An extension of the standard model with a vector like fourth generation

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Abstract

We study a model with an extra family of vector like particles. Particularly they are d-quarks which are $SU(2)$ singlets and also $SU(2)$ doublet leptons and right-handed neutrinos. The mass matrix mix all them with the particles of the Standard Model so it causes some new predictions in phenomenology. One of them is the existence of tree level flavour changing neutral current.

1 Introduction

The Standard model can describe all known phenomenology of elementary particles without gravitational and cosmological effects. But since it was constructed there were many different attempts to extend it. Most of them predict some new kind of particles which are not observed yet, but not excluded by experimentalists. For instance there is no proof that in the Nature there are only three generations of particles. Also there are some more complicated extensions of the SM. For instance supersymmetry can be introduced because it simplifies renormalization. Another possible way to extend the SM is to suppose that its gauge group $SU(3) \times SU(2) \times U(1)$ is a subgroup of a larger gauge group. Also the idea of extra dimensions in the space-time was considered many times.

In my project a supersymmentric extension of the Standard Model with $SO(10)$ gauge group and two extra dimensions is considered.It is described in details in ref. [1, 2, 3]. This model includes a forth generation of vector like d-quarks. It means that these particles have the same quantum numbers as right-handed d-quarks. Also there is the fourth generation of vector like leptons with the same quantum numbers as left-handed lepton doublets and there are right-handed neutrinos. The effects connected with the compactified extra dimensions and larger than in the SM gauge group can be neglected in case of energies much smaller than the GUT scale which is about$10^{15}GeV$. The sypersymmetric partners are not taken into account in this paper though they are in the model. So the effective low-energy theory considered here is the SM with an added forth family particles and right-handed neutrinos. The neutrino masses are obtained via see-saw mechanism. The masses of the extra charged fermions have to be larger than the masses of observable particles since they are not found experimentally.

2 The quarks mass matrix

The superpotential of our model is

$$W = u^L m^u u^R + d^L m^d d^R + e^R m^e e^L + \nu^R m^D \nu^L + \frac{1}{2}\nu^R m^N \nu^R \tag{1}$$

Here m^u and m^N are diagonal 3×3 matrices but m^d, m^e and m^D are 4×4 matrices which look like

$$\begin{pmatrix} \mu_1 & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2 & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3 & \tilde{\mu}_3 \\ M_1 & M_2 & M_3 & M_4 \end{pmatrix} \tag{2}$$

Further any of them will be designated by m. Our purpose is to diagonalize these matrices and to find the expression for the Flavour Changing Neutral Current in this model. This matrix is diagonalized by double-unitary transformation:

$$m = U_4 U_3 D V_3^+ V_4^+ \tag{3}$$

where U_4 and V_4 single out the heaviest eigenstate while U_3 and V_3 act only on the SM flavour indices. Now let us find U_4. To do this we consider $U_4^+ m m^+ U$ which should also have the distinguished heaviest eigenvalue. So

$$mm^+ = \begin{pmatrix} \mid \mu_i \mid^2 \delta_{ij} + \tilde{\mu}_i \tilde{\mu}_j & \mu_i M_i + \tilde{\mu}_i M_4 \\ \mu_i^* M_i + \tilde{\mu}_i M_4 & M^2 \end{pmatrix} \tag{4}$$

Here

$$M^2 = \sum_{\alpha=1}^{4} M_\alpha^2$$

Let's suppose that

$$(U_4)_{\alpha\beta} = (e_\alpha)_\beta$$

U_4 is unitary so

$$(e_\alpha)_\beta (e_\gamma)_\beta = \delta_{\alpha\gamma} \tag{5}$$

Let's consider

$$(U_4^+ m m^+ U_4)_{\alpha\beta}$$

which is equal to

$$\begin{aligned} &\sum_i (e_\beta)_i^* \mid \mu_i \mid^2 (e_\alpha)_i + \sum_{ij} (e_\beta)_i^* \tilde{\mu}_i \tilde{\mu}_j (e_\alpha)_i + \\ &\qquad \sum_i (e_\beta)_i^* (\mu_i M_i + \tilde{\mu}_i M_4)(e_\alpha)_4 + \\ &\sum_i (e_\beta)_4^* (\mu_i^* M_i + \tilde{\mu}_i M_4)(e_\alpha)_i + (e_\beta)_4^* M^2 (e_\alpha)_4 \end{aligned} \tag{6}$$

