Preheating in gravitation coupled quartic inflation

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Introduction The inflationary Universe model explains the Cosmic Microwave Background fluctuations and solves the flatness and horizon problems of the standard big bang cosmology, so it is a subject of careful investigation during the past two decades [1]. After the work of A.Guth [2] who proposed the first inflationary model a lot of theories of inflation was developed. It's difficult to decide which model is more realistic because of lack of information about such early times. In particular the only experimental issues for to understand inflation properties are matter distribution in the Universe and CMB fluctuations data (COBE and WMAP). The relic gravitational waves could be another source of information when detected. So the criteria to determine the likelihood of some model is the spectrum of fluctuations generated after the end of inflation. In this project we discuss preheating in quartic inflation. Firstly we investigate preheating in simple $\lambda\phi^4$-model proposed by A. Linde [3]. We will see that unfortunately the thermalization in this model is forming too slow. Besides, tensor perturbations after inflation in this model are too large and do not correspond to the WMAP data. So this theory is currently out of interest. But there is a way to rehabilitate it. We investigate the simple generalization of this model — the theory with introduced non-minimally coupling with gravitation with a large coupling constant. This model is also a limit of the model recently investigated by F.Bezrukov and M.Shaposhnikov [4] when neglecting the spontaneously broken symmetry.Whey showed that inflaton and the Higgs field could be the same in this model. So we'll study the properties of preheating in renewed model and compare numerical results with simple quartic inflation.

In the first section we briefly describe the chaotic inflation in slow-roll approximation. Then we switch to the $\lambda\phi^4$-model and investigate processes of decaying inflaton field after the end of inflation. We discuss parametrical resonance in the theory and obtain numerically the spectra of fluctuations and discuss its properties. In the next section we introduce non-minimally coupling with gravitation use conformal transformation to bring problem back to Minkowsky space and carry out numerical analysis.All computations were conducted using "LatticeEasy" package [5].

Slow-roll approximation Let's discuss the simplest approximation in the model of chaotic inflation. Consider the evolution of the scalar field ϕ, minimally interacting with gravitation

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right),$$

In the case of homogeneous scalar field in the flat Friedmann-Robertson-Walker Universe the equations of motion take the form

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0,$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right).$$

First equation is similar to the newtonian equation of motion of a particle. Assume that our particle is slowly rolling down its potential. So the friction should be much greater than acceleration and kinetic energy much smaller than potential $\ddot{\phi} \ll 3H\dot{\phi}$, $\dot{\phi}^2 \ll V(\phi)$ The last condition leads to the state equation $p \simeq -\rho$ which is corresponding to vacuum-like matter and enough to inflation. The equations in the "slow-roll" approximation reduce to the following system

$$\dot{\phi} = \frac{V'(\phi)}{3H},$$

$$H = \sqrt{\frac{8\pi G}{3}V(\phi)},$$

It is useful to relate our conditions with properties of the potential. Substituting the second equation into the first we can obtain for the rolling rate

$$\dot{\phi} = -\frac{V'(\phi)}{\sqrt{24\pi G V(\phi)}}.$$

And for acceleration we have

$$\ddot{\phi} = -\frac{1}{8\pi G}\left(\frac{V''}{V} - \frac{1}{2}\left(\frac{V'}{V}\right)^2\right)H\dot{\phi},$$

It is obvious now that slow-roll conditions lead to the following restrictions on potential

$$\epsilon = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2 \ll 1,$$

$$\eta = \frac{1}{8\pi G}\frac{V''}{V} \ll 1.$$

Let's now compute the number of e-foldings in our model. It could be done straightforward using the slow-roll equations

$$N_e \equiv \int_{t_i}^{t_f} H dt = -8\pi G \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$$

Applying the last formula to the power potential $g\phi^n$ we realize that for the successful inflation we should take the superplank fields, for example, if $n = 4$ and $N_e \simeq 60$, ϕ is greater than $3M_{PL}$

Preheating in ϕ^4 theory While inflaton field slowly rolling down, ϵ and η increase and inflation comes to the end when $\epsilon \sim 1$. After a short stage of fast rolling down the inflaton field starts to oscillate near the minimum of potential with very large amplitude about $0.1M_{PL}$. The energy of these oscillations turns into the energy of elementary particles produced. Eventually particles come to a thermal equilibrium interacting with each other. This process is known as reheating. To obtain particles from inflaton oscillations we should put some interaction terms into the lagrangian. The inflaton can decay into bosons $-g^2\phi^2\chi^2$, fermions $-h\bar{\psi}\psi\phi$ or into its own quanta $\lambda\phi^2\delta\phi^2$. Let's review this process in the simple case of inflaton with $\lambda\phi^4/4$ potential decaying on its own quanta. The equation of motion with dependence on $\vec{r}$ is

