Stable de Sitter vacua in Supergravity

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Abstract

We study de Sitter vacua in general Supergravity models with $\mathcal{N}=1$ Supersymmetry in four dimensions. We will review the conditions for metastability of non-supersymmetric vacua and analyse these conditions, reproducing the relevant equations. We go on to looking at particular cases of metastability in large-volume scenarios, where the Kähler potential satisfies the no-scale property. Finally we perform the derived analysis on several string-theoretically relevant examples

1 Introduction

The most accurate theory today to describe physical laws turns out to be the Standard Model, a relativistic Quatum Field Theory. It consists of quantum mechanics and special relativity and evolves around physical laws, wich are *local*. The most important experimental result underlining the Standard Model was the Muon (g-2) Experiment at Brookhaven National Laboratory, which measured the Anomalous Magnetic Moment of the Muon to unprecedented precision.

QFT is necessary for any relativistic system, but also highly useful in systems with many particles. It has great importance in condensed matter, highenergy physics, cosmology, quantum gravity and pure mathematics.

In QFT we quantize the classical field. The fundamental degrees of freedom in QFT are operator valued functions of spacetime. Since there is at least one degree of freedom for every point in space, we have the trouble of dealing with an infinite number of degrees of freedom.

The main principles governing the interactions in QFT end up being: locality, symmetry and renormalization (difference of physics at small and large scales). [2]

We are interested in this symmetry and its features and where it is locally broken. It took a long time to develop the current Standard Model. Now we are interested in what might lie beyond its present ideas. An eventual goal is to see whether certain models of potential are favoured above others. Supersymmetry is the basic idea of extending the Standard Model. In particle theory, Supersymmetry is recognized as necessary to solve the Hierarchy problem of the unification scale and the electroweak scale. It protects the Higgs boson mass against radiative corrections, and can also provide a natural dark matter candidate. SuSy can either be global or local, in the latter case it would be called Supergravity, as it includes gravity, and is most likely to be the version chosen by Nature.

Our believe of the Standard Model of gauge interations to be incomplete is among others based on the facts that the theory has too many parameters, its incapability to describe fermion masses and why the number of generations is three. Neither does it contain the notion of gravity. So we speculate a new symmetry: the yet undiscovered Supersymmetry. It deals with the problems mentioned above, that the current non-supersymmetric field theory is not able to tackle.

Supersymmetry is expected to play a fundamental role during the early evolution of the Universe, particularly during inflation. Invoking scalar fields means that Supersymmetry is involved and it helps for the necessety of the potential to be very flat in the direction of the inflaton.[3]

Today's observations strictly imply that in the case of Supersymmetry, it must be broken spontaneously. This spontaneous breaking occurs when supersymmetric system goes to a non-symmetric vacuum state. Therefore we are interested in the vacuum expectation values of the potential. In the low-energy regime we require the one generator supersymmetry kown as $\mathcal{N}$=1.

Our study focuses on de Sitter vacua relevant in no-scale supergravities and Calabi-Yau string models. It is mostly based on understanding the paper [1] and deriving the main equations and conditions.

We perform a precise analysis of four-dimensional $\mathcal{N}$=1 supergravities by paying particular attention to the mass of scalar superpartners of the Goldstino, which is relevant for metastability of the vacuum. Although the possibility of appropriately choosing the superpotential we can make the scalar fields arbitrarily massive, it becomes impossible in the case of two sGoldstinos. The reason is that within supersymmetry the Goldstino is massless and therefore the sGoldstino cannot accquire its mass from the superpotential, but the mass is generated by the SuSy-breaking mechanism, making it possibly negative. Around this fact the study for locally stable de Sitter vacua is evolving. This strictly implies that for metastability of de Sitter vacua we need a positive sGoldstino mass, which will solely depend on the Kähler Potential, but not on the superpotential. This simplifies the search for stable de Sitter vacua and motivates the following chapters. [1]

2 Conditions for Metastable vacua

We review and extend strategies from refs.[4, 5, 6], following closely ref.[1]. We study the stability of non-supersymmetric vacua with $\mathcal{N}$=1 supersymmetry in four dimensions, centering it on theories with only chiral multiplets.

In supergravity theory we find that the two-derivative Lagrangian is defined by a real function G, depending on Φ^i and $\bar{\Phi}^{\bar{i}}$. Since we are using Planck units G can be written as following:

$$G(\Phi,\bar{\Phi}) = K(\Phi,\bar{\Phi}) + \log W(\Phi) + \log \bar{W}(\bar{\Phi}) \tag{1}$$

The Kähler Potential K and the Superpotential W are defined only up to Kähler transformations, which for an arbitrary holomorphic function f act as $K \to K + f + \bar{f}$ and $W \to We^{-f}$

Before we start a couple of words on notation. Φ^i will generally denote a chiral superfield and $\bar{\Phi}^{\bar{i}}$ its complex conjugate. Derivatives with respect to these fields will be denoted by lower indices i and $\bar{i}$ respectively.

The nabla symbol ∇_i denotes the covariant derivative, which acting on a function $f(\Phi,\bar{\Phi})$ is defined to be ∇_i=$\partial_i - \Gamma^k_{ij}$ and Γ^k_{ij}=$G_{ij\bar{l}}G^{\bar{l}k}$. The associated Riemann Curvature Tensor we accept without further explanation to be $R_{i\bar{j}m\bar{n}}$=$K_{i\bar{j}m\bar{n}} - K_{im\bar{r}}g^{\bar{r}s}K_{s\bar{j}\bar{n}}$

Planck units will be imposed with M_P=1.