We demand that in case of $\beta = 4$, $\alpha = k \neq 4$ this expression should be equal to zero. By expanding e_α over powers of μ_i/M and supposing that $(e_\alpha)^{(0)}_\beta = \delta_{\alpha\beta}$ we get equations:

$$(\mu^*_k M_k + \tilde{\mu}_k M_4)\delta_{ki} + M^2 (e_k)^{(1)}_4 = 0 \tag{7}$$

this equation give us the same result as in the article [1] :

$$(e_k)^{(1)}_4 = -\frac{(\mu^*_k M_k + \tilde{\mu}_k M_4)}{M^2} \tag{8}$$

The second order equations are

$$\sum_i (\mu^*_i M_i + \tilde{\mu}_i M_4)(e_k)^{(1)}_i + M^2 (e_k)^{(2)}_4 = 0 \tag{9}$$

It gives us that we can assume $(e_k)^{(1)}_i = 0$, $(e_k)^{(2)}_4 = 0$ In the third order we have

$$\begin{aligned} \frac{\mu^*_k M_k + \tilde{\mu}_k M_4}{M^2} \mid \mu_k \mid^2 + \sum_i \frac{\mu^*_i M_i + \tilde{\mu}_i M_4}{M^2} \tilde{\mu}_i \tilde{\mu}_k - \\ - \sum_i \frac{\mid \mu_i M_i + \tilde{\mu}_i M_4 \mid^2}{M^2} \frac{\mu^*_k M_k + \tilde{\mu}_k M_4}{M^2} + \\ \sum_i \mu^*_i M_i + \tilde{\mu}_i M_4 (e_k)^2_i + M^2 (e_k)^{(3)}_4 = 0 \end{aligned} \tag{10}$$

From the last equation $(e_k)_i$ can not be found unambiguously so to find it we use the orthogonality condition (5) up to the second order of μ/M. It gives us:

$$(e_l)^{(2)}_k + (e^*_k)^{(2)}_l = -(e^*_k)^{(1)}_4 (e_l)^{(1)}_4 = -\frac{(\mu_k M_k + \tilde{\mu}_k M_4)}{M^2} \frac{(\mu^*_l M_l + \tilde{\mu}_l M_4)}{M^2} \tag{11}$$

$$Re(e_4)^{(2)}_4 = -\frac{1}{2} \sum_k \mid (e_4)^{(1)}_k \mid^2 \tag{12}$$

If we write the matrix $U^+_4 m$ up to the second order $(e_k)_{l(2)}$ does not contribute to it. Also one can check that $(e_4)^{(2)}_4$ does not contribute to $(U^+_4 m m^+ U_4)_{4j}$ if $j \neq 4$.

Then let us find V_4 in the leading order.

$$U^+_4 m = \begin{pmatrix} \mu_i \delta_{ij} - \frac{(\tilde{\mu}_i M_4 + \mu^*_i M_i) M_j}{M^2} & \tilde{\mu}_i - \frac{M_4}{M^2}(\mu^*_i M_i + \tilde{\mu}_i M_4) \\ M_i + \frac{\mu_i M_i + \tilde{\mu}_i M_4}{M^2} \mu_i & \sum_i \frac{\mu_i M_i + \tilde{\mu}_i M_4}{M^2} \tilde{\mu}_i + M_4 \end{pmatrix} \tag{13}$$

$$\begin{aligned} (U^+_4 m V_4)_{4j} = \\ \sum_i \left(\frac{\mu_i M_i + \tilde{\mu}_i M_4}{M^2} \mu_i + M_i \right) V_{ij} + \left(\sum_i \frac{\mu_i M_i + \tilde{\mu}_i M_4}{M^2} \tilde{\mu}_i + M_4 \right) v_{4j} \end{aligned} \tag{14}$$

$$(U_4^+ m V_4)_{i4} = \sum_j \left(\mu_i\delta_{ij} - \frac{(\tilde{\mu}_i M_4 + \mu_i^* M_i) M_j}{M^2}\right) v_{j4} + \left(\tilde{\mu}_i - \frac{M_4}{M^2}(\mu_i^* M_i + \tilde{\mu}_i M_4)\right) v_{44} \quad (15)$$