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\triangle\phi}{a^2} + V'(\phi) = 0.$$

It's convenient to switch to a conformal time $dt = ad\eta$ and redefine the field $\phi = v/a$ We obtain

$$v'' + \triangle v - \frac{a''}{a}v + \lambda v^3 = 0$$

Note that $a''/a = 2/\eta^2$ is negligible during the inflation. Then for space-uniform field we obtain an autonomous equation whose solution is the Jacobi sine

$$v_0 = \pm v_i\, sn(v_i\sqrt{\frac{\lambda}{2}}\eta, -1)$$

Now introducing perturbations on a homogeneous background $v = v_0 + \delta v$ we get linearized equation for Fourier modes

$$\delta v_k'' + (k^2 + 3\lambda v_0^2)\delta v_k = 0,$$

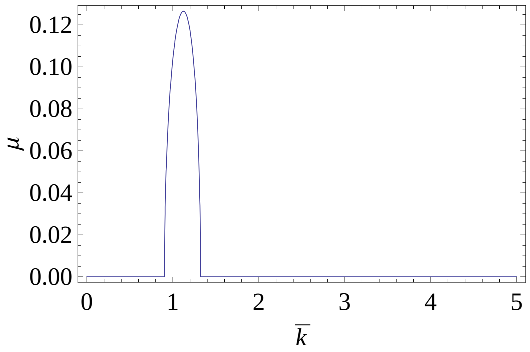

Figure 1: The real part of characteristic exponent μ as a function of rescaled wave number $\bar{k}$

. This is the equation of motion for harmonic oscillator with variable frequency. The Jacobi sine with parameter -1 is very close to ordinary sine so the equation can finally be reduced to the Mathieu equation with the solution

$$\delta v_k = A\, ce\left(\bar{k}^2 + 3, \quad \frac{3}{2}, \quad v_i\sqrt{\frac{\lambda}{2}}\eta\right)$$

with $\bar{k} = 2k^2/\lambda v_i^2$. Note that the form of solution does not depend on the initial value of field. It is known that every solution of Mathieu equation can be presented in a form $f(t) = e^{\mu}F(t)$ where μ is complex characteristic number and $F(t)$ is 2π-periodic. For some specific values of parameters $Re\,\mu \neq 0$ and solution is growing exponentially. Therefore a parametrical resonance occur in this system. It leads to explosive non-thermal particle production for some wave numbers. This process is called preheating to distinguish from a thermal reheating. The characteristic exponent dependence on wave number is shown on Fig[1]. We can see that firstly modes with $\bar{k} \sim 1.1$ are growing.

Results of numerical investigation for $\lambda = 9{*}10^{-13}$ are presented on a figure. We can see that firstly the occupation number for $k \sim 1.27$ starts to grow. This is the parametrical resonance. $\lambda\phi^4$ is conformal invariant theory and all its characteristics are moving with expanding Universe, so the peak is standing on the same value of k. Later more peaks occur due to re-scattering of waves from the first peak. Field quanta strongly interact with each other and finally we obtain some kind of equilibrium spectrum after $\eta = 1000$. It contains two region — the power low k^{-s} with $s \sim 1.5$ and the exponential cut off at higher k. So we see that our system doesn't come to the thermal equilibrium but the particle distribution corresponds to Kolmogorov turbulence. Besides, the motion can be described as a self-similar evolution [7]

