_2\frac{m_{l_i}}{\sqrt{2}m_W\cos\beta}(O_L)_{A2}U_{X,i+3}^{l},
\end{aligned}
$$

where O_L and O_R are 2x2 orthogonal matrices which diagonalize M_C , $O_R M_C O_L^{\mathsf{T}} = (\text{diagonal})$

and O_N is neutralino mass-matrix diagonalizing matrix $O_N M_N O_N^{\mathsf{T}} = \text{diagonal}$

$$
M_N = \begin{pmatrix} M_1 & 0 & -m_Z\sin\theta_W\cos\beta & m_Z\sin\theta_W\sin\beta \\ 0 & M_2 & m_Z\cos\theta_W\cos\beta & -m_Z\cos\theta_W\sin\beta \\ -m_Z\sin\theta_W\cos\beta & m_Z\cos\theta_W\cos\beta & 0 & -\mu \\ m_Z\sin\theta_W\sin\beta & -m_Z\cos\theta_W\sin\beta & -\mu & 0 \end{pmatrix}
$$

$$
M_C = \begin{pmatrix} M_2 & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & \mu \end{pmatrix}
$$

and they come from the following parts of the Lagrangian

$$
-\mathcal{L}_m = \begin{pmatrix} \overline{\tilde{W}_R^-} & \overline{\tilde{H}_{2R}^-} \end{pmatrix} \begin{pmatrix} M_2 & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}_L^- \\ \tilde{H}_{1L}^- \end{pmatrix} + h.c.
$$

$$
-\mathcal{L}_m = \frac{1}{2}\left(\tilde{B}_L\tilde{W}_L^0\tilde{H}_{1L}^0\tilde{H}_{2L}^0\right) M_N \begin{pmatrix} \tilde{B}_L \\ \tilde{W}_L^0 \\ \tilde{H}_{1L}^0 \\ \tilde{H}_{2L}^0 \end{pmatrix} + h.c.,
$$

The decay rate for the process $l_j^- \to l_i^- \ \gamma$ is calculated as

$$
\Gamma(l_j^- \to l_i^- \ \gamma) = \frac{e^2}{16\pi} m_{l_j}^5 (|A_2^L|^2 + |A_2^R|^2).
$$

Even simple calculations are not a trivial because for every parameter change one need to search for eigenvectors of 6x6 matrix, but all calculations could be done in PC numerically, so a special script for Maple was written. MEG experiment has experimental bound of branching ratio up to 1.2×10^{11}

2.2 (g-2) muon.

There are many reasons to believe that the SM is an incomplete description of nature besides the present indications from a_μ. For example, the SM does not explain baryogenesis, dark matter, the ratios of fundamental scales, or the strengths of gauge and Yukawa interactions. Supersymmetry is an appealing theoretical framework that may answer many of the questions unanswerable within the SM. The anomalous magnetic moment of the muon is just one observable out of many that supersymmetry can affect. Using just this one quantity to divine predictions for other observables is difficult for the obvious reason that each observable requires a different set of supersymmetry masses and mixing angles.

The amplitude for the photon-muon-muon magnetic moment coupling in the limit of the photon momentum q
going to zero can be written as

$$\frac{ie}{2m_\mu}F(q^2)\bar{u}(p_f)\sigma_{\mu\nu}q^\mu\epsilon^\nu u(p_i),$$

where $q = p_f - p_i$ and ϵ the polarization vector of external photon. Then, the anomalous magnetic dipole-moment of muon is

$$a_\mu = (g-2)_\mu \equiv 2F(q^2=0).$$

The state of the art calculation of anomalous magnetic dipole-moment of muon within the Standard Model is [5]

$$a_\mu^{\rm SM} = 11\,659\,159.6(6.7)\times 10^{-10}$$

The majority of the uncertainty comes from hadrons in the photon vacuum polarization diagram.The Brookhaven E821 [6] experiment has released a new measurement of a_μ and found

$$a_\mu^{\rm E821} = 11\,659\,202(14)(6)\times 10^{-10}.$$

This result indicates that the anomalous magnetic moment of the muon may need additional contributions beyond the SM to be consistent with the experimental measurement.