Throughout the article we will use the summation convention of repeated indices and the Kähler metric $g_{i\bar{j}}$=$K_{i\bar{j}}$=$\partial_i\partial_{\bar{j}}K$ is used to raise and lower indices. It is assumed to be strictly greater than 0 for the scalars kinetic energy to be positive.

The Potential takes the form:

$$V = e^G(G^iG_i - 3) \tag{2}$$

The auxiliary fields of the chiral multiplets are determined by their equations of motion to be F^i=$m_{3/2}G^i$ with a scale defined by the gravitino mass $m_{3/2}$=$e^{G/2}$. Supersymmetry is spontaneously broken, when F^i is non-vanishing on the vacuum. The Goldstino, which is absorbed by the gravitino during SuSy-breaking, is defined by the direction G^i in the space of chiral fermions.

2.1 Derivation of the Metastability Condition

Assuming that the cosmological constant is not negative metastale susy-breaking vacua are associated to the global minima of the Potential $V \geq 0$, while the auxiliary fields and chiral multiplets are not 0. At the vaccuum expectation value (vev) the conditions for the stationary points are $V' = 0$ (critical point, necessary condition) and the Hessian matrix should be positive definite: $V'' \geq 0$ (Stability).

Let us take first and second order derivatives of V. Since V is a scalar function we find V_i=∇_iV. Then using G^i=$G^{i\bar{j}}G_{\bar{j}}$ and the Product Rule of the covariant Derivative and noting that $\nabla_i G^{i\bar{j}}$=$\nabla_i G_{i\bar{j}}$=0 on (2) we find:

$$\begin{aligned} V_i = \nabla_i V = \nabla_i(e^G)(G^{k\bar{j}}G_k G_{\bar{j}} - 3) + e^G \nabla_i (G^{k\bar{j}}G_k G_{\bar{j}} - 3) \\ = G_i V + e^G[(\nabla_i G^{k\bar{j}} G_k G_{\bar{j}} + G^{k\bar{j}}\nabla_i G_k G_{\bar{j}} + G^{k\bar{j}} G_k \nabla_i G_{\bar{j}}] \\ = G_i V + e^G[G^{k\bar{j}}\nabla_i G_k G_{\bar{j}} + G^{k\bar{j}} G_k \nabla_i G_{\bar{j}}] \\ = G_i V + e^G[G^k \nabla_i G_k + G^{k\bar{j}} G_k G_{i\bar{j}}] \end{aligned}$$

Using the Statioary condition and $G^{k\bar{j}}G_{i\bar{j}}$=δ_i^k we get

$$G_i V + e^G[G^k \nabla_i G_k + G_i] = 0 \tag{3}$$

Now taking higher order Derivatives $V_{i\bar{j}}$ and V_{ij} we get

$$\begin{aligned} V_{i\bar{j}} = \nabla_{\bar{j}}\nabla_i V = \nabla_{\bar{j}}(G_i V + e^G[G^k\nabla_i G_k + G_i]) \\ = e^G(G_{\bar{j}}(G_i + G^k\nabla_i G_k) + G_{i\bar{j}} + \nabla_{\bar{j}}(G^k\nabla_i G_k)) + G_{i\bar{j}}V + G_i \nabla_{\bar{j}} V \end{aligned} \tag{4}$$

By the Stationary Condition (3) we get $e^G[G^k\nabla_i G_k + G_i] = -G_i V$ and $G_i \nabla_{\bar{j}} V = 0$

So

$$V_{i\bar{j}} = e^G(G_{i\bar{j}} + \nabla_{\bar{j}} G^k \nabla_i G_k + G^k \nabla_i \nabla_{\bar{j}} G_k) - G_i G_{\bar{j}} V + G_{i\bar{j}} V \tag{5}$$

Now

$$\begin{aligned} G^k \nabla_i \nabla_{\bar{j}} G_k = G^k \nabla_i [G_{k\bar{j}} - \Gamma^{\bar{m}}_{k\bar{j}} G_{\bar{m}}] \\ = G^k \nabla_i (G_{k\bar{j}} - G_{\bar{k}\bar{j}r} G^{r\bar{m}} G_{\bar{m}i}) \\ = G^k (G_{\bar{k}\bar{j}i} - \nabla i G_{\bar{k}\bar{j}r} G^r - G_{\bar{k}\bar{j}r} G^{r\bar{m}} G_{\bar{m}i}) \end{aligned}$$

Note that $G^{r\bar{m}}G_{\bar{m}i}$=δ_i^r, so that $G_{\bar{k}\bar{j}r}G^{r\bar{m}}G_{\bar{m}i}$=$G_{\bar{k}\bar{j}i}$.
So $G^k\nabla_i\nabla_{\bar{j}}G_k$ becomes

$$\begin{aligned} G^k \nabla_i \nabla_{\bar{j}} G_k = -G^k G^r \nabla_i G_{\bar{k}\bar{j}r} = G^k G^r (G_{\bar{k}\bar{j}ri} - \Gamma^m_{ir} G_{m\bar{k}\bar{j}}) \\ = -G^k G^r (G_{\bar{k}\bar{j}ri} - G_{ir\bar{s}} G^{\bar{s}m} G_{m\bar{k}\bar{j}}) \\ = -G^k G^r (K_{\bar{k}\bar{j}ri} - K_{ir\bar{s}} g^{\bar{s}m} K_{m\bar{k}\bar{j}}) = -G^{\bar{k}} G^r R_{i\bar{j}r\bar{k}} \end{aligned} \tag{6}$$