$$n(k, \eta) = \eta^{-q} n_0(k\eta^{-p})$$

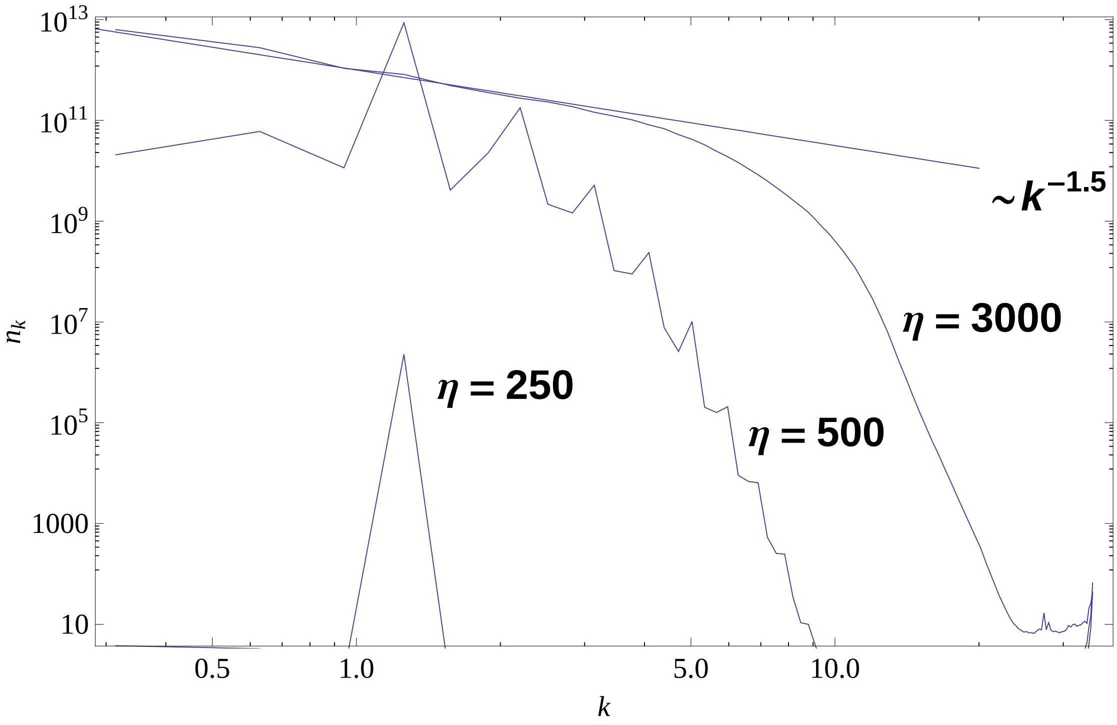

Figure 2: Spectra evolution in $\lambda\phi^4$

with $q \simeq 3.5p$ and $p \simeq 1/5$. So, the equilibration time for this model is veri large and it is not the only problem of $\lambda\phi^4$ inflation. The predicted value of the tensor to scalar perturbation ratio in this theory lies outside the range allowed by current WMAP data. Therefore in spite of conformal invariance this theory cannot be accepted as good theory of inflation. But introducing decoupling to other fields we can avoid this problems. There is however one more way to improve the theory.

ϕ^4 theory non-minimally coupled with gravitation. Let's now modify our model by introducing interaction between inflaton and gravitational field. This is the limit of Bezrukov and Shaposhnikov model [4], when the amplitude of field fluctuations is large and we can let the vacuum value $v \simeq 0$. Such type of models have been investigated for a long time [6]. If we put $M_{Pl}/\sqrt{8\pi} = 1$ the action takes the form

$$S = \int d^4x\sqrt{-g}\left(-\frac{1+\xi\phi^2}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4\right),$$

Here ξ is the arbitrary parameter to be chosen. If ξ is small the new theory is close to the minimally coupled one and the preheating process is sufficiently the same. So we are interesting in large values of ξ.

It is useful to make a conformal transformation from the Jordan frame to the Einstein frame for to get rid of non-minimal coupling to gravity.

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = 1 + \xi\phi^2$$

This transformation leads to a complex kinetic term for ϕ. But we can change variables following [8]

$$\left(\frac{d\chi}{d\phi}\right)^2 = \frac{\Omega^2 + 6\xi^2\phi^2}{\Omega^2}, \qquad U(\chi) = \frac{\lambda\phi(\chi)^4}{4\Omega^2}$$

For $\xi \gg 1$ first equation has a solution

$$\phi = \sqrt{\frac{1}{\xi}\left(e^{\pm\chi\sqrt{\frac{2}{3}}} - 1\right)}$$

Now we collect all our transformations and write down new action for χ

$$S = \int d^4x\sqrt{-\hat{g}}\left(-\frac{1}{2}\hat{R} + \frac{1}{2}\partial_\mu\chi\partial_\nu\chi - \frac{\lambda}{4\xi^2}\tanh^2\frac{\chi}{\sqrt{6}}\right),$$

In the region of small χ values the new potential vanishes as χ^2 and when χ is large $U(\chi)$ is exponentially flat so the successful chaotic inflation is possible. We can use the slow-roll approximation and express the slow-roll parameters in terms of ξ [4]

$$\epsilon = \frac{1}{2}\left(\frac{U'(\chi)}{U(\chi)}\right)^2 \simeq \frac{4}{3\xi^2\phi^4}$$

$$\eta = \frac{U''(\chi)}{U(\chi)} \simeq \frac{4}{3\xi\phi^2}.$$

The inflation ends when $\epsilon \simeq 1$ at the field value in Plank masses $\phi_e \simeq 1.07/\sqrt{\xi}$. The number of e-foldings

$$N = \int_{\phi_e}^{\phi_i} \frac{U}{dU/d\phi}\left(\frac{d\chi}{d\phi}\right)^2 d\phi \simeq \frac{6}{8}\xi(\phi_i^2 - \phi_e^2)$$