We need both these expressions be equal to zero. The last equation in the leading order over $\frac{\mu_i}{M}$ has a solution

$$v_{\alpha 4} = \frac{M_i}{M} \quad (16)$$

Here we have taken into account that $v_{\alpha\beta}$ wshould be normalized. The equation (14) in the leading order gives us that vectors $v_{\alpha i}$ should be orthogonal to $\frac{M_i}{M}$. For example we can take them in the following form

$$v_1 = \frac{1}{\sqrt{M_1^2 + M_4^2}} \begin{pmatrix} -M_4 \\ 0 \\ 0 \\ M_1 \end{pmatrix} \quad (17)$$

$$v_2 = \frac{1}{\sqrt{M_2^2 + M_3^2}} \begin{pmatrix} 0 \\ M_3 \\ -M_2 \\ 0 \end{pmatrix} \quad (18)$$

$$v_3 = \begin{pmatrix} -M_1\sqrt{\frac{M_2^2+M_3^2}{M_1^2+M_4^2}} \\ M_2\sqrt{\frac{M_1^2+M_4^2}{M_2^2+M_3^2}} \\ M_3\sqrt{\frac{M_1^2+M_4^2}{M_2^2+M_3^2}} \\ -M_4\sqrt{\frac{M_2^2+M_3^2}{M_1^2+M_4^2}} \end{pmatrix} \quad (19)$$

Finally in the leading order

$$m' = U_4^+ m V_4 = \begin{pmatrix} \hat{m} & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} \frac{\mu_1 M_4 - \tilde{\mu}_1 M_1}{\sqrt{M_1^2+M_4^2}} & 0 & -\left(\frac{\mu_1 M_1 + \tilde{\mu}_1 M_4}{M}\right)\sqrt{\frac{M_2^2+M_3^2}{M_1^2+M_4^2}} & 0 \\ -\frac{\mu_2 M_1}{\sqrt{M_1^2+M_4^2}} & \frac{\mu_2 M_3}{\sqrt{M_2^2+M_3^2}} & \frac{1}{M}\left(\mu_2 M_2\sqrt{\frac{M_1^2+M_4^2}{M_2^2+M_3^2}} - \tilde{\mu}_2 M_4\sqrt{\frac{M_2^2+M_3^2}{M_1^2+M_4^2}}\right) & 0 \\ -\frac{\mu_3 M_1}{\sqrt{M_1^2+M_4^2}} & -\frac{\mu_3 M_2}{\sqrt{M_2^2+M_3^2}} & \frac{1}{M}\left(\mu_3 M_2\sqrt{\frac{M_1^2+M_4^2}{M_2^2+M_3^2}} - \tilde{\mu}_3 M_4\sqrt{\frac{M_2^2+M_3^2}{M_1^2+M_4^2}}\right) & 0 \\ 0 & 0 & 0 & M \end{pmatrix} \quad (20)$$

Then we have to find the matrix U_3 and to diagonalize $\hat{m}$. To do it let us at first transform it into upper triangular form by multiplying to $\hat{V}$:

$$\bar{m} = \hat{m}\hat{V} = \begin{pmatrix} \gamma\bar{\mu}_1 & \bar{\mu}_1 & \beta\mu_1 \\ 0 & \bar{\mu}_2 & \alpha\bar{\mu}_2 \\ 0 & 0 & \bar{\mu}_3 \end{pmatrix} \tag{21}$$

If we represent the matrix $\hat{m}$ as three vectors:

$$\hat{m} = \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \end{pmatrix} \tag{22}$$

our transformation is just the change of the basis of these vectors. So if we orthogonalize the set of $\bar{v}_1, \bar{v}_2, \bar{v}_3$ and use the obtained vectors $\bar{e}_1, \bar{e}_2, \bar{e}_3$ as a new basis for the first vectors we get $\bar{m}$ in the form we need. Particularly

$$\begin{aligned} \bar{e}_3 &= \frac{\bar{v}_3}{\|\bar{v}_3\|} \\ \bar{e}_2 &= \frac{\bar{v}_2}{\|\bar{v}_2\|} - \frac{\bar{e}_3\bar{v}_2}{\|v_2\|}\bar{e}_3 \\ \bar{e}_1 &= \bar{e}_2 \times \bar{e}_3 \end{aligned} \tag{23}$$

One can find that

$$\bar{\mu}_3 = \sqrt{|\mu_3|^2 + \tilde{\mu}_3^2 - \frac{|\mu_3 M_3 + \tilde{\mu}_3 M_4|^2}{M^2}} \tag{24}$$

$$\alpha\bar{\mu}_2 = \frac{\tilde{\mu}_2\tilde{\mu}_3}{\bar{m}u_3} - \frac{(\mu_2 M_2 + \tilde{\mu}_2 M_4)(\mu_3^* M_3 + \tilde{\mu}_3 M_4)}{\bar{\mu}_3 M^2} \tag{25}$$