Hence from (5) we find

$$V_{i\bar{j}} = e^G(G_{i\bar{j}} + \nabla_i G^k \nabla_{\bar{j}} G_k - G^{\bar{k}} G^r R_{i\bar{j}r\bar{k}}) - (G_i G_{\bar{j}} + G_{i\bar{j}})V \tag{7}$$

Similarly we can calculate V_{ij}=$\nabla_i\nabla_j V$:

$$\begin{aligned}V_{ij} &= \nabla_j(e^G[G^k\nabla_i G_k + G_i] + G_i V)\\
&= e^G(G_j(G_i + G^k\nabla_i G_k) + \nabla_j G_i + \nabla_j G^k\nabla_i G_k + G^k\nabla_j\nabla_i G_k)\\
&\quad +\nabla_j G_i V + G_i\nabla_j V\\
&= e^G(G_j(-G_iV)e^{-G} + \nabla_j G_i + G^{k\bar{m}}\nabla_j G_{\bar{m}}\nabla_j G_k + G^k\nabla_j\nabla_i G_k) + \nabla_j G_i V\\
&= e^G(\nabla_j G_i + G^{k\bar{m}}G_{\bar{m}j}\nabla_j G_k + G^k\nabla_j\nabla_i G_k) + (\nabla_j G_i - G_jG_i)V\end{aligned}$$

since $G^{k\bar{m}}G_{\bar{m}j}{=}\delta^j_k$ and $\nabla_j G_i{=}G_{ij} - \Gamma^k_{ij}G_k{=}G_{ji} - \Gamma^k_{ji}G_k{=}\nabla_i G_j$ we conclude

$$V_{ij} = e^G(2\nabla_j G_i + G^k\nabla_j\nabla_i G_k) + (\nabla_j G_i - G_jG_i)V \tag{8}$$

So to ensure metastability we need the Hessian Matrix M^2 to be positive definite, where

$$\mathbf{M^2} = \begin{pmatrix} V_{i\bar{j}} & V_{ij} \\ V_{ij} & V_{\bar{i}\bar{j}} \end{pmatrix} \tag{9}$$

Recall that derivatives of G with mixed holomorphic and antiholomorphic indices depend only on K, unlike the K- and W-dependent G_i, $\nabla_i G_j$ and $\nabla_i\nabla_j G_k$. Fixing K and varying W we can tune V_{ij} to 0 by adjusting $\nabla_i\nabla_j G_k$. Also we make most eigenvalues of $V_{i\bar{j}}$ positive by adjusting $\nabla_i G_j$. Further we can arbitrarily choose G_i, not forgetting to keep G^i fixed as imposed by the Stationary Condition. Therefore to analyse metastability it suffices to study the projection of the diagonal Block Matrix $V_{i\bar{j}}$ along G^i. After rescaling the quantity by the convenient $m^2_{3/2}$ (see[1]) we consider the following parameter:

$$\lambda = e^{-G}V_{i\bar{j}}G^iG^{\bar{j}} \tag{10}$$

Using equations (7) we find:

$$\lambda = ((G_{i\bar{j}} + \nabla_i G_k\nabla_{\bar{j}}G^k - G^kG^rR_{i\bar{j}r\bar{k}}) - (G_iG_{\bar{j}} + G_{i\bar{j}})Ve^{-G})G^iG^{\bar{j}} \tag{11}$$

Define $A{:=}G^kG_k{=}G^{i\bar{j}}G_{\bar{j}}G_i{=}G_{i\bar{j}}G^iG^{\bar{j}}$. Then:

$$\begin{aligned}\lambda &= G_{i\bar{j}}G^iG^{\bar{j}} + (G^i\nabla_i G_k)(G^{\bar{j}}\nabla_{\bar{j}}G^k) - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}} - (G_{i\bar{j}}G^iG^{\bar{j}} + G_iG^iG_{\bar{j}}G^{\bar{j}})(G_kG^k - 3)\\
&= A + (G^i\nabla_k Gi)(G^{\bar{j}}\nabla_{\bar{j}}G^k) - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}} + (A - A^2)(A-3)\\
&= A + (G^i\nabla_k G_i)(G^{\bar{j}}G^{k\bar{s}}\nabla_{\bar{j}}G_{\bar{s}}) + (A - A^2)(A-3) - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}}\end{aligned} \tag{12}$$

Now $G^i\nabla_k G_i{=}{-}G_kVe^{-G} - G_k{=}{-}G_k(A-3) - G_k$ and λ becomes:

$$\begin{aligned}\lambda &= A + (-G_k(A-3) - G_k)(G^{k\bar{s}}(-G_{\bar{s}}(A-3) - G_{\bar{s}}) + (A - A^2)(A-3) - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}}\\
&= A + (G_k(A-3) + G_k)(G^k(A-3) + G^k) + (A - A^2)(A-3) - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}}\\
&= A + A(A-3)^2 + A(A-3) + A(A-3) + A + (A - A^2)(A-3) - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}}\\
&= 2A - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}}\\
&\Leftrightarrow \lambda = 2g_{i\bar{j}}G^iG^{\bar{j}} - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}}\end{aligned} \tag{13}$$

e^{-G} is positive definite, for the Hessian to be positive definite we need $V_{i\bar{j}}z^i z^{\bar{j}}>0$ for all non-zero vectors z^i and $z^{\bar{j}}$. Taking $z^i=G^i$ the condition on the Hessian to be positive definite results in the condition [4]:

$$\lambda > 0 \tag{14}$$

We would like to rewrite λ:

$$\begin{aligned}\lambda &= 2g_{i\bar{j}}G^iG^{\bar{j}} - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}} \\ &= -\frac{2}{3}(A-3)A + \frac{1}{3}(g_{i\bar{j}}G^iG^{\bar{j}}g_{m\bar{n}}G^mG^{\bar{n}} + g_{i\bar{n}}G^iG^{\bar{n}}g_{m\bar{j}}G^mG^{\bar{j}}) - R_{i\bar{j}r\bar{k}}G^{\bar{k}}G^rG^iG^{\bar{j}} \\ &= -\frac{2}{3}(A-3)A + [\frac{1}{3}(g_{i\bar{j}}g_{m\bar{n}} + g_{i\bar{n}}g_{m\bar{j}}) - R_{i\bar{j}r\bar{k}}]G^{\bar{k}}G^rG^iG^{\bar{j}}\end{aligned}$$