Let's compare it with the COBE data. $N_{COBE} \simeq 62$ so $\phi_{COBE} \simeq 9.4/\sqrt{\xi}$ therefore using the COBE normalization $U/\epsilon = (0.027M_{Pl})^4$ we find for ξ

$$\xi \simeq \sqrt{\frac{\lambda}{3}\frac{N_{COBE}}{0.027^4}} = 49000\sqrt{\lambda}$$

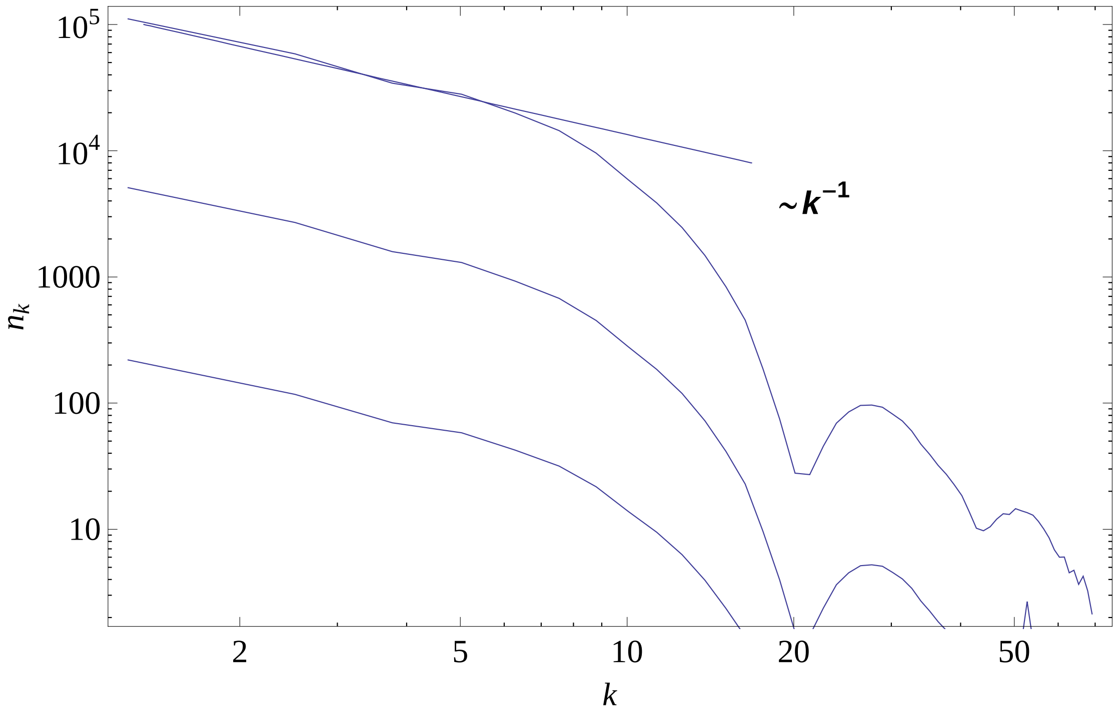

Figure 3: Spectra evolution in $\lambda\phi^4$ non-minimally coupled with gravitation

The numerical simulation of processes in new theory is shown on a figure 3. New theory is not scale invariant and solution depends on initial value of inflaton field. New potential is close to ϕ^2 near its minimum. So the effective preheating occur right after the end of inflation on a first few fluctuations of inflaton when amplitude is large. Then nonlinearity is negligible and the evolution is similar to an evolution of the free field. So we should start with large amplitude for to generate enough particles in the first few fluctuations. As we see from simulation the thermal spectra occur in the model. But the Universe is expanding very fast and after few fluctuations the spectrum freezes and the inflaton behaves as free field. We can compute the temperature of the Universe after preheating

$$T = \sqrt{\frac{12\xi^2}{\lambda}\langle\triangle\chi\rangle^2} \simeq 10^{-5} M_{PL}$$

Conclusion We discussed properties of preheating in the quartic inflation. We obtained the spectrum of created particles number density and discuss it following [7]. Then we examined non-minimally gravitation coupled generalization and showed that successive preheating in improved model occurs much faster and the temperature lies at the GUT-scale and enough to further evolution of the hot Universe. The detailed studying of the properties of obtained spectra will be carried out shortly.

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