$$\bar{\mu}_2 = \sqrt{|\mu_2|^2 + \tilde{\mu}_2^2 - |\alpha\mu_2|^2} \tag{26}$$

$$\beta\bar{\mu}_1 = \frac{\tilde{\mu}_1\tilde{\mu}_3}{\bar{m}u_3} - \frac{(\mu_1 M_1 + \tilde{\mu}_1 M_4)(\mu_3^* M_3 + \tilde{\mu}_3 M_4)}{\bar{\mu}_3 M^2} \tag{27}$$

$$\bar{\mu}_1 = \tilde{\mu}_1(\frac{\tilde{\mu}_2}{\bar{\mu}_2} - \alpha^*\frac{\tilde{\mu}_3}{\bar{\mu}_3}) - \frac{\mu_1 M_1 + \tilde{\mu}_1 M_4}{M}\left(\frac{M_4}{M}(\frac{\tilde{\mu}_2}{\bar{\mu}_2} - \alpha^*\frac{\tilde{\mu}_3}{\bar{\mu}_3}) + \frac{\tilde{\mu}_2^* M_2}{\bar{\mu}_2 M} - \alpha^*\frac{\tilde{\mu}_3^* M_3}{\bar{\mu}_3 M}\right) \tag{28}$$

$$\gamma\mu_1 = \sqrt{|\mu_1|^2 + \tilde{\mu}_1^2 - \frac{|\mu_1 M_1 + \tilde{\mu}_1 M_4|^2}{M^2} - |\bar{\mu}_1|^2(1+\beta^2)} \tag{29}$$

The diagonalization of $\bar{m}$ is made by the transformation

$$D = U_3^+ \bar{m} V_3'.$$

Since $\bar{m}$ is not Hermitian to find eigenvalues it is necessary to consider $\bar{m}^+\bar{m}$ to solve the characteristic equation for it. After expanding over γ the eigenvalues are

$$m_d \approx |\gamma\bar{\mu}_1| \tag{30}$$

$$m_s \approx \bar{\mu}_2 \tag{31}$$

$$m_b \approx \bar{\mu}_3 \tag{32}$$

$$V_3' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{|\bar{\mu}_2|^2\alpha+|\bar{\mu}_1|^2\beta}{|\bar{\mu}_3|^2} \\ 0 & -\frac{|\bar{\mu}_2|^2\alpha^*+|\bar{\mu}_1|^2\beta^*}{|\bar{\mu}_3|^2} & 1 \end{pmatrix} \tag{33}$$

$$U_3 = \begin{pmatrix} 1 & \frac{\bar{\mu}_1}{|\bar{\mu}_2|} & \frac{\beta\bar{\mu}_1}{|\bar{\mu}_3|} \\ -\frac{\bar{\mu}_1^*}{|\bar{\mu}_2|} & 1 & \frac{\bar{\mu}_2\alpha}{|\bar{\mu}_3|} \\ -\frac{\beta^*\bar{\mu}_1^*}{|\bar{\mu}_3|} & \frac{\bar{\mu}_2^*\alpha^*}{|\bar{\mu}_3|} & 1 \end{pmatrix} \tag{34}$$

If we take into account that the mass matrix for u-quarks is diagonal we can see that

$$V_{CKM} = U_3$$

3 Interaction between d-quarks and Z boson

One of the features of the model with vector like forth generation is the presence of Flavour Changing Neutral Current (FCNC) at tree level. It allows different processes where the flavour of a fermion changes but its charge remains invariant. In this model it appears in the terms responsible for interaction with Z boson and neutral Higgs. Let us find FCNC interacting with Z. In the Lagrangian of the Standard Model the weak interaction terms are

$$L_{weak} = \bar{Q}_L\sigma^\mu(\frac{g}{2}\tau_a W_{\mu,a} + \frac{g'}{6}B_\mu)Q_L - \bar{d}_R\sigma^\mu\frac{g'B_\mu}{3}d_R + \bar{u}_R\sigma^\mu\frac{2g'B_\mu}{3}u_R$$

Observable variables are:

$$Z = W_3\cos\theta_W - B\sin\theta_W$$

$$A = W_3\sin\theta_W + B\cos\theta_W$$

The terms in the Lagrangian related to neutral current are

$$\begin{aligned} \bar{u}_L\sigma^\mu(\frac{g^2 - g'^2/3}{2\sqrt{g^2+g'^2}}Z_\mu + \frac{2gg'}{3\sqrt{g^2+g'^2}}A_\mu)u_L &+ \\ \bar{d}_L\sigma^\mu(-\frac{g^2 + g'^2/3}{2\sqrt{g^2+g'^2}}Z_\mu - \frac{gg'}{3\sqrt{g^2+g'^2}}A_\mu)d_L &+ \\ \bar{u}_R\sigma^\mu\frac{2g'}{3\sqrt{g^2+g'^2}}(-g'Z_\mu + gA_\mu)u_R &- \\ \bar{d}_R\sigma^\mu\frac{g'}{3\sqrt{g^2+g'^2}}(-g'Z_\mu + gA_\mu)d_R & \end{aligned} \tag{35}$$