We could go on to studying the implications of the metastability condition $\lambda>0$ for models with a fixed cosmological constant, as done in refs. [4, 5]. Instead we will require the cosmological constant to be non-negative by having some G^i=O(1) so that te quadratic and quartic terms of λ compete and its sign would strongly depend on the curvature tensor $R_{i\bar{j}r\bar{k}}$.

Defining $\sigma:=[\frac{1}{3}(g_{i\bar{j}}g_{m\bar{n}} + g_{i\bar{n}}g_{m\bar{j}}) - R_{i\bar{j}r\bar{k}}]G^{\bar{k}}G^rG^iG^{\bar{j}}$ we find:

$$\lambda = -\frac{2}{3}e^{-G}V(e^{-G}V+3) + \sigma \tag{15}$$

with

$$\sigma = S_{i\bar{j}r\bar{k}}G^iG^{\bar{j}}G^rG^{\bar{k}} \tag{16}$$

where

$$S_{i\bar{j}r\bar{k}} = \frac{1}{3}(g_{i\bar{j}}g_{m\bar{n}} + g_{i\bar{n}}g_{m\bar{j}}) - R_{i\bar{j}r\bar{k}} \tag{17}$$

We identify the condition of $\lambda>0$ with $\sigma>0$ because for $V>0$ the first term of (15) is clearly negative and its value depends only on the length of G^i. The second term depends on the orientation of G^i, but not on its length. More precisely the reason is that with an arbitrary G^i giving $\sigma(G^i)>0$ we can always adjust the superpotential W to get G^i rescaled for some real r resulting in V(rG^i)=0. This implies λ(rG^i)>0, showing the possibility of Minkowski vacua. By increasing r we can achieve both V(rG^i)>0 and λ(rG^i)>0, which would give us the existence of de Sitter vacua. Keeping the gravitino mass scale $m_{3/2}=e^{G/2}$ fixed the size of the cosmological constant is implied by keeping $\lambda>0$ depending on σ in the situation of V(G^i)=0. Similarly it is impossible to get V≥0 and $\lambda>0$ simultaneously provided $\sigma<0$. Therefore the existence of viable de Sitter vacua is only possible for

$$\sigma > 0 \tag{18}$$

In the following section we will analyse some examples arising from string theory, finding σ and discussing its implication.

3 Simple Kähler-Potential Models

This section is completely determined to applying the discussed method to some interesting examples. We will follow step-by-step the way of obtaining σ. The Models will generally consist of one modulus T and its conjugate $\bar{T}$ as well as up to one matter field Φ with conjugate

$\bar{\Phi}$. Partial derivatives with respect to any of these fields will be denoted by writing it as an index.

Let us derive a general formula for σ, which depends on one mudulus and one matter field and their conjugates.

$$\begin{aligned}\sigma = & S_{T\bar{T}T\bar{T}}G^TG^{\bar{T}}G^TG^{\bar{T}} + S_{T\bar{T}\Phi\bar{T}}G^TG^{\bar{T}}G^{\Phi}G^{\bar{T}} + S_{T\bar{\Phi}T\bar{\Phi}}G^TG^{\bar{\Phi}}G^TG^{\bar{\Phi}} + S_{T\bar{\Phi}\Phi\bar{T}}G^TG^{\bar{\Phi}}G^{\Phi}G^{\bar{T}} \\ & + S_{T\bar{T}T\bar{\Phi}}G^TG^{\bar{T}}G^TG^{\bar{\Phi}} + S_{T\bar{T}\Phi\bar{\Phi}}G^TG^{\bar{T}}G^{\Phi}G^{\bar{\Phi}} + S_{T\bar{\Phi}T\bar{T}}G^TG^{\bar{\Phi}}G^TG^{\bar{T}} + S_{T\bar{\Phi}\Phi\bar{\Phi}}G^TG^{\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}} \\ & + S_{\Phi\bar{T}T\bar{T}}G^{\Phi}G^{\bar{T}}G^TG^{\bar{T}} + S_{\Phi\bar{T}\Phi\bar{T}}G^{\Phi}G^{\bar{T}}G^{\Phi}G^{\bar{T}} + S_{\Phi\bar{\Phi}T\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}}G^TG^{\bar{\Phi}} + S_{\Phi\bar{\Phi}\Phi\bar{T}}G^{\Phi}G^{\bar{\Phi}}G^{\Phi}G^{\bar{T}} \\ & + S_{\Phi\bar{T}T\bar{\Phi}}G^{\Phi}G^{\bar{T}}G^TG^{\bar{\Phi}} + S_{\Phi\bar{T}\Phi\bar{\Phi}}G^{\Phi}G^{\bar{T}}G^{\Phi}G^{\bar{\Phi}} + S_{\Phi\bar{\Phi}T\bar{T}}G^{\Phi}G^{\bar{\Phi}}G^TG^{\bar{T}} + S_{\Phi\bar{\Phi}\Phi\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}} \end{aligned} \quad (19)$$