In [1] SUSY contribution to (g-2) one can write as $(g-2)_\mu^{SUSY} = (g^{(C)}+g^{(N)})_\mu$

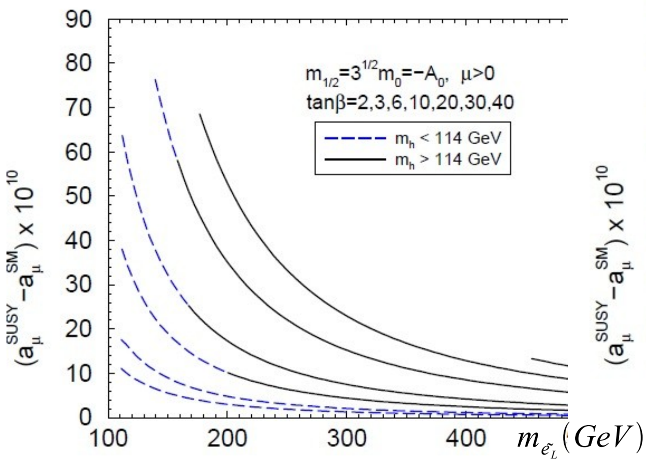
$$\begin{aligned} g_\mu^{(C)} &= \frac{1}{48\pi^2}\frac{m_\mu^2}{m_{\tilde{\nu}_X}^2}|C_{2AX}^{L(l)}|^2\frac{2+3x_{AX}-6x_{AX}^2+x_{AX}^3+6x_{AX}\ln x_{AX}}{(1-x_{AX})^4} \\ &+\frac{1}{16\pi^2}\frac{m_\mu M_{\tilde{\chi}_A^-}}{m_{\tilde{\nu}_X}^2}C_{2AX}^{L(l)}C_{2AX}^{R(l)*}\frac{-3+4x_{AX}-x_{AX}^2-2\ln x_{AX}}{(1-x_{AX})^3} \\ &+(L\leftrightarrow R) \end{aligned}$$

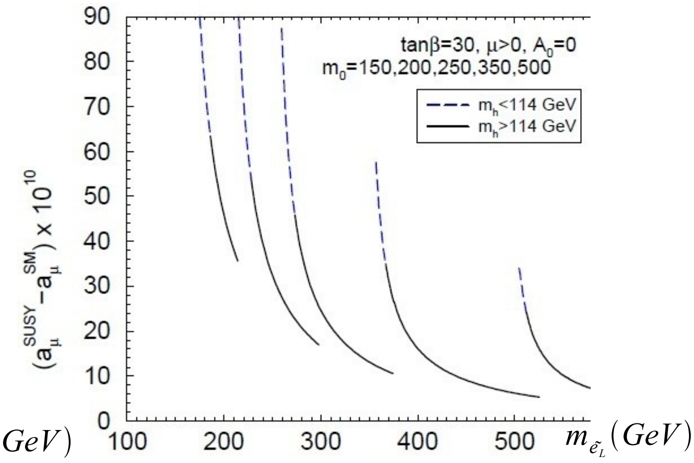
where $x_{AX} = M_{\tilde{\chi}_A^-}^2/m_{\tilde{\nu}_X}^2$.

$$\begin{aligned} g_\mu^{(N)} &= -\frac{1}{48\pi^2}\frac{m_\mu^2}{m_{\tilde{l}_X}^2}|N_{2AX}^{L(l)}|^2\frac{1-6x_{AX}+3x_{AX}^2+2x_{AX}^3-6x_{AX}^2\ln x_{AX}}{(1-x_{AX})^4} \\ &-\frac{1}{16\pi^2}\frac{m_\mu M_{\tilde{\chi}_A^0}}{m_{\tilde{l}_X}^2}N_{2AX}^{L(l)}N_{2AX}^{R(l)*}\frac{1-x_{AX}^2+2x_{AX}\ln x_{AX}}{(1-x_{AX})^3} \\ &+(L\leftrightarrow R), \end{aligned}$$

where $x_{AX} = M_{\tilde{\chi}_A^0}^2/m_{\tilde{l}_X}^2$.

Meaning of C and N are the same as in the previous section 2.1





Plots for SUSY contribution to anomalous magnetic dipole-moment of muon as a function of the left-handed selectron mass. In the l.h.s., tan β varied from 2 to 40 (highest curve). In the r.h.s, left cruve is for $m_0=150$ and the right-most curve is for $m_0=500$

One can get other constraints for selectron mass from LHC, PAMELA data (because some models that describes positron flux are SUSY-models).

Conclusion.

There are many reasons to believe that the SM is an incomplete description of nature besides the present indications from a_μ. For example, the SM does not explain baryogenesis, dark matter, the ratios of fundamental scales, or the strengths of gauge and Yukawa interactions. Supersymmetry is an appealing theoretical framework that may answer many of the questions unanswerable within the SM**.** Landau wrote, that if one has two or more free parameters, he can describe everything. In some modern MSSM frameworks there are eight. So, according to Landau, SUSY can describe a very big part of everything. We saw some plots for different MSSM modern frameworks. Future experiments help us to know will whether it a SUSY or some other mechanism which answers the many open questions.

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