The terms related to the forth generation are

$$-\bar{d}_{R,4}\sigma^\mu\frac{g'}{3\sqrt{g^2+g'^2}}(-g'Z_\mu + gA_\mu)d_{R,4} - \bar{d}_{L,4}\sigma^\mu\frac{g'}{3\sqrt{g^2+g'^2}}(-g'Z_\mu + gA_\mu)d_{L,4}$$

The weak interaction Lagrangian for d-quarks can be rewritten as

$$L_{d,Z} =$$

$$\bar{d}_L\sigma^\mu(-\frac{g^2+g'^2/3}{2\sqrt{g^2+g'^2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}+\frac{g'^2}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix})Z_\mu d_L -$$

$$\bar{d}_R\sigma^\mu(-\frac{g'^2}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}-\frac{g'^2}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix})Z_\mu d_R -$$

$$\bar{d}_L\sigma^\mu(\frac{gg'}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}+\frac{gg'}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix})A_\mu d_L -$$

$$\bar{d}_R\sigma^\mu(\frac{gg'}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}+\frac{gg'}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix})A_\mu d_R \quad (36)$$

Here d_L , d_R are columns or rows. Each component of them corresponds to each generation. To find FCNC one need go into mass basis. The transformation of d_l is

$$d_L^{mass} = U_3^T U_4^T d_L$$

.

One can see that the only non-trivial FCNC appears in case of interaction between d_l and Z.

$I^\mu_{FCNC} =$

$$\bar{d}_L^{mass}\sigma^\mu U_3^T U_4^T(-\frac{g^2+g'^2/3}{2\sqrt{g^2+g'^2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}+\frac{g'^2}{3\sqrt{g^2+g'^2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix})U_4^* U_3^* d_L^{mass}$$

In the leading order

$$U_4 = \begin{pmatrix} 1 & 0 & 0 & u_1 \\ 0 & 1 & 0 & u_2 \\ 0 & 0 & 1 & u_3 \\ -u_1^* & -u_2^* & -u_3^* & 1 \end{pmatrix}$$

where $u_i = \frac{\tilde{M}_4\tilde{\mu}_i+\mu_i^*\tilde{M}_i}{\tilde{M}^2}$

$$U_3 = \begin{pmatrix} V_{CKM} & 0 \\ 0 & 1 \end{pmatrix}$$

So

$$I^\mu_{FCNC} = \bar{d}_L^{mass}\sigma^\mu(-\frac{g^2+g'^2/3}{2\sqrt{g^2+g'^2}}\begin{pmatrix} 1 & V_{CKM}^T u^* \\ u^T V_{CKM}^* & 0 \end{pmatrix}+$$

$$\frac{g'^2}{3\sqrt{g^2+g'^2}}\left(\begin{array}{cc} 0 & -V_{CKM}^T u^* \\ -u^T V_{CKM}^* & 1 \end{array}\right)) d_L^{mass} \quad (37)$$

The weak interaction Lagrangian can be rewritten using four component spinors. To do it one has to change σ^μ into γ^μ and to substitute $d_L = \frac{1+\gamma^5}{2} d$. In these terms

$$L_{d,Z} = \bar{d}_\alpha \gamma^\mu Z_\mu \frac{\sqrt{g^2+g'^2}}{2}(v_{\alpha\beta} - a_{\alpha\beta\gamma^5}) d_\beta \quad (38)$$

where

$$v_{\alpha\beta} = -\frac{1}{2}\left(\begin{array}{cc} 1 & V_{CKM}^T u^* \\ u^T V_{CKM}^* & 0 \end{array}\right)_{\alpha\beta} + \frac{2}{3}\frac{g'^2}{g^2+g'^2}\delta_{\alpha\beta}$$
$$a_{\alpha\beta} = \frac{1}{2}\left(\begin{array}{cc} 1 & V_{CKM}^T u^* \\ u^T V_{CKM}^* & 0 \end{array}\right)_{\alpha\beta} \quad (39)$$