It is obvious that $S_{i\bar{j}r\bar{k}}$ is invariant under permutation of holomorphic and antiholomorphic indices if K depends only T,$\bar{T}$,Φ and $\bar{\Phi}$. σ then becomes:

$$\begin{aligned}\sigma = & S_{T\bar{T}T\bar{T}}G^TG^{\bar{T}}G^TG^{\bar{T}} + S_{\Phi\bar{\Phi}\Phi\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}} + S_{T\bar{\Phi}T\bar{\Phi}}G^TG^{\bar{\Phi}}G^TG^{\bar{\Phi}} + S_{\Phi\bar{T}\Phi\bar{T}}G^{\Phi}G^{\bar{T}}G^{\Phi}G^{\bar{T}} \\ & + 2S_{\Phi\bar{T}T\bar{T}}G^{\Phi}G^{\bar{T}}G^TG^{\bar{T}} + 2S_{\Phi\bar{\Phi}T\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}}G^TG^{\bar{\Phi}} + 2S_{T\bar{\Phi}\Phi\bar{\Phi}}G^TG^{\bar{\Phi}}G^{\Phi}G^{\bar{\Phi}} + 2S_{T\bar{\Phi}T\bar{T}}G^TG^{\bar{\Phi}}G^TG^{\bar{T}} \\ & + 4S_{T\bar{T}\Phi\bar{\Phi}}G^TG^{\bar{T}}G^{\Phi}G^{\bar{\Phi}} \end{aligned} \quad (20)$$

We shall use this formula in the following cases.

3.1 A Separable Model Goverened by a Single Modulus and One Matter Field

The following Example arises from string compactifications described by a single modulus T and a Kähler Potential of the following form:

$$K = -3\log(T+\bar{T}) - |\Phi|^2 \quad (21)$$

$$\Leftrightarrow K = -3\log(T+\bar{T}) - \Phi\bar{\Phi} \quad (22)$$

If assuming W to be also separable implying solely gravitational interaction between the two fields it would be possible to uplift any would-be supersummetric minimum in the T sector with a Φ sector that breaks supersymmetry spontaneously well below the Planck scale [4]. For a generalization of to a certain type of non-seperable W consult [9], and specific examples are discussed in [10, 11].

We find

$$K_T = K_{\bar{T}} = -\frac{3}{T+\bar{T}}$$

$$K_{T\bar{T}} = g_{T\bar{T}} = \frac{3}{(T+\bar{T})^2} \quad (23)$$

$$K_{TT\bar{T}} = K_{T\bar{T}\bar{T}} = -\frac{6}{(T+\bar{T})^3}$$

$$K_{T\bar{T}T\bar{T}} = \frac{18}{(T+\bar{T})^4}$$

Also

$$K_\Phi = \bar{\Phi} \qquad K_{\bar{\Phi}} = \Phi$$

$$K_{\Phi\bar{\Phi}} = 1$$

We find all other derivatives up to fourth order to vanish. Hence we can see from (23) that $K^{T\bar{T}} = \frac{(T+\bar{T})^2}{3}$.

Hence from plugging the values into (17)

$$S_{T\bar{T}T\bar{T}} = S_{\Phi\bar{T}T\bar{T}} = S_{T\bar{\Phi}t\bar{T}} = S_{\Phi\bar{T}\Phi\bar{\Phi}} = S_{\Phi\bar{T}\Phi\bar{T}} = S_{T\bar{\Phi}T\bar{\Phi}} = 0$$

$$S_{T\bar{T}\Phi\bar{\Phi}} = \frac{1}{(T+\bar{T})^2}$$

$$S_{\Phi\bar{\Phi}\Phi\bar{\Phi}} = \frac{2}{3}$$

which immediately implies

$$\begin{aligned} \sigma &= \frac{4}{(T+\bar{T})^2} G^T G^{\bar{T}} G^\Phi G^{\bar{\Phi}} + \frac{2}{3}(G^\Phi G^{\bar{\Phi}})^2 \\ &= \frac{4}{(T+\bar{T})^2}(G^{T\bar{T}})^2 (G^{\Phi\bar{\Phi}})^2 G_T G_{\bar{T}} G_\Phi G_{\bar{\Phi}} + \frac{2}{3}((G^{\Phi\bar{\Phi}})^2 (G_\Phi G_{\bar{\Phi}})^2)^2 \\ &= \frac{4}{(T+\bar{T})^2}(G^{T\bar{T}})^2 (G^{\Phi\bar{\Phi}})^2 G_T \bar{G_T} G_\Phi \bar{G_\Phi} + \frac{2}{3}(G^{\Phi\bar{\Phi}})^4 |G_\Phi|^4 \end{aligned}$$

Plugging in $(G^{T\bar{T}})^2 = (K^{T\bar{T}})^2 = (\frac{(T+\bar{T})^2}{3})^2$ and $(G^{\Phi\bar{\Phi}})^2 = 1$ we conclude:

$$\sigma = \frac{4(T+\bar{T})^2}{9}|G_T|^2|G_\Phi|^2 + \frac{2}{3}|G_\Phi|^4 \tag{24}$$

Provided that $G^\Phi \neq 0$, meaning we need only the matter fields to break SuSy, σ consists of positive definite summands, hence $\sigma > 0$. Note also that the sign of σ is independent of G^T, so the existence of stable de Sitter vacua for the considered model depends only on the matter fields.

3.2 A Non-Separable No-Scale Kähler Potential with One Modulus and One Matter Field

Let us consider the following Model:

$$\begin{aligned} K &= -3\log(T+\bar{T}-|\Phi|^2) \quad (25) \\ &= -3\log(T+\bar{T}-\Phi\bar{\Phi}) \quad (26) \end{aligned}$$

On side note the fact that this Kähler Potential is a no-scale model, i.e. it satisfies the condition [14]:

$$K^i K_i = 3 \tag{27}$$

Noting this property the analysis can be largely simplified and calculations easily manipulated. As for our example we continue with the regular investigation and show that in this example $\sigma = 0$

indepentdently of G^T and G^Φ excluding possible dS vacua. More general models with numerous matter fields are considered in [12] and a recent general study of this type of uplifting in [13].[1]

We find

$$K_T = K_{\bar{T}} = -\frac{3}{T+\bar{T}-\Phi\bar{\Phi}} \qquad K_\Phi = \frac{3\bar{\Phi}}{T+\bar{T}-\Phi\bar{\Phi}} \qquad K_{\bar{\Phi}} = \frac{3\Phi}{T+\bar{T}-\Phi\bar{\Phi}}$$

$$K_{T\bar{T}} = K_{TT} = \frac{3}{(T+\bar{T}-\Phi\bar{\Phi})^2} \tag{28}$$

$$K_{T\bar{\Phi}} = K_{\bar{T}\bar{\Phi}} = -\frac{3\Phi}{(T+\bar{T}-\Phi\bar{\Phi})^2} \qquad K_{T\Phi} = K_{\bar{T}\Phi} = -\frac{3\bar{\Phi}}{(T+\bar{T}-\Phi\bar{\Phi})^2}$$

$$K_{\Phi\bar{\Phi}} = \frac{3(T+\bar{T})}{(T+\bar{T}-\Phi\bar{\Phi})^2} \qquad K_{\Phi\Phi} = \frac{3(\bar{\Phi})^2}{(T+\bar{T}-\Phi\bar{\Phi})^2}$$

So

$$\mathbf{K_{i\bar{j}}} = \begin{pmatrix} K_{T\bar{T}} & K_{T\bar{\Phi}} \\ K_{\Phi\bar{T}} & K_{\Phi\bar{\Phi}} \end{pmatrix} = \begin{pmatrix} \frac{3}{(T+\bar{T}-\Phi\bar{\Phi})^2} & \frac{-3\Phi}{(T+\bar{T}-\Phi\bar{\Phi})^2} \\ \frac{-3\bar{\Phi}}{(T+\bar{T}-\Phi\bar{\Phi})^2} & \frac{3(T+\bar{T})}{(T+\bar{T}-\Phi\bar{\Phi})^2} \end{pmatrix} \tag{29}$$

Hence

$$\mathbf{K^{i\bar{j}}} = \begin{pmatrix} K^{T\bar{T}} & K^{T\bar{\Phi}} \\ K^{\Phi\bar{T}} & K^{\Phi\bar{\Phi}} \end{pmatrix} = \begin{pmatrix} \frac{(T+\bar{T})(T+\bar{T}-\Phi\bar{\Phi})}{3} & \frac{\Phi(T+\bar{T}-\Phi\bar{\Phi})}{3} \\ \frac{\bar{\Phi}(T+\bar{T}-\Phi\bar{\Phi})}{3} & \frac{T+\bar{T}-\Phi\bar{\Phi}}{3} \end{pmatrix} \tag{30}$$

$$K_{TT\bar{T}} = K_{T\bar{T}\bar{T}} = -\frac{6}{(T+\bar{T}-\Phi\bar{\Phi})^3} \qquad K_{\Phi\Phi\bar{\Phi}} = \frac{6(T+\bar{T})\bar{\Phi}}{(T+\bar{T}-\Phi\bar{\Phi})^3} \qquad K_{\bar{\Phi}\bar{\Phi}\Phi} = \frac{6(T+\bar{T})\Phi}{(T+\bar{T}-\Phi\bar{\Phi})^3}$$

$$K_{T\bar{T}\Phi} = \frac{6\bar{\Phi}}{(T+\bar{T}-\Phi\bar{\Phi})^3} \qquad K_{T\bar{T}\bar{\Phi}} = \frac{6\Phi}{(T+\bar{T}-\Phi\bar{\Phi})^3} \qquad K_{\Phi\bar{\Phi}T} = K_{\Phi\bar{\Phi}\bar{T}} = \frac{3(T+\bar{T}+\Phi\bar{\Phi})}{(T+\bar{T}-\Phi\bar{\Phi})^3}$$

$$K_{T\bar{T}T\bar{T}} = \frac{18}{(T+\bar{T}-\Phi\bar{\Phi})^4} \qquad K_{\Phi\bar{\Phi}\Phi\bar{\Phi}} = \frac{6(T+\bar{T})(T+\bar{T}+2|\Phi|^2)}{(T+\bar{T}-\Phi\bar{\Phi})^4}$$

$$K_{T\bar{\Phi}T\bar{\Phi}} = \frac{18\Phi^2}{(T+\bar{T}-\Phi\bar{\Phi})^4} \qquad K_{\Phi\bar{T}\Phi\bar{T}} = \frac{18\bar{\Phi}^2}{(T+\bar{T}-\Phi\bar{\Phi})^4} \qquad K_{\Phi\bar{T}T\bar{T}} = -\frac{18\bar{\Phi}}{(T+\bar{T}-\Phi\bar{\Phi})^4}$$

$$K_{\Phi\bar{\Phi}T\bar{\Phi}} = K_{T\bar{\Phi}\Phi\bar{\Phi}} = -\frac{6\Phi((2T+2\bar{T}+\Phi\bar{\Phi})}{(T+\bar{T}-\Phi\bar{\Phi})^4}$$

$$K_{T\bar{\Phi}T\bar{T}} = -\frac{18\Phi}{(T+\bar{T}-\Phi\bar{\Phi})^4} \qquad K_{T\bar{T}\Phi\bar{\Phi}} = \frac{6(T+\bar{T}+2\Phi\bar{\Phi})}{(T+\bar{T}-\Phi\bar{\Phi})^4}$$