4 Interaction between d-quarks and Higgs

In case of energies far less GUT scale the mass matrix of d-quarks is derived from the following terms:

$$L_{H,d} = \sum_{i=1}^{3} H_d^a \epsilon_{ab} Q_i^{Lb} h_{ii} d_i^R + \sum_{i=1}^{3} H_d^a \epsilon_{ab} Q_i^{Lb} \frac{g_i^d v_N}{M*} d_4^R + \sum_{i=1}^{3} d_L^4 f_i v_N d_i^R + d_4^L M_d d_4^R \quad (40)$$

Here

$$Q_i = \left(\begin{array}{c} u_i \\ d_i \end{array}\right),$$

H_d is Higgs field, v_N is vacuum expectation of the field responsible for $SO(10)$ breaking, h_{ii}, g_i^d, M^*, f_i, M_d are constants. The vacuum expectation of Higgs is

$$H = \left(\begin{array}{c} 0 \\ v_1 \end{array}\right),$$

so the mass matrix for d-quarks is

$$\left(\begin{array}{cccc} h_{11}v_1 & 0 & 0 & \frac{g_1^d v_N}{M^*} v_1 \\ 0 & h_{22}v_1 & 0 & \frac{g_2^d v_N}{M^*} v_1 \\ 0 & 0 & h_{33}v_1 & \frac{g_3^d v_N}{M^*} v_1 \\ f_1 v_N & f_2 v_N & f_3 v_N & M^d \end{array}\right).$$

Its components can be designated according to (2). One can expand the Higgs field over its vacuum:

$$H_d = \left(\begin{array}{c} 0 \\ v_1 \end{array}\right) + \left(\begin{array}{c} 0 \\ H_d^0 \end{array}\right) + \left(\begin{array}{c} H_d^- \\ 0 \end{array}\right). \quad (41)$$

If fermions are in mass basis the terms interacting with Higgs are

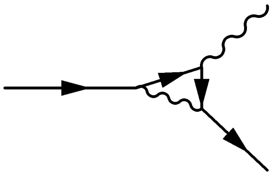
$$L_{d,H_d^0} = H_d^0 d^L \frac{1}{v_1} \begin{pmatrix} m_d & 0 & 0 & -M(V_{CKM}^{+}u)_1 \\ 0 & m_s & 0 & -M(V_{CKM}^{+}u)_2 \\ 0 & 0 & m_b & -M(V_{CKM}^{+}u)_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} d^R \tag{42}$$

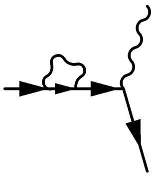
$$L_{d,H_d^-} = H_d^- d^L \frac{1}{v_1} \left(\begin{array}{cc} V_{CKM}\tilde{m}_d & -Mu \end{array} \right) d^R, \tag{43}$$

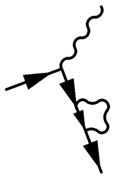
where $\tilde{m}_d = diag(m_d, m_s, m_b)$. So these terms also contain FCNC.

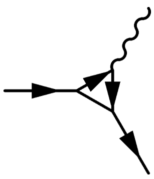
5 Phenomenology

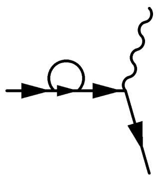
One feature of the model with the vector like forth generation is FCNC at tree level. It gives new diagrams which can contribute to different processes. One possible process is a decay $b \to s\gamma$ via the following channels:

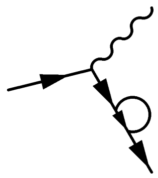












On these digrams the fermion b changes its flavour and finally turns into s or d by emitting and absorbing Z or Higgs boson.Also it emits a photon.

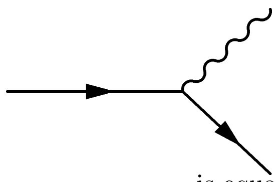
These processes can be calculated in different gauges. In the Feynman gauge the propagator of Z is equal to

$$\frac{-ig_{\mu\nu}}{p^2 - M_Z^2}$$

but in this case we also have to take into account Goldstone bosons which can be instead of Z. In R_ξ gauge when $\xi \to \infty$ the Z propagator is written as

$$\frac{-i(g_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2})}{p^2 - M_Z^2}.$$