[1] A simpler case without matter fields is discussed in [7]. See also [8]

We expect the de Sitter vacua to occur at vanishing vev of the matter fields, so setting Φ=0$\Leftrightarrow\bar{\Phi}$=0 we have the only nonvanishing derivatives up to fourth order being:

$$K_T = K_{\bar{T}} = -\frac{3}{T+\bar{T}} \qquad K_{\Phi\bar{\Phi}} = -\frac{3}{T+\bar{T}} \qquad K_{T\bar{T}} = \frac{3}{(T+\bar{T})^2}$$

$$\Leftrightarrow K^{\Phi\bar{\Phi}} = -\frac{T+\bar{T}}{3} \qquad K_{T\bar{T}} = \frac{(T+\bar{T})^2}{3}$$

$$K_{\Phi\bar{\Phi}T} = -\frac{3}{(T+\bar{T})^2} \qquad K_{T\bar{T}\bar{T}} = K_{\bar{T}TT} = -\frac{6}{(T+\bar{T})^3}$$

$$K_{T\bar{T}T\bar{T}} = \frac{18}{(T+\bar{T})^4} \qquad K_{T\bar{\Phi}\Phi\bar{T}} = \frac{6}{(T+\bar{T})^3} \qquad K_{\Phi\bar{\Phi}\Phi\bar{\Phi}} = \frac{6}{(T+\bar{T})^2}$$

Plugging all these into $S_{i\bar{j}r\bar{k}}$ we find it to be 0 for any indices. So σ is 0, independently of G^T or G^Φ, so we can see that de Sitter vacua are excluded in the considered non-separable case with one modulus and one matter field.

3.3 A more general case

Any holomorphic function can be expressed in terms of a Taylor Expasion around Φ=0. Suppose we have an expansion of the form

$$K = -3\log(T+\bar{T}) + A(T,\bar{T})|\Phi|^2 + B(T,\bar{T})|\Phi|^4 + h.o.t. \tag{31}$$

We observe that σ in this case would be dependent on $A(T,\bar{T})$ and $B(T,\bar{T})$ (though not on the h.o.t.). However, there is a way of reducing the Problem to a single function by writing

$$K = -3\log(T+\bar{T} - f(T,\bar{T})|\Phi|^2) \tag{32}$$

with $f(T,\bar{T})$ such that $K_{\Phi\bar{\Phi}}(\Phi=0)$=$A(T,\bar{T})$ and $K_{\Phi\bar{\Phi}\Phi\bar{\Phi}}(\Phi=0)$=$4B(T,\bar{T})$. Considering all dervatives up to fourth order we find:

$$K_T = K_{\bar{T}} = -\frac{3}{T+\bar{T}-f(T,\bar{T})|\Phi|^2} + O(\Phi\bar{\Phi}) \qquad K_\Phi = O(\bar{\Phi}) \qquad K_{\bar{\Phi}} = O(\Phi)$$

$$K_{T\bar{T}} = K_{TT} = \frac{3}{(T+\bar{T}-f(T,\bar{T})\Phi\bar{\Phi})^2} + O(\Phi\bar{\Phi}) \qquad K_{T\bar{\Phi}} = K_{\bar{T}\bar{\Phi}} = O(\Phi)$$

$$K_{T\Phi} = K_{\bar{T}\Phi} = O(\bar{\Phi}) \qquad K_{\Phi\bar{\Phi}} = \frac{3f(T,\bar{T})(T+\bar{T})}{(T+\bar{T}-f(T,\bar{T})\Phi\bar{\Phi})^2} + O(\Phi\bar{\Phi}) \qquad K_{\Phi\Phi} = O(\bar{\Phi}^2)$$

$$K_{TT\bar{T}} = K_{T\bar{T}\bar{T}} = -\frac{6}{(T+\bar{T}-f(T,\bar{T})\Phi\bar{\Phi})^3} + O(\Phi\bar{\Phi}) \qquad K_{\Phi\Phi\bar{\Phi}} = O(\bar{\Phi}) \qquad K_{\bar{\Phi}\bar{\Phi}\Phi} = O(\Phi)$$

$$K_{T\bar{T}\Phi} = O(\bar{\Phi}) \qquad K_{T\bar{T}\bar{\Phi}} = O(\Phi) \qquad K_{\Phi\bar{\Phi}T} = K_{\Phi\bar{\Phi}\bar{T}} = \frac{3(f_T(T+\bar{T}-f(T,\bar{T})|\Phi|^2) - f)}{(T+\bar{T}-f(T,\bar{T})|\Phi|^2)^2} + O(\Phi\bar{\Phi})$$

$$K_{T\bar{T}T\bar{T}} = \frac{18}{(T+\bar{T}-f(T,\bar{T})|\Phi|^2)^4} + O(\Phi\bar{\Phi}) \qquad K_{\Phi\bar{\Phi}\Phi\bar{\Phi}} = \frac{6f^2(T+\bar{T})(T+\bar{T}-f(T,\bar{T})\Phi\bar{\Phi})}{(T+\bar{T}-f(T,\bar{T})|\Phi|^2)^4} + O(\Phi)$$