In this case we have not to take into account Goldstone bosons. Let us calculate the diagram by the first way. The vertex function for interaction between fermions and Z-boson



is equal to

$$-i\frac{\sqrt{g^2 + g'^2}}{2}\gamma^\mu(v_{\alpha\beta} - a_{\alpha\beta}\gamma^5) \tag{44}$$

where $v_{\alpha\beta}$,$a_{\alpha\beta}$ are derived according to (39) The vertex for fermions and photons is assosiated with the function

$$i\frac{gg'}{3\sqrt{g^2 + g'^2}} \tag{45}$$

The fermion propagator is

$$\frac{i(\hat{p} + m)}{p^2 - m_\alpha^2} \tag{46}$$

So the amplitude is

$$'iM = \frac{1}{3!}\int \frac{d^4p}{(2\pi)^4}\frac{gg'\sqrt{g^2+g'^2}}{12}\bar{s}\gamma^\mu(v_{s\alpha}-a_{s\alpha}\gamma^5)\frac{\hat{p}+m_\alpha}{p^2-m_\alpha^2}\gamma^\lambda$$
$$\frac{\hat{k}+\hat{p}+m_\alpha}{(k+p)^2-m_\alpha^2}\gamma_\mu(v_{\alpha b}-a_{\alpha b}\gamma^5)\frac{1}{(q-p)^2-M_Z^2}b \quad (47)$$

Here p is the loop momentum, k is the momentum of the photon, q is the momentum of the s-quark. By taking into account that for any function$F$ depending only on the module of the momentum

$$c\int d^4pp_\rho p_\sigma F(p^2) = \frac{g_{\rho\sigma}}{4}\int d^4pp^2F(p^2)$$

this expression can be transformed to

$$iM =$$
$$\frac{1}{3!}\frac{gg'\sqrt{g^2+g'^2}}{12}\bar{s}(v_{s\alpha}+a_{s\alpha}\gamma^5)(\gamma^\lambda(I_2-2m_\alpha^2I)+4k^\lambda m_\alpha I)(v_{\alpha b}-a_{\alpha b}\gamma^5)b \quad (48)$$

where

$$I = \int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-m_\alpha^2)((k+p)^2-m_\alpha^2)((q-p)^2)-M_Z^2} \quad (49)$$
$$I_2 = \int\frac{d^4p}{(2\pi)^4}\frac{p^2}{(p^2-m_\alpha^2)((k+p)^2-m_\alpha^2)((q-p)^2)-M_Z^2} \quad (50)$$

Both integrals are calculated in the Appendix and the second of them is divergent. To make an explicit calculation it is necessary to summarize all possible diagram but unfortunately this part is not done completely by the author of this project.

6 Conclusions

In this report we have described one possible construction of an extension of the Standard Model with extra vectorlike fermions. In [1] it is shown that the parameters of this model, particularly the components of the mass matrix (2) can be chosen in such way that they fit the experimental values of d-quarks masses and the components of CKM matrix. A peculiarity of this model is the presence of tree level FCNC which can contribute to different processes, such as a decay $b \to s/d\gamma$. Also we have shown the idea of calculating such processes. A similar problem is also considered in [4]. In higher orders over μ/M FCNC can also contribute to some other processes, for example $B_0-\bar{B}_0$ mixing, decays $Z \to d_\alpha d_\beta$, $b \to s/dl^+l^-$ etc.

In the present moment the calculation of the process $b \to s/d\gamma$ is not finished by the author of this report. He hopes to continue this work further and finally to get numerical constraints for the masses of the forth generation particles and for their mixing with observable particles.

I would like to thank my supervisors Laura Covi and Jae-Hyeon Park for fruitful help in doing this project.

7 Appendix. Calculation of the integrals in the diagrams.

At first the integrals are transformed by introducing the Feynman parameters x,y via formula:

$$\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} dy \frac{1}{(A \times +By + C(1-x-y))^3} \tag{51}$$

After the change $p + kx - qy \to p$ the first integral looks like

$$I = \int \frac{d^4p}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{2}{(p^2 + k^2x + q^2y - (kx - qy)^2 - m_\alpha^2(1-y) - M_Z^2 y)^3} \tag{52}$$

Then we make Wick transformation which change Minkovski space into Euclidian:

$$\begin{aligned} p_0 &\to ip_0 \\ p_i &\to p_i \end{aligned} \tag{53}$$

therefore $p^2 \to -p^2$.