$$K_{T\bar{T}\Phi\bar{\Phi}} = \frac{3(f_{TT}(T+\bar{T}-f(T,\bar{T})|\Phi|)^2 - 3(f_T+f_{\bar{T}})(T+\bar{T}-f(T,\bar{T})|\Phi|^2)+2f}{(T+\bar{T}-f(T,\bar{T})|\Phi|^2)^3} + O(\Phi\bar{\Phi})$$

$$\begin{array}{lll} K_{T\bar{\Phi}T\bar{\Phi}} = O(\Phi^2) & K_{\Phi\bar{T}\Phi\bar{T}} = O(\bar{\Phi}) & K_{\Phi\bar{T}T\bar{T}} = O(\bar{\Phi}) \\ K_{\Phi\bar{T}\Phi\bar{\Phi}} = O(\bar{\Phi}) & K_{\Phi\bar{\Phi}T\bar{\Phi}} = O(\Phi) & K_{T\bar{\Phi}T\bar{T}} = O(\Phi) \end{array}$$

Hence at vanishing matter fields, i.e. Φ=$\bar{\Phi}$=0

$$\mathbf{K^{i\bar{j}}} = \begin{pmatrix} K^{T\bar{T}} & K^{T\bar{\Phi}} \\ K^{\Phi\bar{T}} & K^{\Phi\bar{\Phi}} \end{pmatrix} = \begin{pmatrix} \frac{T+\bar{T}-f(T,\bar{T})\Phi\bar{\Phi}}{3} & 0 \\ 0 & \frac{(T+\bar{T}-f(T,\bar{T})\Phi\bar{\Phi})^2}{3f(T,\bar{T})(T+\bar{T})} \end{pmatrix} \tag{33}$$

So at Φ=0 we find the only non-vanishing $S_{i\bar{j}r\bar{k}}$ to be:

$$S_{T\bar{T}\Phi\bar{\Phi}} = \frac{-3(f*f_{T\bar{T}} - f_T f_{\bar{T}})(T+\bar{T}) + 6f(f_T+f_{\bar{T}})}{(T+\bar{T})^2} \tag{34}$$

Therefore

$$\begin{aligned} \sigma &= \frac{-3(f*f_{T\bar{T}} - f_T f_{\bar{T}})(T+\bar{T}) + 6f(f_T+f_{\bar{T}})}{(T+\bar{T})^2} (G^{\bar{T}T})^2 (G^{\bar{\Phi}\Phi})^2 |G_T|^2 |G_\Phi|^2 \\ &= \frac{-3(f*f_{T\bar{T}} - f_T f_{\bar{T}})(T+\bar{T}) + 6f(f_T+f_{\bar{T}})(T+\bar{T})}{3f(T,\bar{T})} |G_T|^2 |G_\Phi|^2 \end{aligned} \tag{35}$$

It is obvious that $|G_T|^2$ and $|G_\Phi|^2$ are positive definite provided $G_T \neq 0$ and $G_\Phi \neq 0$. Therefore it is necessary for the moduli and the matter fields to break SuSy in order to have de Sitter vacua. So the sign of σ at Φ=0 depends only on $\frac{-3(f*f_{T\bar{T}}-f_T f_{\bar{T}})(T+\bar{T})+16f(f_T+f_{\bar{T}})(T+\bar{T})}{3f(T,\bar{T})}$. Provided it is greater than 0 we achieve de Sitter vacua at Φ=0. Looking at the Taylor expansion we can rewrite this condition simply in terms of $A(T,\bar{T})$.

4 Conclusion

We have studied the Stability arising in Supergravity Theories and the circumstances for a symmetry-breaking minimum of the Scalar Potential implying de Sitter vacua. Although the investigation of the Eigenvalues of the Hessian Matrix is very important, we were able to derive a rather simple necessary condition depending only on the Kähler Potential and independent of the form, that the Superpotential W might take. These conditions need to be satisfied to make sure the possibility of de Sitter vacua to exist. Nevertheless it is not to forget that we did not find the sufficient condition and the existence of the vacua can not be guaranteed, but has to be determined by the particular form of the Kähler Potential and the Superpotential. It becomes sufficent as soon as we can assume complete freedom of W [4].

Summarizing our results we find:

- K=$-3\log(T+\bar{T}) - |\Phi|^2$ has σ>0 if the matter fields break SuSy, i.e. $G^\Phi \neq 0$, implying possible exitence of de Sitter vacua.
- K=$-3\log(T+\bar{T}-|\Phi|^2)$ has σ=0 idependently of G^T and G^Φ, so de Sitter vacua are excluded.

- $K=-3\log(T+\bar{T}-f(T,\bar{T})|\Phi|^2)$ we have $\mathrm{sgn}(\sigma)=\mathrm{sgn}(\frac{-3(f*f_{T\bar{T}}-f_Tf_{\bar{T}})(T+\bar{T})+16f(f_T+f_{\bar{T}})(T+\bar{T})}{3f(T,\bar{T})})$, where sgn is the sign function, if G^T and G^Φ are non-zero simultaneously. Therefore de Sitter vacua can only exist if $\frac{-3(f*f_{T\bar{T}}-f_Tf_{\bar{T}})(T+\bar{T})+16f(f_T+f_{\bar{T}})(T+\bar{T})}{3f(T,\bar{T})}>0$ and the moduli and matter fields both break SuSy.

The obtained results are relevant in order to identify promising models for the potential in String Theory. Further study should involve more complicated cases and examples where more moduli and matter fields are involved, particularly vector superfields. We leave this for future analysis.

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