Also we introduce dimensional regularization and change the measure of integration according to

$$\int d^dp = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int_0^\infty dpp^{d-1}$$

Here $d = 4 - 2\epsilon$ After these transformations

$$\begin{gathered} I = -\frac{4\pi^{2-\epsilon} i}{\Gamma(2-\epsilon)(2\pi)^4} \int dpp^{3-2\epsilon} \int_0^1 dx \int_0^{1-x} dy \\ \frac{1}{(p^2 - k^2x - q^2y + (kx - qy)^2 + m_\alpha^2(1-y) + M_Z^2 y)^3} \end{gathered} \tag{54}$$

The integral over p is calculated by using the formula

$$\int_0^\infty \frac{p^{d-1}dp}{(p^2 + D)^L} = \frac{D^{d/2-L}}{2} \frac{\Gamma(d/2)\Gamma(L - d/2)}{\Gamma(L)}. \tag{55}$$

So if $\epsilon \to 0$

$$I = -\frac{\pi^2 i}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{1}{-k^2x - q^2y + (kx - qy)^2 + m_\alpha^2(1-y) + M_Z^2 y}. \tag{56}$$

It can be simplified by taking into account that

$$
\begin{aligned}
k^2 &= 0 \\
q^2 &= m_s^2 \\
2(kq) &= m_b^2 - m_s^2 \approx m_b^2
\end{aligned}
\tag{57}
$$

The final answer is

$$
I = -\frac{\pi^2 i}{(2\pi)^4} \frac{1}{M_Z^2 - m_{\alpha^2}} \left(-1 + \frac{1}{r_\alpha - 1} \ln r_\alpha \right) \tag{58}
$$

where $r_\alpha = \frac{m_\alpha^2}{M_Z^2}$.

I_2 is calculated in the similar way. It can be transformed to

$$
I_2 = I_2^{(1)} + I_2^{(2)}
$$

where

$$
I_2^{(1)} = \int \frac{d^4 p}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{2(kx - qy)^2}{(p^2 + k^2 x + q^2 y - (kx - qy)^2 - m_\alpha^2 (1-y) - M_Z^2 y)^3} \tag{59}
$$

$$
I_2^{(2)} = \int \frac{d^4 p}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{2p^2}{(p^2 + k^2 x + q^2 y - (kx - qy)^2 - m_\alpha^2 (1-y) - M_Z^2 y)^3} \tag{60}
$$

$I_2^{(1)}$ is convergent and equal to

$$
I_2^{(1)} = \frac{\pi^2 i}{(2\pi)^4} \frac{m_b^2}{2(M_Z^2 - m_{\alpha^2})} \left((\frac{r_\alpha}{1 - r_\alpha})^2 + \frac{3}{2} \frac{r_\alpha}{1 - r_\alpha} + \frac{1}{3} + \frac{r_\alpha}{(1 - r_\alpha)^3} \ln r_\alpha \right) \tag{61}
$$

But $I_2^{(2)}$ is divergent. After introducing dimensional regularization it is equal to

$$
I_2^{(2)} = \frac{4 i \pi^2}{(2\pi)^4 \Gamma(2 - \epsilon)} \frac{\Gamma(\epsilon)}{2} \int_0^1 dx \int_0^{1-x} dy (-k^2 x - q^2 y + (kx - qy)^2 + m_\alpha^2 (1-y) + M_Z^2 y)^{-\epsilon} \tag{62}
$$

After expanding overϵ and taking into account that

$$
\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma_e + o(\epsilon)
$$

where γ_e is the Euler constant the integral turns into

$$
\frac{2 i \pi^2}{(2\pi)^4} \left(\frac{1}{2\epsilon} + const - I_3 \right) \tag{63}
$$

where

$$
I_3 = \int_0^1 dx \int_0^{1-x} dy \ln(-k^2 x - q^2 y + (kx - qy)^2 + m_\alpha^2 (1-y) + M_Z^2 y) \tag{64}
$$

Finally

$$I_3 = \ln M_Z - \frac{r_\alpha(2-r_\alpha)}{2(1-r_\alpha)^2}\ln r_\alpha + \frac{1+r_\alpha}{4(r_\alpha-1)} - \frac{1}{2} \tag{65}$$

and

$$I_2 = \frac{\pi^2 i}{(2\pi)^4}\frac{m_b^2}{2(M_Z^2 - m_{\alpha^2})}\left((\frac{r_\alpha}{1-r_\alpha})^2 + \frac{3}{2}\frac{r_\alpha}{1-r_\alpha} + \frac{1}{3} + \frac{r_\alpha}{(1-r_\alpha)^3}\ln r_\alpha\right) \tag{66}$$

$$+\frac{2i\pi^2}{(2\pi)^4}\left(\frac{1}{2\epsilon} + const - \ln M_Z + \frac{r_\alpha(2-r_\alpha)}{2(1-r_\alpha)^2}\ln r_\alpha - \frac{1+r_\alpha}{4(r_\alpha-1)} + \frac{1}{2}\right) \tag{67}$$

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